

# System G3i

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## 1 Height preserving admissibility of weakening

- Case(s) rule  $\top_R$

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top_R \rightarrow \frac{}{\bullet h_1 : \Delta_2, F_0 \vdash \top} \top_R$$

- Case(s) rule  $\rightarrow_R$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \rightarrow F_4} \rightarrow_R \rightarrow \frac{\frac{}{h_1 : \Delta_2, F_3 \vdash F_4} \text{ax}}{h_1 : \Delta_2, F_0, F_3 \vdash F_4} \text{IH}}{\bullet h_1 : \Delta_2, F_0 \vdash F_3 \rightarrow F_4} \rightarrow_R$$

- Case(s) rule  $\wedge_R$

$$\frac{h_1 : \Delta_2 \vdash F_3 \quad h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \wedge F_4} \wedge_R \rightarrow \frac{\frac{}{h_1 : \Delta_2 \vdash F_3} \text{ax} \quad \frac{}{h_1 : \Delta_2 \vdash F_4} \text{ax}}{h_1 : \Delta_2, F_0 \vdash F_3 \quad h_1 : \Delta_2, F_0 \vdash F_4} \text{IH}}{\bullet h_1 : \Delta_2, F_0 \vdash F_3 \wedge F_4} \wedge_R$$

- Case(s) rule  $\vee_1$

$$\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3 \vee F_4} \vee_1 \rightarrow \frac{\frac{}{h_1 : \Delta_2 \vdash F_3} \text{ax}}{h_1 : \Delta_2, F_0 \vdash F_3} \text{IH}}{\bullet h_1 : \Delta_2, F_0 \vdash F_3 \vee F_4} \vee_1$$

- Case(s) rule  $\vee_2$

$$\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \vee F_4} \vee_2 \rightarrow \frac{\frac{}{h_1 : \Delta_2 \vdash F_4} \text{ax}}{h_1 : \Delta_2, F_0 \vdash F_4} \text{IH}}{\bullet h_1 : \Delta_2, F_0 \vdash F_3 \vee F_4} \vee_2$$

- Case(s) rule  $\rightarrow_L$

$$\frac{h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_5} \rightarrow_L \rightarrow \frac{\frac{}{h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_3} \text{ax} \quad \frac{}{h_1 : \Delta_2, F_4 \vdash F_5} \text{ax}}{h_1 : \Delta_2, F_0, F_3 \rightarrow F_4 \vdash F_3 \quad h_1 : \Delta_2, F_0, F_4 \vdash F_5} \text{IH}}{\bullet h_1 : \Delta_2, F_0, F_3 \rightarrow F_4 \vdash F_5} \rightarrow_L$$

- Case(s) rule  $\wedge_L$

$$\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash F_5} \wedge_L \rightarrow \frac{\frac{}{h_1 : \Delta_2, F_3, F_4 \vdash F_5} \text{ax}}{h_1 : \Delta_2, F_0, F_3, F_4 \vdash F_5} \text{IH}}{\bullet h_1 : \Delta_2, F_0, F_3 \wedge F_4 \vdash F_5} \wedge_L$$

- Case(s) rule  $\vee_L$

$$\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5 \quad h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_5} \vee_L}{\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, F_0, F_3 \vdash F_5} \text{IH} \quad \frac{h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_0, F_4 \vdash F_5} \text{IH}}{\bullet h_1 : \Delta_2, F_0, F_3 \vee F_4 \vdash F_5} \vee_L} \rightarrow$$

- Case(s) rule  $\perp_L$

$$\frac{}{\bullet h_1 : \perp, \Delta_2 \vdash F_3} \perp_L \rightarrow \frac{}{\bullet h_1 : \perp, \Delta_2, F_0 \vdash F_3} \perp_L$$

- Case(s) rule  $I$

$$\frac{}{\bullet h_1 : \Delta_2, P_3 \vdash P_3} I \rightarrow \frac{}{\bullet h_1 : \Delta_2, F_0, P_3 \vdash P_3} I$$

- Case(s) rule  $\top_L$

$$\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \top, \Delta_2 \vdash F_3} \top_L \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2, F_0 \vdash F_3} \text{IH}}{\bullet h_1 : \top, \Delta_2, F_0 \vdash F_3} \top_L} \rightarrow$$

## 2 Measure of derivations

- Case(s) rule  $\top_R$

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top_R \rightarrow \frac{}{\bullet \bullet h_1 : \Delta_2 \vdash \top} \top_R$$

- Case(s) rule  $\rightarrow_R$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \rightarrow F_4} \rightarrow_R \rightarrow \frac{\frac{}{h_1 : \Delta_2, F_3 \vdash F_4} \text{ax}}{\bullet h_1 : \Delta_2, F_3 \vdash F_4} \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash F_3 \rightarrow F_4} \rightarrow_R$$

- Case(s) rule  $\wedge_R$

$$\frac{h_1 : \Delta_2 \vdash F_3 \quad h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \wedge F_4} \wedge_R \rightarrow \frac{\frac{}{h_1 : \Delta_2 \vdash F_3} \text{ax}}{\bullet h_1 : \Delta_2 \vdash F_3} \text{IH} \quad \frac{}{h_1 : \Delta_2 \vdash F_4} \text{ax}}{\bullet \bullet h_1 : \Delta_2 \vdash F_3 \wedge F_4} \wedge_R$$

- Case(s) rule  $\vee_1$

$$\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3 \vee F_4} \vee_1 \rightarrow \frac{\frac{}{h_1 : \Delta_2 \vdash F_3} \text{ax}}{\bullet h_1 : \Delta_2 \vdash F_3} \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash F_3 \vee F_4} \vee_1$$

- Case(s) rule  $\vee_2$

$$\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \vee F_4} \vee_2 \rightarrow \frac{\frac{}{h_1 : \Delta_2 \vdash F_4} \text{ax}}{\bullet h_1 : \Delta_2 \vdash F_4} \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash F_3 \vee F_4} \vee_2$$

- Case(s) rule  $\rightarrow_L$

$$\frac{h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_5} \rightarrow_L \rightarrow \frac{\frac{}{h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_3} \text{ax}}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_3} \text{IH} \quad \frac{}{h_1 : \Delta_2, F_4 \vdash F_5} \text{ax}}{\bullet \bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_5} \rightarrow_L$$

- Case(s) rule  $\wedge_L$

$$\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash F_5} \wedge_L \rightarrow \frac{\frac{}{h_1 : \Delta_2, F_3, F_4 \vdash F_5} \text{ax}}{\bullet h_1 : \Delta_2, F_3, F_4 \vdash F_5} \text{IH}}{\bullet \bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash F_5} \wedge_L$$

- Case(s) rule  $\vee_L$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_5 \quad h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_5} \vee_L \rightarrow \frac{\frac{}{h_1 : \Delta_2, F_3 \vdash F_5} \text{ax}}{\bullet h_1 : \Delta_2, F_3 \vdash F_5} \text{IH} \quad \frac{}{h_1 : \Delta_2, F_4 \vdash F_5} \text{ax}}{\bullet \bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_5} \vee_L$$

- Case(s) rule  $\perp_L$

$$\frac{}{\bullet h_1 : \perp, \Delta_2 \vdash F_3} \perp_L \rightarrow \frac{}{\bullet \bullet h_1 : \perp, \Delta_2 \vdash F_3} \perp_L$$

- Case(s) rule  $I$

$$\frac{}{\bullet h_1 : \Delta_2, P_3 \vdash P_3} I \rightarrow \frac{}{\bullet \bullet h_1 : \Delta_2, P_3 \vdash P_3} I$$

- Case(s) rule  $\top_L$

$$\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \top, \Delta_2 \vdash F_3} \top_L \rightarrow \frac{\frac{}{h_1 : \Delta_2 \vdash F_3} \text{ax}}{\bullet h_1 : \Delta_2 \vdash F_3} \text{IH}}{\bullet \bullet h_1 : \top, \Delta_2 \vdash F_3} \top_L$$

### 3 Invertibility of Rules

#### 3.1 Status of $\top_R$ : : Invertible

- Case rule  $\top_R$

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top_R \rightarrow \text{trivial}$$

- Case rule  $\rightarrow_R$

- Case rule  $\wedge_R$

- Case rule  $\vee_1$

- Case rule  $\vee_2$

- Case rule  $\rightarrow_L$

$$\frac{h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash \top}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash \top} \rightarrow_L \rightarrow \text{trivial}$$

- Case rule  $\wedge_L$

$$\frac{h_1 : \Delta_2, F_3, F_4 \vdash \top}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash \top} \wedge_L \rightarrow \text{trivial}$$

- Case rule  $\vee_L$

$$\frac{h_1 : \Delta_2, F_3 \vdash \top \quad h_1 : \Delta_2, F_4 \vdash \top}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \top} \vee_L \rightarrow \text{trivial}$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_1 : \perp, \Delta_2 \vdash \top} \perp_L \rightarrow \text{trivial}$$

- Case rule  $I$

- Case rule  $\top_L$

$$\frac{h_1 : \Delta_2 \vdash \top}{\bullet h_1 : \top, \Delta_2 \vdash \top} \top_L \rightarrow \text{trivial}$$

### 3.2 Status of $\rightarrow_R$ : : Invertible

- Case rule  $\top_R$
- Case rule  $\rightarrow_R$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \rightarrow F_4} \rightarrow_R \rightarrow \frac{\overline{h_1 : \Delta_2, F_3 \vdash F_4}^{\text{ax}}}{\bullet h_1 : \Delta_2, F_3 \vdash F_4} \text{H}$$

- Case rule  $\wedge_R$
- Case rule  $\vee_1$
- Case rule  $\vee_2$
- Case rule  $\rightarrow_L$

$$\frac{\frac{h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash F_5 \quad h_3 : \Delta_4, F_6 \vdash F_1 \rightarrow F_2}{\bullet h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash F_1 \rightarrow F_2} \rightarrow_L}{\bullet h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash F_1 \rightarrow F_2} \rightarrow_L \rightarrow \frac{\overline{h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash F_5}^{\text{ax}}}{h_3 : \Delta_4, F_1, F_5 \rightarrow F_6 \vdash F_5} W \frac{\overline{h_3 : \Delta_4, F_1, F_6 \vdash F_2}^{\text{ax/ind}}}{\bullet h_3 : \Delta_4, F_1, F_5 \rightarrow F_6 \vdash F_2} \rightarrow_L$$

- Case rule  $\wedge_L$

$$\frac{\frac{h_3 : \Delta_4, F_5, F_6 \vdash F_1 \rightarrow F_2}{\bullet h_3 : \Delta_4, F_5 \wedge F_6 \vdash F_1 \rightarrow F_2} \wedge_L}{\bullet h_3 : \Delta_4, F_5 \wedge F_6 \vdash F_1 \rightarrow F_2} \wedge_L \rightarrow \frac{\overline{h_3 : \Delta_4, F_1, F_5, F_6 \vdash F_2}^{\text{ax/ind}}}{\bullet h_3 : \Delta_4, F_1, F_5 \wedge F_6 \vdash F_2} \wedge_L$$

- Case rule  $\vee_L$

$$\frac{\frac{h_3 : \Delta_4, F_5 \vdash F_1 \rightarrow F_2 \quad h_3 : \Delta_4, F_6 \vdash F_1 \rightarrow F_2}{\bullet h_3 : \Delta_4, F_5 \vee F_6 \vdash F_1 \rightarrow F_2} \vee_L}{\bullet h_3 : \Delta_4, F_5 \vee F_6 \vdash F_1 \rightarrow F_2} \vee_L \rightarrow \frac{\overline{h_3 : \Delta_4, F_1, F_5 \vdash F_2}^{\text{ax/ind}}}{\bullet h_3 : \Delta_4, F_1, F_5 \vee F_6 \vdash F_2} \vee_L \frac{\overline{h_3 : \Delta_4, F_1, F_6 \vdash F_2}^{\text{ax/ind}}}{\bullet h_3 : \Delta_4, F_1, F_5 \vee F_6 \vdash F_2} \vee_L$$

- Case rule  $\perp_L$

$$\frac{\overline{\bullet h_3 : \perp, \Delta_4 \vdash F_1 \rightarrow F_2}}{\bullet h_3 : \perp, \Delta_4 \vdash F_1 \rightarrow F_2} \perp_L \rightarrow \frac{\overline{\bullet h_3 : \perp, \Delta_4, F_1 \vdash F_2}}{\bullet h_3 : \perp, \Delta_4, F_1 \vdash F_2} \perp_L$$

- Case rule  $I$

- Case rule  $\top_L$

$$\frac{\frac{h_3 : \Delta_4 \vdash F_1 \rightarrow F_2}{\bullet h_3 : \top, \Delta_4 \vdash F_1 \rightarrow F_2} \top_L}{\bullet h_3 : \top, \Delta_4 \vdash F_1 \rightarrow F_2} \top_L \rightarrow \frac{\overline{h_3 : \Delta_4, F_1 \vdash F_2}^{\text{ax/ind}}}{\bullet h_3 : \top, \Delta_4, F_1 \vdash F_2} \top_L$$

### 3.3 Status of $\wedge_R$ : (Left Premise): Invertible

- Case rule  $\top_R$
- Case rule  $\rightarrow_R$
- Case rule  $\wedge_R$

$$\frac{h_1 : \Delta_2 \vdash F_3 \quad h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \wedge F_4} \wedge_R \rightarrow \frac{\overline{h_1 : \Delta_2 \vdash F_3}^{\text{ax}}}{\bullet h_1 : \Delta_2 \vdash F_3}^{\text{H}}$$

- Case rule  $\vee_1$
- Case rule  $\vee_2$
- Case rule  $\rightarrow_L$

$$\frac{h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash F_5 \quad h_3 : \Delta_4, F_6 \vdash F_1 \wedge F_2}{\bullet h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash F_1 \wedge F_2} \rightarrow_L \rightarrow \frac{\overline{h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash F_5}^{\text{ax}} \quad \overline{h_3 : \Delta_4, F_6 \vdash F_1}^{\text{ax/ind}}}{\bullet h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash F_1} \rightarrow_L$$

- Case rule  $\wedge_L$

$$\frac{h_3 : \Delta_4, F_5, F_6 \vdash F_1 \wedge F_2}{\bullet h_3 : \Delta_4, F_5 \wedge F_6 \vdash F_1 \wedge F_2} \wedge_L \rightarrow \frac{\overline{h_3 : \Delta_4, F_5, F_6 \vdash F_1}^{\text{ax/ind}}}{\bullet h_3 : \Delta_4, F_5 \wedge F_6 \vdash F_1} \wedge_L$$

- Case rule  $\vee_L$

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \wedge F_2 \quad h_3 : \Delta_4, F_6 \vdash F_1 \wedge F_2}{\bullet h_3 : \Delta_4, F_5 \vee F_6 \vdash F_1 \wedge F_2} \vee_L \rightarrow \frac{\overline{h_3 : \Delta_4, F_5 \vdash F_1}^{\text{ax/ind}} \quad \overline{h_3 : \Delta_4, F_6 \vdash F_1}^{\text{ax/ind}}}{\bullet h_3 : \Delta_4, F_5 \vee F_6 \vdash F_1} \vee_L$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_3 : \perp, \Delta_4 \vdash F_1 \wedge F_2} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_4 \vdash F_1} \perp_L$$

- Case rule  $I$

- Case rule  $\top_L$

$$\frac{h_3 : \Delta_4 \vdash F_1 \wedge F_2}{\bullet h_3 : \top, \Delta_4 \vdash F_1 \wedge F_2} \top_L \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash F_1}^{\text{ax/ind}}}{\bullet h_3 : \top, \Delta_4 \vdash F_1} \top_L$$

### 3.4 Status of $\wedge_R$ (Right Premise): : Invertible

- Case rule  $\top_R$
- Case rule  $\rightarrow_R$
- Case rule  $\wedge_R$

$$\frac{\frac{h_1 : \Delta_2 \vdash F_3 \quad h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \wedge F_4}}{\wedge_R} \rightarrow \frac{\frac{\frac{}{h_1 : \Delta_2 \vdash F_4} \text{ax}}{h_1 : \Delta_2 \vdash F_4} \text{H}}{\bullet h_1 : \Delta_2 \vdash F_4} \text{H}}$$

- Case rule  $\vee_1$
- Case rule  $\vee_2$
- Case rule  $\rightarrow_L$

$$\frac{\frac{h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash F_5 \quad h_3 : \Delta_4, F_6 \vdash F_1 \wedge F_2}{\bullet h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash F_1 \wedge F_2}}{\rightarrow_L} \rightarrow \frac{\frac{\frac{}{h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash F_5} \text{ax} \quad \frac{}{h_3 : \Delta_4, F_6 \vdash F_2} \text{ax/ind}}{\bullet h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash F_2}}{\rightarrow_L}}$$

- Case rule  $\wedge_L$

$$\frac{\frac{h_3 : \Delta_4, F_5, F_6 \vdash F_1 \wedge F_2}{\bullet h_3 : \Delta_4, F_5 \wedge F_6 \vdash F_1 \wedge F_2}}{\wedge_L} \rightarrow \frac{\frac{\frac{}{h_3 : \Delta_4, F_5, F_6 \vdash F_2} \text{ax/ind}}{\bullet h_3 : \Delta_4, F_5 \wedge F_6 \vdash F_2} \wedge_L}}{\wedge_L}$$

- Case rule  $\vee_L$

$$\frac{\frac{h_3 : \Delta_4, F_5 \vdash F_1 \wedge F_2 \quad h_3 : \Delta_4, F_6 \vdash F_1 \wedge F_2}{\bullet h_3 : \Delta_4, F_5 \vee F_6 \vdash F_1 \wedge F_2}}{\vee_L} \rightarrow \frac{\frac{\frac{}{h_3 : \Delta_4, F_5 \vdash F_2} \text{ax/ind} \quad \frac{}{h_3 : \Delta_4, F_6 \vdash F_2} \text{ax/ind}}{\bullet h_3 : \Delta_4, F_5 \vee F_6 \vdash F_2}}{\vee_L}}$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_3 : \perp, \Delta_4 \vdash F_1 \wedge F_2} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_4 \vdash F_2} \perp_L$$

- Case rule  $I$

- Case rule  $\top_L$

$$\frac{\frac{h_3 : \Delta_4 \vdash F_1 \wedge F_2}{\bullet h_3 : \top, \Delta_4 \vdash F_1 \wedge F_2}}{\top_L} \rightarrow \frac{\frac{\frac{}{h_3 : \Delta_4 \vdash F_2} \text{ax/ind}}{\bullet h_3 : \top, \Delta_4 \vdash F_2}}{\top_L}}$$



### 3.5 Status of $\vee_1$ : : Non invertible

- Case rule  $\top_R$
- Case rule  $\rightarrow_R$
- Case rule  $\wedge_R$
- Case rule  $\vee_1$

$$\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3 \vee F_4} \vee_1 \rightarrow \frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3} \text{ax} \text{H}$$

- Case rule  $\vee_2$

$$\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \vee F_4} \vee_2 \rightarrow \frac{}{\bullet h_1 : \Delta_2 \vdash F_3} \text{fail}$$

- Case rule  $\rightarrow_L$

$$\frac{h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash F_5 \quad h_3 : \Delta_4, F_6 \vdash F_1 \vee F_2}{\bullet h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash F_1 \vee F_2} \rightarrow_L \rightarrow \frac{h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash F_5 \quad h_3 : \Delta_4, F_6 \vdash F_1}{\bullet h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash F_1} \text{ax/ind} \text{ax/ind} \rightarrow_L$$

- Case rule  $\wedge_L$

$$\frac{h_3 : \Delta_4, F_5, F_6 \vdash F_1 \vee F_2}{\bullet h_3 : \Delta_4, F_5 \wedge F_6 \vdash F_1 \vee F_2} \wedge_L \rightarrow \frac{h_3 : \Delta_4, F_5, F_6 \vdash F_1}{\bullet h_3 : \Delta_4, F_5 \wedge F_6 \vdash F_1} \text{ax/ind} \wedge_L$$

- Case rule  $\vee_L$

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \vee F_2 \quad h_3 : \Delta_4, F_6 \vdash F_1 \vee F_2}{\bullet h_3 : \Delta_4, F_5 \vee F_6 \vdash F_1 \vee F_2} \vee_L \rightarrow \frac{h_3 : \Delta_4, F_5 \vdash F_1 \quad h_3 : \Delta_4, F_6 \vdash F_1}{\bullet h_3 : \Delta_4, F_5 \vee F_6 \vdash F_1} \text{ax/ind} \text{ax/ind} \vee_L$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_3 : \perp, \Delta_4 \vdash F_1 \vee F_2} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_4 \vdash F_1} \perp_L$$

- Case rule  $I$

- Case rule  $\top_L$

$$\frac{h_3 : \Delta_4 \vdash F_1 \vee F_2}{\bullet h_3 : \top, \Delta_4 \vdash F_1 \vee F_2} \top_L \rightarrow \frac{h_3 : \Delta_4 \vdash F_1}{\bullet h_3 : \top, \Delta_4 \vdash F_1} \text{ax/ind} \top_L$$

### 3.6 Status of $\vee_2$ : : Non invertible

- Case rule  $\top_R$
- Case rule  $\rightarrow_R$
- Case rule  $\wedge_R$
- Case rule  $\vee_1$

$$\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3 \vee F_4} \vee_1 \rightarrow \frac{}{\bullet h_1 : \Delta_2 \vdash F_4} \text{fail}$$

- Case rule  $\vee_2$

$$\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \vee F_4} \vee_2 \rightarrow \frac{}{\bullet h_1 : \Delta_2 \vdash F_4} \begin{matrix} \text{ax} \\ \text{H} \end{matrix}$$

- Case rule  $\rightarrow_L$

$$\frac{h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash F_5 \quad h_3 : \Delta_4, F_6 \vdash F_1 \vee F_2}{\bullet h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash F_1 \vee F_2} \rightarrow_L \rightarrow \frac{}{h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash F_5} \text{ax} \quad \frac{}{h_3 : \Delta_4, F_6 \vdash F_2} \text{ax/ind} \rightarrow_L$$

- Case rule  $\wedge_L$

$$\frac{h_3 : \Delta_4, F_5, F_6 \vdash F_1 \vee F_2}{\bullet h_3 : \Delta_4, F_5 \wedge F_6 \vdash F_1 \vee F_2} \wedge_L \rightarrow \frac{}{h_3 : \Delta_4, F_5, F_6 \vdash F_2} \text{ax/ind} \wedge_L$$

- Case rule  $\vee_L$

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \vee F_2 \quad h_3 : \Delta_4, F_6 \vdash F_1 \vee F_2}{\bullet h_3 : \Delta_4, F_5 \vee F_6 \vdash F_1 \vee F_2} \vee_L \rightarrow \frac{}{h_3 : \Delta_4, F_5 \vdash F_2} \text{ax/ind} \quad \frac{}{h_3 : \Delta_4, F_6 \vdash F_2} \text{ax/ind} \vee_L$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_3 : \perp, \Delta_4 \vdash F_1 \vee F_2} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_4 \vdash F_2} \perp_L$$

- Case rule  $I$

- Case rule  $\top_L$

$$\frac{h_3 : \Delta_4 \vdash F_1 \vee F_2}{\bullet h_3 : \top, \Delta_4 \vdash F_1 \vee F_2} \top_L \rightarrow \frac{}{h_3 : \Delta_4 \vdash F_2} \text{ax/ind} \top_L$$

### 3.7 Status of $\rightarrow_L$ : (Left Premise): Non invertible

- Case rule  $\top_R$

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \top} \top_R \rightarrow \frac{}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_2} \text{fail}$$

- Case rule  $\rightarrow_R$

$$\frac{h_4 : \Delta_1, F_5, F_2 \rightarrow F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_5 \rightarrow F_6} \rightarrow_R \rightarrow \frac{}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_2} \text{fail}$$

- Case rule  $\wedge_R$

$$\frac{h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_5 \quad h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_5 \wedge F_6} \wedge_R \rightarrow \frac{h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_2}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_2} \text{ax/ind} \quad \text{H}$$

- Case rule  $\vee_1$

$$\frac{h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_5}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_5 \vee F_6} \vee_1 \rightarrow \frac{h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_2}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_2} \text{ax/ind} \quad \text{H}$$

- Case rule  $\vee_2$

$$\frac{h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_5 \vee F_6} \vee_2 \rightarrow \frac{h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_2}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_2} \text{ax/ind} \quad \text{H}$$

- Case rule  $\rightarrow_L$

$$\frac{h_3 : \Delta_7, F_1 \rightarrow F_2, F_4 \rightarrow F_5 \vdash F_4 \quad h_3 : \Delta_7, F_5, F_1 \rightarrow F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \rightarrow F_5 \vdash F_6} \rightarrow_L \rightarrow \frac{h_3 : \Delta_7, F_1 \rightarrow F_2, F_4 \rightarrow F_5 \vdash F_1}{\bullet h_3 : \Delta_7, F_1 \rightarrow F_2, F_4 \rightarrow F_5 \vdash F_1} \text{ax/ind} \quad \text{H}$$

$$\frac{h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_5} \rightarrow_L \rightarrow \frac{h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_3}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_3} \text{ax/ind} \quad \text{H}$$

- Case rule  $\wedge_L$

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \rightarrow F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \wedge F_5 \vdash F_6} \wedge_L \rightarrow \frac{h_3 : \Delta_7, F_4, F_5, F_1 \rightarrow F_2 \vdash F_1}{\bullet h_3 : \Delta_7, F_1 \rightarrow F_2, F_4 \wedge F_5 \vdash F_1} \text{ax/ind} \quad \wedge_L$$

- Case rule  $\vee_L$

$$\frac{h_3 : \Delta_7, F_4, F_1 \rightarrow F_2 \vdash F_6 \quad h_3 : \Delta_7, F_5, F_1 \rightarrow F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \vee F_5 \vdash F_6} \vee_L \rightarrow \frac{h_3 : \Delta_7, F_4, F_1 \rightarrow F_2 \vdash F_1}{\bullet h_3 : \Delta_7, F_1 \rightarrow F_2, F_4 \vee F_5 \vdash F_1} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_5, F_1 \rightarrow F_2 \vdash F_1}{\vee_L}$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \rightarrow F_2 \vdash F_4} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \rightarrow F_2 \vdash F_1} \perp_L$$

- Case rule  $I$

$$\frac{}{\bullet h_3 : p_4, \Delta_5, F_1 \rightarrow F_2 \vdash p_4} I \rightarrow \frac{}{\bullet h_3 : \Delta_5, p_4, F_1 \rightarrow F_2 \vdash F_1} \text{fail}$$

- Case rule  $\top_L$

$$\frac{h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash F_4}{\bullet h_3 : \top, \Delta_5, F_1 \rightarrow F_2 \vdash F_4} \top_L \rightarrow \frac{}{h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash F_1} \text{ax/ind} \top_L$$

### 3.8 Status of $\rightarrow_L$ (Right Premise): : Invertible

- Case rule  $\top_R$

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \top} \top_R \rightarrow \frac{}{\bullet h_4 : \Delta_1, F_3 \vdash \top} \top_R$$

- Case rule  $\rightarrow_R$

$$\frac{h_4 : \Delta_1, F_5, F_2 \rightarrow F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_5 \rightarrow F_6} \rightarrow_R \rightarrow \frac{}{h_4 : \Delta_1, F_3, F_5 \vdash F_6} \text{ax/ind} \rightarrow_R$$

- Case rule  $\wedge_R$

$$\frac{h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_5 \quad h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_5 \wedge F_6} \wedge_R \rightarrow \frac{}{h_4 : \Delta_1, F_3 \vdash F_5} \text{ax/ind} \frac{}{h_4 : \Delta_1, F_3 \vdash F_6} \text{ax/ind} \wedge_R$$

- Case rule  $\vee_1$

$$\frac{h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_5}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_5 \vee F_6} \vee_1 \rightarrow \frac{}{h_4 : \Delta_1, F_3 \vdash F_5} \text{ax/ind} \vee_1$$

- Case rule  $\vee_2$

$$\frac{h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash F_5 \vee F_6} \vee_2 \rightarrow \frac{}{h_4 : \Delta_1, F_3 \vdash F_6} \text{ax/ind} \vee_2$$

- Case rule  $\rightarrow_L$

$$\frac{h_3 : \Delta_7, F_1 \rightarrow F_2, F_4 \rightarrow F_5 \vdash F_4 \quad h_3 : \Delta_7, F_5, F_1 \rightarrow F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \rightarrow F_5 \vdash F_6} \rightarrow_L \rightarrow \frac{}{h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash F_4} \text{ax/ind} \frac{}{h_3 : \Delta_7, F_2, F_5 \vdash F_6} \text{ax/ind} \rightarrow_L$$

$$\frac{h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_5} \rightarrow_L \rightarrow \frac{}{h_1 : \Delta_2, F_4 \vdash F_5} \text{ax} \text{H}$$

- Case rule  $\wedge_L$

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \rightarrow F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \wedge F_5 \vdash F_6} \wedge_L \rightarrow \frac{}{h_3 : \Delta_7, F_2, F_4, F_5 \vdash F_6} \text{ax/ind} \wedge_L$$

- Case rule  $\vee_L$

$$\frac{\frac{h_3 : \Delta_7, F_4, F_1 \rightarrow F_2 \vdash F_6 \quad h_3 : \Delta_7, F_5, F_1 \rightarrow F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \vee F_5 \vdash F_6} \vee_L}{\bullet h_3 : \Delta_7, F_2, F_4 \vee F_5 \vdash F_6} \rightarrow \frac{\frac{h_3 : \Delta_7, F_2, F_4 \vdash F_6}{\bullet h_3 : \Delta_7, F_2, F_4 \vee F_5 \vdash F_6} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_2, F_5 \vdash F_6}{\bullet h_3 : \Delta_7, F_2, F_4 \vee F_5 \vdash F_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \vee F_5 \vdash F_6} \vee_L$$

- Case rule  $\perp_L$

$$\frac{\bullet h_3 : \perp, \Delta_5, F_1 \rightarrow F_2 \vdash F_4}{\bullet h_3 : \perp, \Delta_5, F_2 \vdash F_4} \perp_L \rightarrow \frac{\bullet h_3 : \perp, \Delta_5, F_2 \vdash F_4}{\bullet h_3 : \perp, \Delta_5, F_2 \vdash F_4} \perp_L$$

- Case rule  $I$

$$\frac{\bullet h_3 : p_4, \Delta_5, F_1 \rightarrow F_2 \vdash p_4}{\bullet h_3 : \Delta_5, F_2, p_4 \vdash p_4} I \rightarrow \frac{\bullet h_3 : \Delta_5, F_2, p_4 \vdash p_4}{\bullet h_3 : \Delta_5, F_2, p_4 \vdash p_4} I$$

- Case rule  $\top_L$

$$\frac{\frac{h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash F_4}{\bullet h_3 : \top, \Delta_5, F_1 \rightarrow F_2 \vdash F_4} \top_L}{\bullet h_3 : \top, \Delta_5, F_2 \vdash F_4} \rightarrow \frac{\frac{h_3 : \Delta_5, F_2 \vdash F_4}{\bullet h_3 : \top, \Delta_5, F_2 \vdash F_4} \text{ax/ind}}{\bullet h_3 : \top, \Delta_5, F_2 \vdash F_4} \top_L$$

### 3.9 Status of $\wedge_L$ : : Invertible

- Case rule  $\top_R$

$$\frac{\bullet h_4 : \Delta_1, F_2 \wedge F_3 \vdash \top}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \top} \top_R \rightarrow \frac{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \top}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \top} \top_R$$

- Case rule  $\rightarrow_R$

$$\frac{\frac{h_4 : \Delta_1, F_5, F_2 \wedge F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \wedge F_3 \vdash F_5 \rightarrow F_6} \rightarrow_R}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \rightarrow F_6} \rightarrow \frac{\frac{h_4 : \Delta_1, F_2, F_3, F_5 \vdash F_6}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \rightarrow F_6} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \rightarrow F_6} \rightarrow_R$$

- Case rule  $\wedge_R$

$$\frac{\frac{h_4 : \Delta_1, F_2 \wedge F_3 \vdash F_5 \quad h_4 : \Delta_1, F_2 \wedge F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \wedge F_3 \vdash F_5 \wedge F_6} \wedge_R}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \wedge F_6} \rightarrow \frac{\frac{h_4 : \Delta_1, F_2, F_3 \vdash F_5}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \wedge F_6} \text{ax/ind} \quad \frac{h_4 : \Delta_1, F_2, F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \wedge F_6} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \wedge F_6} \wedge_R$$

- Case rule  $\vee_1$

$$\frac{\frac{h_4 : \Delta_1, F_2 \wedge F_3 \vdash F_5}{\bullet h_4 : \Delta_1, F_2 \wedge F_3 \vdash F_5 \vee F_6} \vee_1}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \vee F_6} \rightarrow \frac{\frac{h_4 : \Delta_1, F_2, F_3 \vdash F_5}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \vee F_6} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \vee F_6} \vee_1$$

- Case rule  $\vee_2$

$$\frac{\frac{h_4 : \Delta_1, F_2 \wedge F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \wedge F_3 \vdash F_5 \vee F_6} \vee_2}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \vee F_6} \rightarrow \frac{\frac{h_4 : \Delta_1, F_2, F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \vee F_6} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \vee F_6} \vee_2$$

- Case rule  $\rightarrow_L$

$$\frac{\frac{h_3 : \Delta_7, F_4 \rightarrow F_5, F_1 \wedge F_2 \vdash F_4 \quad h_3 : \Delta_7, F_5, F_1 \wedge F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \wedge F_2), F_4 \rightarrow F_5 \vdash F_6} \rightarrow_L}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \rightarrow F_5 \vdash F_6} \text{ax/ind} \rightarrow_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_2, F_4 \rightarrow F_5 \vdash F_4}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \rightarrow F_5 \vdash F_6} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_1, F_2, F_5 \vdash F_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \rightarrow F_5 \vdash F_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \rightarrow F_5 \vdash F_6} \rightarrow_L$$

- Case rule  $\wedge_L$

$$\frac{\frac{h_3 : \Delta_7, F_4, F_5, F_1 \wedge F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \wedge F_2), F_4 \wedge F_5 \vdash F_6} \wedge_L}{\bullet h_1 : \Delta_2, F_3, F_4 \vdash F_5} \wedge_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_2, F_4, F_5 \vdash F_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \wedge F_5 \vdash F_6} \text{ax/ind}}{\bullet h_1 : \Delta_2, F_3, F_4 \vdash F_5} \wedge_L$$

$$\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash F_5} \wedge_L}{\bullet h_1 : \Delta_2, F_3, F_4 \vdash F_5} \wedge_L \rightarrow \frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3, F_4 \vdash F_5} \text{ax}}{\bullet h_1 : \Delta_2, F_3, F_4 \vdash F_5} \text{H}$$

- Case rule  $\vee_L$

$$\frac{\frac{h_3 : \Delta_7, F_4, F_1 \wedge F_2 \vdash F_6 \quad h_3 : \Delta_7, F_5, F_1 \wedge F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \wedge F_2), F_4 \vee F_5 \vdash F_6} \vee_L}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \vee F_5 \vdash F_6} \text{ax/ind} \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_2, F_4 \vdash F_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \vee F_5 \vdash F_6} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_1, F_2, F_5 \vdash F_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \vee F_5 \vdash F_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \vee F_5 \vdash F_6} \vee_L$$

- Case rule  $\perp_L$

$$\frac{\bullet h_3 : \perp, \Delta_5, F_1 \wedge F_2 \vdash F_4}{\bullet h_3 : \perp, \Delta_5, F_1, F_2 \vdash F_4} \perp_L \rightarrow \frac{\bullet h_3 : \perp, \Delta_5, F_1, F_2 \vdash F_4}{\bullet h_3 : \perp, \Delta_5, F_1, F_2 \vdash F_4} \perp_L$$

- Case rule  $I$

$$\frac{\bullet h_3 : p_4, \Delta_5, F_1 \wedge F_2 \vdash p_4}{\bullet h_3 : \Delta_5, F_1, F_2, p_4 \vdash p_4} I \rightarrow \frac{\bullet h_3 : \Delta_5, F_1, F_2, p_4 \vdash p_4}{\bullet h_3 : \Delta_5, F_1, F_2, p_4 \vdash p_4} I$$

- Case rule  $\top_L$

$$\frac{\frac{h_3 : \Delta_5, F_1 \wedge F_2 \vdash F_4}{\bullet h_3 : \top, \Delta_5, F_1 \wedge F_2 \vdash F_4} \top_L}{\bullet h_3 : \top, \Delta_5, F_1, F_2 \vdash F_4} \top_L \rightarrow \frac{\frac{h_3 : \Delta_5, F_1, F_2 \vdash F_4}{\bullet h_3 : \top, \Delta_5, F_1, F_2 \vdash F_4} \text{ax/ind}}{\bullet h_3 : \top, \Delta_5, F_1, F_2 \vdash F_4} \top_L$$

### 3.10 Status of $\vee_L$ : (Left Premise): Invertible

- Case rule  $\top_R$

$$\frac{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \top}{\bullet h_4 : \Delta_1, F_2 \vdash \top} \top_R \rightarrow \frac{\bullet h_4 : \Delta_1, F_2 \vdash \top}{\bullet h_4 : \Delta_1, F_2 \vdash \top} \top_R$$

- Case rule  $\rightarrow_R$

$$\frac{\frac{h_4 : \Delta_1, F_5, F_2 \vee F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash F_5 \rightarrow F_6} \rightarrow_R}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \rightarrow F_6} \rightarrow_R \rightarrow \frac{\frac{h_4 : \Delta_1, F_2, F_5 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \rightarrow F_6} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \rightarrow F_6} \rightarrow_R$$

- Case rule  $\wedge_R$

$$\frac{\frac{h_4 : \Delta_1, F_2 \vee F_3 \vdash F_5 \quad h_4 : \Delta_1, F_2 \vee F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash F_5 \wedge F_6} \wedge_R}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \wedge F_6} \wedge_R \rightarrow \frac{\frac{h_4 : \Delta_1, F_2 \vdash F_5}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \wedge F_6} \text{ax/ind} \quad \frac{h_4 : \Delta_1, F_2 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \wedge F_6} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \wedge F_6} \wedge_R$$

- Case rule  $\vee_1$

$$\frac{h_4 : \Delta_1, F_2 \vee F_3 \vdash F_5}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash F_5 \vee F_6} \vee_1 \rightarrow \frac{\overline{h_4 : \Delta_1, F_2 \vdash F_5} \text{ ax/ind}}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \vee F_6} \vee_1$$

- Case rule  $\vee_2$

$$\frac{h_4 : \Delta_1, F_2 \vee F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash F_5 \vee F_6} \vee_2 \rightarrow \frac{\overline{h_4 : \Delta_1, F_2 \vdash F_6} \text{ ax/ind}}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \vee F_6} \vee_2$$

- Case rule  $\rightarrow_L$

$$\frac{h_3 : \Delta_7, F_4 \rightarrow F_5, F_1 \vee F_2 \vdash F_4 \quad h_3 : \Delta_7, F_5, F_1 \vee F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \rightarrow F_5 \vdash F_6} \rightarrow_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_4 \rightarrow F_5 \vdash F_4} \text{ ax/ind} \quad \overline{h_3 : \Delta_7, F_1, F_5 \vdash F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \rightarrow F_5 \vdash F_6} \rightarrow_L$$

- Case rule  $\wedge_L$

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \vee F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \wedge F_5 \vdash F_6} \wedge_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_4, F_5 \vdash F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \wedge F_5 \vdash F_6} \wedge_L$$

- Case rule  $\vee_L$

$$\frac{h_3 : \Delta_7, F_4, F_1 \vee F_2 \vdash F_6 \quad h_3 : \Delta_7, F_5, F_1 \vee F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \vee F_5 \vdash F_6} \vee_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_4 \vdash F_6} \text{ ax/ind} \quad \overline{h_3 : \Delta_7, F_1, F_5 \vdash F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \vee F_5 \vdash F_6} \vee_L$$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_5 \quad h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_5} \vee_L \rightarrow \frac{\overline{h_1 : \Delta_2, F_3 \vdash F_5} \text{ ax}}{\bullet h_1 : \Delta_2, F_3 \vdash F_5} \text{ H}$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \vee F_2 \vdash F_4} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \vdash F_4} \perp_L$$

- Case rule  $I$

$$\frac{}{\bullet h_3 : p_4, \Delta_5, F_1 \vee F_2 \vdash p_4} I \rightarrow \frac{}{\bullet h_3 : \Delta_5, F_1, p_4 \vdash p_4} I$$

- Case rule  $\top_L$

$$\frac{h_3 : \Delta_5, F_1 \vee F_2 \vdash F_4}{\bullet h_3 : \top, \Delta_5, F_1 \vee F_2 \vdash F_4} \top_L \rightarrow \frac{\overline{h_3 : \Delta_5, F_1 \vdash F_4} \text{ ax/ind}}{\bullet h_3 : \top, \Delta_5, F_1 \vdash F_4} \top_L$$

### 3.11 Status of $\vee_L$ (Right Premise): : Invertible

- Case rule  $\top_R$

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \top} \top_R \rightarrow \frac{}{\bullet h_4 : \Delta_1, F_3 \vdash \top} \top_R$$

- Case rule  $\rightarrow_R$

$$\frac{h_4 : \Delta_1, F_5, F_2 \vee F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash F_5 \rightarrow F_6} \rightarrow_R \rightarrow \frac{\frac{}{h_4 : \Delta_1, F_3, F_5 \vdash F_6} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \rightarrow F_6} \rightarrow_R$$

- Case rule  $\wedge_R$

$$\frac{\frac{h_4 : \Delta_1, F_2 \vee F_3 \vdash F_5 \quad h_4 : \Delta_1, F_2 \vee F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash F_5 \wedge F_6} \wedge_R}{\frac{\frac{}{h_4 : \Delta_1, F_3 \vdash F_5} \text{ax/ind} \quad \frac{}{h_4 : \Delta_1, F_3 \vdash F_6} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \wedge F_6} \wedge_R}$$

- Case rule  $\vee_1$

$$\frac{h_4 : \Delta_1, F_2 \vee F_3 \vdash F_5}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash F_5 \vee F_6} \vee_1 \rightarrow \frac{\frac{}{h_4 : \Delta_1, F_3 \vdash F_5} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \vee F_6} \vee_1$$

- Case rule  $\vee_2$

$$\frac{h_4 : \Delta_1, F_2 \vee F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash F_5 \vee F_6} \vee_2 \rightarrow \frac{\frac{}{h_4 : \Delta_1, F_3 \vdash F_6} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \vee F_6} \vee_2$$

- Case rule  $\rightarrow_L$

$$\frac{\frac{h_3 : \Delta_7, F_4 \rightarrow F_5, F_1 \vee F_2 \vdash F_4 \quad h_3 : \Delta_7, F_5, F_1 \vee F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \rightarrow F_5 \vdash F_6} \rightarrow_L}{\frac{\frac{}{h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash F_4} \text{ax/ind} \quad \frac{}{h_3 : \Delta_7, F_2, F_5 \vdash F_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash F_6} \rightarrow_L}$$

- Case rule  $\wedge_L$

$$\frac{\frac{h_3 : \Delta_7, F_4, F_5, F_1 \vee F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \wedge F_5 \vdash F_6} \wedge_L}{\frac{\frac{}{h_3 : \Delta_7, F_2, F_4, F_5 \vdash F_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \wedge F_5 \vdash F_6} \wedge_L}$$

- Case rule  $\vee_L$

$$\frac{\frac{h_3 : \Delta_7, F_4, F_1 \vee F_2 \vdash F_6 \quad h_3 : \Delta_7, F_5, F_1 \vee F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \vee F_5 \vdash F_6} \vee_L}{\frac{\frac{}{h_3 : \Delta_7, F_2, F_4 \vdash F_6} \text{ax/ind} \quad \frac{}{h_3 : \Delta_7, F_2, F_5 \vdash F_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \vee F_5 \vdash F_6} \vee_L}$$

$$\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5 \quad h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_5} \vee_L}{\frac{\frac{}{h_1 : \Delta_2, F_4 \vdash F_5} \text{ax}}{\bullet h_1 : \Delta_2, F_4 \vdash F_5} \text{H}}$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \vee F_2 \vdash F_4} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5, F_2 \vdash F_4} \perp_L$$



- Case rule  $I$

$$\frac{}{\bullet h_3 : P_4, \Delta_5, F_1 \vee F_2 \vdash P_4} I \quad \rightarrow \quad \frac{}{\bullet h_3 : \Delta_5, F_2, P_4 \vdash P_4} I$$

- Case rule  $\top_L$

$$\frac{h_3 : \Delta_5, F_1 \vee F_2 \vdash F_4}{\bullet h_3 : \top, \Delta_5, F_1 \vee F_2 \vdash F_4} \top_L \quad \rightarrow \quad \frac{\frac{}{h_3 : \Delta_5, F_2 \vdash F_4} \text{ax/ind}}{\bullet h_3 : \top, \Delta_5, F_2 \vdash F_4} \top_L$$

### 3.12 Status of $\perp_L$ : : Invertible

- Case rule  $\top_R$

$$\frac{}{\bullet h_2 : \perp, \Delta_1 \vdash \top} \top_R \quad \rightarrow \quad \text{trivial}$$

- Case rule  $\rightarrow_R$

$$\frac{h_2 : \perp, \Delta_1, F_3 \vdash F_4}{\bullet h_2 : \perp, \Delta_1 \vdash F_3 \rightarrow F_4} \rightarrow_R \quad \rightarrow \quad \text{trivial}$$

- Case rule  $\wedge_R$

$$\frac{h_2 : \perp, \Delta_1 \vdash F_3 \quad h_2 : \perp, \Delta_1 \vdash F_4}{\bullet h_2 : \perp, \Delta_1 \vdash F_3 \wedge F_4} \wedge_R \quad \rightarrow \quad \text{trivial}$$

- Case rule  $\vee_1$

$$\frac{h_2 : \perp, \Delta_1 \vdash F_3}{\bullet h_2 : \perp, \Delta_1 \vdash F_3 \vee F_4} \vee_1 \quad \rightarrow \quad \text{trivial}$$

- Case rule  $\vee_2$

$$\frac{h_2 : \perp, \Delta_1 \vdash F_4}{\bullet h_2 : \perp, \Delta_1 \vdash F_3 \vee F_4} \vee_2 \quad \rightarrow \quad \text{trivial}$$

- Case rule  $\rightarrow_L$

$$\frac{h_1 : \perp, \Delta_5, F_2 \rightarrow F_3 \vdash F_2 \quad h_1 : \perp, \Delta_5, F_3 \vdash F_4}{\bullet h_1 : (\perp, \Delta_5), F_2 \rightarrow F_3 \vdash F_4} \rightarrow_L \quad \rightarrow \quad \text{trivial}$$

- Case rule  $\wedge_L$

$$\frac{h_1 : \perp, \Delta_5, F_2, F_3 \vdash F_4}{\bullet h_1 : (\perp, \Delta_5), F_2 \wedge F_3 \vdash F_4} \wedge_L \quad \rightarrow \quad \text{trivial}$$

- Case rule  $\vee_L$

$$\frac{h_1 : \perp, \Delta_5, F_2 \vdash F_4 \quad h_1 : \perp, \Delta_5, F_3 \vdash F_4}{\bullet h_1 : (\perp, \Delta_5), F_2 \vee F_3 \vdash F_4} \vee_L \quad \rightarrow \quad \text{trivial}$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_1 : \perp, \Delta_2 \vdash F_3} \perp_L \rightarrow \text{trivial}$$

- Case rule  $I$

$$\frac{}{\bullet h_1 : p_2, \perp, \Delta_3 \vdash p_2} I \rightarrow \text{trivial}$$

- Case rule  $\top_L$

$$\frac{h_1 : \perp, \Delta_3 \vdash F_2}{\bullet h_1 : \top, \perp, \Delta_3 \vdash F_2} \top_L \rightarrow \text{trivial}$$

### 3.13 Status of $I$ : : Invertible

- Case rule  $\top_R$

- Case rule  $\rightarrow_R$

- Case rule  $\wedge_R$

- Case rule  $\vee_1$

- Case rule  $\vee_2$

- Case rule  $\rightarrow_L$

$$\frac{h_2 : \Delta_5, p_1, F_3 \rightarrow F_4 \vdash F_3 \quad h_2 : \Delta_5, F_4, p_1 \vdash p_1}{\bullet h_2 : (\Delta_5, p_1), F_3 \rightarrow F_4 \vdash p_1} \rightarrow_L \rightarrow \text{trivial}$$

- Case rule  $\wedge_L$

$$\frac{h_2 : \Delta_5, F_3, F_4, p_1 \vdash p_1}{\bullet h_2 : (\Delta_5, p_1), F_3 \wedge F_4 \vdash p_1} \wedge_L \rightarrow \text{trivial}$$

- Case rule  $\vee_L$

$$\frac{h_2 : \Delta_5, F_3, p_1 \vdash p_1 \quad h_2 : \Delta_5, F_4, p_1 \vdash p_1}{\bullet h_2 : (\Delta_5, p_1), F_3 \vee F_4 \vdash p_1} \vee_L \rightarrow \text{trivial}$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_2 : \perp, \Delta_3, p_1 \vdash p_1} \perp_L \rightarrow \text{trivial}$$

- Case rule  $I$

$$\frac{}{\bullet h_1 : p_3, \Delta_2 \vdash p_3} I \rightarrow \text{trivial}$$

- Case rule  $\top_L$

$$\frac{h_2 : \Delta_3, p_1 \vdash p_1}{\bullet h_2 : \top, \Delta_3, p_1 \vdash p_1} \top_L \rightarrow \text{trivial}$$

### 3.14 Status of $\top_L$ : : Invertible

- Case rule  $\top_R$

$$\frac{}{\bullet h_2 : \top, \Delta_1 \vdash \top} \top_R \rightarrow \frac{}{\bullet h_2 : \Delta_1 \vdash \top} \top_R$$

- Case rule  $\rightarrow_R$

$$\frac{h_2 : \top, \Delta_1, F_3 \vdash F_4}{\bullet h_2 : \top, \Delta_1 \vdash F_3 \rightarrow F_4} \rightarrow_R \rightarrow \frac{\frac{}{h_2 : \Delta_1, F_3 \vdash F_4} \text{ax/ind}}{\bullet h_2 : \Delta_1 \vdash F_3 \rightarrow F_4} \rightarrow_R}{\bullet h_2 : \Delta_1 \vdash F_3 \rightarrow F_4} \rightarrow_R$$

- Case rule  $\wedge_R$

$$\frac{\frac{h_2 : \top, \Delta_1 \vdash F_3 \quad h_2 : \top, \Delta_1 \vdash F_4}{\bullet h_2 : \top, \Delta_1 \vdash F_3 \wedge F_4} \wedge_R}{\bullet h_2 : \Delta_1 \vdash F_3 \wedge F_4} \wedge_R \rightarrow \frac{\frac{\frac{}{h_2 : \Delta_1 \vdash F_3} \text{ax/ind} \quad \frac{}{h_2 : \Delta_1 \vdash F_4} \text{ax/ind}}{\bullet h_2 : \Delta_1 \vdash F_3 \wedge F_4} \wedge_R}{\bullet h_2 : \Delta_1 \vdash F_3 \wedge F_4} \wedge_R$$

- Case rule  $\vee_1$

$$\frac{h_2 : \top, \Delta_1 \vdash F_3}{\bullet h_2 : \top, \Delta_1 \vdash F_3 \vee F_4} \vee_1 \rightarrow \frac{\frac{}{h_2 : \Delta_1 \vdash F_3} \text{ax/ind}}{\bullet h_2 : \Delta_1 \vdash F_3 \vee F_4} \vee_1$$

- Case rule  $\vee_2$

$$\frac{h_2 : \top, \Delta_1 \vdash F_4}{\bullet h_2 : \top, \Delta_1 \vdash F_3 \vee F_4} \vee_2 \rightarrow \frac{\frac{}{h_2 : \Delta_1 \vdash F_4} \text{ax/ind}}{\bullet h_2 : \Delta_1 \vdash F_3 \vee F_4} \vee_2$$

- Case rule  $\rightarrow_L$

$$\frac{\frac{h_1 : \top, \Delta_5, F_2 \rightarrow F_3 \vdash F_2 \quad h_1 : \top, \Delta_5, F_3 \vdash F_4}{\bullet h_1 : (\top, \Delta_5), F_2 \rightarrow F_3 \vdash F_4} \rightarrow_L}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_4} \rightarrow_L \rightarrow \frac{\frac{\frac{}{h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_2} \text{ax/ind} \quad \frac{}{h_1 : \Delta_5, F_3 \vdash F_4} \text{ax/ind}}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_4} \rightarrow_L}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash F_4} \rightarrow_L$$

- Case rule  $\wedge_L$

$$\frac{\frac{h_1 : \top, \Delta_5, F_2, F_3 \vdash F_4}{\bullet h_1 : (\top, \Delta_5), F_2 \wedge F_3 \vdash F_4} \wedge_L}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash F_4} \wedge_L \rightarrow \frac{\frac{\frac{}{h_1 : \Delta_5, F_2, F_3 \vdash F_4} \text{ax/ind}}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash F_4} \wedge_L}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash F_4} \wedge_L$$

- Case rule  $\vee_L$

$$\frac{\frac{h_1 : \top, \Delta_5, F_2 \vdash F_4}{\bullet h_1 : (\top, \Delta_5), F_2 \vee F_3 \vdash F_4} \vee_L \quad \frac{h_1 : \top, \Delta_5, F_3 \vdash F_4}{\bullet h_1 : (\top, \Delta_5), F_2 \vee F_3 \vdash F_4} \vee_L}{\frac{h_1 : \Delta_5, F_2 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_4} \text{ax/ind} \quad \frac{h_1 : \Delta_5, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash F_4} \text{ax/ind}}{\vee_L} \rightarrow$$

- Case rule  $\perp_L$

$$\frac{}{\bullet h_1 : \perp, \top, \Delta_3 \vdash F_2} \perp_L \rightarrow \frac{}{\bullet h_1 : \perp, \Delta_3 \vdash F_2} \perp_L$$

- Case rule  $I$

$$\frac{}{\bullet h_1 : p_2, \top, \Delta_3 \vdash p_2} I \rightarrow \frac{}{\bullet h_1 : \Delta_3, p_2 \vdash p_2} I$$

- Case rule  $\top_L$

$$\frac{\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \top, \Delta_2 \vdash F_3} \top_L}{\bullet h_1 : \Delta_2 \vdash F_3} \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3} \text{ax}}{\bullet h_1 : \Delta_2 \vdash F_3} \text{H}$$

## 4 Height preserving admissibility of contraction

- Case(s) rule  $\top_R$

$$\frac{}{\bullet h_3 : \Delta_1, F_2, F_2 \vdash \top} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_1, F_2 \vdash \top} \top_R$$

- Case(s) rule  $\rightarrow_R$

$$\frac{h_3 : \Delta_1, F_2, F_2, F_4 \vdash F_5}{\bullet h_3 : \Delta_1, F_2, F_2 \vdash F_4 \rightarrow F_5} \rightarrow_R \rightarrow \frac{\frac{h_3 : \Delta_1, F_2, F_2, F_4 \vdash F_5}{\bullet h_3 : \Delta_1, F_2, F_2 \vdash F_4 \rightarrow F_5} \text{ ax}}{h_3 : \Delta_1, F_2, F_2, F_4 \vdash F_5} \text{ IH} \rightarrow_R$$

- Case(s) rule  $\wedge_R$

$$\frac{h_3 : \Delta_1, F_2, F_2 \vdash F_4 \quad h_3 : \Delta_1, F_2, F_2 \vdash F_5}{\bullet h_3 : \Delta_1, F_2, F_2 \vdash F_4 \wedge F_5} \wedge_R \rightarrow \frac{\frac{h_3 : \Delta_1, F_2, F_2 \vdash F_4}{\bullet h_3 : \Delta_1, F_2 \vdash F_4} \text{ ax} \quad \frac{h_3 : \Delta_1, F_2, F_2 \vdash F_5}{\bullet h_3 : \Delta_1, F_2 \vdash F_5} \text{ ax}}{h_3 : \Delta_1, F_2 \vdash F_4 \wedge F_5} \text{ IH} \wedge_R$$

- Case(s) rule  $\vee_1$

$$\frac{h_3 : \Delta_1, F_2, F_2 \vdash F_4}{\bullet h_3 : \Delta_1, F_2, F_2 \vdash F_4 \vee F_5} \vee_1 \rightarrow \frac{\frac{h_3 : \Delta_1, F_2, F_2 \vdash F_4}{\bullet h_3 : \Delta_1, F_2 \vdash F_4} \text{ ax}}{h_3 : \Delta_1, F_2 \vdash F_4 \vee F_5} \text{ IH} \vee_1$$

- Case(s) rule  $\vee_2$

$$\frac{h_3 : \Delta_1, F_2, F_2 \vdash F_5}{\bullet h_3 : \Delta_1, F_2, F_2 \vdash F_4 \vee F_5} \vee_2 \rightarrow \frac{\frac{h_3 : \Delta_1, F_2, F_2 \vdash F_5}{\bullet h_3 : \Delta_1, F_2 \vdash F_5} \text{ ax}}{h_3 : \Delta_1, F_2 \vdash F_4 \vee F_5} \text{ IH} \vee_2$$

- Case(s) rule  $\rightarrow_L$

$$\frac{h_2 : \Delta_1, F_3 \rightarrow F_4, F_3 \rightarrow F_4 \vdash F_3 \quad h_2 : \Delta_1, F_4, F_3 \rightarrow F_4 \vdash F_5}{\bullet h_2 : \Delta_1, F_3 \rightarrow F_4, F_3 \rightarrow F_4 \vdash F_5} \rightarrow_L \rightarrow \frac{\frac{h_2 : \Delta_1, F_3 \rightarrow F_4, F_3 \rightarrow F_4 \vdash F_3}{\bullet h_2 : \Delta_1, F_3 \rightarrow F_4 \vdash F_3} \text{ ax} \quad \frac{h_2 : \Delta_1, F_4, F_3 \rightarrow F_4 \vdash F_5}{\bullet h_2 : \Delta_1, F_4 \vdash F_5} \text{ inv-th/ax}}{h_2 : \Delta_1, F_3 \rightarrow F_4 \vdash F_5} \text{ IH} \rightarrow_L$$

$$\frac{h_2 : \Delta_6, F_1, F_1, F_3 \rightarrow F_4 \vdash F_3 \quad h_2 : \Delta_6, F_1, F_1, F_4 \vdash F_5}{\bullet h_2 : (\Delta_6, F_3 \rightarrow F_4), F_1, F_1 \vdash F_5} \rightarrow_L \rightarrow \frac{\frac{h_2 : \Delta_6, F_1, F_1, F_3 \rightarrow F_4 \vdash F_3}{\bullet h_2 : \Delta_6, F_1, F_3 \rightarrow F_4 \vdash F_3} \text{ ax} \quad \frac{h_2 : \Delta_6, F_1, F_1, F_4 \vdash F_5}{\bullet h_2 : \Delta_6, F_1, F_4 \vdash F_5} \text{ ax}}{h_2 : \Delta_6, F_1, F_3 \rightarrow F_4 \vdash F_5} \text{ IH} \rightarrow_L$$

- Case(s) rule  $\wedge_L$

$$\frac{h_2 : \Delta_1, F_3, F_4, F_3 \wedge F_4 \vdash F_5}{\bullet h_2 : \Delta_1, F_3 \wedge F_4, F_3 \wedge F_4 \vdash F_5} \wedge_L \rightarrow \frac{\frac{h_2 : \Delta_1, F_3, F_3, F_4 \vdash F_5}{\bullet h_2 : \Delta_1, F_3, F_3, F_4 \vdash F_5} \text{ inv-th/ax} \quad \frac{h_2 : \Delta_1, F_3, F_4 \vdash F_5}{\bullet h_2 : \Delta_1, F_3, F_4 \vdash F_5} \text{ IH}}{h_2 : \Delta_1, F_3, F_3, F_4 \vdash F_5} \text{ IH} \wedge_L$$

$$\frac{h_2 : \Delta_6, F_1, F_1, F_3, F_4 \vdash F_5}{\bullet h_2 : (\Delta_6, F_3 \wedge F_4), F_1, F_1 \vdash F_5} \wedge_L \rightarrow \frac{\frac{h_2 : \Delta_6, F_1, F_1, F_3, F_4 \vdash F_5}{\bullet h_2 : \Delta_6, F_1, F_3, F_4 \vdash F_5} \text{ ax} \quad \frac{h_2 : \Delta_6, F_1, F_1, F_4 \vdash F_5}{\bullet h_2 : \Delta_6, F_1, F_4 \vdash F_5} \text{ ax}}{h_2 : \Delta_6, F_1, F_3, F_4 \vdash F_5} \text{ IH} \wedge_L$$

- Case(s) rule  $\vee_L$

$$\frac{h_2 : \Delta_1, F_3, F_3 \vee F_4 \vdash F_5 \quad h_2 : \Delta_1, F_4, F_3 \vee F_4 \vdash F_5}{\bullet h_2 : \Delta_1, F_3 \vee F_4, F_3 \vee F_4 \vdash F_5} \vee_L \rightarrow \frac{\frac{h_2 : \Delta_1, F_3, F_3 \vdash F_5}{\bullet h_2 : \Delta_1, F_3 \vdash F_5} \text{ inv-th/ax} \quad \frac{h_2 : \Delta_1, F_4, F_3 \vee F_4 \vdash F_5}{\bullet h_2 : \Delta_1, F_4 \vdash F_5} \text{ inv-th/ax}}{h_2 : \Delta_1, F_3 \vee F_4 \vdash F_5} \text{ IH} \vee_L$$

$$\frac{h_2 : \Delta_6, F_1, F_1, F_3 \vdash F_5 \quad h_2 : \Delta_6, F_1, F_1, F_4 \vdash F_5}{\bullet h_2 : (\Delta_6, F_3 \vee F_4), F_1, F_1 \vdash F_5} \vee_L \rightarrow \frac{\frac{h_2 : \Delta_6, F_1, F_1, F_3 \vdash F_5}{\bullet h_2 : \Delta_6, F_1, F_3 \vdash F_5} \text{ ax} \quad \frac{h_2 : \Delta_6, F_1, F_1, F_4 \vdash F_5}{\bullet h_2 : \Delta_6, F_1, F_4 \vdash F_5} \text{ ax}}{h_2 : \Delta_6, F_1, F_3 \vee F_4 \vdash F_5} \text{ IH} \vee_L$$

- Case(s) rule  $\perp_L$

$$\frac{\bullet h_2 : \Delta_1, \perp, \perp \vdash F_3}{\bullet h_2 : \Delta_1, \perp, \perp \vdash F_3} \perp_L \rightarrow \frac{\bullet h_2 : \perp, \Delta_1 \vdash F_3}{\bullet h_2 : \perp, \Delta_1 \vdash F_3} \perp_L$$

$$\frac{\bullet h_2 : (\perp, \Delta_4), F_1, F_1 \vdash F_3}{\bullet h_2 : (\perp, \Delta_4), F_1, F_1 \vdash F_3} \perp_L \rightarrow \frac{\bullet h_2 : \perp, \Delta_4, F_1 \vdash F_3}{\bullet h_2 : \perp, \Delta_4, F_1 \vdash F_3} \perp_L$$

- Case(s) rule  $I$

$$\frac{\bullet h_2 : \Delta_1, P_3, P_3 \vdash P_3}{\bullet h_2 : \Delta_1, P_3, P_3 \vdash P_3} I \rightarrow \frac{\bullet h_2 : \Delta_1, P_3 \vdash P_3}{\bullet h_2 : \Delta_1, P_3 \vdash P_3} I$$

$$\frac{\bullet h_2 : (\Delta_4, P_3), F_1, F_1 \vdash P_3}{\bullet h_2 : (\Delta_4, P_3), F_1, F_1 \vdash P_3} I \rightarrow \frac{\bullet h_2 : \Delta_4, F_1, P_3 \vdash P_3}{\bullet h_2 : \Delta_4, F_1, P_3 \vdash P_3} I$$

- Case(s) rule  $\top_L$

$$\frac{h_2 : \top, \Delta_1 \vdash F_3}{\bullet h_2 : \Delta_1, \top, \top \vdash F_3} \top_L \rightarrow \frac{h_2 : \Delta_1 \vdash F_3}{\bullet h_2 : \top, \Delta_1 \vdash F_3} \text{inv-th/ax} \top_L$$

$$\frac{h_2 : \Delta_4, F_1, F_1 \vdash F_3}{\bullet h_2 : (\top, \Delta_4), F_1, F_1 \vdash F_3} \top_L \rightarrow \frac{\frac{h_2 : \Delta_4, F_1, F_1 \vdash F_3}{h_2 : \Delta_4, F_1 \vdash F_3} \text{ax}}{\bullet h_2 : \top, \Delta_4, F_1 \vdash F_3} \text{IH} \top_L$$

## 5 Identity-Expansion

$$\frac{\frac{\overline{- : F_0 \vdash F_0} \text{ IH}}{- : F_0 \vdash F_0 \vee F_1} \vee_1 \quad \frac{\overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_1 \vdash F_0 \vee F_1} \vee_2}{- : F_0 \vee F_1 \vdash F_0 \vee F_1} \vee_L$$

$$\frac{\frac{\overline{- : F_0 \vdash F_0} \text{ IH}}{- : F_0, F_1 \vdash F_0} W \quad \frac{\overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_0, F_1 \vdash F_1} W}{- : F_0, F_1 \vdash F_0 \wedge F_1} \wedge_R}{- : F_0 \wedge F_1 \vdash F_0 \wedge F_1} \wedge_L$$

$$\frac{\frac{\overline{- : F_0 \vdash F_0} \text{ IH}}{- : F_0, F_0 \rightarrow F_1 \vdash F_0} W \quad \frac{\overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_0, F_1 \vdash F_1} W}{- : F_0, F_0 \rightarrow F_1 \vdash F_1} \rightarrow_L}{- : F_0 \rightarrow F_1 \vdash F_0 \rightarrow F_1} \rightarrow_R$$

$$\frac{}{- : \top \vdash \top} \top_R$$

$$\frac{}{- : \perp \vdash \perp} \perp_L$$

## 6 Cut-Elimination

### 6.1 Status of $\top_R$ : OK

- Case rule  $\top_R$

$$\frac{\frac{\overline{\bullet h_1 : \Delta_3 \vdash \top}}{\top_R} \quad \frac{\overline{\bullet h_4 : \Delta_3, \top \vdash \top}}{\top_R}}{- : \Delta_3 \vdash \top} \text{Cut} \rightarrow \frac{\overline{- : \Delta_3 \vdash \top}}{\top_R}$$

- Case rule  $\rightarrow_R$

$$\frac{\frac{\overline{\bullet h_1 : \Delta_3 \vdash \top}}{\top_R} \quad \frac{\frac{h_4 : \top, \Delta_3, F_5 \vdash F_6}{\rightarrow_R} \quad \frac{\bullet h_4 : \Delta_3, \top \vdash F_5 \rightarrow F_6}{\text{Cut}}}{- : \Delta_3 \vdash F_5 \rightarrow F_6} \text{Cut} \rightarrow \frac{\frac{\bullet h_1 : \Delta_3, F_5 \vdash \top}{\top_R} \quad \frac{h_4 : \top, \Delta_3, F_5 \vdash F_6}{\text{ax/W}}}{- : \Delta_3, F_5 \vdash F_6} \text{hCut} \rightarrow_R \frac{\overline{- : \Delta_3 \vdash F_5 \rightarrow F_6}}{\rightarrow_R}$$

- Case rule  $\wedge_R$

$$\frac{\frac{\overline{\bullet h_1 : \Delta_3 \vdash \top}}{\top_R} \quad \frac{\frac{h_4 : \top, \Delta_3 \vdash F_5 \quad h_4 : \top, \Delta_3 \vdash F_6}{\wedge_R} \quad \frac{\bullet h_4 : \Delta_3, \top \vdash F_5 \wedge F_6}{\text{Cut}}}{- : \Delta_3 \vdash F_5 \wedge F_6} \text{Cut} \rightarrow \frac{\frac{\bullet h_1 : \Delta_3 \vdash \top}{\text{ax/W}} \quad \frac{h_4 : \top, \Delta_3 \vdash F_5}{\text{hCut}} \quad \frac{\bullet h_1 : \Delta_3 \vdash \top}{\text{ax/W}} \quad \frac{h_4 : \top, \Delta_3 \vdash F_6}{\text{hCut}}}{- : \Delta_3 \vdash F_5 \wedge F_6} \wedge_R$$

- Case rule  $\vee_1$

$$\frac{\frac{\overline{\bullet h_1 : \Delta_3 \vdash \top}}{\top_R} \quad \frac{\frac{h_4 : \top, \Delta_3 \vdash F_5}{\vee_1} \quad \frac{\bullet h_4 : \Delta_3, \top \vdash F_5 \vee F_6}{\text{Cut}}}{- : \Delta_3 \vdash F_5 \vee F_6} \text{Cut} \rightarrow \frac{\frac{\bullet h_1 : \Delta_3 \vdash \top}{\text{ax/W}} \quad \frac{h_4 : \top, \Delta_3 \vdash F_5}{\text{hCut}}}{- : \Delta_3 \vdash F_5} \vee_1 \rightarrow \frac{\overline{- : \Delta_3 \vdash F_5 \vee F_6}}{\vee_1}$$

- Case rule  $\vee_2$

$$\frac{\frac{\overline{\bullet h_1 : \Delta_3 \vdash \top}}{\top_R} \quad \frac{\frac{h_4 : \top, \Delta_3 \vdash F_6}{\vee_2} \quad \frac{\bullet h_4 : \Delta_3, \top \vdash F_5 \vee F_6}{\text{Cut}}}{- : \Delta_3 \vdash F_5 \vee F_6} \text{Cut} \rightarrow \frac{\frac{\bullet h_1 : \Delta_3 \vdash \top}{\top_R} \quad \frac{h_4 : \top, \Delta_3 \vdash F_6}{\text{hCut}}}{- : \Delta_3 \vdash F_6} \vee_2 \rightarrow \frac{\overline{- : \Delta_3 \vdash F_5 \vee F_6}}{\vee_2}$$

- Case rule  $\rightarrow_L$

$$\frac{\frac{\overline{\bullet h_1 : \Delta_7, F_4 \rightarrow F_5 \vdash \top}}{\top_R} \quad \frac{\frac{h_3 : \top, \Delta_7, F_4 \rightarrow F_5 \vdash F_4 \quad h_3 : \top, \Delta_7, F_5 \vdash F_6}{\rightarrow_L} \quad \frac{\bullet h_3 : (\Delta_7, F_4 \rightarrow F_5), \top \vdash F_6}{\text{Cut}}}{- : \Delta_7, F_4 \rightarrow F_5 \vdash F_6} \text{Cut} \rightarrow \frac{\frac{\bullet h_1 : \Delta_7, F_4 \rightarrow F_5 \vdash \top}{\text{ax/W}} \quad \frac{h_3 : \top, \Delta_7, F_4 \rightarrow F_5 \vdash F_4}{\text{hCut}} \quad \frac{\bullet h_1 : \Delta_7, F_5 \vdash \top}{\top_R} \quad \frac{h_3 : \top, \Delta_7, F_5 \vdash F_6}{\text{hCut}}}{- : \Delta_7, F_4 \rightarrow F_5 \vdash F_6} \rightarrow_L$$



- Case rule  $\wedge_L$

$$\frac{\frac{\frac{}{\bullet h_1 : \Delta_7, F_4 \wedge F_5 \vdash \top} \top_R \quad \frac{h_3 : \top, \Delta_7, F_4, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, F_4 \wedge F_5), \top \vdash F_6} \wedge_L}{- : \Delta_7, F_4 \wedge F_5 \vdash F_6} \text{Cut}}{\rightarrow} \quad \frac{\frac{\frac{}{\bullet h_1 : \Delta_7, F_4, F_5 \vdash \top} \top_R \quad \frac{h_3 : \top, \Delta_7, F_4, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, F_4 \wedge F_5), \top \vdash F_6} \wedge_L}{- : \Delta_7, F_4 \wedge F_5 \vdash F_6} \text{hCut}}{- : \Delta_7, F_4 \wedge F_5 \vdash F_6} \wedge_L$$

- Case rule  $\vee_L$

$$\frac{\frac{\frac{}{\bullet h_1 : \Delta_7, F_4 \vee F_5 \vdash \top} \top_R \quad \frac{h_3 : \top, \Delta_7, F_4 \vdash F_6 \quad h_3 : \top, \Delta_7, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, F_4 \vee F_5), \top \vdash F_6} \vee_L}{- : \Delta_7, F_4 \vee F_5 \vdash F_6} \text{Cut}}{\rightarrow} \quad \frac{\frac{\frac{}{\bullet h_1 : \Delta_7, F_4 \vdash \top} \top_R \quad \frac{h_3 : \top, \Delta_7, F_4 \vdash F_6}{\bullet h_3 : (\Delta_7, F_4 \vee F_5), \top \vdash F_6} \vee_L}{- : \Delta_7, F_4 \vdash F_6} \text{hCut}}{- : \Delta_7, F_4 \vee F_5 \vdash F_6} \vee_L \quad \frac{\frac{\frac{}{\bullet h_1 : \Delta_7, F_5 \vdash \top} \top_R \quad \frac{h_3 : \top, \Delta_7, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, F_4 \vee F_5), \top \vdash F_6} \vee_L}{- : \Delta_7, F_5 \vdash F_6} \text{hCut}}{- : \Delta_7, F_4 \vee F_5 \vdash F_6} \vee_L$$

- Case rule  $\perp_L$

$$\frac{\frac{\frac{}{\bullet h_1 : \perp, \Delta_5 \vdash \top} \top_R \quad \frac{}{\bullet h_3 : (\perp, \Delta_5), \top \vdash F_4} \perp_L}{- : \perp, \Delta_5 \vdash F_4} \text{Cut}}{\rightarrow} \quad \frac{}{- : \perp, \Delta_5 \vdash F_4} \perp_L$$

- Case rule  $I$

$$\frac{\frac{\frac{}{\bullet h_1 : \Delta_5, p_4 \vdash \top} \top_R \quad \frac{}{\bullet h_3 : (\Delta_5, p_4), \top \vdash p_4} I}{- : \Delta_5, p_4 \vdash p_4} \text{Cut}}{\rightarrow} \quad \frac{}{- : \Delta_5, p_4 \vdash p_4} I$$

- Case rule  $\top_L$

$$\frac{\frac{\frac{}{\bullet h_1 : \Delta_4 \vdash \top} \top_R \quad \frac{h_3 : \Delta_4 \vdash F_5}{\bullet h_3 : \Delta_4, \top \vdash F_5} \top_L}{- : \Delta_4 \vdash F_5} \text{Cut}}{\rightarrow} \quad \frac{}{- : \Delta_4 \vdash F_5} \text{ax/W}$$

## 6.2 Status of $\rightarrow_R$ : OK

- Case rule  $\top_R$

$$\frac{\frac{h_1 : \Delta_5, F_6 \vdash F_7}{\bullet h_1 : \Delta_5 \vdash F_6 \rightarrow F_7} \rightarrow_R \quad \frac{}{\bullet h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash \top} \top_R}{- : \Delta_5 \vdash \top} \text{Cut}}{\rightarrow} \quad \frac{}{- : \Delta_5 \vdash \top} \top_R$$

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_6 \vdash F_7}{\bullet h_1 : \Delta_5 \vdash F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_5, F_9, F_6 \rightarrow F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash F_9 \rightarrow F_{10}} \rightarrow_R \\
\text{Cut} \\
\frac{}{- : \Delta_5 \vdash F_9 \rightarrow F_{10}} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_5, F_9 \vdash F_6 \rightarrow F_7 \quad \text{ax/W} \quad h_8 : \Delta_5, F_9, F_6 \rightarrow F_7 \vdash F_{10}}{- : \Delta_5, F_9 \vdash F_{10}} \text{hCut} \\
\frac{}{- : \Delta_5 \vdash F_9 \rightarrow F_{10}} \rightarrow_R
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_6 \vdash F_7}{\bullet h_1 : \Delta_5 \vdash F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash F_9 \quad h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash F_9 \wedge F_{10}} \wedge_R \\
\text{Cut} \\
\frac{}{- : \Delta_5 \vdash F_9 \wedge F_{10}} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_5 \vdash F_6 \rightarrow F_7 \quad \text{ax/W} \quad h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash F_9 \quad \text{ax/W}}{- : \Delta_5 \vdash F_9} \text{hCut} \quad \frac{\bullet h_1 : \Delta_5 \vdash F_6 \rightarrow F_7 \quad \text{ax/W} \quad h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash F_{10}}{- : \Delta_5 \vdash F_{10}} \text{hCut} \\
\frac{}{- : \Delta_5 \vdash F_9 \wedge F_{10}} \wedge_R
\end{array}$$

- Case rule  $\vee_1$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_6 \vdash F_7}{\bullet h_1 : \Delta_5 \vdash F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash F_9}{\bullet h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash F_9 \vee F_{10}} \vee_1 \\
\text{Cut} \\
\frac{}{- : \Delta_5 \vdash F_9 \vee F_{10}} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_5 \vdash F_6 \rightarrow F_7 \quad \text{ax/W} \quad h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash F_9}{- : \Delta_5 \vdash F_9} \text{hCut} \\
\frac{}{- : \Delta_5 \vdash F_9 \vee F_{10}} \vee_1
\end{array}$$

- Case rule  $\vee_2$

$$\begin{array}{c}
\frac{h_1 : \Delta_5, F_6 \vdash F_7}{\bullet h_1 : \Delta_5 \vdash F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash F_9 \vee F_{10}} \vee_2 \\
\text{Cut} \\
\frac{}{- : \Delta_5 \vdash F_9 \vee F_{10}} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_5 \vdash F_6 \rightarrow F_7 \quad \text{ax/W} \quad h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash F_{10}}{- : \Delta_5 \vdash F_{10}} \text{hCut} \\
\frac{}{- : \Delta_5 \vdash F_9 \vee F_{10}} \vee_2
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_1 : (\Delta_{11}, F_8 \rightarrow F_9), F_5 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{h_7 : \Delta_{11}, F_5 \rightarrow F_6, F_8 \rightarrow F_9 \vdash F_8 \quad h_7 : \Delta_{11}, F_9, F_5 \rightarrow F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \rightarrow F_9), F_5 \rightarrow F_6 \vdash F_{10}} \rightarrow_L \\
\text{Cut} \\
\frac{}{- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_{10}} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_5 \rightarrow F_6 \quad \text{ax/W} \quad h_7 : \Delta_{11}, F_5 \rightarrow F_6, F_8 \rightarrow F_9 \vdash F_8 \quad \text{ax/W}}{- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_8} \text{hCut} \quad \frac{h_1 : \Delta_{11}, F_5, F_9 \vdash F_6 \quad \text{inv-th/ax}}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \rightarrow F_6 \vdash F_{10}}{- : \Delta_{11}, F_9 \vdash F_{10}} \rightarrow_L \\
\frac{}{- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_{10}} \\
\rightarrow \\
\frac{h_1 : \Delta_6, F_7 \vdash F_8}{\bullet h_1 : \Delta_6 \vdash F_7 \rightarrow F_8} \rightarrow_R \quad \frac{h_5 : \Delta_6, F_7 \rightarrow F_8 \vdash F_7 \quad h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \rightarrow F_8 \vdash F_9} \rightarrow_L \\
\text{Cut} \\
\frac{}{- : \Delta_6 \vdash F_9} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_6 \vdash F_7 \rightarrow F_8 \quad \text{ax/W} \quad h_5 : \Delta_6, F_7 \rightarrow F_8 \vdash F_7 \quad \text{ax/W}}{- : \Delta_6 \vdash F_7} \text{hCut} \quad \frac{- : \Delta_6, F_7 \vdash F_8 \quad \text{ax/W} \quad - : \Delta_6, F_7, F_8 \vdash F_9}{- : \Delta_6, F_7 \vdash F_9} \text{sCut} \\
\frac{}{- : \Delta_6 \vdash F_9}
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_1 : (\Delta_{11}, F_8 \wedge F_9), F_5 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8 \wedge F_9 \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \rightarrow F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \wedge F_9), F_5 \rightarrow F_6 \vdash F_{10}} \wedge_L \\
\hline
- : \Delta_{11}, F_8 \wedge F_9 \vdash F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{11}, F_5, F_8, F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8, F_9 \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{\text{inv-th/ax}}{h_7 : \Delta_{11}, F_8, F_9, F_5 \rightarrow F_6 \vdash F_{10}}}{- : \Delta_{11}, F_8, F_9 \vdash F_{10}} \text{ax/W} \\
\hline
- : \Delta_{11}, F_8 \wedge F_9 \vdash F_{10} \quad \wedge_L \quad \text{hCut}
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{h_1 : (\Delta_{11}, F_8 \vee F_9), F_5 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8 \vee F_9 \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \rightarrow F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \rightarrow F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_5 \rightarrow F_6 \vdash F_{10}} \vee_L \\
\hline
- : \Delta_{11}, F_8 \vee F_9 \vdash F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_{11}, F_5, F_8 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8 \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{\text{inv-th/ax}}{h_7 : \Delta_{11}, F_8, F_5 \rightarrow F_6 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, F_8 \vdash F_{10}} \text{hCut} \quad \frac{\frac{h_1 : \Delta_{11}, F_5, F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{\text{inv-th/ax}}{h_7 : \Delta_{11}, F_9, F_5 \rightarrow F_6 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, F_8 \vee F_9 \vdash F_{10}} \vee_L
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{h_1 : (\perp, \Delta_9), F_5 \vdash F_6}{\bullet h_1 : \perp, \Delta_9 \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{}{\bullet h_7 : (\perp, \Delta_9), F_5 \rightarrow F_6 \vdash F_8} \perp_L \\
\hline
- : \perp, \Delta_9 \vdash F_8 \quad \text{Cut} \\
\rightarrow \\
- : \perp, \Delta_9 \vdash F_8 \quad \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{h_1 : (\Delta_9, p_8), F_5 \vdash F_6}{\bullet h_1 : \Delta_9, p_8 \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{}{\bullet h_7 : (\Delta_9, p_8), F_5 \rightarrow F_6 \vdash p_8} I \\
\hline
- : \Delta_9, p_8 \vdash p_8 \quad \text{Cut} \\
\rightarrow \\
- : \Delta_9, p_8 \vdash p_8 \quad I
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{h_1 : (\top, \Delta_9), F_5 \vdash F_6}{\bullet h_1 : \top, \Delta_9 \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{h_7 : \Delta_9, F_5 \rightarrow F_6 \vdash F_8}{\bullet h_7 : (\top, \Delta_9), F_5 \rightarrow F_6 \vdash F_8} \top_L \\
\hline
- : \top, \Delta_9 \vdash F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \top, \Delta_9 \vdash F_5 \rightarrow F_6}{- : \top, \Delta_9 \vdash F_6} \text{ax/W} \quad \frac{h_7 : \top, \Delta_9, F_5 \rightarrow F_6 \vdash F_8}{- : \top, \Delta_9 \vdash F_8} \text{ax/W}}{- : \top, \Delta_9 \vdash F_8} \text{hCut}
\end{array}$$

### 6.3 Status of $\wedge_R$ : OK

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_5 \vdash F_6 \quad h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_5 \vdash F_6 \wedge F_7} \wedge_R \quad \frac{}{\bullet h_8 : \Delta_5, F_6 \wedge F_7 \vdash \top} \top_R \\
\hline
- : \Delta_5 \vdash \top \quad \text{Cut} \\
\rightarrow \\
- : \Delta_5 \vdash \top \quad \top_R
\end{array}$$

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_5 \vdash F_6 \quad h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_5 \vdash F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_5, F_9, F_6 \wedge F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_9 \rightarrow F_{10}} \rightarrow_R \\
\hline
- : \Delta_5 \vdash F_9 \rightarrow F_{10} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_5, F_9 \vdash F_6 \wedge F_7 \quad \text{ax/W} \quad h_8 : \Delta_5, F_9, F_6 \wedge F_7 \vdash F_{10} \quad \text{ax/W}}{- : \Delta_5, F_9 \vdash F_{10}} \text{hCut} \\
\hline
- : \Delta_5 \vdash F_9 \rightarrow F_{10} \quad \rightarrow_R
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_5 \vdash F_6 \quad h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_5 \vdash F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_9 \quad h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_9 \wedge F_{10}} \wedge_R \\
\hline
- : \Delta_5 \vdash F_9 \wedge F_{10} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_5 \vdash F_6 \wedge F_7 \quad \text{ax/W} \quad h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_9 \quad \text{ax/W} \quad \bullet h_1 : \Delta_5 \vdash F_6 \wedge F_7 \quad \text{ax/W} \quad h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_{10} \quad \text{ax/W}}{- : \Delta_5 \vdash F_9} \text{hCut} \quad \frac{\bullet h_1 : \Delta_5 \vdash F_6 \wedge F_7 \quad \text{ax/W} \quad h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_{10} \quad \text{ax/W}}{- : \Delta_5 \vdash F_{10}} \text{hCut} \\
\hline
- : \Delta_5 \vdash F_9 \wedge F_{10} \quad \wedge_R
\end{array}$$

- Case rule  $\vee_1$

$$\begin{array}{c}
\frac{h_1 : \Delta_5 \vdash F_6 \quad h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_5 \vdash F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_9}{\bullet h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_9 \vee F_{10}} \vee_1 \\
\hline
- : \Delta_5 \vdash F_9 \vee F_{10} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_5 \vdash F_6 \wedge F_7 \quad \text{ax/W} \quad h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_9 \quad \text{ax/W}}{- : \Delta_5 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_5 \vdash F_9 \vee F_{10} \quad \vee_1
\end{array}$$

- Case rule  $\vee_2$

$$\begin{array}{c}
\frac{h_1 : \Delta_5 \vdash F_6 \quad h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_5 \vdash F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_9 \vee F_{10}} \vee_2 \\
\hline
- : \Delta_5 \vdash F_9 \vee F_{10} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_5 \vdash F_6 \wedge F_7 \quad \text{ax/W} \quad h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_{10} \quad \text{ax/W}}{- : \Delta_5 \vdash F_{10}} \text{hCut} \\
\hline
- : \Delta_5 \vdash F_9 \vee F_{10} \quad \vee_2
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_5 \quad h_1 : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{h_7 : \Delta_{11}, F_8 \rightarrow F_9, F_5 \wedge F_6 \vdash F_8 \quad h_7 : \Delta_{11}, F_9, F_5 \wedge F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \rightarrow F_9), F_5 \wedge F_6 \vdash F_{10}} \rightarrow_L \\
\hline
- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_5 \wedge F_6 \quad \text{ax/W} \quad h_7 : \Delta_{11}, F_8 \rightarrow F_9, F_5 \wedge F_6 \vdash F_8 \quad \text{ax/W}}{- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_8} \text{hCut} \quad \frac{h_1 : \Delta_{11}, F_9 \vdash F_5 \quad \text{inv-th/ax} \quad h_1 : \Delta_{11}, F_9 \vdash F_6 \quad \text{inv-th/ax}}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \wedge F_6 \vdash F_{10}}{- : \Delta_{11}, F_9 \vdash F_{10}} \rightarrow_L \\
\hline
- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_{10}
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_8 \wedge F_9 \vdash F_5 \quad h_1 : \Delta_{11}, F_8 \wedge F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8 \wedge F_9 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \wedge F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \wedge F_9), F_5 \wedge F_6 \vdash F_{10}} \wedge_L \\
\hline
- : \Delta_{11}, F_8 \wedge F_9 \vdash F_{10} \\
\rightarrow \\
\frac{h_1 : \Delta_{11}, F_8, F_9 \vdash F_5 \quad \text{inv-th/ax} \quad h_1 : \Delta_{11}, F_8, F_9 \vdash F_6 \quad \text{inv-th/ax}}{\bullet h_1 : \Delta_{11}, F_8, F_9 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \wedge F_6 \vdash F_{10}}{- : \Delta_{11}, F_8, F_9 \vdash F_{10}} \text{ax/W} \quad \text{hCut} \\
\hline
- : \Delta_{11}, F_8 \wedge F_9 \vdash F_{10} \quad \wedge_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_6 \vdash F_7 \quad h_1 : \Delta_6 \vdash F_8}{\bullet h_1 : \Delta_6 \vdash F_7 \wedge F_8} \wedge_R \quad \frac{h_5 : \Delta_6, F_7, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \wedge F_8 \vdash F_9} \wedge_L \\
\hline
- : \Delta_6 \vdash F_9 \\
\rightarrow \\
\frac{- : \Delta_6 \vdash F_7 \quad \text{ax/W} \quad - : \Delta_6, F_7 \vdash F_8 \quad \text{ax/W} \quad - : \Delta_6, F_7, F_8 \vdash F_9 \quad \text{ax/W}}{- : \Delta_6 \vdash F_9} \text{sCut}
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_8 \vee F_9 \vdash F_5 \quad h_1 : \Delta_{11}, F_8 \vee F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8 \vee F_9 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \wedge F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \wedge F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_5 \wedge F_6 \vdash F_{10}} \wedge_L \\
\hline
- : \Delta_{11}, F_8 \vee F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{11}, F_8 \vdash F_5 \quad \text{inv-th/ax}}{\bullet h_1 : \Delta_{11}, F_8 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{h_1 : \Delta_{11}, F_8 \vdash F_6 \quad \text{inv-th/ax}}{\bullet h_1 : \Delta_{11}, F_8 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \wedge F_6 \vdash F_{10} \quad \text{ax/W}}{h_7 : \Delta_{11}, F_8, F_5 \wedge F_6 \vdash F_{10}} \text{hCut} \quad \frac{h_1 : \Delta_{11}, F_9 \vdash F_5 \quad \text{inv-th/ax} \quad h_1 : \Delta_{11}, F_9 \vdash F_6 \quad \text{inv-th/ax}}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5 \wedge F_6} \wedge_R}{- : \Delta_{11}, F_8 \vee F_9 \vdash F_{10}} \text{Cut}
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{h_1 : \perp, \Delta_9 \vdash F_5 \quad h_1 : \perp, \Delta_9 \vdash F_6}{\bullet h_1 : \perp, \Delta_9 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{\bullet h_7 : (\perp, \Delta_9), F_5 \wedge F_6 \vdash F_8}{\bullet h_7 : (\perp, \Delta_9), F_5 \wedge F_6 \vdash F_8} \perp_L \\
\hline
- : \perp, \Delta_9 \vdash F_8 \\
\rightarrow \\
- : \perp, \Delta_9 \vdash F_8 \quad \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{h_1 : \Delta_9, p_8 \vdash F_5 \quad h_1 : \Delta_9, p_8 \vdash F_6}{\bullet h_1 : \Delta_9, p_8 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{\bullet h_7 : (\Delta_9, p_8), F_5 \wedge F_6 \vdash p_8}{\bullet h_7 : (\Delta_9, p_8), F_5 \wedge F_6 \vdash p_8} I \\
\hline
- : \Delta_9, p_8 \vdash p_8 \\
\rightarrow \\
- : \Delta_9, p_8 \vdash p_8 \quad I
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{h_1 : \top, \Delta_9 \vdash F_5 \quad h_1 : \top, \Delta_9 \vdash F_6}{\bullet h_1 : \top, \Delta_9 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{h_7 : \Delta_9, F_5 \wedge F_6 \vdash F_8}{\bullet h_7 : (\top, \Delta_9), F_5 \wedge F_6 \vdash F_8} \top_L \\
\hline
- : \top, \Delta_9 \vdash F_8 \\
\rightarrow \\
\frac{\bullet h_1 : \top, \Delta_9 \vdash F_5 \wedge F_6 \quad \text{ax/W} \quad h_7 : \top, \Delta_9, F_5 \wedge F_6 \vdash F_8 \quad \text{ax/W}}{- : \top, \Delta_9 \vdash F_8} \text{hCut}
\end{array}$$

## 6.4 Status of $\vee_1$ : OK

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_5 \vdash F_6}{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7} \vee_1 \quad \frac{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash \top}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash \top} \top_R \\
\hline
- : \Delta_5 \vdash \top \\
\rightarrow \\
- : \Delta_5 \vdash \top \quad \top_R
\end{array}$$

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_5 \vdash F_6}{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7} \vee_1 \quad \frac{h_8 : \Delta_5, F_9, F_6 \vee F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \rightarrow F_{10}} \rightarrow_R}{- : \Delta_5 \vdash F_9 \rightarrow F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5, F_9 \vdash F_6 \vee F_7}{- : \Delta_5, F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_9, F_6 \vee F_7 \vdash F_{10}}{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}} \text{ax/W}}{- : \Delta_5 \vdash F_9 \rightarrow F_{10}} \text{hCut} \rightarrow_R \\
- : \Delta_5 \vdash F_9 \rightarrow F_{10}
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_5 \vdash F_6}{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7} \vee_1 \quad \frac{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \quad h_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \wedge F_{10}} \wedge_R}{- : \Delta_5 \vdash F_9 \wedge F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7}{- : \Delta_5 \vdash F_9} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9}{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9} \text{ax/W}}{- : \Delta_5 \vdash F_9} \text{hCut} \quad \frac{\frac{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7}{- : \Delta_5 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}}{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}} \text{ax/W}}{- : \Delta_5 \vdash F_{10}} \text{hCut} \wedge_R \\
- : \Delta_5 \vdash F_9 \wedge F_{10}
\end{array}$$

- Case rule  $\vee_1$

$$\begin{array}{c}
\frac{h_1 : \Delta_5 \vdash F_6}{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7} \vee_1 \quad \frac{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \vee F_{10}} \vee_1}{- : \Delta_5 \vdash F_9 \vee F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7}{- : \Delta_5 \vdash F_9} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9}{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9} \text{ax/W}}{- : \Delta_5 \vdash F_9} \text{hCut} \vee_1 \\
- : \Delta_5 \vdash F_9 \vee F_{10}
\end{array}$$

- Case rule  $\vee_2$

$$\begin{array}{c}
\frac{h_1 : \Delta_5 \vdash F_6}{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7} \vee_1 \quad \frac{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \vee F_{10}} \vee_2}{- : \Delta_5 \vdash F_9 \vee F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7}{- : \Delta_5 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}}{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}} \text{ax/W}}{- : \Delta_5 \vdash F_{10}} \text{hCut} \vee_2 \\
- : \Delta_5 \vdash F_9 \vee F_{10}
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_5}{\bullet h_1 : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_5 \vee F_6} \vee_1 \quad \frac{h_7 : \Delta_{11}, F_8 \rightarrow F_9, F_5 \vee F_6 \vdash F_8 \quad h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \rightarrow F_9), F_5 \vee F_6 \vdash F_{10}} \rightarrow_L}{- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_5 \vee F_6}{- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8 \rightarrow F_9, F_5 \vee F_6 \vdash F_8}{h_7 : \Delta_{11}, F_8 \rightarrow F_9, F_5 \vee F_6 \vdash F_8} \text{ax/W}}{- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_8} \text{hCut} \quad \frac{\frac{h_1 : \Delta_{11}, F_9 \vdash F_5}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5 \vee F_6} \text{inv-th/ax} \vee_1 \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, F_9 \vdash F_{10}} \text{hCut} \rightarrow_L \\
- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_{10}
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_8 \wedge F_9 \vdash F_5}{\bullet h_1 : \Delta_{11}, F_8 \wedge F_9 \vdash F_5 \vee F_6} \vee_1 \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \wedge F_9), F_5 \vee F_6 \vdash F_{10}} \wedge_L}{- : \Delta_{11}, F_8 \wedge F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{11}, F_8, F_9 \vdash F_5}{\bullet h_1 : \Delta_{11}, F_8, F_9 \vdash F_5 \vee F_6} \text{inv-th/ax} \vee_1 \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \vee F_6 \vdash F_{10}}{h_7 : \Delta_{11}, F_8, F_9, F_5 \vee F_6 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, F_8, F_9 \vdash F_{10}} \text{hCut} \wedge_L \\
- : \Delta_{11}, F_8 \wedge F_9 \vdash F_{10}
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{11}, F_8 \vee F_9 \vdash F_5}{\bullet h_1 : \Delta_{11}, F_8 \vee F_9 \vdash F_5 \vee F_6} \vee_1 \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \vee F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_5 \vee F_6 \vdash F_{10}} \vee_L}{- : \Delta_{11}, F_8 \vee F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_{11}, F_8 \vdash F_5}{\bullet h_1 : \Delta_{11}, F_8 \vdash F_5 \vee F_6} \vee_1 \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \vee F_6 \vdash F_{10}}{h_7 : \Delta_{11}, F_8, F_5 \vee F_6 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, F_8 \vdash F_{10}} \text{hCut} \quad \frac{\frac{h_1 : \Delta_{11}, F_9 \vdash F_5}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5 \vee F_6} \vee_1 \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, F_8 \vee F_9 \vdash F_{10}} \vee_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_6 \vdash F_7}{\bullet h_1 : \Delta_6 \vdash F_7 \vee F_8} \vee_1 \quad \frac{h_5 : \Delta_6, F_7 \vdash F_9 \quad h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \vee F_8 \vdash F_9} \vee_L}{- : \Delta_6 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_6 \vdash F_7}{- : \Delta_6 \vdash F_7} \text{ax/W} \quad \frac{- : \Delta_6, F_7 \vdash F_9}{- : \Delta_6, F_7 \vdash F_9} \text{ax/W}}{- : \Delta_6 \vdash F_9} \text{sCut}
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \perp, \Delta_9 \vdash F_5}{\bullet h_1 : \perp, \Delta_9 \vdash F_5 \vee F_6} \vee_1 \quad \frac{}{\bullet h_7 : (\perp, \Delta_9), F_5 \vee F_6 \vdash F_8} \perp_L}{- : \perp, \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_9 \vdash F_8} \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_9, p_8 \vdash F_5}{\bullet h_1 : \Delta_9, p_8 \vdash F_5 \vee F_6} \vee_1 \quad \frac{}{\bullet h_7 : (\Delta_9, p_8), F_5 \vee F_6 \vdash p_8} I}{- : \Delta_9, p_8 \vdash p_8} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_9, p_8 \vdash p_8} I
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \top, \Delta_9 \vdash F_5}{\bullet h_1 : \top, \Delta_9 \vdash F_5 \vee F_6} \vee_1 \quad \frac{h_7 : \Delta_9, F_5 \vee F_6 \vdash F_8}{\bullet h_7 : (\top, \Delta_9), F_5 \vee F_6 \vdash F_8} \top_L}{- : \top, \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \top, \Delta_9 \vdash F_5 \vee F_6}{- : \top, \Delta_9 \vdash F_5 \vee F_6} \text{ax/W} \quad \frac{h_7 : \top, \Delta_9, F_5 \vee F_6 \vdash F_8}{h_7 : \top, \Delta_9, F_5 \vee F_6 \vdash F_8} \text{ax/W}}{- : \top, \Delta_9 \vdash F_8} \text{hCut}
\end{array}$$

## 6.5 Status of $\vee_2$ : OK

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7} \vee_2 \quad \frac{}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash \top} \top_R}{- : \Delta_5 \vdash \top} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_5 \vdash \top} \top_R
\end{array}$$

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7} \vee_2 \quad \frac{h_8 : \Delta_5, F_9, F_6 \vee F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \rightarrow F_{10}} \rightarrow_R \\
\text{Cut} \\
\hline
- : \Delta_5 \vdash F_9 \rightarrow F_{10} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_5, F_9 \vdash F_6 \vee F_7}{- : \Delta_5, F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_9, F_6 \vee F_7 \vdash F_{10}}{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}} \text{ax/W} \\
\text{hCut} \\
\hline
- : \Delta_5 \vdash F_9 \rightarrow F_{10} \rightarrow_R \\
- : \Delta_5 \vdash F_9 \rightarrow F_{10}
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7} \vee_2 \quad \frac{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \quad h_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \wedge F_{10}} \wedge_R \\
\text{Cut} \\
\hline
- : \Delta_5 \vdash F_9 \wedge F_{10} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7}{- : \Delta_5 \vdash F_9} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9}{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9} \text{ax/W} \\
\text{hCut} \quad \frac{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7}{- : \Delta_5 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}}{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}} \text{ax/W} \\
\text{hCut} \\
\hline
- : \Delta_5 \vdash F_9 \wedge F_{10} \wedge_R \\
- : \Delta_5 \vdash F_9 \wedge F_{10}
\end{array}$$

- Case rule  $\vee_1$

$$\begin{array}{c}
\frac{h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7} \vee_2 \quad \frac{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \vee F_{10}} \vee_1 \\
\text{Cut} \\
\hline
- : \Delta_5 \vdash F_9 \vee F_{10} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7}{- : \Delta_5 \vdash F_9} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9}{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9} \text{ax/W} \\
\text{hCut} \\
\hline
- : \Delta_5 \vdash F_9 \vee F_{10} \vee_1 \\
- : \Delta_5 \vdash F_9 \vee F_{10}
\end{array}$$

- Case rule  $\vee_2$

$$\begin{array}{c}
\frac{h_1 : \Delta_5 \vdash F_7}{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7} \vee_2 \quad \frac{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \vee F_{10}} \vee_2 \\
\text{Cut} \\
\hline
- : \Delta_5 \vdash F_9 \vee F_{10} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_5 \vdash F_6 \vee F_7}{- : \Delta_5 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}}{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}} \text{ax/W} \\
\text{hCut} \\
\hline
- : \Delta_5 \vdash F_{10} \vee_2 \\
- : \Delta_5 \vdash F_9 \vee F_{10}
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_5 \vee F_6} \vee_2 \quad \frac{h_7 : \Delta_{11}, F_8 \rightarrow F_9, F_5 \vee F_6 \vdash F_8 \quad h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \rightarrow F_9), F_5 \vee F_6 \vdash F_{10}} \rightarrow_L \\
\text{Cut} \\
\hline
- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_5 \vee F_6}{- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8 \rightarrow F_9, F_5 \vee F_6 \vdash F_8}{h_7 : \Delta_{11}, F_8 \rightarrow F_9, F_5 \vee F_6 \vdash F_8} \text{ax/W} \\
\text{hCut} \quad \frac{h_1 : \Delta_{11}, F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5 \vee F_6} \text{inv-th/ax} \vee_2 \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}} \text{ax/W} \\
\text{hCut} \\
\hline
- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_{10} \rightarrow_L \\
- : \Delta_{11}, F_8 \rightarrow F_9 \vdash F_{10}
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, F_8 \wedge F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8 \wedge F_9 \vdash F_5 \vee F_6} \vee_2 \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \wedge F_9), F_5 \vee F_6 \vdash F_{10}} \wedge_L \\
\text{Cut} \\
\hline
- : \Delta_{11}, F_8 \wedge F_9 \vdash F_{10} \\
\rightarrow \\
\frac{h_1 : \Delta_{11}, F_8, F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8, F_9 \vdash F_5 \vee F_6} \text{inv-th/ax} \vee_2 \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \vee F_6 \vdash F_{10}}{h_7 : \Delta_{11}, F_8, F_9, F_5 \vee F_6 \vdash F_{10}} \text{ax/W} \\
\text{hCut} \\
\hline
- : \Delta_{11}, F_8, F_9 \vdash F_{10} \wedge_L \\
- : \Delta_{11}, F_8 \wedge F_9 \vdash F_{10}
\end{array}$$



- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_{11}, F_8 \vee F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8 \vee F_9 \vdash F_5 \vee F_6} \vee_2 \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \vee F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_5 \vee F_6 \vdash F_{10}} \vee_L}{- : \Delta_{11}, F_8 \vee F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_{11}, F_8 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_8 \vdash F_5 \vee F_6} \text{inv-th/ax} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \vee F_6 \vdash F_{10}}{h_7 : \Delta_{11}, F_8, F_5 \vee F_6 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, F_8 \vdash F_{10}} \vee_2 \quad \frac{\frac{h_1 : \Delta_{11}, F_9 \vdash F_6}{\bullet h_1 : \Delta_{11}, F_9 \vdash F_5 \vee F_6} \text{inv-th/ax} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, F_9 \vdash F_{10}} \vee_2}{- : \Delta_{11}, F_8 \vee F_9 \vdash F_{10}} \vee_L \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_6 \vdash F_8}{\bullet h_1 : \Delta_6 \vdash F_7 \vee F_8} \vee_2 \quad \frac{h_5 : \Delta_6, F_7 \vdash F_9 \quad h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \vee F_8 \vdash F_9} \vee_L}{- : \Delta_6 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_6 \vdash F_8}{- : \Delta_6 \vdash F_8} \text{ax/W} \quad \frac{- : \Delta_6, F_8 \vdash F_9}{- : \Delta_6, F_8 \vdash F_9} \text{ax/W}}{- : \Delta_6 \vdash F_9} \text{sCut}
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \perp, \Delta_9 \vdash F_6}{\bullet h_1 : \perp, \Delta_9 \vdash F_5 \vee F_6} \vee_2 \quad \frac{}{\bullet h_7 : (\perp, \Delta_9), F_5 \vee F_6 \vdash F_8} \perp_L}{- : \perp, \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_9 \vdash F_8} \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_9, p_8 \vdash F_6}{\bullet h_1 : \Delta_9, p_8 \vdash F_5 \vee F_6} \vee_2 \quad \frac{}{\bullet h_7 : (\Delta_9, p_8), F_5 \vee F_6 \vdash p_8} I}{- : \Delta_9, p_8 \vdash p_8} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_9, p_8 \vdash p_8} I
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \top, \Delta_9 \vdash F_6}{\bullet h_1 : \top, \Delta_9 \vdash F_5 \vee F_6} \vee_2 \quad \frac{h_7 : \Delta_9, F_5 \vee F_6 \vdash F_8}{\bullet h_7 : (\top, \Delta_9), F_5 \vee F_6 \vdash F_8} \top_L}{- : \top, \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \top, \Delta_9 \vdash F_5 \vee F_6}{- : \top, \Delta_9 \vdash F_5 \vee F_6} \text{ax/W} \quad \frac{h_7 : \top, \Delta_9, F_5 \vee F_6 \vdash F_8}{h_7 : \top, \Delta_9, F_5 \vee F_6 \vdash F_8} \text{ax/W}}{- : \top, \Delta_9 \vdash F_8} \text{hCut}
\end{array}$$

## 6.6 Status of $\rightarrow_L$ : OK

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_6, F_8 \rightarrow F_9 \vdash F_8 \quad h_1 : \Delta_6, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \rightarrow F_9 \vdash F_7} \rightarrow_L \quad \frac{}{\bullet h_{10} : (\Delta_6, F_8 \rightarrow F_9), F_7 \vdash \top} \top_R}{- : \Delta_6, F_8 \rightarrow F_9 \vdash \top} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_6, F_8 \rightarrow F_9 \vdash \top} \top_R
\end{array}$$

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_6, F_8 \rightarrow F_9 \vdash F_8 \quad h_1 : \Delta_6, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \rightarrow F_9 \vdash F_7} \rightarrow_L \quad \frac{h_{10} : \Delta_6, F_7, F_{11}, F_8 \rightarrow F_9 \vdash F_{12}}{\bullet h_{10} : (\Delta_6, F_8 \rightarrow F_9), F_7 \vdash F_{11} \rightarrow F_{12}} \rightarrow_R \\
\hline
- : \Delta_6, F_8 \rightarrow F_9 \vdash F_{11} \rightarrow F_{12} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_6, F_{11}, F_8 \rightarrow F_9 \vdash F_7}{- : \Delta_6, F_{11}, F_8 \rightarrow F_9 \vdash F_{12}} \text{ ax/W} \quad \frac{h_{10} : \Delta_6, F_{11}, F_7, F_8 \rightarrow F_9 \vdash F_{12}}{- : \Delta_6, F_{11}, F_8 \rightarrow F_9 \vdash F_{12}} \text{ ax/W}}{- : \Delta_6, F_8 \rightarrow F_9 \vdash F_{11} \rightarrow F_{12}} \text{ hCut} \\
\rightarrow_R \\
- : \Delta_6, F_8 \rightarrow F_9 \vdash F_{11} \rightarrow F_{12}
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_6, F_8 \rightarrow F_9 \vdash F_8 \quad h_1 : \Delta_6, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \rightarrow F_9 \vdash F_7} \rightarrow_L \quad \frac{h_{10} : \Delta_6, F_7, F_8 \rightarrow F_9 \vdash F_{11} \quad h_{10} : \Delta_6, F_7, F_8 \rightarrow F_9 \vdash F_{12}}{\bullet h_{10} : (\Delta_6, F_8 \rightarrow F_9), F_7 \vdash F_{11} \wedge F_{12}} \text{ Cut} \\
\hline
- : \Delta_6, F_8 \rightarrow F_9 \vdash F_{11} \wedge F_{12} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_6, F_8 \rightarrow F_9 \vdash F_7}{- : \Delta_6, F_8 \rightarrow F_9 \vdash F_{11}} \text{ ax/W} \quad \frac{h_{10} : \Delta_6, F_7, F_8 \rightarrow F_9 \vdash F_{11}}{- : \Delta_6, F_7, F_8 \rightarrow F_9 \vdash F_{11}} \text{ ax/W}}{- : \Delta_6, F_8 \rightarrow F_9 \vdash F_{11}} \text{ hCut} \quad \frac{\frac{\bullet h_1 : \Delta_6, F_8 \rightarrow F_9 \vdash F_7}{- : \Delta_6, F_8 \rightarrow F_9 \vdash F_{12}} \text{ ax/W} \quad \frac{h_{10} : \Delta_6, F_7, F_8 \rightarrow F_9 \vdash F_{12}}{- : \Delta_6, F_7, F_8 \rightarrow F_9 \vdash F_{12}} \text{ ax/W}}{- : \Delta_6, F_8 \rightarrow F_9 \vdash F_{12}} \text{ hCut} \\
\wedge_R \\
- : \Delta_6, F_8 \rightarrow F_9 \vdash F_{11} \wedge F_{12}
\end{array}$$

- Case rule  $\vee_1$

$$\begin{array}{c}
\frac{h_1 : \Delta_6, F_8 \rightarrow F_9 \vdash F_8 \quad h_1 : \Delta_6, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \rightarrow F_9 \vdash F_7} \rightarrow_L \quad \frac{h_{10} : \Delta_6, F_7, F_8 \rightarrow F_9 \vdash F_{11}}{\bullet h_{10} : (\Delta_6, F_8 \rightarrow F_9), F_7 \vdash F_{11} \vee F_{12}} \vee_1 \\
\hline
- : \Delta_6, F_8 \rightarrow F_9 \vdash F_{11} \vee F_{12} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_6, F_8 \rightarrow F_9 \vdash F_7}{- : \Delta_6, F_8 \rightarrow F_9 \vdash F_{11}} \text{ ax/W} \quad \frac{h_{10} : \Delta_6, F_7, F_8 \rightarrow F_9 \vdash F_{11}}{- : \Delta_6, F_7, F_8 \rightarrow F_9 \vdash F_{11}} \text{ ax/W}}{- : \Delta_6, F_8 \rightarrow F_9 \vdash F_{11}} \text{ hCut} \\
\vee_1 \\
- : \Delta_6, F_8 \rightarrow F_9 \vdash F_{11} \vee F_{12}
\end{array}$$

- Case rule  $\vee_2$

$$\begin{array}{c}
\frac{h_1 : \Delta_6, F_8 \rightarrow F_9 \vdash F_8 \quad h_1 : \Delta_6, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \rightarrow F_9 \vdash F_7} \rightarrow_L \quad \frac{h_{10} : \Delta_6, F_7, F_8 \rightarrow F_9 \vdash F_{12}}{\bullet h_{10} : (\Delta_6, F_8 \rightarrow F_9), F_7 \vdash F_{11} \vee F_{12}} \vee_2 \\
\hline
- : \Delta_6, F_8 \rightarrow F_9 \vdash F_{11} \vee F_{12} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_6, F_8 \rightarrow F_9 \vdash F_7}{- : \Delta_6, F_8 \rightarrow F_9 \vdash F_{12}} \text{ ax/W} \quad \frac{h_{10} : \Delta_6, F_7, F_8 \rightarrow F_9 \vdash F_{12}}{- : \Delta_6, F_7, F_8 \rightarrow F_9 \vdash F_{12}} \text{ ax/W}}{- : \Delta_6, F_8 \rightarrow F_9 \vdash F_{12}} \text{ hCut} \\
\vee_2 \\
- : \Delta_6, F_8 \rightarrow F_9 \vdash F_{11} \vee F_{12}
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_1 : (\Delta_{13}, F_{10} \rightarrow F_{11}), F_7 \rightarrow F_8 \vdash F_7 \quad h_1 : (\Delta_{13}, F_{10} \rightarrow F_{11}), F_8 \vdash F_6}{\bullet h_1 : (\Delta_{13}, F_{10} \rightarrow F_{11}), F_7 \rightarrow F_8 \vdash F_6} \rightarrow_L \quad \frac{h_9 : \Delta_{13}, F_6, F_7 \rightarrow F_8, F_{10} \rightarrow F_{11} \vdash F_{10} \quad h_9 : \Delta_{13}, F_6, F_{11}, F_7 \rightarrow F_8 \vdash F_{12}}{\bullet h_9 : ((\Delta_{13}, F_{10} \rightarrow F_{11}), F_7 \rightarrow F_8), F_6 \vdash F_{12}} \text{ Cut} \\
\hline
- : (\Delta_{13}, F_{10} \rightarrow F_{11}), F_7 \rightarrow F_8 \vdash F_{12} \\
\rightarrow \\
\frac{\frac{- : \Delta_{13}, F_{10} \rightarrow F_{11}, F_7 \rightarrow F_8 \vdash F_7}{- : \Delta_{13}, F_{10} \rightarrow F_{11}, F_7 \rightarrow F_8 \vdash F_7} \text{ ax/W} \quad \frac{h_1 : \Delta_{13}, F_8, F_{10} \rightarrow F_{11} \vdash F_6}{- : \Delta_{13}, F_8, F_{10} \rightarrow F_{11} \vdash F_6} \text{ ax/W} \quad \frac{h_9 : \Delta_{13}, F_6, F_8, F_{10} \rightarrow F_{11} \vdash F_{10}}{\bullet h_9 : \Delta_{13}, F_6, F_8, F_{10} \rightarrow F_{11} \vdash F_{12}} \text{ inv-th/ax}}{- : \Delta_{13}, F_8, F_{10} \rightarrow F_{11} \vdash F_{12}} \text{ hCut} \\
\rightarrow_L \\
- : \Delta_{13}, F_{10} \rightarrow F_{11}, F_7 \rightarrow F_8 \vdash F_{12} \\
\rightarrow \\
\frac{h_1 : \Delta_6, F_7 \rightarrow F_8 \vdash F_7 \quad h_1 : \Delta_6, F_8 \vdash F_{10} \rightarrow F_{11}}{\bullet h_1 : \Delta_6, F_7 \rightarrow F_8 \vdash F_{10} \rightarrow F_{11}} \rightarrow_L \quad \frac{h_9 : \Delta_6, F_7 \rightarrow F_8, F_{10} \rightarrow F_{11} \vdash F_{10} \quad h_9 : \Delta_6, F_{11}, F_7 \rightarrow F_8 \vdash F_{12}}{\bullet h_9 : (\Delta_6, F_7 \rightarrow F_8), F_{10} \rightarrow F_{11} \vdash F_{12}} \text{ Cut} \\
\hline
- : \Delta_6, F_7 \rightarrow F_8 \vdash F_{12} \\
\rightarrow \\
\frac{\frac{- : \Delta_6, F_7 \rightarrow F_8 \vdash F_7}{- : \Delta_6, F_7 \rightarrow F_8 \vdash F_7} \text{ ax/W} \quad \frac{h_1 : \Delta_6, F_8 \vdash F_{10} \rightarrow F_{11}}{- : \Delta_6, F_8 \vdash F_{10} \rightarrow F_{11}} \text{ ax/W} \quad \frac{h_9 : \Delta_6, F_8, F_{10} \rightarrow F_{11} \vdash F_{10}}{\bullet h_9 : \Delta_6, F_8, F_{10} \rightarrow F_{11} \vdash F_{12}} \text{ inv-th/ax}}{- : \Delta_6, F_8 \vdash F_{10} \rightarrow F_{11} \vdash F_{12}} \text{ hCut} \\
\rightarrow_L \\
- : \Delta_6, F_7 \rightarrow F_8 \vdash F_{12}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_6, F_9 \rightarrow F_{10} \vdash F_9 \quad h_1 : \Delta_6, F_{10} \vdash F_7 \quad \rightarrow_L \quad h_8 : \Delta_6, F_7, F_9 \rightarrow F_{10} \vdash F_9 \quad h_8 : \Delta_6, F_7, F_{10} \vdash F_{11} \quad \rightarrow_L}{\frac{\bullet h_1 : \Delta_6, F_9 \rightarrow F_{10} \vdash F_7}{- : \Delta_6, F_9 \rightarrow F_{10} \vdash F_7} \quad \frac{\bullet h_8 : (\Delta_6, F_9 \rightarrow F_{10}), F_7 \vdash F_{11}}{- : \Delta_6, F_9 \rightarrow F_{10} \vdash F_{11}} \text{Cut}}{- : \Delta_6, F_9 \rightarrow F_{10} \vdash F_{11}} \rightarrow \\
\frac{\frac{- : \Delta_6, F_9 \rightarrow F_{10} \vdash F_9 \quad \text{ax/W} \quad \frac{h_1 : \Delta_6, F_{10} \vdash F_7 \quad \text{ax/W}}{- : \Delta_6, F_{10} \vdash F_7} \quad \frac{h_8 : \Delta_6, F_{10}, F_7 \vdash F_{11} \quad \text{ax/W}}{\bullet h_8 : \Delta_6, F_{10}, F_7 \vdash F_{11}} \text{H}}{- : \Delta_6, F_{10} \vdash F_{11}} \text{hCut}}{- : \Delta_6, F_9 \rightarrow F_{10} \vdash F_{11}} \rightarrow_L
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_1 : (\Delta_{13}, F_{10} \wedge F_{11}), F_7 \rightarrow F_8 \vdash F_7 \quad h_1 : (\Delta_{13}, F_{10} \wedge F_{11}), F_8 \vdash F_6 \quad \rightarrow_L \quad \frac{h_9 : \Delta_{13}, F_6, F_{10}, F_{11}, F_7 \rightarrow F_8 \vdash F_{12}}{\bullet h_9 : ((\Delta_{13}, F_{10} \wedge F_{11}), F_7 \rightarrow F_8), F_6 \vdash F_{12}} \wedge_L}{\frac{\bullet h_1 : (\Delta_{13}, F_{10} \wedge F_{11}), F_7 \rightarrow F_8 \vdash F_6}{- : (\Delta_{13}, F_{10} \wedge F_{11}), F_7 \rightarrow F_8 \vdash F_6} \text{Cut}}{- : (\Delta_{13}, F_{10} \wedge F_{11}), F_7 \rightarrow F_8 \vdash F_{12}} \rightarrow \\
\frac{\frac{- : \Delta_{13}, F_7 \rightarrow F_8, F_{10} \wedge F_{11} \vdash F_7 \quad \text{ax/W} \quad \frac{h_1 : \Delta_{13}, F_8, F_{10} \wedge F_{11} \vdash F_6 \quad \text{ax/W}}{- : \Delta_{13}, F_8, F_{10} \wedge F_{11} \vdash F_6} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11}, F_6, F_8 \vdash F_{12} \quad \text{inv-th/ax}}{\bullet h_9 : \Delta_{13}, F_6, F_8, F_{10} \wedge F_{11} \vdash F_{12}} \wedge_L}{- : \Delta_{13}, F_7 \rightarrow F_8, F_{10} \wedge F_{11} \vdash F_{12}} \text{hCut}}{- : \Delta_{13}, F_7 \rightarrow F_8, F_{10} \wedge F_{11} \vdash F_{12}} \rightarrow_L \\
\frac{h_1 : \Delta_6, F_7 \rightarrow F_8 \vdash F_7 \quad h_1 : \Delta_6, F_8 \vdash F_{10} \wedge F_{11} \quad \rightarrow_L \quad \frac{h_9 : \Delta_6, F_{10}, F_{11}, F_7 \rightarrow F_8 \vdash F_{12}}{\bullet h_9 : (\Delta_6, F_7 \rightarrow F_8), F_{10} \wedge F_{11} \vdash F_{12}} \wedge_L}{\frac{\bullet h_1 : \Delta_6, F_7 \rightarrow F_8 \vdash F_{10} \wedge F_{11}}{- : \Delta_6, F_7 \rightarrow F_8 \vdash F_{10} \wedge F_{11}} \text{Cut}}{- : \Delta_6, F_7 \rightarrow F_8 \vdash F_{12}} \rightarrow \\
\frac{\frac{- : \Delta_6, F_7 \rightarrow F_8 \vdash F_7 \quad \text{ax/W} \quad \frac{h_1 : \Delta_6, F_8 \vdash F_{10} \wedge F_{11} \quad \text{ax/W}}{- : \Delta_6, F_8 \vdash F_{10} \wedge F_{11}} \quad \frac{h_9 : \Delta_6, F_{10}, F_{11}, F_8 \vdash F_{12} \quad \text{inv-th/ax}}{\bullet h_9 : \Delta_6, F_8, F_{10} \wedge F_{11} \vdash F_{12}} \wedge_L}{- : \Delta_6, F_7 \rightarrow F_8 \vdash F_{12}} \text{hCut}}{- : \Delta_6, F_7 \rightarrow F_8 \vdash F_{12}} \rightarrow_L
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{h_1 : (\Delta_{13}, F_{10} \vee F_{11}), F_7 \rightarrow F_8 \vdash F_7 \quad h_1 : (\Delta_{13}, F_{10} \vee F_{11}), F_8 \vdash F_6 \quad \rightarrow_L \quad \frac{h_9 : \Delta_{13}, F_6, F_{10}, F_7 \rightarrow F_8 \vdash F_{12} \quad h_9 : \Delta_{13}, F_6, F_{11}, F_7 \rightarrow F_8 \vdash F_{12}}{\bullet h_9 : ((\Delta_{13}, F_{10} \vee F_{11}), F_7 \rightarrow F_8), F_6 \vdash F_{12}} \vee_L}{\frac{\bullet h_1 : (\Delta_{13}, F_{10} \vee F_{11}), F_7 \rightarrow F_8 \vdash F_6}{- : (\Delta_{13}, F_{10} \vee F_{11}), F_7 \rightarrow F_8 \vdash F_6} \text{Cut}}{- : (\Delta_{13}, F_{10} \vee F_{11}), F_7 \rightarrow F_8 \vdash F_{12}} \rightarrow \\
\frac{\frac{- : \Delta_{13}, F_7 \rightarrow F_8, F_{10} \vee F_{11} \vdash F_7 \quad \text{ax/W} \quad \frac{h_1 : \Delta_{13}, F_8, F_{10} \vee F_{11} \vdash F_6 \quad \text{ax/W}}{- : \Delta_{13}, F_8, F_{10} \vee F_{11} \vdash F_6} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_6, F_8 \vdash F_{12} \quad \text{inv-th/ax}}{\bullet h_9 : \Delta_{13}, F_6, F_8, F_{10} \vee F_{11} \vdash F_{12}} \wedge_L \quad \frac{h_9 : \Delta_{13}, F_{11}, F_6, F_8 \vdash F_{12} \quad \text{inv-th/ax}}{\bullet h_9 : \Delta_{13}, F_6, F_8, F_{10} \vee F_{11} \vdash F_{12}} \wedge_L}{- : \Delta_{13}, F_7 \rightarrow F_8, F_{10} \vee F_{11} \vdash F_{12}} \text{hCut}}{- : \Delta_{13}, F_7 \rightarrow F_8, F_{10} \vee F_{11} \vdash F_{12}} \rightarrow_L \\
\frac{h_1 : \Delta_6, F_7 \rightarrow F_8 \vdash F_7 \quad h_1 : \Delta_6, F_8 \vdash F_{10} \vee F_{11} \quad \rightarrow_L \quad \frac{h_9 : \Delta_6, F_{10}, F_7 \rightarrow F_8 \vdash F_{12} \quad h_9 : \Delta_6, F_{11}, F_7 \rightarrow F_8 \vdash F_{12}}{\bullet h_9 : (\Delta_6, F_7 \rightarrow F_8), F_{10} \vee F_{11} \vdash F_{12}} \vee_L}{\frac{\bullet h_1 : \Delta_6, F_7 \rightarrow F_8 \vdash F_{10} \vee F_{11}}{- : \Delta_6, F_7 \rightarrow F_8 \vdash F_{10} \vee F_{11}} \text{Cut}}{- : \Delta_6, F_7 \rightarrow F_8 \vdash F_{12}} \rightarrow \\
\frac{\frac{- : \Delta_6, F_7 \rightarrow F_8 \vdash F_7 \quad \text{ax/W} \quad \frac{h_1 : \Delta_6, F_8 \vdash F_{10} \vee F_{11} \quad \text{ax/W}}{- : \Delta_6, F_8 \vdash F_{10} \vee F_{11}} \quad \frac{h_9 : \Delta_6, F_{10}, F_8 \vdash F_{12} \quad \text{inv-th/ax}}{\bullet h_9 : \Delta_6, F_8, F_{10} \vee F_{11} \vdash F_{12}} \wedge_L \quad \frac{h_9 : \Delta_6, F_{11}, F_8 \vdash F_{12} \quad \text{inv-th/ax}}{\bullet h_9 : \Delta_6, F_8, F_{10} \vee F_{11} \vdash F_{12}} \wedge_L}{- : \Delta_6, F_7 \rightarrow F_8 \vdash F_{12}} \text{hCut}}{- : \Delta_6, F_7 \rightarrow F_8 \vdash F_{12}} \rightarrow_L
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_6, F_7 \rightarrow F_8 \vdash F_7 \quad h_1 : \Delta_6, F_8 \vdash \perp \quad \rightarrow_L \quad \frac{\bullet h_9 : (\Delta_6, F_7 \rightarrow F_8), \perp \vdash F_{10}}{\bullet h_9 : (\Delta_6, F_7 \rightarrow F_8), \perp \vdash F_{10}} \perp_L}{\frac{\bullet h_1 : \Delta_6, F_7 \rightarrow F_8 \vdash \perp}{- : \Delta_6, F_7 \rightarrow F_8 \vdash \perp} \text{Cut}}{- : \Delta_6, F_7 \rightarrow F_8 \vdash F_{10}} \rightarrow \\
\frac{\frac{- : \Delta_6, F_7 \rightarrow F_8 \vdash F_7 \quad \text{ax/W} \quad \frac{h_1 : \Delta_6, F_8 \vdash \perp \quad \text{ax/W}}{- : \Delta_6, F_8 \vdash \perp} \quad \frac{\bullet h_9 : \perp, \Delta_6, F_8 \vdash F_{10}}{\bullet h_9 : \perp, \Delta_6, F_8 \vdash F_{10}} \perp_L}{- : \Delta_6, F_7 \rightarrow F_8 \vdash F_{10}} \text{hCut}}{- : \Delta_6, F_7 \rightarrow F_8 \vdash F_{10}} \rightarrow_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : (\perp, \Delta_{11}), F_7 \rightarrow F_8 \vdash F_7 \quad h_1 : (\perp, \Delta_{11}), F_8 \vdash F_6}{\bullet h_1 : (\perp, \Delta_{11}), F_7 \rightarrow F_8 \vdash F_6} \rightarrow_L \quad \frac{\bullet h_9 : ((\perp, \Delta_{11}), F_7 \rightarrow F_8), F_6 \vdash F_{10}}{\bullet h_9 : ((\perp, \Delta_{11}), F_7 \rightarrow F_8), F_6 \vdash F_{10}} \perp_L \\
\frac{\bullet h_1 : (\perp, \Delta_{11}), F_7 \rightarrow F_8 \vdash F_6 \quad \bullet h_9 : ((\perp, \Delta_{11}), F_7 \rightarrow F_8), F_6 \vdash F_{10}}{- : (\perp, \Delta_{11}), F_7 \rightarrow F_8 \vdash F_{10}} \text{Cut} \\
\frac{- : (\perp, \Delta_{11}), F_7 \rightarrow F_8 \vdash F_{10}}{- : \perp, \Delta_{11}, F_7 \rightarrow F_8 \vdash F_{10}} \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{h_1 : (\Delta_{11}, P_{10}), F_7 \rightarrow F_8 \vdash F_7 \quad h_1 : (\Delta_{11}, P_{10}), F_8 \vdash F_6}{\bullet h_1 : (\Delta_{11}, P_{10}), F_7 \rightarrow F_8 \vdash F_6} \rightarrow_L \quad \frac{\bullet h_9 : ((\Delta_{11}, P_{10}), F_7 \rightarrow F_8), F_6 \vdash P_{10}}{\bullet h_9 : ((\Delta_{11}, P_{10}), F_7 \rightarrow F_8), F_6 \vdash P_{10}} I \\
\frac{\bullet h_1 : (\Delta_{11}, P_{10}), F_7 \rightarrow F_8 \vdash F_6 \quad \bullet h_9 : ((\Delta_{11}, P_{10}), F_7 \rightarrow F_8), F_6 \vdash P_{10}}{- : (\Delta_{11}, P_{10}), F_7 \rightarrow F_8 \vdash P_{10}} \text{Cut} \\
\frac{- : (\Delta_{11}, P_{10}), F_7 \rightarrow F_8 \vdash P_{10}}{- : \Delta_{11}, P_{10}, F_7 \rightarrow F_8 \vdash P_{10}} I \\
\frac{h_1 : \Delta_6, F_7 \rightarrow F_8 \vdash F_7 \quad h_1 : \Delta_6, F_8 \vdash P_{10}}{\bullet h_1 : \Delta_6, F_7 \rightarrow F_8 \vdash P_{10}} \rightarrow_L \quad \frac{\bullet h_9 : (\Delta_6, F_7 \rightarrow F_8), P_{10} \vdash P_{10}}{\bullet h_9 : (\Delta_6, F_7 \rightarrow F_8), P_{10} \vdash P_{10}} I \\
\frac{\bullet h_1 : \Delta_6, F_7 \rightarrow F_8 \vdash P_{10} \quad \bullet h_9 : (\Delta_6, F_7 \rightarrow F_8), P_{10} \vdash P_{10}}{- : \Delta_6, F_7 \rightarrow F_8 \vdash P_{10}} \text{Cut} \\
\frac{- : \Delta_6, F_7 \rightarrow F_8 \vdash P_{10}}{- : \Delta_6, F_7 \rightarrow F_8 \vdash P_{10}} \text{ax/W}
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_6, F_7 \rightarrow F_8 \vdash F_7 \quad h_1 : \Delta_6, F_8 \vdash \top}{\bullet h_1 : \Delta_6, F_7 \rightarrow F_8 \vdash \top} \rightarrow_L \quad \frac{h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash F_{10}}{\bullet h_9 : (\Delta_6, F_7 \rightarrow F_8), \top \vdash F_{10}} \top_L \\
\frac{\bullet h_1 : \Delta_6, F_7 \rightarrow F_8 \vdash \top \quad \bullet h_9 : (\Delta_6, F_7 \rightarrow F_8), \top \vdash F_{10}}{- : \Delta_6, F_7 \rightarrow F_8 \vdash F_{10}} \text{Cut} \\
\frac{- : \Delta_6, F_7 \rightarrow F_8 \vdash F_{10}}{- : \Delta_6, F_7 \rightarrow F_8 \vdash F_{10}} \text{ax/W} \\
\frac{h_1 : (\top, \Delta_{11}), F_7 \rightarrow F_8 \vdash F_7 \quad h_1 : (\top, \Delta_{11}), F_8 \vdash F_6}{\bullet h_1 : (\top, \Delta_{11}), F_7 \rightarrow F_8 \vdash F_6} \rightarrow_L \quad \frac{h_9 : \Delta_{11}, F_6, F_7 \rightarrow F_8 \vdash F_{10}}{\bullet h_9 : ((\top, \Delta_{11}), F_7 \rightarrow F_8), F_6 \vdash F_{10}} \top_L \\
\frac{\bullet h_1 : (\top, \Delta_{11}), F_7 \rightarrow F_8 \vdash F_6 \quad \bullet h_9 : ((\top, \Delta_{11}), F_7 \rightarrow F_8), F_6 \vdash F_{10}}{- : (\top, \Delta_{11}), F_7 \rightarrow F_8 \vdash F_{10}} \text{Cut} \\
\frac{- : (\top, \Delta_{11}), F_7 \rightarrow F_8 \vdash F_{10}}{- : \top, \Delta_{11}, F_7 \rightarrow F_8 \vdash F_{10}} \text{ax/W} \\
\frac{\bullet h_1 : \top, \Delta_{11}, F_7 \rightarrow F_8 \vdash F_6 \quad \text{ax/W} \quad h_9 : \top, \Delta_{11}, F_6, F_7 \rightarrow F_8 \vdash F_{10}}{- : \top, \Delta_{11}, F_7 \rightarrow F_8 \vdash F_{10}} \text{hCut}
\end{array}$$

## 6.7 Status of $\wedge_L$ : OK

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_6, F_8, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \wedge F_9 \vdash F_7} \wedge_L \quad \frac{\bullet h_{10} : (\Delta_6, F_8 \wedge F_9), F_7 \vdash \top}{\bullet h_{10} : (\Delta_6, F_8 \wedge F_9), F_7 \vdash \top} \top_R \\
\frac{\bullet h_1 : \Delta_6, F_8 \wedge F_9 \vdash F_7 \quad \bullet h_{10} : (\Delta_6, F_8 \wedge F_9), F_7 \vdash \top}{- : \Delta_6, F_8 \wedge F_9 \vdash \top} \text{Cut} \\
\frac{- : \Delta_6, F_8 \wedge F_9 \vdash \top}{- : \Delta_6, F_8 \wedge F_9 \vdash \top} \top_R
\end{array}$$

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_6, F_8, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \wedge F_9 \vdash F_7} \wedge_L \quad \frac{h_{10} : \Delta_6, F_7, F_{11}, F_8 \wedge F_9 \vdash F_{12}}{\bullet h_{10} : (\Delta_6, F_8 \wedge F_9), F_7 \vdash F_{11} \rightarrow F_{12}} \rightarrow_R \\
\frac{\bullet h_1 : \Delta_6, F_8 \wedge F_9 \vdash F_7 \quad \bullet h_{10} : (\Delta_6, F_8 \wedge F_9), F_7 \vdash F_{11} \rightarrow F_{12}}{- : \Delta_6, F_8 \wedge F_9 \vdash F_{11} \rightarrow F_{12}} \text{Cut} \\
\frac{- : \Delta_6, F_8 \wedge F_9 \vdash F_{11} \rightarrow F_{12}}{- : \Delta_6, F_8 \wedge F_9 \vdash F_{11} \rightarrow F_{12}} \rightarrow_R \\
\frac{\bullet h_1 : \Delta_6, F_{11}, F_8 \wedge F_9 \vdash F_7 \quad \text{ax/W} \quad h_{10} : \Delta_6, F_{11}, F_7, F_8 \wedge F_9 \vdash F_{12}}{- : \Delta_6, F_{11}, F_8 \wedge F_9 \vdash F_{12}} \text{ax/W} \\
\frac{- : \Delta_6, F_{11}, F_8 \wedge F_9 \vdash F_{12}}{- : \Delta_6, F_8 \wedge F_9 \vdash F_{11} \rightarrow F_{12}} \text{hCut}
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_6, F_8, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \wedge F_9 \vdash F_7} \wedge_L \quad \frac{h_{10} : \Delta_6, F_7, F_8 \wedge F_9 \vdash F_{11} \quad h_{10} : \Delta_6, F_7, F_8 \wedge F_9 \vdash F_{12}}{\bullet h_{10} : (\Delta_6, F_8 \wedge F_9), F_7 \vdash F_{11} \wedge F_{12}} \wedge_R \\
\hline
- : \Delta_6, F_8 \wedge F_9 \vdash F_{11} \wedge F_{12} \quad \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_6, F_8, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_8, F_9 \vdash F_7} \text{ax/W} \quad \frac{h_{10} : \Delta_6, F_7, F_8, F_9 \vdash F_{11} \quad \text{inv-th/ax}}{\bullet h_{10} : \Delta_6, F_7, F_8, F_9 \vdash F_{11} \wedge F_{12}} \text{inv-th/ax} \\
\hline
- : \Delta_6, F_8, F_9 \vdash F_{11} \wedge F_{12} \quad \text{hCut} \\
\hline
- : \Delta_6, F_8 \wedge F_9 \vdash F_{11} \wedge F_{12} \quad \wedge_L
\end{array}$$

- Case rule  $\vee_1$

$$\begin{array}{c}
\frac{h_1 : \Delta_6, F_8, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \wedge F_9 \vdash F_7} \wedge_L \quad \frac{h_{10} : \Delta_6, F_7, F_8 \wedge F_9 \vdash F_{11}}{\bullet h_{10} : (\Delta_6, F_8 \wedge F_9), F_7 \vdash F_{11} \vee F_{12}} \vee_1 \\
\hline
- : \Delta_6, F_8 \wedge F_9 \vdash F_{11} \vee F_{12} \quad \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_6, F_8, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \wedge F_9 \vdash F_7} \text{ax/W} \quad \frac{h_{10} : \Delta_6, F_7, F_8 \wedge F_9 \vdash F_{11}}{\bullet h_{10} : \Delta_6, F_7, F_8 \wedge F_9 \vdash F_{11}} \text{ax/W} \\
\hline
- : \Delta_6, F_8 \wedge F_9 \vdash F_{11} \quad \text{hCut} \\
\hline
- : \Delta_6, F_8 \wedge F_9 \vdash F_{11} \vee F_{12} \quad \vee_1
\end{array}$$

- Case rule  $\vee_2$

$$\begin{array}{c}
\frac{h_1 : \Delta_6, F_8, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \wedge F_9 \vdash F_7} \wedge_L \quad \frac{h_{10} : \Delta_6, F_7, F_8 \wedge F_9 \vdash F_{12}}{\bullet h_{10} : (\Delta_6, F_8 \wedge F_9), F_7 \vdash F_{11} \vee F_{12}} \vee_2 \\
\hline
- : \Delta_6, F_8 \wedge F_9 \vdash F_{11} \vee F_{12} \quad \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_6, F_8, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \wedge F_9 \vdash F_7} \text{ax/W} \quad \frac{h_{10} : \Delta_6, F_7, F_8 \wedge F_9 \vdash F_{12}}{\bullet h_{10} : \Delta_6, F_7, F_8 \wedge F_9 \vdash F_{12}} \text{ax/W} \\
\hline
- : \Delta_6, F_8 \wedge F_9 \vdash F_{12} \quad \text{hCut} \\
\hline
- : \Delta_6, F_8 \wedge F_9 \vdash F_{11} \vee F_{12} \quad \vee_2
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_1 : (\Delta_{13}, F_{10} \rightarrow F_{11}), F_7, F_8 \vdash F_6}{\bullet h_1 : (\Delta_{13}, F_{10} \rightarrow F_{11}), F_7 \wedge F_8 \vdash F_6} \wedge_L \quad \frac{h_9 : \Delta_{13}, F_6, F_{10} \rightarrow F_{11}, F_7 \wedge F_8 \vdash F_{10} \quad h_9 : \Delta_{13}, F_6, F_{11}, F_7 \wedge F_8 \vdash F_{12}}{\bullet h_9 : ((\Delta_{13}, F_{10} \rightarrow F_{11}), F_7 \wedge F_8), F_6 \vdash F_{12}} \rightarrow_L \\
\hline
- : (\Delta_{13}, F_{10} \rightarrow F_{11}), F_7 \wedge F_8 \vdash F_{12} \quad \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_{13}, F_7, F_8, F_{10} \rightarrow F_{11} \vdash F_6}{\bullet h_1 : \Delta_{13}, F_7, F_8, F_{10} \rightarrow F_{11} \vdash F_6} \text{ax/W} \quad \frac{h_9 : \Delta_{13}, F_6, F_7, F_8, F_{10} \rightarrow F_{11} \vdash F_{10} \quad \text{inv-th/ax}}{\bullet h_9 : \Delta_{13}, F_6, F_7, F_8, F_{10} \rightarrow F_{11} \vdash F_{12}} \text{inv-th/ax} \\
\hline
- : \Delta_{13}, F_7, F_8, F_{10} \rightarrow F_{11} \vdash F_{12} \quad \text{hCut} \\
\hline
- : \Delta_{13}, F_{10} \rightarrow F_{11}, F_7 \wedge F_8 \vdash F_{12} \quad \wedge_L \\
\rightarrow \\
\frac{h_1 : \Delta_6, F_7, F_8 \vdash F_{10} \rightarrow F_{11}}{\bullet h_1 : \Delta_6, F_7 \wedge F_8 \vdash F_{10} \rightarrow F_{11}} \wedge_L \quad \frac{h_9 : \Delta_6, F_{10} \rightarrow F_{11}, F_7 \wedge F_8 \vdash F_{10} \quad h_9 : \Delta_6, F_{11}, F_7 \wedge F_8 \vdash F_{12}}{\bullet h_9 : (\Delta_6, F_7 \wedge F_8), F_{10} \rightarrow F_{11} \vdash F_{12}} \rightarrow_L \\
\hline
- : \Delta_6, F_7 \wedge F_8 \vdash F_{12} \quad \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_6, F_7, F_8 \vdash F_{10} \rightarrow F_{11}}{\bullet h_1 : \Delta_6, F_7, F_8 \vdash F_{10} \rightarrow F_{11}} \text{ax/W} \quad \frac{h_9 : \Delta_6, F_7, F_8, F_{10} \rightarrow F_{11} \vdash F_{10} \quad \text{inv-th/ax}}{\bullet h_9 : \Delta_6, F_7, F_8, F_{10} \rightarrow F_{11} \vdash F_{12}} \text{inv-th/ax} \\
\hline
- : \Delta_6, F_7, F_8 \vdash F_{12} \quad \text{hCut} \\
\hline
- : \Delta_6, F_7 \wedge F_8 \vdash F_{12} \quad \wedge_L
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_1 : (\Delta_{13}, F_{10} \wedge F_{11}), F_7, F_8 \vdash F_6}{\bullet h_1 : (\Delta_{13}, F_{10} \wedge F_{11}), F_7 \wedge F_8 \vdash F_6} \wedge_L \quad \frac{h_9 : \Delta_{13}, F_6, F_{10}, F_{11}, F_7 \wedge F_8 \vdash F_{12}}{\bullet h_9 : ((\Delta_{13}, F_{10} \wedge F_{11}), F_7 \wedge F_8), F_6 \vdash F_{12}} \wedge_L \\
\hline
- : (\Delta_{13}, F_{10} \wedge F_{11}), F_7 \wedge F_8 \vdash F_{12} \quad \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_{13}, F_{10}, F_{11}, F_7, F_8 \vdash F_6}{\bullet h_1 : \Delta_{13}, F_{10}, F_{11}, F_7 \wedge F_8 \vdash F_6} \text{inv-th/ax} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_{11}, F_6, F_7 \wedge F_8 \vdash F_{12}}{\bullet h_9 : \Delta_{13}, F_{10}, F_{11}, F_6, F_7 \wedge F_8 \vdash F_{12}} \text{inv-th/ax} \\
\hline
- : \Delta_{13}, F_{10}, F_{11}, F_7 \wedge F_8 \vdash F_{12} \quad \text{hCut} \\
\hline
- : \Delta_{13}, F_{10} \wedge F_{11}, F_7 \wedge F_8 \vdash F_{12} \quad \wedge_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_6, F_7, F_8 \vdash F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_6, F_7 \wedge F_8 \vdash F_{10} \wedge F_{11}} \wedge_L \quad \frac{h_9 : \Delta_6, F_{10}, F_{11}, F_7 \wedge F_8 \vdash F_{12}}{\bullet h_9 : (\Delta_6, F_7 \wedge F_8), F_{10} \wedge F_{11} \vdash F_{12}} \wedge_L \\
\hline
- : \Delta_6, F_7 \wedge F_8 \vdash F_{12} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_6, F_7, F_8 \vdash F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_6, F_7, F_8 \vdash F_{10} \wedge F_{11}} \text{ ax/W} \quad \frac{h_9 : \Delta_6, F_{10}, F_{11}, F_7, F_8 \vdash F_{12}}{\bullet h_9 : \Delta_6, F_7, F_8, F_{10} \wedge F_{11} \vdash F_{12}} \text{ inv-th/ax} \wedge_L}{- : \Delta_6, F_7, F_8 \vdash F_{12}} \wedge_L \quad \frac{}{- : \Delta_6, F_7 \wedge F_8 \vdash F_{12}} \text{ hCut} \\
\hline
\frac{h_1 : \Delta_6, F_9, F_{10} \vdash F_7}{\bullet h_1 : \Delta_6, F_9 \wedge F_{10} \vdash F_7} \wedge_L \quad \frac{h_8 : \Delta_6, F_7, F_9, F_{10} \vdash F_{11}}{\bullet h_8 : (\Delta_6, F_9 \wedge F_{10}), F_7 \vdash F_{11}} \wedge_L \\
\hline
- : \Delta_6, F_9 \wedge F_{10} \vdash F_{11} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_6, F_{10}, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_{10}, F_9 \vdash F_7} \text{ ax/W} \quad \frac{h_8 : \Delta_6, F_{10}, F_7, F_9 \vdash F_{11}}{\bullet h_8 : \Delta_6, F_{10}, F_7, F_9 \vdash F_{11}} \text{ ax/W} \quad \frac{}{- : \Delta_6, F_{10}, F_9 \vdash F_{11}} \text{ H}}{- : \Delta_6, F_9 \wedge F_{10} \vdash F_{11}} \wedge_L \quad \text{hCut}
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{h_1 : (\Delta_{13}, F_{10} \vee F_{11}), F_7, F_8 \vdash F_6}{\bullet h_1 : (\Delta_{13}, F_{10} \vee F_{11}), F_7 \wedge F_8 \vdash F_6} \wedge_L \quad \frac{h_9 : \Delta_{13}, F_6, F_{10}, F_7 \wedge F_8 \vdash F_{12}}{\bullet h_9 : ((\Delta_{13}, F_{10} \vee F_{11}), F_7 \wedge F_8), F_6 \vdash F_{12}} \wedge_L \quad \frac{h_9 : \Delta_{13}, F_6, F_{11}, F_7 \wedge F_8 \vdash F_{12}}{\bullet h_9 : ((\Delta_{13}, F_{10} \vee F_{11}), F_7 \wedge F_8), F_6 \vdash F_{12}} \wedge_L \\
\hline
- : (\Delta_{13}, F_{10} \vee F_{11}), F_7 \wedge F_8 \vdash F_{12} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{13}, F_7, F_8, F_{10} \vee F_{11} \vdash F_6}{\bullet h_1 : \Delta_{13}, F_7, F_8, F_{10} \vee F_{11} \vdash F_6} \text{ ax/W} \quad \frac{h_9 : \Delta_{13}, F_{10}, F_6, F_7, F_8 \vdash F_{12}}{\bullet h_9 : \Delta_{13}, F_6, F_7, F_8, F_{10} \vee F_{11} \vdash F_{12}} \text{ inv-th/ax} \wedge_L \quad \frac{h_9 : \Delta_{13}, F_{11}, F_6, F_7, F_8 \vdash F_{12}}{\bullet h_9 : \Delta_{13}, F_{11}, F_6, F_7, F_8 \vdash F_{12}} \text{ inv-th/ax} \wedge_L}{- : \Delta_{13}, F_7, F_8, F_{10} \vee F_{11} \vdash F_{12}} \wedge_L \quad \text{hCut} \\
\hline
\frac{h_1 : \Delta_6, F_7, F_8 \vdash F_{10} \vee F_{11}}{\bullet h_1 : \Delta_6, F_7 \wedge F_8 \vdash F_{10} \vee F_{11}} \wedge_L \quad \frac{h_9 : \Delta_6, F_{10}, F_7 \wedge F_8 \vdash F_{12}}{\bullet h_9 : (\Delta_6, F_7 \wedge F_8), F_{10} \vee F_{11} \vdash F_{12}} \wedge_L \quad \frac{h_9 : \Delta_6, F_{11}, F_7 \wedge F_8 \vdash F_{12}}{\bullet h_9 : (\Delta_6, F_7 \wedge F_8), F_{10} \vee F_{11} \vdash F_{12}} \wedge_L \\
\hline
- : \Delta_6, F_7 \wedge F_8 \vdash F_{12} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_6, F_7, F_8 \vdash F_{10} \vee F_{11}}{\bullet h_1 : \Delta_6, F_7, F_8 \vdash F_{10} \vee F_{11}} \text{ ax/W} \quad \frac{h_9 : \Delta_6, F_{10}, F_7, F_8 \vdash F_{12}}{\bullet h_9 : \Delta_6, F_7, F_8, F_{10} \vee F_{11} \vdash F_{12}} \text{ inv-th/ax} \wedge_L \quad \frac{h_9 : \Delta_6, F_{11}, F_7, F_8 \vdash F_{12}}{\bullet h_9 : \Delta_6, F_{11}, F_7, F_8 \vdash F_{12}} \text{ inv-th/ax} \wedge_L}{- : \Delta_6, F_7, F_8 \vdash F_{12}} \wedge_L \quad \text{hCut} \\
\hline
- : \Delta_6, F_7 \wedge F_8 \vdash F_{12}
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_6, F_7, F_8 \vdash \perp}{\bullet h_1 : \Delta_6, F_7 \wedge F_8 \vdash \perp} \wedge_L \quad \frac{}{\bullet h_9 : (\Delta_6, F_7 \wedge F_8), \perp \vdash F_{10}} \perp_L \\
\hline
- : \Delta_6, F_7 \wedge F_8 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_6, F_7, F_8 \vdash \perp}{\bullet h_1 : \Delta_6, F_7, F_8 \vdash \perp} \text{ ax/W} \quad \frac{}{\bullet h_9 : \perp, \Delta_6, F_7, F_8 \vdash F_{10}} \perp_L}{- : \Delta_6, F_7, F_8 \vdash F_{10}} \wedge_L \quad \text{hCut} \\
\hline
- : \Delta_6, F_7 \wedge F_8 \vdash F_{10} \\
\rightarrow \\
\frac{h_1 : (\perp, \Delta_{11}), F_7, F_8 \vdash F_6}{\bullet h_1 : (\perp, \Delta_{11}), F_7 \wedge F_8 \vdash F_6} \wedge_L \quad \frac{}{\bullet h_9 : ((\perp, \Delta_{11}), F_7 \wedge F_8), F_6 \vdash F_{10}} \perp_L \\
\hline
- : (\perp, \Delta_{11}), F_7 \wedge F_8 \vdash F_{10} \\
\rightarrow \\
- : \perp, \Delta_{11}, F_7 \wedge F_8 \vdash F_{10} \quad \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{h_1 : (\Delta_{11}, p_{10}), F_7, F_8 \vdash F_6}{\bullet h_1 : (\Delta_{11}, p_{10}), F_7 \wedge F_8 \vdash F_6} \wedge_L \quad \frac{}{\bullet h_9 : ((\Delta_{11}, p_{10}), F_7 \wedge F_8), F_6 \vdash p_{10}} I \\
\hline
- : (\Delta_{11}, p_{10}), F_7 \wedge F_8 \vdash p_{10} \\
\rightarrow \\
- : \Delta_{11}, p_{10}, F_7 \wedge F_8 \vdash p_{10} \quad I
\end{array}$$

$$\frac{\frac{h_1 : \Delta_6, F_7, F_8 \vdash P_{10}}{\bullet h_1 : \Delta_6, F_7 \wedge F_8 \vdash P_{10}} \wedge_L \quad \frac{}{\bullet h_9 : (\Delta_6, F_7 \wedge F_8), P_{10} \vdash P_{10}} I}{\frac{}{- : \Delta_6, F_7 \wedge F_8 \vdash P_{10}} \text{Cut}}{\rightarrow} \frac{}{- : \Delta_6, F_7 \wedge F_8 \vdash P_{10}} \text{ax/W}$$

- Case rule  $\top_L$

$$\frac{\frac{h_1 : \Delta_6, F_7, F_8 \vdash \top}{\bullet h_1 : \Delta_6, F_7 \wedge F_8 \vdash \top} \wedge_L \quad \frac{h_9 : \Delta_6, F_7 \wedge F_8 \vdash F_{10}}{\bullet h_9 : (\Delta_6, F_7 \wedge F_8), \top \vdash F_{10}} \top_L}{\frac{}{- : \Delta_6, F_7 \wedge F_8 \vdash F_{10}} \text{Cut}}{\rightarrow} \frac{}{- : \Delta_6, F_7 \wedge F_8 \vdash F_{10}} \text{ax/W}$$

$$\frac{\frac{h_1 : (\top, \Delta_{11}), F_7, F_8 \vdash F_6}{\bullet h_1 : (\top, \Delta_{11}), F_7 \wedge F_8 \vdash F_6} \wedge_L \quad \frac{h_9 : \Delta_{11}, F_6, F_7 \wedge F_8 \vdash F_{10}}{\bullet h_9 : ((\top, \Delta_{11}), F_7 \wedge F_8), F_6 \vdash F_{10}} \top_L}{\frac{}{- : (\top, \Delta_{11}), F_7 \wedge F_8 \vdash F_{10}} \text{Cut}}{\rightarrow} \frac{}{- : (\top, \Delta_{11}), F_7 \wedge F_8 \vdash F_{10}} \text{ax/W}$$

$$\frac{\frac{}{\bullet h_1 : \top, \Delta_{11}, F_7 \wedge F_8 \vdash F_6} \text{ax/W} \quad \frac{h_9 : \top, \Delta_{11}, F_6, F_7 \wedge F_8 \vdash F_{10}}{\bullet h_9 : \top, \Delta_{11}, F_7 \wedge F_8 \vdash F_{10}} \text{ax/W}}{\frac{}{- : \top, \Delta_{11}, F_7 \wedge F_8 \vdash F_{10}} \text{hCut}}$$

## 6.8 Status of $\vee_L$ : OK

- Case rule  $\top_R$

$$\frac{\frac{h_1 : \Delta_6, F_8 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \vee F_9 \vdash F_7} \vee_L \quad \frac{h_1 : \Delta_6, F_9 \vdash F_7}{\bullet h_{10} : (\Delta_6, F_8 \vee F_9), F_7 \vdash \top} \top_R}{\frac{}{- : \Delta_6, F_8 \vee F_9 \vdash \top} \text{Cut}}{\rightarrow} \frac{}{- : \Delta_6, F_8 \vee F_9 \vdash \top} \top_R$$

- Case rule  $\rightarrow_R$

$$\frac{\frac{h_1 : \Delta_6, F_8 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \vee F_9 \vdash F_7} \vee_L \quad \frac{h_{10} : \Delta_6, F_7, F_{11}, F_8 \vee F_9 \vdash F_{12}}{\bullet h_{10} : (\Delta_6, F_8 \vee F_9), F_7 \vdash F_{11} \rightarrow F_{12}} \rightarrow_R}{\frac{}{- : \Delta_6, F_8 \vee F_9 \vdash F_{11} \rightarrow F_{12}} \text{Cut}}{\rightarrow} \frac{}{- : \Delta_6, F_8 \vee F_9 \vdash F_{11} \rightarrow F_{12}} \text{ax/W}$$

$$\frac{\frac{}{\bullet h_1 : \Delta_6, F_{11}, F_8 \vee F_9 \vdash F_7} \text{ax/W} \quad \frac{h_{10} : \Delta_6, F_{11}, F_7, F_8 \vee F_9 \vdash F_{12}}{\bullet h_{10} : \Delta_6, F_{11}, F_7, F_8 \vee F_9 \vdash F_{12}} \text{ax/W}}{\frac{}{- : \Delta_6, F_{11}, F_8 \vee F_9 \vdash F_{12}} \rightarrow_R} \frac{}{- : \Delta_6, F_8 \vee F_9 \vdash F_{11} \rightarrow F_{12}} \text{hCut}}$$

- Case rule  $\wedge_R$

$$\frac{\frac{h_1 : \Delta_6, F_8 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \vee F_9 \vdash F_7} \vee_L \quad \frac{h_{10} : \Delta_6, F_7, F_8 \vee F_9 \vdash F_{11} \quad h_{10} : \Delta_6, F_7, F_8 \vee F_9 \vdash F_{12}}{\bullet h_{10} : (\Delta_6, F_8 \vee F_9), F_7 \vdash F_{11} \wedge F_{12}} \wedge_R}{\frac{}{- : \Delta_6, F_8 \vee F_9 \vdash F_{11} \wedge F_{12}} \text{Cut}}{\rightarrow} \frac{}{- : \Delta_6, F_8 \vee F_9 \vdash F_{11} \wedge F_{12}} \text{ax/W}$$

$$\frac{\frac{}{\bullet h_1 : \Delta_6, F_8 \vee F_9 \vdash F_7} \text{ax/W} \quad \frac{h_{10} : \Delta_6, F_7, F_8 \vee F_9 \vdash F_{11}}{\bullet h_{10} : \Delta_6, F_7, F_8 \vee F_9 \vdash F_{11}} \text{ax/W}}{\frac{}{- : \Delta_6, F_8 \vee F_9 \vdash F_{11}} \wedge_R} \frac{}{- : \Delta_6, F_8 \vee F_9 \vdash F_{11} \wedge F_{12}} \text{hCut}}$$

- Case rule  $\vee_1$

$$\frac{\frac{h_1 : \Delta_6, F_8 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \vee F_9 \vdash F_7} \vee_L \quad \frac{h_{10} : \Delta_6, F_7, F_8 \vee F_9 \vdash F_{11}}{\bullet h_{10} : (\Delta_6, F_8 \vee F_9), F_7 \vdash F_{11} \vee F_{12}} \vee_1}{\frac{}{- : \Delta_6, F_8 \vee F_9 \vdash F_{11} \vee F_{12}} \text{Cut}}{\rightarrow} \frac{}{- : \Delta_6, F_8 \vee F_9 \vdash F_{11} \vee F_{12}} \text{ax/W}$$

$$\frac{\frac{}{\bullet h_1 : \Delta_6, F_8 \vee F_9 \vdash F_7} \text{ax/W} \quad \frac{h_{10} : \Delta_6, F_7, F_8 \vee F_9 \vdash F_{11}}{\bullet h_{10} : \Delta_6, F_7, F_8 \vee F_9 \vdash F_{11}} \text{ax/W}}{\frac{}{- : \Delta_6, F_8 \vee F_9 \vdash F_{11}} \vee_1} \frac{}{- : \Delta_6, F_8 \vee F_9 \vdash F_{11} \vee F_{12}} \text{hCut}}$$

- Case rule  $\vee_2$

$$\begin{array}{c}
\frac{h_1 : \Delta_6, F_8 \vdash F_7 \quad h_1 : \Delta_6, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_8 \vee F_9 \vdash F_7} \vee_L \quad \frac{h_{10} : \Delta_6, F_7, F_8 \vee F_9 \vdash F_{12}}{\bullet h_{10} : (\Delta_6, F_8 \vee F_9), F_7 \vdash F_{11} \vee F_{12}} \vee_2 \\
\frac{}{- : \Delta_6, F_8 \vee F_9 \vdash F_{11} \vee F_{12}}{\rightarrow} \text{Cut} \\
\frac{\frac{}{\bullet h_1 : \Delta_6, F_8 \vee F_9 \vdash F_7} \text{ax/W} \quad \frac{}{h_{10} : \Delta_6, F_7, F_8 \vee F_9 \vdash F_{12}} \text{ax/W}}{- : \Delta_6, F_8 \vee F_9 \vdash F_{12}} \vee_2}{- : \Delta_6, F_8 \vee F_9 \vdash F_{11} \vee F_{12}} \text{hCut}
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_1 : (\Delta_{13}, F_{10} \rightarrow F_{11}), F_7 \vdash F_6 \quad h_1 : (\Delta_{13}, F_{10} \rightarrow F_{11}), F_8 \vdash F_6}{\bullet h_1 : (\Delta_{13}, F_{10} \rightarrow F_{11}), F_7 \vee F_8 \vdash F_6} \vee_L \quad \frac{h_9 : \Delta_{13}, F_6, F_{10} \rightarrow F_{11}, F_7 \vee F_8 \vdash F_{10} \quad h_9 : \Delta_{13}, F_6, F_{11}, F_7 \vee F_8 \vdash F_{12}}{\bullet h_9 : ((\Delta_{13}, F_{10} \rightarrow F_{11}), F_7 \vee F_8), F_6 \vdash F_{12}} \vee_L \\
\frac{}{- : (\Delta_{13}, F_{10} \rightarrow F_{11}), F_7 \vee F_8 \vdash F_{12}}{\rightarrow} \text{Cut} \\
\frac{\frac{}{\bullet h_1 : \Delta_{13}, F_{10} \rightarrow F_{11}, F_7 \vee F_8 \vdash F_6} \text{ax/W} \quad \frac{}{h_9 : \Delta_{13}, F_6, F_{10} \rightarrow F_{11}, F_7 \vee F_8 \vdash F_{10}} \text{ax/W}}{- : \Delta_{13}, F_{10} \rightarrow F_{11}, F_7 \vee F_8 \vdash F_{10}} \text{hCut} \quad \frac{\frac{}{h_1 : \Delta_{13}, F_{11}, F_7 \vdash F_6} \text{inv-th/ax} \quad \frac{}{h_1 : \Delta_{13}, F_{11}, F_8 \vdash F_6} \text{inv-th/ax}}{\bullet h_1 : \Delta_{13}, F_{11}, F_7 \vee F_8 \vdash F_6} \vee_L}{- : \Delta_{13}, F_{11}, F_7 \vee F_8 \vdash F_{12}} \text{hCut} \\
\frac{}{- : \Delta_{13}, F_{10} \rightarrow F_{11}, F_7 \vee F_8 \vdash F_{12}}{\rightarrow} \text{Cut} \\
\frac{h_1 : \Delta_6, F_7 \vdash F_{10} \rightarrow F_{11} \quad h_1 : \Delta_6, F_8 \vdash F_{10} \rightarrow F_{11}}{\bullet h_1 : \Delta_6, F_7 \vee F_8 \vdash F_{10} \rightarrow F_{11}} \vee_L \quad \frac{h_9 : \Delta_6, F_{10} \rightarrow F_{11}, F_7 \vee F_8 \vdash F_{10} \quad h_9 : \Delta_6, F_{11}, F_7 \vee F_8 \vdash F_{12}}{\bullet h_9 : (\Delta_6, F_7 \vee F_8), F_{10} \rightarrow F_{11} \vdash F_{12}} \vee_L \\
\frac{}{- : \Delta_6, F_7 \vee F_8 \vdash F_{12}}{\rightarrow} \text{Cut} \\
\frac{\frac{}{h_1 : \Delta_6, F_7 \vdash F_{10} \rightarrow F_{11}} \text{ax/W} \quad \frac{}{h_9 : \Delta_6, F_7, F_{10} \rightarrow F_{11} \vdash F_{10}} \text{inv-th/ax}}{\bullet h_9 : \Delta_6, F_7, F_{10} \rightarrow F_{11} \vdash F_{12}} \text{hCut} \quad \frac{}{h_9 : \Delta_6, F_{11}, F_7 \vdash F_{12}} \text{inv-th/ax} \rightarrow_L \quad \frac{}{h_1 : \Delta_6, F_8 \vdash F_{10} \rightarrow F_{11}} \text{ax/W} \quad \frac{}{h_9 : \Delta_6, F_8 \vdash F_{10} \rightarrow F_{11}} \text{ax/W}}{- : \Delta_6, F_7 \vee F_8 \vdash F_{12}} \text{hCut}
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_1 : (\Delta_{13}, F_{10} \wedge F_{11}), F_7 \vdash F_6 \quad h_1 : (\Delta_{13}, F_{10} \wedge F_{11}), F_8 \vdash F_6}{\bullet h_1 : (\Delta_{13}, F_{10} \wedge F_{11}), F_7 \vee F_8 \vdash F_6} \vee_L \quad \frac{h_9 : \Delta_{13}, F_6, F_{10}, F_{11}, F_7 \vee F_8 \vdash F_{12}}{\bullet h_9 : ((\Delta_{13}, F_{10} \wedge F_{11}), F_7 \vee F_8), F_6 \vdash F_{12}} \vee_L \\
\frac{}{- : (\Delta_{13}, F_{10} \wedge F_{11}), F_7 \vee F_8 \vdash F_{12}}{\rightarrow} \text{Cut} \\
\frac{\frac{}{h_1 : \Delta_{13}, F_{10}, F_{11}, F_7 \vdash F_6} \text{inv-th/ax} \quad \frac{}{h_1 : \Delta_{13}, F_{10}, F_{11}, F_8 \vdash F_6} \text{inv-th/ax}}{\bullet h_1 : \Delta_{13}, F_{10}, F_{11}, F_7 \vee F_8 \vdash F_6} \vee_L \quad \frac{}{h_9 : \Delta_{13}, F_{10}, F_{11}, F_6, F_7 \vee F_8 \vdash F_{12}} \text{ax/W}}{- : \Delta_{13}, F_{10}, F_{11}, F_7 \vee F_8 \vdash F_{12}} \wedge_L}{- : \Delta_{13}, F_{10} \wedge F_{11}, F_7 \vee F_8 \vdash F_{12}} \text{hCut} \\
\frac{h_1 : \Delta_6, F_7 \vdash F_{10} \wedge F_{11} \quad h_1 : \Delta_6, F_8 \vdash F_{10} \wedge F_{11}}{\bullet h_1 : \Delta_6, F_7 \vee F_8 \vdash F_{10} \wedge F_{11}} \vee_L \quad \frac{h_9 : \Delta_6, F_{10}, F_{11}, F_7 \vee F_8 \vdash F_{12}}{\bullet h_9 : (\Delta_6, F_7 \vee F_8), F_{10} \wedge F_{11} \vdash F_{12}} \wedge_L \\
\frac{}{- : \Delta_6, F_7 \vee F_8 \vdash F_{12}}{\rightarrow} \text{Cut} \\
\frac{\frac{}{h_1 : \Delta_6, F_7 \vdash F_{10} \wedge F_{11}} \text{ax/W} \quad \frac{}{h_9 : \Delta_6, F_{10}, F_{11}, F_7 \vdash F_{12}} \text{inv-th/ax}}{\bullet h_9 : \Delta_6, F_7, F_{10} \wedge F_{11} \vdash F_{12}} \wedge_L \quad \frac{}{h_1 : \Delta_6, F_8 \vdash F_{10} \wedge F_{11}} \text{ax/W} \quad \frac{}{h_9 : \Delta_6, F_{10}, F_{11}, F_8 \vdash F_{12}} \text{inv-th/ax}}{\bullet h_9 : \Delta_6, F_8, F_{10} \wedge F_{11} \vdash F_{12}} \wedge_L}{- : \Delta_6, F_7 \vee F_8 \vdash F_{12}} \vee_L}{- : \Delta_6, F_7 \vee F_8 \vdash F_{12}} \text{hCut}
\end{array}$$

- Case rule  $\vee_L$



$$\begin{array}{c}
\frac{h_1 : (\Delta_{13}, F_{10} \vee F_{11}), F_7 \vdash F_6 \quad h_1 : (\Delta_{13}, F_{10} \vee F_{11}), F_8 \vdash F_6}{\bullet h_1 : (\Delta_{13}, F_{10} \vee F_{11}), F_7 \vee F_8 \vdash F_6} \vee_L \quad \frac{h_9 : \Delta_{13}, F_6, F_{10}, F_7 \vee F_8 \vdash F_{12} \quad h_9 : \Delta_{13}, F_6}{\bullet h_9 : ((\Delta_{13}, F_{10} \vee F_{11}), F_7 \vee F_8), F_6 \vdash F_{12}} \text{Cut} \\
\hline
- : (\Delta_{13}, F_{10} \vee F_{11}), F_7 \vee F_8 \vdash F_{12} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{13}, F_{10}, F_7 \vdash F_6}{\bullet h_1 : \Delta_{13}, F_{10}, F_7 \vee F_8 \vdash F_6} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{13}, F_{10}, F_8 \vdash F_6}{\bullet h_1 : \Delta_{13}, F_{10}, F_8 \vee F_7 \vdash F_6} \text{inv-th/ax}}{\bullet h_1 : \Delta_{13}, F_{10}, F_7 \vee F_8 \vdash F_6} \vee_L \quad \frac{h_9 : \Delta_{13}, F_{10}, F_6, F_7 \vee F_8 \vdash F_{12}}{\bullet h_9 : \Delta_{13}, F_{10}, F_6, F_7 \vee F_8 \vdash F_{12}} \text{ax/W}}{- : \Delta_{13}, F_{10}, F_7 \vee F_8 \vdash F_{12}} \text{hCut} \\
\hline
- : \Delta_{13}, F_{10} \vee F_{11}, F_7 \vee F_8 \vdash F_{12} \\
\rightarrow \\
\frac{h_1 : \Delta_6, F_7 \vdash F_{10} \vee F_{11} \quad h_1 : \Delta_6, F_8 \vdash F_{10} \vee F_{11}}{\bullet h_1 : \Delta_6, F_7 \vee F_8 \vdash F_{10} \vee F_{11}} \vee_L \quad \frac{h_9 : \Delta_6, F_{10}, F_7 \vee F_8 \vdash F_{12} \quad h_9 : \Delta_6, F_{11}, F_7 \vee F_8 \vdash F_{12}}{\bullet h_9 : (\Delta_6, F_7 \vee F_8), F_{10} \vee F_{11} \vdash F_{12}} \text{Cut} \\
\hline
- : \Delta_6, F_7 \vee F_8 \vdash F_{12} \\
\rightarrow \\
\frac{h_1 : \Delta_6, F_7 \vdash F_{10} \vee F_{11}}{\bullet h_1 : \Delta_6, F_7 \vdash F_{10} \vee F_{11}} \text{ax/W} \quad \frac{h_9 : \Delta_6, F_{10}, F_7 \vdash F_{12}}{\bullet h_9 : \Delta_6, F_7, F_{10} \vee F_{11} \vdash F_{12}} \text{inv-th/ax} \quad \frac{h_9 : \Delta_6, F_{11}, F_7 \vdash F_{12}}{\bullet h_9 : \Delta_6, F_{11}, F_7 \vee F_8 \vdash F_{12}} \text{inv-th/ax}}{- : \Delta_6, F_7 \vdash F_{12}} \text{hCut} \\
\hline
- : \Delta_6, F_7 \vee F_8 \vdash F_{12} \\
\rightarrow \\
\frac{h_1 : \Delta_6, F_9 \vdash F_7 \quad h_1 : \Delta_6, F_{10} \vdash F_7}{\bullet h_1 : \Delta_6, F_9 \vee F_{10} \vdash F_7} \vee_L \quad \frac{h_8 : \Delta_6, F_7, F_9 \vdash F_{11} \quad h_8 : \Delta_6, F_7, F_{10} \vdash F_{11}}{\bullet h_8 : (\Delta_6, F_9 \vee F_{10}), F_7 \vdash F_{11}} \text{Cut} \\
\hline
- : \Delta_6, F_9 \vee F_{10} \vdash F_{11} \\
\rightarrow \\
\frac{h_1 : \Delta_6, F_9 \vdash F_7}{\bullet h_1 : \Delta_6, F_9 \vdash F_7} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7, F_9 \vdash F_{11}}{\bullet h_8 : \Delta_6, F_7, F_9 \vdash F_{11}} \text{H}}{- : \Delta_6, F_9 \vdash F_{11}} \text{hCut} \quad \frac{h_1 : \Delta_6, F_{10} \vdash F_7}{\bullet h_1 : \Delta_6, F_{10} \vdash F_7} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_{10}, F_7 \vdash F_{11}}{\bullet h_8 : \Delta_6, F_{10}, F_7 \vdash F_{11}} \text{H}}{- : \Delta_6, F_{10} \vdash F_{11}} \text{hCut} \\
\hline
- : \Delta_6, F_9 \vee F_{10} \vdash F_{11} \\
\vee_L
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_6, F_7 \vdash \perp \quad h_1 : \Delta_6, F_8 \vdash \perp}{\bullet h_1 : \Delta_6, F_7 \vee F_8 \vdash \perp} \vee_L \quad \frac{}{\bullet h_9 : (\Delta_6, F_7 \vee F_8), \perp \vdash F_{10}} \perp_L \\
\hline
- : \Delta_6, F_7 \vee F_8 \vdash F_{10} \\
\rightarrow \\
\frac{h_1 : \Delta_6, F_7 \vdash \perp}{\bullet h_1 : \Delta_6, F_7 \vdash \perp} \text{ax/W} \quad \frac{h_9 : \perp, \Delta_6, F_7 \vdash F_{10}}{\bullet h_9 : \perp, \Delta_6, F_7 \vdash F_{10}} \perp_L}{- : \Delta_6, F_7 \vdash F_{10}} \text{hCut} \quad \frac{h_1 : \Delta_6, F_8 \vdash \perp}{\bullet h_1 : \Delta_6, F_8 \vdash \perp} \text{ax/W} \quad \frac{h_9 : \perp, \Delta_6, F_8 \vdash F_{10}}{\bullet h_9 : \perp, \Delta_6, F_8 \vdash F_{10}} \perp_L}{- : \Delta_6, F_8 \vdash F_{10}} \text{hCut} \\
\hline
- : \Delta_6, F_7 \vee F_8 \vdash F_{10} \\
\vee_L \\
\rightarrow \\
\frac{h_1 : (\perp, \Delta_{11}), F_7 \vdash F_6 \quad h_1 : (\perp, \Delta_{11}), F_8 \vdash F_6}{\bullet h_1 : (\perp, \Delta_{11}), F_7 \vee F_8 \vdash F_6} \vee_L \quad \frac{}{\bullet h_9 : ((\perp, \Delta_{11}), F_7 \vee F_8), F_6 \vdash F_{10}} \perp_L \\
\hline
- : (\perp, \Delta_{11}), F_7 \vee F_8 \vdash F_{10} \\
\rightarrow \\
- : \perp, \Delta_{11}, F_7 \vee F_8 \vdash F_{10} \quad \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{h_1 : (\Delta_{11}, P_{10}), F_7 \vdash F_6 \quad h_1 : (\Delta_{11}, P_{10}), F_8 \vdash F_6}{\bullet h_1 : (\Delta_{11}, P_{10}), F_7 \vee F_8 \vdash F_6} \vee_L \quad \frac{}{\bullet h_9 : ((\Delta_{11}, P_{10}), F_7 \vee F_8), F_6 \vdash P_{10}} I \\
\hline
- : (\Delta_{11}, P_{10}), F_7 \vee F_8 \vdash P_{10} \\
\rightarrow \\
- : \Delta_{11}, P_{10}, F_7 \vee F_8 \vdash P_{10} \quad I \\
\rightarrow \\
\frac{h_1 : \Delta_6, F_7 \vdash P_{10} \quad h_1 : \Delta_6, F_8 \vdash P_{10}}{\bullet h_1 : \Delta_6, F_7 \vee F_8 \vdash P_{10}} \vee_L \quad \frac{}{\bullet h_9 : (\Delta_6, F_7 \vee F_8), P_{10} \vdash P_{10}} I \\
\hline
- : \Delta_6, F_7 \vee F_8 \vdash P_{10} \\
\rightarrow \\
- : \Delta_6, F_7 \vee F_8 \vdash P_{10} \quad \text{ax/W}
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_6, F_7 \vdash \top \quad h_1 : \Delta_6, F_8 \vdash \top}{\bullet h_1 : \Delta_6, F_7 \vee F_8 \vdash \top} \vee_L \quad \frac{h_9 : \Delta_6, F_7 \vee F_8 \vdash F_{10}}{\bullet h_9 : (\Delta_6, F_7 \vee F_8), \top \vdash F_{10}} \top_L}{\frac{- : \Delta_6, F_7 \vee F_8 \vdash F_{10}}{\rightarrow} \text{Cut}} \text{Cut} \\
\frac{}{- : \Delta_6, F_7 \vee F_8 \vdash F_{10}}{\text{ax/W}} \\
\frac{}{- : \Delta_6, F_7 \vee F_8 \vdash F_{10}}{\text{ax/W}}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : (\top, \Delta_{11}), F_7 \vdash F_6 \quad h_1 : (\top, \Delta_{11}), F_8 \vdash F_6}{\bullet h_1 : (\top, \Delta_{11}), F_7 \vee F_8 \vdash F_6} \vee_L \quad \frac{h_9 : \Delta_{11}, F_6, F_7 \vee F_8 \vdash F_{10}}{\bullet h_9 : ((\top, \Delta_{11}), F_7 \vee F_8), F_6 \vdash F_{10}} \top_L}{\frac{- : (\top, \Delta_{11}), F_7 \vee F_8 \vdash F_{10}}{\rightarrow} \text{Cut}} \text{Cut} \\
\frac{}{- : (\top, \Delta_{11}), F_7 \vee F_8 \vdash F_{10}}{\text{ax/W}} \quad \frac{}{h_9 : \top, \Delta_{11}, F_6, F_7 \vee F_8 \vdash F_{10}}{\text{ax/W}} \\
\frac{}{- : \top, \Delta_{11}, F_7 \vee F_8 \vdash F_{10}}{\text{hCut}}
\end{array}$$

## 6.9 Status of $\perp_L$ : OK

- Case rule  $\top_R$

$$\frac{\frac{}{\bullet h_1 : \perp, \Delta_4 \vdash F_5} \perp_L \quad \frac{}{\bullet h_6 : (\perp, \Delta_4), F_5 \vdash \top} \top_R}{\frac{- : \perp, \Delta_4 \vdash \top}{\rightarrow} \text{Cut}} \text{Cut} \\
\frac{}{- : \perp, \Delta_4 \vdash \top} \top_R$$

- Case rule  $\rightarrow_R$

$$\frac{\frac{}{\bullet h_1 : \perp, \Delta_4 \vdash F_5} \perp_L \quad \frac{h_6 : \perp, \Delta_4, F_5, F_7 \vdash F_8}{\bullet h_6 : (\perp, \Delta_4), F_5 \vdash F_7 \rightarrow F_8} \rightarrow_R}{\frac{- : \perp, \Delta_4 \vdash F_7 \rightarrow F_8}{\rightarrow} \text{Cut}} \text{Cut} \\
\frac{}{- : \perp, \Delta_4 \vdash F_7 \rightarrow F_8} \perp_L$$

- Case rule  $\wedge_R$

$$\frac{\frac{}{\bullet h_1 : \perp, \Delta_4 \vdash F_5} \perp_L \quad \frac{h_6 : \perp, \Delta_4, F_5 \vdash F_7 \quad h_6 : \perp, \Delta_4, F_5 \vdash F_8}{\bullet h_6 : (\perp, \Delta_4), F_5 \vdash F_7 \wedge F_8} \wedge_R}{\frac{- : \perp, \Delta_4 \vdash F_7 \wedge F_8}{\rightarrow} \text{Cut}} \text{Cut} \\
\frac{}{- : \perp, \Delta_4 \vdash F_7 \wedge F_8} \perp_L$$

- Case rule  $\vee_1$

$$\frac{\frac{}{\bullet h_1 : \perp, \Delta_4 \vdash F_5} \perp_L \quad \frac{h_6 : \perp, \Delta_4, F_5 \vdash F_7}{\bullet h_6 : (\perp, \Delta_4), F_5 \vdash F_7 \vee F_8} \vee_1}{\frac{- : \perp, \Delta_4 \vdash F_7 \vee F_8}{\rightarrow} \text{Cut}} \text{Cut} \\
\frac{}{- : \perp, \Delta_4 \vdash F_7 \vee F_8} \perp_L$$

- Case rule  $\vee_2$

$$\frac{\frac{}{\bullet h_1 : \perp, \Delta_4 \vdash F_5} \perp_L \quad \frac{h_6 : \perp, \Delta_4, F_5 \vdash F_8}{\bullet h_6 : (\perp, \Delta_4), F_5 \vdash F_7 \vee F_8} \vee_2}{\frac{- : \perp, \Delta_4 \vdash F_7 \vee F_8}{\rightarrow} \text{Cut}} \text{Cut} \\
\frac{}{- : \perp, \Delta_4 \vdash F_7 \vee F_8} \perp_L$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \perp, \Delta_9, F_6 \rightarrow F_7 \vdash F_4} \perp_L \quad \frac{h_5 : \perp, \Delta_9, F_4, F_6 \rightarrow F_7 \vdash F_6 \quad h_5 : \perp, \Delta_9, F_4, F_7 \vdash F_8}{\bullet h_5 : (\perp, \Delta_9, F_6 \rightarrow F_7), F_4 \vdash F_8} \rightarrow_L}{\frac{}{- : \perp, \Delta_9, F_6 \rightarrow F_7 \vdash F_8} \text{Cut}}{\rightarrow} \perp_L \\
\frac{}{- : \perp, \Delta_9, F_6 \rightarrow F_7 \vdash F_8} \rightarrow \\
\frac{}{- : \perp, \Delta_9, F_6 \rightarrow F_7 \vdash F_8} \perp_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \perp, \Delta_4 \vdash F_6 \rightarrow F_7} \perp_L \quad \frac{h_5 : \perp, \Delta_4, F_6 \rightarrow F_7 \vdash F_6 \quad h_5 : \perp, \Delta_4, F_7 \vdash F_8}{\bullet h_5 : (\perp, \Delta_4), F_6 \rightarrow F_7 \vdash F_8} \rightarrow_L}{\frac{}{- : \perp, \Delta_4 \vdash F_8} \text{Cut}}{\rightarrow} \perp_L \\
\frac{}{- : \perp, \Delta_4 \vdash F_8} \rightarrow \\
\frac{}{- : \perp, \Delta_4 \vdash F_8} \perp_L
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \perp, \Delta_9, F_6 \wedge F_7 \vdash F_4} \perp_L \quad \frac{h_5 : \perp, \Delta_9, F_4, F_6, F_7 \vdash F_8}{\bullet h_5 : (\perp, \Delta_9, F_6 \wedge F_7), F_4 \vdash F_8} \wedge_L}{\frac{}{- : \perp, \Delta_9, F_6 \wedge F_7 \vdash F_8} \text{Cut}}{\rightarrow} \perp_L \\
\frac{}{- : \perp, \Delta_9, F_6 \wedge F_7 \vdash F_8} \rightarrow \\
\frac{}{- : \perp, \Delta_9, F_6 \wedge F_7 \vdash F_8} \perp_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \perp, \Delta_4 \vdash F_6 \wedge F_7} \perp_L \quad \frac{h_5 : \perp, \Delta_4, F_6, F_7 \vdash F_8}{\bullet h_5 : (\perp, \Delta_4), F_6 \wedge F_7 \vdash F_8} \wedge_L}{\frac{}{- : \perp, \Delta_4 \vdash F_8} \text{Cut}}{\rightarrow} \perp_L \\
\frac{}{- : \perp, \Delta_4 \vdash F_8} \rightarrow \\
\frac{}{- : \perp, \Delta_4 \vdash F_8} \perp_L
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \perp, \Delta_9, F_6 \vee F_7 \vdash F_4} \perp_L \quad \frac{h_5 : \perp, \Delta_9, F_4, F_6 \vdash F_8 \quad h_5 : \perp, \Delta_9, F_4, F_7 \vdash F_8}{\bullet h_5 : (\perp, \Delta_9, F_6 \vee F_7), F_4 \vdash F_8} \vee_L}{\frac{}{- : \perp, \Delta_9, F_6 \vee F_7 \vdash F_8} \text{Cut}}{\rightarrow} \perp_L \\
\frac{}{- : \perp, \Delta_9, F_6 \vee F_7 \vdash F_8} \rightarrow \\
\frac{}{- : \perp, \Delta_9, F_6 \vee F_7 \vdash F_8} \perp_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \perp, \Delta_4 \vdash F_6 \vee F_7} \perp_L \quad \frac{h_5 : \perp, \Delta_4, F_6 \vdash F_8 \quad h_5 : \perp, \Delta_4, F_7 \vdash F_8}{\bullet h_5 : (\perp, \Delta_4), F_6 \vee F_7 \vdash F_8} \vee_L}{\frac{}{- : \perp, \Delta_4 \vdash F_8} \text{Cut}}{\rightarrow} \perp_L \\
\frac{}{- : \perp, \Delta_4 \vdash F_8} \rightarrow \\
\frac{}{- : \perp, \Delta_4 \vdash F_8} \perp_L
\end{array}$$

- Case rule  $\perp_L$

$$\frac{\frac{}{\bullet h_1 : \perp, \Delta_4 \vdash F_5} \perp_L \quad \frac{}{\bullet h_6 : (\perp, \Delta_4), F_5 \vdash F_7} \perp_L}{\frac{}{- : \perp, \Delta_4 \vdash F_7} \text{Cut}}{\rightarrow} \perp_L \\
\frac{}{- : \perp, \Delta_4 \vdash F_7} \rightarrow \\
\frac{}{- : \perp, \Delta_4 \vdash F_7} \perp_L$$

- Case rule  $I$

$$\frac{\frac{}{\bullet h_1 : \perp, \Delta_7, p_6 \vdash F_4} \perp_L \quad \frac{}{\bullet h_5 : (\perp, \Delta_7, p_6), F_4 \vdash p_6} I}{\frac{}{- : \perp, \Delta_7, p_6 \vdash p_6} \text{Cut}}{\rightarrow} \perp_L \\
\frac{}{- : \perp, \Delta_7, p_6 \vdash p_6} \rightarrow \\
\frac{}{- : \perp, \Delta_7, p_6 \vdash p_6} \perp_L$$

$$\frac{\frac{}{\bullet h_1 : \perp, \Delta_4 \vdash p_6} \perp_L \quad \frac{}{\bullet h_5 : (\perp, \Delta_4), p_6 \vdash p_6} I}{\frac{}{- : \perp, \Delta_4 \vdash p_6} \text{Cut}}{\rightarrow} \perp_L \\
\frac{}{- : \perp, \Delta_4 \vdash p_6} \rightarrow \\
\frac{}{- : \perp, \Delta_4 \vdash p_6} \perp_L$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \perp, \Delta_4 \vdash \top} \perp_L \quad \frac{h_5 : \perp, \Delta_4 \vdash F_6}{\bullet h_5 : (\perp, \Delta_4), \top \vdash F_6} \top_L}{\frac{}{- : \perp, \Delta_4 \vdash F_6} \text{Cut}}{\rightarrow} \perp_L \\
\frac{}{- : \perp, \Delta_4 \vdash F_6} \perp_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \perp, \top, \Delta_7 \vdash F_4} \perp_L \quad \frac{h_5 : \perp, \Delta_7, F_4 \vdash F_6}{\bullet h_5 : (\perp, \top, \Delta_7), F_4 \vdash F_6} \top_L}{\frac{}{- : \perp, \top, \Delta_7 \vdash F_6} \text{Cut}}{\rightarrow} \perp_L \\
\frac{}{- : \perp, \top, \Delta_7 \vdash F_6} \perp_L
\end{array}$$

## 6.10 Status of $I$ : OK

- Case rule  $\top_R$

$$\frac{\frac{}{\bullet h_1 : \Delta_4, P_5 \vdash P_5} I \quad \frac{}{\bullet h_6 : (\Delta_4, P_5), P_5 \vdash \top} \top_R}{\frac{}{- : \Delta_4, P_5 \vdash \top} \text{Cut}}{\rightarrow} \top_R \\
\frac{}{- : \Delta_4, P_5 \vdash \top} \top_R$$

- Case rule  $\rightarrow_R$

$$\frac{\frac{}{\bullet h_1 : \Delta_4, P_5 \vdash P_5} I \quad \frac{h_6 : \Delta_4, F_7, P_5, P_5 \vdash F_8}{\bullet h_6 : (\Delta_4, P_5), P_5 \vdash F_7 \rightarrow F_8} \rightarrow_R}{\frac{}{- : \Delta_4, P_5 \vdash F_7 \rightarrow F_8} \text{Cut}}{\rightarrow} \rightarrow_R \\
\frac{\frac{}{\bullet h_1 : \Delta_4, F_7, P_5 \vdash P_5} I \quad \frac{h_6 : \Delta_4, F_7, P_5, P_5 \vdash F_8}{\bullet h_6 : \Delta_4, F_7, P_5, P_5 \vdash F_8} \text{ax/w}}{\frac{}{- : \Delta_4, F_7, P_5 \vdash F_8} \text{hCut}}{\rightarrow} \rightarrow_R \\
\frac{}{- : \Delta_4, P_5 \vdash F_7 \rightarrow F_8} \rightarrow_R$$

- Case rule  $\wedge_R$

$$\frac{\frac{}{\bullet h_1 : \Delta_4, P_5 \vdash P_5} I \quad \frac{h_6 : \Delta_4, P_5, P_5 \vdash F_7 \quad h_6 : \Delta_4, P_5, P_5 \vdash F_8}{\bullet h_6 : (\Delta_4, P_5), P_5 \vdash F_7 \wedge F_8} \wedge_R}{\frac{}{- : \Delta_4, P_5 \vdash F_7 \wedge F_8} \text{Cut}}{\rightarrow} \wedge_R \\
\frac{\frac{}{\bullet h_1 : \Delta_4, P_5 \vdash P_5} \text{ax/w} \quad \frac{h_6 : \Delta_4, P_5, P_5 \vdash F_7}{\bullet h_6 : \Delta_4, P_5, P_5 \vdash F_7} \text{ax/w}}{\frac{}{- : \Delta_4, P_5 \vdash F_7} \text{hCut}}{\rightarrow} \wedge_R \\
\frac{\frac{}{\bullet h_1 : \Delta_4, P_5 \vdash P_5} \text{ax/w} \quad \frac{h_6 : \Delta_4, P_5, P_5 \vdash F_8}{\bullet h_6 : \Delta_4, P_5, P_5 \vdash F_8} \text{ax/w}}{\frac{}{- : \Delta_4, P_5 \vdash F_8} \text{hCut}}{\rightarrow} \wedge_R \\
\frac{}{- : \Delta_4, P_5 \vdash F_7 \wedge F_8} \wedge_R$$

- Case rule  $\vee_1$

$$\frac{\frac{}{\bullet h_1 : \Delta_4, P_5 \vdash P_5} I \quad \frac{h_6 : \Delta_4, P_5, P_5 \vdash F_7}{\bullet h_6 : (\Delta_4, P_5), P_5 \vdash F_7 \vee F_8} \vee_1}{\frac{}{- : \Delta_4, P_5 \vdash F_7 \vee F_8} \text{Cut}}{\rightarrow} \vee_1 \\
\frac{\frac{}{\bullet h_1 : \Delta_4, P_5 \vdash P_5} \text{ax/w} \quad \frac{h_6 : \Delta_4, P_5, P_5 \vdash F_7}{\bullet h_6 : \Delta_4, P_5, P_5 \vdash F_7} \text{ax/w}}{\frac{}{- : \Delta_4, P_5 \vdash F_7} \text{hCut}}{\rightarrow} \vee_1 \\
\frac{}{- : \Delta_4, P_5 \vdash F_7 \vee F_8} \vee_1$$

- Case rule  $\vee_2$

$$\begin{array}{c}
\frac{\bullet h_1 : \Delta_4, p_5 \vdash p_5}{- : \Delta_4, p_5 \vdash F_7 \vee F_8} I \quad \frac{h_6 : \Delta_4, p_5, p_5 \vdash F_8}{\bullet h_6 : (\Delta_4, p_5), p_5 \vdash F_7 \vee F_8} \vee_2 \\
\text{Cut} \\
\frac{}{- : \Delta_4, p_5 \vdash F_7 \vee F_8} \rightarrow \\
\frac{\bullet h_1 : \Delta_4, p_5 \vdash p_5 \quad \frac{\text{ax/W}}{h_6 : \Delta_4, p_5, p_5 \vdash F_8} \text{ax/W}}{- : \Delta_4, p_5 \vdash F_8} \text{hCut} \\
\frac{}{- : \Delta_4, p_5 \vdash F_7 \vee F_8} \vee_2
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{\bullet h_1 : (\Delta_9, F_6 \rightarrow F_7), p_4 \vdash p_4}{- : (\Delta_9, F_6 \rightarrow F_7), p_4 \vdash F_8} I \quad \frac{h_5 : \Delta_9, p_4, p_4, F_6 \rightarrow F_7 \vdash F_6 \quad h_5 : \Delta_9, F_7, p_4, p_4 \vdash F_8}{\bullet h_5 : ((\Delta_9, F_6 \rightarrow F_7), p_4), p_4 \vdash F_8} \rightarrow_L \\
\text{Cut} \\
\frac{}{- : (\Delta_9, F_6 \rightarrow F_7), p_4 \vdash F_8} \rightarrow \\
\frac{\bullet h_1 : \Delta_9, p_4, F_6 \rightarrow F_7 \vdash p_4 \quad \frac{\text{ax/W}}{h_5 : \Delta_9, p_4, p_4, F_6 \rightarrow F_7 \vdash F_6} \text{ax/W}}{- : \Delta_9, p_4, F_6 \rightarrow F_7 \vdash F_6} \text{hCut} \quad \frac{\bullet h_1 : \Delta_9, F_7, p_4 \vdash p_4 \quad \frac{I}{h_5 : \Delta_9, F_7, p_4, p_4 \vdash F_8} \text{ax/W}}{- : \Delta_9, F_7, p_4 \vdash F_8} \text{hCut} \\
\frac{}{- : \Delta_9, p_4, F_6 \rightarrow F_7 \vdash F_8} \rightarrow_L
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{\bullet h_1 : (\Delta_9, F_6 \wedge F_7), p_4 \vdash p_4}{- : (\Delta_9, F_6 \wedge F_7), p_4 \vdash F_8} I \quad \frac{h_5 : \Delta_9, F_6, F_7, p_4, p_4 \vdash F_8}{\bullet h_5 : ((\Delta_9, F_6 \wedge F_7), p_4), p_4 \vdash F_8} \wedge_L \\
\text{Cut} \\
\frac{}{- : (\Delta_9, F_6 \wedge F_7), p_4 \vdash F_8} \rightarrow \\
\frac{\bullet h_1 : \Delta_9, F_6, F_7, p_4 \vdash p_4 \quad \frac{I}{h_5 : \Delta_9, F_6, F_7, p_4, p_4 \vdash F_8} \text{ax/W}}{- : \Delta_9, F_6, F_7, p_4 \vdash F_8} \text{hCut} \\
\frac{}{- : \Delta_9, p_4, F_6 \wedge F_7 \vdash F_8} \wedge_L
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{\bullet h_1 : (\Delta_9, F_6 \vee F_7), p_4 \vdash p_4}{- : (\Delta_9, F_6 \vee F_7), p_4 \vdash F_8} I \quad \frac{h_5 : \Delta_9, F_6, p_4, p_4 \vdash F_8 \quad h_5 : \Delta_9, F_7, p_4, p_4 \vdash F_8}{\bullet h_5 : ((\Delta_9, F_6 \vee F_7), p_4), p_4 \vdash F_8} \vee_L \\
\text{Cut} \\
\frac{}{- : (\Delta_9, F_6 \vee F_7), p_4 \vdash F_8} \rightarrow \\
\frac{\bullet h_1 : \Delta_9, F_6, p_4 \vdash p_4 \quad \frac{I}{h_5 : \Delta_9, F_6, p_4, p_4 \vdash F_8} \text{ax/W}}{- : \Delta_9, F_6, p_4 \vdash F_8} \text{hCut} \quad \frac{\bullet h_1 : \Delta_9, F_7, p_4 \vdash p_4 \quad \frac{I}{h_5 : \Delta_9, F_7, p_4, p_4 \vdash F_8} \text{ax/W}}{- : \Delta_9, F_7, p_4 \vdash F_8} \text{hCut} \\
\frac{}{- : \Delta_9, p_4, F_6 \vee F_7 \vdash F_8} \vee_L
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{\bullet h_1 : (\perp, \Delta_7), p_4 \vdash p_4}{- : (\perp, \Delta_7), p_4 \vdash F_6} I \quad \frac{\bullet h_5 : ((\perp, \Delta_7), p_4), p_4 \vdash F_6}{\text{Cut}} \perp_L \\
\frac{}{- : (\perp, \Delta_7), p_4 \vdash F_6} \rightarrow \\
\frac{}{- : \perp, \Delta_7, p_4 \vdash F_6} \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{\bullet h_1 : \Delta_4, p_6 \vdash p_6}{- : \Delta_4, p_6 \vdash p_6} I \quad \frac{\bullet h_5 : (\Delta_4, p_6), p_6 \vdash p_6}{\text{Cut}} I \\
\frac{}{- : \Delta_4, p_6 \vdash p_6} \rightarrow \\
\frac{}{- : \Delta_4, p_6 \vdash p_6} I \\
\frac{\bullet h_1 : (\Delta_7, p_6), p_4 \vdash p_4 \quad \frac{I}{\bullet h_5 : ((\Delta_7, p_6), p_4), p_4 \vdash p_6} I}{- : (\Delta_7, p_6), p_4 \vdash p_6} \text{Cut} \\
\frac{}{- : \Delta_7, p_4, p_6 \vdash p_6} I
\end{array}$$

- Case rule  $\top_L$

$$\frac{\frac{\frac{}{\bullet h_1 : (\top, \Delta_7), P_4 \vdash P_4} I \quad \frac{h_5 : \Delta_7, P_4, P_4 \vdash F_6}{\bullet h_5 : ((\top, \Delta_7), P_4), P_4 \vdash F_6} \top_L}{- : (\top, \Delta_7), P_4 \vdash F_6} \text{Cut}}{\frac{\frac{\bullet h_1 : \top, \Delta_7, P_4 \vdash P_4}{\text{ax/W}} \quad \frac{h_5 : \top, \Delta_7, P_4, P_4 \vdash F_6}{\text{hCut}}}{- : \top, \Delta_7, P_4 \vdash F_6} \rightarrow} \text{Cut}$$

## 6.11 Status of $\top_L$ : OK

- Case rule  $\top_R$

$$\frac{\frac{\frac{h_1 : \Delta_4 \vdash F_5}{\bullet h_1 : \top, \Delta_4 \vdash F_5} \top_L \quad \frac{}{\bullet h_6 : (\top, \Delta_4), F_5 \vdash \top} \top_R}{- : \top, \Delta_4 \vdash \top} \text{Cut}}{\frac{}{- : \top, \Delta_4 \vdash \top} \top_R} \rightarrow$$

- Case rule  $\rightarrow_R$

$$\frac{\frac{\frac{h_1 : \Delta_4 \vdash F_5}{\bullet h_1 : \top, \Delta_4 \vdash F_5} \top_L \quad \frac{h_6 : \top, \Delta_4, F_5, F_7 \vdash F_8}{\bullet h_6 : (\top, \Delta_4), F_5 \vdash F_7 \rightarrow F_8} \rightarrow_R}{- : \top, \Delta_4 \vdash F_7 \rightarrow F_8} \text{Cut}}{\frac{\frac{h_1 : \top, \Delta_4 \vdash F_5}{\text{ax/W}} \quad \frac{\bullet h_6 : \top, \Delta_4, F_5 \vdash F_7 \rightarrow F_8}{\text{hCut}}}{- : \top, \Delta_4 \vdash F_7 \rightarrow F_8} \rightarrow} \text{Cut}$$

- Case rule  $\wedge_R$

$$\frac{\frac{\frac{h_1 : \Delta_4 \vdash F_5}{\bullet h_1 : \top, \Delta_4 \vdash F_5} \top_L \quad \frac{h_6 : \top, \Delta_4, F_5 \vdash F_7 \quad h_6 : \top, \Delta_4, F_5 \vdash F_8}{\bullet h_6 : (\top, \Delta_4), F_5 \vdash F_7 \wedge F_8} \wedge_R}{- : \top, \Delta_4 \vdash F_7 \wedge F_8} \text{Cut}}{\frac{\frac{h_1 : \top, \Delta_4 \vdash F_5}{\text{ax/W}} \quad \frac{\bullet h_6 : \top, \Delta_4, F_5 \vdash F_7 \wedge F_8}{\text{hCut}}}{- : \top, \Delta_4 \vdash F_7 \wedge F_8} \rightarrow} \text{Cut}$$

- Case rule  $\vee_1$

$$\frac{\frac{\frac{h_1 : \Delta_4 \vdash F_5}{\bullet h_1 : \top, \Delta_4 \vdash F_5} \top_L \quad \frac{h_6 : \top, \Delta_4, F_5 \vdash F_7}{\bullet h_6 : (\top, \Delta_4), F_5 \vdash F_7 \vee F_8} \vee_1}{- : \top, \Delta_4 \vdash F_7 \vee F_8} \text{Cut}}{\frac{\frac{h_1 : \top, \Delta_4 \vdash F_5}{\text{ax/W}} \quad \frac{\bullet h_6 : \top, \Delta_4, F_5 \vdash F_7 \vee F_8}{\text{hCut}}}{- : \top, \Delta_4 \vdash F_7 \vee F_8} \rightarrow} \text{Cut}$$

- Case rule  $\vee_2$

$$\frac{\frac{\frac{h_1 : \Delta_4 \vdash F_5}{\bullet h_1 : \top, \Delta_4 \vdash F_5} \top_L \quad \frac{h_6 : \top, \Delta_4, F_5 \vdash F_8}{\bullet h_6 : (\top, \Delta_4), F_5 \vdash F_7 \vee F_8} \vee_2}{- : \top, \Delta_4 \vdash F_7 \vee F_8} \text{Cut}}{\frac{\frac{h_1 : \top, \Delta_4 \vdash F_5}{\text{ax/W}} \quad \frac{\bullet h_6 : \top, \Delta_4, F_5 \vdash F_7 \vee F_8}{\text{hCut}}}{- : \top, \Delta_4 \vdash F_7 \vee F_8} \rightarrow} \text{Cut}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_9, F_6 \rightarrow F_7 \vdash F_4}{\bullet h_1 : \top, \Delta_9, F_6 \rightarrow F_7 \vdash F_4} \top_L \quad \frac{h_5 : \top, \Delta_9, F_4, F_6 \rightarrow F_7 \vdash F_6 \quad h_5 : \top, \Delta_9, F_4, F_7 \vdash F_8}{\bullet h_5 : (\top, \Delta_9, F_6 \rightarrow F_7), F_4 \vdash F_8} \rightarrow_L}{- : \top, \Delta_9, F_6 \rightarrow F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_9, F_6 \rightarrow F_7 \vdash F_4}{- : \top, \Delta_9, F_6 \rightarrow F_7 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_9, F_4, F_6 \rightarrow F_7 \vdash F_8}{- : \top, \Delta_9, F_6 \rightarrow F_7 \vdash F_8} \text{ax/W}}{- : \top, \Delta_9, F_6 \rightarrow F_7 \vdash F_8} \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4 \vdash F_6 \rightarrow F_7}{\bullet h_1 : \top, \Delta_4 \vdash F_6 \rightarrow F_7} \top_L \quad \frac{h_5 : \top, \Delta_4, F_6 \rightarrow F_7 \vdash F_6 \quad h_5 : \top, \Delta_4, F_7 \vdash F_8}{\bullet h_5 : (\top, \Delta_4), F_6 \rightarrow F_7 \vdash F_8} \rightarrow_L}{- : \top, \Delta_4 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_4 \vdash F_6 \rightarrow F_7}{- : \top, \Delta_4 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_4, F_6 \rightarrow F_7 \vdash F_8}{- : \top, \Delta_4 \vdash F_8} \text{ax/W}}{- : \top, \Delta_4 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_9, F_6 \wedge F_7 \vdash F_4}{\bullet h_1 : \top, \Delta_9, F_6 \wedge F_7 \vdash F_4} \top_L \quad \frac{h_5 : \top, \Delta_9, F_4, F_6, F_7 \vdash F_8}{\bullet h_5 : (\top, \Delta_9, F_6 \wedge F_7), F_4 \vdash F_8} \wedge_L}{- : \top, \Delta_9, F_6 \wedge F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_9, F_6 \wedge F_7 \vdash F_4}{- : \top, \Delta_9, F_6 \wedge F_7 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_9, F_4, F_6 \wedge F_7 \vdash F_8}{- : \top, \Delta_9, F_6 \wedge F_7 \vdash F_8} \text{ax/W}}{- : \top, \Delta_9, F_6 \wedge F_7 \vdash F_8} \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4 \vdash F_6 \wedge F_7}{\bullet h_1 : \top, \Delta_4 \vdash F_6 \wedge F_7} \top_L \quad \frac{h_5 : \top, \Delta_4, F_6, F_7 \vdash F_8}{\bullet h_5 : (\top, \Delta_4), F_6 \wedge F_7 \vdash F_8} \wedge_L}{- : \top, \Delta_4 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_4 \vdash F_6 \wedge F_7}{- : \top, \Delta_4 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_4, F_6 \wedge F_7 \vdash F_8}{- : \top, \Delta_4 \vdash F_8} \text{ax/W}}{- : \top, \Delta_4 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_9, F_6 \vee F_7 \vdash F_4}{\bullet h_1 : \top, \Delta_9, F_6 \vee F_7 \vdash F_4} \top_L \quad \frac{h_5 : \top, \Delta_9, F_4, F_6 \vdash F_8 \quad h_5 : \top, \Delta_9, F_4, F_7 \vdash F_8}{\bullet h_5 : (\top, \Delta_9, F_6 \vee F_7), F_4 \vdash F_8} \vee_L}{- : \top, \Delta_9, F_6 \vee F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_9, F_6 \vee F_7 \vdash F_4}{- : \top, \Delta_9, F_6 \vee F_7 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_9, F_4, F_6 \vee F_7 \vdash F_8}{- : \top, \Delta_9, F_6 \vee F_7 \vdash F_8} \text{ax/W}}{- : \top, \Delta_9, F_6 \vee F_7 \vdash F_8} \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4 \vdash F_6 \vee F_7}{\bullet h_1 : \top, \Delta_4 \vdash F_6 \vee F_7} \top_L \quad \frac{h_5 : \top, \Delta_4, F_6 \vdash F_8 \quad h_5 : \top, \Delta_4, F_7 \vdash F_8}{\bullet h_5 : (\top, \Delta_4), F_6 \vee F_7 \vdash F_8} \vee_L}{- : \top, \Delta_4 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_4 \vdash F_6 \vee F_7}{- : \top, \Delta_4 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_5 : \top, \Delta_4, F_6 \vee F_7 \vdash F_8}{- : \top, \Delta_4 \vdash F_8} \text{ax/W}}{- : \top, \Delta_4 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4 \vdash \perp}{\bullet h_1 : \top, \Delta_4 \vdash \perp} \top_L \quad \frac{}{\bullet h_5 : (\top, \Delta_4), \perp \vdash F_6} \perp_L}{- : \top, \Delta_4 \vdash F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_4 \vdash \perp}{- : \top, \Delta_4 \vdash F_6} \text{ax/W} \quad \frac{\bullet h_5 : \perp, \top, \Delta_4 \vdash F_6}{- : \top, \Delta_4 \vdash F_6} \perp_L}{- : \top, \Delta_4 \vdash F_6} \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \perp, \Delta_7 \vdash F_4}{\bullet h_1 : \top, \perp, \Delta_7 \vdash F_4} \top_L \quad \frac{}{\bullet h_5 : (\top, \perp, \Delta_7), F_4 \vdash F_6} \perp_L}{- : \top, \perp, \Delta_7 \vdash F_6} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \top, \Delta_7 \vdash F_6} \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_7, P_6 \vdash F_4}{\bullet h_1 : \top, \Delta_7, P_6 \vdash F_4} \top_L \quad \frac{}{\bullet h_5 : (\top, \Delta_7, P_6), F_4 \vdash P_6} I}{- : \top, \Delta_7, P_6 \vdash P_6} \text{Cut} \\
\rightarrow \\
\frac{}{- : \top, \Delta_7, P_6 \vdash P_6} I
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4 \vdash P_6}{\bullet h_1 : \top, \Delta_4 \vdash P_6} \top_L \quad \frac{}{\bullet h_5 : (\top, \Delta_4), P_6 \vdash P_6} I}{- : \top, \Delta_4 \vdash P_6} \text{Cut} \\
\rightarrow \\
\frac{}{- : \top, \Delta_4 \vdash P_6} \text{ax/W}
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_4 \vdash F_5}{\bullet h_1 : \top, \Delta_4 \vdash F_5} \top_L \quad \frac{h_6 : \Delta_4, F_5 \vdash F_7}{\bullet h_6 : (\top, \Delta_4), F_5 \vdash F_7} \top_L}{- : \top, \Delta_4 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \top, \Delta_4 \vdash F_5}{- : \top, \Delta_4 \vdash F_5} \text{ax/W} \quad \frac{h_6 : \top, \Delta_4, F_5 \vdash F_7}{- : \top, \Delta_4 \vdash F_7} \text{ax/W}}{- : \top, \Delta_4 \vdash F_7} \text{hCut}
\end{array}$$



## 7 Cut-Elimination

### 7.1 Status of $\top_R$ : OK

- Case rule  $\top_R$

$$\frac{\frac{\overline{\bullet h_1 : \Delta_2 \vdash \top}}{\top_R} \quad \frac{\overline{\bullet h_4 : \Delta_3, \top \vdash \top}}{\top_R}}{- : \Delta_2, \Delta_3 \vdash \top} \text{Cut}}{\rightarrow} \frac{\overline{- : \Delta_2, \Delta_3 \vdash \top}}{\top_R}$$

- Case rule  $\rightarrow_R$

$$\frac{\frac{\overline{\bullet h_1 : \Delta_2 \vdash \top}}{\top_R} \quad \frac{\frac{h_4 : \top, \Delta_3, F_5 \vdash F_6}{\rightarrow_R} \quad \frac{\overline{\bullet h_4 : \Delta_3, \top \vdash F_5 \rightarrow F_6}}{\text{Cut}}}{- : \Delta_2, \Delta_3 \vdash F_5 \rightarrow F_6} \text{Cut}}{\rightarrow} \frac{\overline{\bullet h_1 : * \vdash \top}}{\top_R} \quad \frac{\overline{h_4 : \top, \Delta_2, \Delta_3, F_5 \vdash F_6}}{\text{ax/W}}}{- : \Delta_2, \Delta_3, F_5 \vdash F_6} \text{hCut}}{- : \Delta_2, \Delta_3 \vdash F_5 \rightarrow F_6} \rightarrow_R$$

- Case rule  $\wedge_R$

$$\frac{\frac{\overline{\bullet h_1 : \Delta_2 \vdash \top}}{\top_R} \quad \frac{\frac{h_4 : \top, \Delta_3 \vdash F_5} \quad \frac{h_4 : \top, \Delta_3 \vdash F_6}{\wedge_R}}{\overline{\bullet h_4 : \Delta_3, \top \vdash F_5 \wedge F_6}} \text{Cut}}{- : \Delta_2, \Delta_3 \vdash F_5 \wedge F_6} \text{Cut}}{\rightarrow} \frac{\overline{\bullet h_1 : * \vdash \top}}{\top_R} \quad \frac{\overline{h_4 : \top, \Delta_2, \Delta_3 \vdash F_5}}{\text{ax/W}}}{- : \Delta_2, \Delta_3 \vdash F_5} \text{hCut}}{\frac{\overline{\bullet h_1 : * \vdash \top}}{\top_R} \quad \frac{\overline{h_4 : \top, \Delta_2, \Delta_3 \vdash F_6}}{\text{ax/W}}}{- : \Delta_2, \Delta_3 \vdash F_6} \text{hCut}}{- : \Delta_2, \Delta_3 \vdash F_5 \wedge F_6} \wedge_R$$

- Case rule  $\vee_1$

$$\frac{\frac{\overline{\bullet h_1 : \Delta_2 \vdash \top}}{\top_R} \quad \frac{\frac{h_4 : \top, \Delta_3 \vdash F_5}{\vee_1} \quad \frac{\overline{\bullet h_4 : \Delta_3, \top \vdash F_5 \vee F_6}}{\text{Cut}}}{- : \Delta_2, \Delta_3 \vdash F_5 \vee F_6} \text{Cut}}{\rightarrow} \frac{\overline{\bullet h_1 : * \vdash \top}}{\top_R} \quad \frac{\overline{h_4 : \top, \Delta_2, \Delta_3 \vdash F_5}}{\text{ax/W}}}{- : \Delta_2, \Delta_3 \vdash F_5} \text{hCut}}{- : \Delta_2, \Delta_3 \vdash F_5 \vee F_6} \vee_1$$

- Case rule  $\vee_2$

$$\frac{\frac{\overline{\bullet h_1 : \Delta_2 \vdash \top}}{\top_R} \quad \frac{\frac{h_4 : \top, \Delta_3 \vdash F_6}{\vee_2} \quad \frac{\overline{\bullet h_4 : \Delta_3, \top \vdash F_5 \vee F_6}}{\text{Cut}}}{- : \Delta_2, \Delta_3 \vdash F_5 \vee F_6} \text{Cut}}{\rightarrow} \frac{\overline{\bullet h_1 : * \vdash \top}}{\top_R} \quad \frac{\overline{h_4 : \top, \Delta_2, \Delta_3 \vdash F_6}}{\text{ax/W}}}{- : \Delta_2, \Delta_3 \vdash F_6} \text{hCut}}{- : \Delta_2, \Delta_3 \vdash F_5 \vee F_6} \vee_2$$

- Case rule  $\rightarrow_L$

$$\frac{\frac{\overline{\bullet h_1 : \Delta_2 \vdash \top}}{\top_R} \quad \frac{\frac{h_3 : \top, \Delta_7, F_4 \rightarrow F_5 \vdash F_4} \quad \frac{h_3 : \top, \Delta_7, F_5 \vdash F_6}{\rightarrow_L}}{\overline{\bullet h_3 : (\Delta_7, F_4 \rightarrow F_5), \top \vdash F_6}} \text{Cut}}{- : \Delta_2, \Delta_7, F_4 \rightarrow F_5 \vdash F_6} \text{Cut}}{\rightarrow} \frac{\overline{\bullet h_1 : * \vdash \top}}{\top_R} \quad \frac{\overline{h_3 : \top, \Delta_2, \Delta_7, F_4 \rightarrow F_5 \vdash F_4}}{\text{ax/W}}}{- : \Delta_2, \Delta_7, F_4 \rightarrow F_5 \vdash F_4} \text{hCut}}{\frac{\overline{\bullet h_1 : * \vdash \top}}{\top_R} \quad \frac{\overline{h_3 : \top, \Delta_2, \Delta_7, F_5 \vdash F_6}}{\text{ax/W}}}{- : \Delta_2, \Delta_7, F_5 \vdash F_6} \text{hCut}}{- : \Delta_2, \Delta_7, F_4 \rightarrow F_5 \vdash F_6} \rightarrow_L$$

- Case rule  $\wedge_L$

$$\frac{\frac{\bullet h_1 : \Delta_2 \vdash \top}{\vdash : \Delta_2, \Delta_7, F_4 \wedge F_5 \vdash F_6} \top_R \quad \frac{h_3 : \top, \Delta_7, F_4, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, F_4 \wedge F_5), \top \vdash F_6} \wedge_L}{\vdash : \Delta_2, \Delta_7, F_4 \wedge F_5 \vdash F_6} \text{Cut}$$

$$\xrightarrow{\quad}$$

$$\frac{\bullet h_1 : * \vdash \top}{\vdash : \Delta_2, \Delta_7, F_4, F_5 \vdash F_6} \top_R \quad \frac{h_3 : \top, \Delta_2, \Delta_7, F_4, F_5 \vdash F_6}{\vdash : \Delta_2, \Delta_7, F_4 \wedge F_5 \vdash F_6} \wedge_L}{\vdash : \Delta_2, \Delta_7, F_4, F_5 \vdash F_6} \text{ax/w hCut}$$

- Case rule  $\vee_L$

$$\frac{\frac{\bullet h_1 : \Delta_2 \vdash \top}{\vdash : \Delta_2, \Delta_7, F_4 \vee F_5 \vdash F_6} \top_R \quad \frac{h_3 : \top, \Delta_7, F_4 \vdash F_6 \quad h_3 : \top, \Delta_7, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, F_4 \vee F_5), \top \vdash F_6} \vee_L}{\vdash : \Delta_2, \Delta_7, F_4 \vee F_5 \vdash F_6} \text{Cut}$$

$$\xrightarrow{\quad}$$

$$\frac{\bullet h_1 : * \vdash \top}{\vdash : \Delta_2, \Delta_7, F_4 \vdash F_6} \top_R \quad \frac{h_3 : \top, \Delta_2, \Delta_7, F_4 \vdash F_6}{\vdash : \Delta_2, \Delta_7, F_4 \vee F_5 \vdash F_6} \vee_L \quad \frac{\bullet h_1 : * \vdash \top}{\vdash : \Delta_2, \Delta_7, F_5 \vdash F_6} \top_R \quad \frac{h_3 : \top, \Delta_2, \Delta_7, F_5 \vdash F_6}{\vdash : \Delta_2, \Delta_7, F_4 \vee F_5 \vdash F_6} \vee_L}{\vdash : \Delta_2, \Delta_7, F_4 \vee F_5 \vdash F_6} \text{ax/w hCut hCut}$$

- Case rule  $\perp_L$

$$\frac{\frac{\bullet h_1 : \Delta_2 \vdash \top}{\vdash : \Delta_2, \perp, \Delta_5 \vdash F_4} \top_R \quad \frac{\bullet h_3 : (\perp, \Delta_5), \top \vdash F_4}{\vdash : \Delta_2, \perp, \Delta_5 \vdash F_4} \perp_L}{\vdash : \Delta_2, \perp, \Delta_5 \vdash F_4} \text{Cut}$$

$$\xrightarrow{\quad}$$

$$\frac{\vdash : \perp, \Delta_2, \Delta_5 \vdash F_4}{\vdash : \perp, \Delta_2, \Delta_5 \vdash F_4} \perp_L$$

- Case rule  $I$

$$\frac{\frac{\bullet h_1 : \Delta_2 \vdash \top}{\vdash : \Delta_2, \Delta_5, p_4 \vdash p_4} \top_R \quad \frac{\bullet h_3 : (\Delta_5, p_4), \top \vdash p_4}{\vdash : \Delta_2, \Delta_5, p_4 \vdash p_4} I}{\vdash : \Delta_2, \Delta_5, p_4 \vdash p_4} \text{Cut}$$

$$\xrightarrow{\quad}$$

$$\frac{\vdash : \Delta_2, \Delta_5, p_4 \vdash p_4}{\vdash : \Delta_2, \Delta_5, p_4 \vdash p_4} I$$

- Case rule  $\top_L$

$$\frac{\frac{\bullet h_1 : \Delta_2 \vdash \top}{\vdash : \Delta_2, \Delta_4 \vdash F_5} \top_R \quad \frac{h_3 : \Delta_4 \vdash F_5}{\bullet h_3 : \Delta_4, \top \vdash F_5} \top_L}{\vdash : \Delta_2, \Delta_4 \vdash F_5} \text{Cut}$$

$$\xrightarrow{\quad}$$

$$\frac{\vdash : \Delta_2, \Delta_4 \vdash F_5}{\vdash : \Delta_2, \Delta_4 \vdash F_5} \text{ax/w}$$

## 7.2 Status of $\rightarrow_R$ : OK

- Case rule  $\top_R$

$$\frac{\frac{h_1 : \Delta_2, F_6 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \rightarrow F_7} \rightarrow_R \quad \frac{\bullet h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash \top}{\vdash : \Delta_2, \Delta_5 \vdash \top} \top_R}{\vdash : \Delta_2, \Delta_5 \vdash \top} \text{Cut}$$

$$\xrightarrow{\quad}$$

$$\frac{\vdash : \Delta_2, \Delta_5 \vdash \top}{\vdash : \Delta_2, \Delta_5 \vdash \top} \top_R$$

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_6 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_5, F_9, F_6 \rightarrow F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash F_9 \rightarrow F_{10}} \rightarrow_R \\
\text{Cut} \\
\frac{}{- : \Delta_2, \Delta_5 \vdash F_9 \rightarrow F_{10}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_6 \rightarrow F_7}{- : \Delta_2, \Delta_5 \vdash F_9 \rightarrow F_{10}} \text{ ax/W} \quad \frac{h_8 : \Delta_5, F_9, F_6 \rightarrow F_7 \vdash F_{10}}{h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash F_9 \rightarrow F_{10}} \text{ ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9 \rightarrow F_{10}} \text{ hCut} \\
\rightarrow_R \\
\frac{}{- : \Delta_2, \Delta_5 \vdash F_9 \rightarrow F_{10}}
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_6 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash F_9 \quad h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash F_9 \wedge F_{10}} \wedge_R \\
\text{Cut} \\
\frac{}{- : \Delta_2, \Delta_5 \vdash F_9 \wedge F_{10}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_6 \rightarrow F_7}{- : \Delta_2, \Delta_5 \vdash F_9} \text{ ax/W} \quad \frac{h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash F_9}{h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash F_9 \wedge F_{10}} \text{ ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9 \wedge F_{10}} \text{ hCut} \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_6 \rightarrow F_7}{- : \Delta_2, \Delta_5 \vdash F_9} \text{ ax/W} \quad \frac{h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash F_{10}}{h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash F_9 \wedge F_{10}} \text{ ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9 \wedge F_{10}} \text{ hCut} \\
\wedge_R \\
\frac{}{- : \Delta_2, \Delta_5 \vdash F_9 \wedge F_{10}}
\end{array}$$

- Case rule  $\vee_1$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_6 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash F_9}{\bullet h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash F_9 \vee F_{10}} \vee_1 \\
\text{Cut} \\
\frac{}{- : \Delta_2, \Delta_5 \vdash F_9 \vee F_{10}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_6 \rightarrow F_7}{- : \Delta_2, \Delta_5 \vdash F_9} \text{ ax/W} \quad \frac{h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash F_9}{h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash F_9 \vee F_{10}} \text{ ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9 \vee F_{10}} \text{ hCut} \\
\vee_1 \\
\frac{}{- : \Delta_2, \Delta_5 \vdash F_9 \vee F_{10}}
\end{array}$$

- Case rule  $\vee_2$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_6 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash F_9 \vee F_{10}} \vee_2 \\
\text{Cut} \\
\frac{}{- : \Delta_2, \Delta_5 \vdash F_9 \vee F_{10}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_6 \rightarrow F_7}{- : \Delta_2, \Delta_5 \vdash F_{10}} \text{ ax/W} \quad \frac{h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash F_{10}}{h_8 : \Delta_5, F_6 \rightarrow F_7 \vdash F_9 \vee F_{10}} \text{ ax/W}}{- : \Delta_2, \Delta_5 \vdash F_{10}} \text{ hCut} \\
\vee_2 \\
\frac{}{- : \Delta_2, \Delta_5 \vdash F_9 \vee F_{10}}
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{h_7 : \Delta_{11}, F_5 \rightarrow F_6, F_8 \rightarrow F_9 \vdash F_8 \quad h_7 : \Delta_{11}, F_9, F_5 \rightarrow F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \rightarrow F_9), F_5 \rightarrow F_6 \vdash F_{10}} \rightarrow_L \\
\text{Cut} \\
\frac{}{- : \Delta_2, \Delta_{11}, F_8 \rightarrow F_9 \vdash F_{10}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6}{- : \Delta_{11}, \Delta_2, F_8 \rightarrow F_9 \vdash F_8} \text{ ax/W} \quad \frac{h_7 : \Delta_{11}, F_5 \rightarrow F_6, F_8 \rightarrow F_9 \vdash F_8}{h_7 : \Delta_{11}, F_9, F_5 \rightarrow F_6 \vdash F_{10}} \text{ ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \rightarrow F_9 \vdash F_{10}} \text{ hCut} \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{ ax/W} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \rightarrow F_6 \vdash F_{10}}{h_7 : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{ ax/W}}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{ hCut} \\
\rightarrow_L \\
\frac{}{- : \Delta_{11}, \Delta_2, F_8 \rightarrow F_9 \vdash F_{10}} \\
\rightarrow \\
\frac{h_1 : \Delta_2, F_7 \vdash F_8}{\bullet h_1 : \Delta_2 \vdash F_7 \rightarrow F_8} \rightarrow_R \quad \frac{h_5 : \Delta_6, F_7 \rightarrow F_8 \vdash F_7 \quad h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \rightarrow F_8 \vdash F_9} \rightarrow_L \\
\text{Cut} \\
\frac{}{- : \Delta_2, \Delta_6 \vdash F_9} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_7 \rightarrow F_8}{- : \Delta_2, \Delta_6 \vdash F_7} \text{ ax/W} \quad \frac{h_5 : \Delta_6, F_7 \rightarrow F_8 \vdash F_7}{h_5 : \Delta_6, F_7 \rightarrow F_8 \vdash F_9} \text{ ax/W}}{- : \Delta_2, \Delta_6 \vdash F_7} \text{ hCut} \\
\frac{- : \Delta_2, F_7 \vdash F_8 \quad - : \Delta_6, F_8 \vdash F_9}{- : \Delta_2, \Delta_6, F_7 \vdash F_9} \text{ sCut} \\
\text{sCut} \\
\frac{}{- : \Delta_2, \Delta_6 \vdash F_9} \\
C \\
\frac{}{- : \Delta_2, \Delta_6 \vdash F_9}
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \rightarrow F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \wedge F_9), F_5 \rightarrow F_6 \vdash F_{10}} \wedge_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \wedge F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6 \quad \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \rightarrow F_6 \vdash F_{10}}{\text{hCut}} \quad \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8, F_9 \vdash F_{10}} \wedge_L \\
\hline
- : \Delta_{11}, \Delta_2, F_8 \wedge F_9 \vdash F_{10} \quad \wedge_L
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \rightarrow F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \rightarrow F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_5 \rightarrow F_6 \vdash F_{10}} \vee_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \vee F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6 \quad \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \rightarrow F_6 \vdash F_{10}}{\text{hCut}} \quad \text{ax/W} \quad \frac{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6 \quad \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \rightarrow F_6 \vdash F_{10}}{\text{hCut}} \quad \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \vee_L}{- : \Delta_{11}, \Delta_2, F_8 \vee F_9 \vdash F_{10}} \vee_L
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{}{\bullet h_7 : (\perp, \Delta_9), F_5 \rightarrow F_6 \vdash F_8} \perp_L \\
\hline
- : \Delta_2, \perp, \Delta_9 \vdash F_8 \\
\rightarrow \\
\frac{}{- : \perp, \Delta_2, \Delta_9 \vdash F_8} \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{}{\bullet h_7 : (\Delta_9, p_8), F_5 \rightarrow F_6 \vdash p_8} I \\
\hline
- : \Delta_2, \Delta_9, p_8 \vdash p_8 \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_9, p_8 \vdash p_8} I
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6} \rightarrow_R \quad \frac{h_7 : \Delta_9, F_5 \rightarrow F_6 \vdash F_8}{\bullet h_7 : (\top, \Delta_9), F_5 \rightarrow F_6 \vdash F_8} \top_L \\
\hline
- : \Delta_2, \top, \Delta_9 \vdash F_8 \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \rightarrow F_6 \quad \text{ax/W} \quad \frac{h_7 : \top, \Delta_9, F_5 \rightarrow F_6 \vdash F_8}{\text{hCut}} \quad \text{ax/W}}{- : \top, \Delta_2, \Delta_9 \vdash F_8} \text{hCut}
\end{array}$$

### 7.3 Status of $\wedge_R$ : OK

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \wedge F_7} \wedge_R \quad \frac{}{\bullet h_8 : \Delta_5, F_6 \wedge F_7 \vdash \top} \top_R \\
\hline
- : \Delta_2, \Delta_5 \vdash \top \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_5 \vdash \top} \top_R
\end{array}$$

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_5, F_9, F_6 \wedge F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_9 \rightarrow F_{10}} \rightarrow_R \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \rightarrow F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_6 \wedge F_7}{- : \Delta_2, \Delta_5 \vdash F_9} \text{ ax/W} \quad \frac{h_8 : \Delta_5, F_9, F_6 \wedge F_7 \vdash F_{10}}{h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_9 \rightarrow F_{10}} \text{ ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9 \rightarrow F_{10}} \text{ hCut} \\
\rightarrow_R \\
- : \Delta_2, \Delta_5 \vdash F_9 \rightarrow F_{10}
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_9 \quad h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_9 \wedge F_{10}} \wedge_R \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \wedge F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_6 \wedge F_7}{- : \Delta_2, \Delta_5 \vdash F_9} \text{ ax/W} \quad \frac{h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_9}{h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_9 \wedge F_{10}} \text{ ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9 \wedge F_{10}} \text{ hCut} \\
\wedge_R \\
- : \Delta_2, \Delta_5 \vdash F_9 \wedge F_{10}
\end{array}$$

- Case rule  $\vee_1$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_9}{\bullet h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_9 \vee F_{10}} \vee_1 \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \vee F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_6 \wedge F_7}{- : \Delta_2, \Delta_5 \vdash F_9} \text{ ax/W} \quad \frac{h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_9}{h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_9 \vee F_{10}} \text{ ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9 \vee F_{10}} \text{ hCut} \\
\vee_1 \\
- : \Delta_2, \Delta_5 \vdash F_9 \vee F_{10}
\end{array}$$

- Case rule  $\vee_2$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_9 \vee F_{10}} \vee_2 \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \vee F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_6 \wedge F_7}{- : \Delta_2, \Delta_5 \vdash F_{10}} \text{ ax/W} \quad \frac{h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_{10}}{h_8 : \Delta_5, F_6 \wedge F_7 \vdash F_9 \vee F_{10}} \text{ ax/W}}{- : \Delta_2, \Delta_5 \vdash F_{10}} \text{ hCut} \\
\vee_2 \\
- : \Delta_2, \Delta_5 \vdash F_9 \vee F_{10}
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{h_7 : \Delta_{11}, F_8 \rightarrow F_9, F_5 \wedge F_6 \vdash F_8 \quad h_7 : \Delta_{11}, F_9, F_5 \wedge F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \rightarrow F_9), F_5 \wedge F_6 \vdash F_{10}} \rightarrow_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \rightarrow F_9 \vdash F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6}{- : \Delta_{11}, \Delta_2, F_8 \rightarrow F_9 \vdash F_8} \text{ ax/W} \quad \frac{h_7 : \Delta_{11}, F_8 \rightarrow F_9, F_5 \wedge F_6 \vdash F_8}{h_7 : \Delta_{11}, F_9, F_5 \wedge F_6 \vdash F_{10}} \text{ ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \rightarrow F_9 \vdash F_{10}} \text{ hCut} \\
\rightarrow_L \\
- : \Delta_{11}, \Delta_2, F_8 \rightarrow F_9 \vdash F_{10}
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \wedge F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \wedge F_9), F_5 \wedge F_6 \vdash F_{10}} \wedge_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \wedge F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6}{- : \Delta_{11}, \Delta_2, F_8, F_9 \vdash F_{10}} \text{ ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \wedge F_6 \vdash F_{10}}{h_7 : (\Delta_{11}, F_8 \wedge F_9), F_5 \wedge F_6 \vdash F_{10}} \text{ ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \wedge F_9 \vdash F_{10}} \text{ hCut} \\
\wedge_L \\
- : \Delta_{11}, \Delta_2, F_8 \wedge F_9 \vdash F_{10}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_7 \quad h_1 : \Delta_2 \vdash F_8}{\bullet h_1 : \Delta_2 \vdash F_7 \wedge F_8} \wedge_R \quad \frac{h_5 : \Delta_6, F_7, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \wedge F_8 \vdash F_9} \wedge_L \\
\hline
- : \Delta_2, \Delta_6 \vdash F_9 \\
\rightarrow \\
\frac{- : \Delta_2 \vdash F_8 \quad \text{ax/W} \quad - : \Delta_6, \Delta_6, F_7, F_8 \vdash F_9 \quad \text{ax/W}}{- : \Delta_2, \Delta_6, \Delta_6, F_7 \vdash F_9} \text{sCut} \\
\hline
\frac{- : \Delta_2 \vdash F_7 \quad \text{ax/W} \quad - : \Delta_2, \Delta_6, \Delta_6, F_7 \vdash F_9}{- : \Delta_2, \Delta_2, \Delta_6, \Delta_6 \vdash F_9} \text{sCut} \\
\hline
- : \Delta_2, \Delta_6 \vdash F_9 \quad C
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \wedge F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \wedge F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_5 \wedge F_6 \vdash F_{10}} \vee_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \vee F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6 \quad \text{ax/W} \quad h_7 : \Delta_{11}, F_8, F_5 \wedge F_6 \vdash F_{10} \quad \text{ax/W} \quad \bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6 \quad \text{ax/W} \quad h_7 : \Delta_{11}, F_9, F_5 \wedge F_6 \vdash F_{10} \quad \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10} \quad \text{hCut} \quad - : \Delta_{11}, \Delta_2, F_9 \vdash F_{10} \quad \text{hCut}} \\
\hline
- : \Delta_{11}, \Delta_2, F_8 \vee F_9 \vdash F_{10} \quad \vee_L
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{}{\bullet h_7 : (\perp, \Delta_9), F_5 \wedge F_6 \vdash F_8} \perp_L \\
\hline
- : \Delta_2, \perp, \Delta_9 \vdash F_8 \\
\rightarrow \\
- : \perp, \Delta_2, \Delta_9 \vdash F_8 \quad \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{}{\bullet h_7 : (\Delta_9, p_8), F_5 \wedge F_6 \vdash p_8} I \\
\hline
- : \Delta_2, \Delta_9, p_8 \vdash p_8 \\
\rightarrow \\
- : \Delta_2, \Delta_9, p_8 \vdash p_8 \quad I
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6} \wedge_R \quad \frac{h_7 : \Delta_9, F_5 \wedge F_6 \vdash F_8}{\bullet h_7 : (\top, \Delta_9), F_5 \wedge F_6 \vdash F_8} \top_L \\
\hline
- : \Delta_2, \top, \Delta_9 \vdash F_8 \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \wedge F_6 \quad \text{ax/W} \quad h_7 : \top, \Delta_9, F_5 \wedge F_6 \vdash F_8 \quad \text{ax/W}}{- : \top, \Delta_2, \Delta_9 \vdash F_8} \text{hCut}
\end{array}$$

## 7.4 Status of $\vee_1$ : OK

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_6 \vee F_7} \vee_1 \quad \frac{}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash \top} \top_R \\
\hline
- : \Delta_2, \Delta_5 \vdash \top \\
\rightarrow \\
- : \Delta_2, \Delta_5 \vdash \top \quad \top_R
\end{array}$$

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_6 \vee F_7} \vee_1 \quad \frac{h_8 : \Delta_5, F_9, F_6 \vee F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \rightarrow F_{10}} \rightarrow_R \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \rightarrow F_{10} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_6 \vee F_7 \quad \text{ax/W} \quad h_8 : \Delta_5, F_9, F_6 \vee F_7 \vdash F_{10} \quad \text{ax/W}}{- : \Delta_2, \Delta_5, F_9 \vdash F_{10}} \text{hCut} \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \rightarrow F_{10} \rightarrow_R
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_6 \vee F_7} \vee_1 \quad \frac{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \quad h_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \wedge F_{10}} \wedge_R \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \wedge F_{10} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_6 \vee F_7 \quad \text{ax/W} \quad h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \quad \text{ax/W} \quad \bullet h_1 : \Delta_2 \vdash F_6 \vee F_7 \quad \text{ax/W} \quad h_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10} \quad \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9 \wedge F_{10}} \text{hCut}
\end{array}$$

- Case rule  $\vee_1$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_6 \vee F_7} \vee_1 \quad \frac{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \vee F_{10}} \vee_1 \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \vee F_{10} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_6 \vee F_7 \quad \text{ax/W} \quad h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \quad \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \vee F_{10} \vee_1
\end{array}$$

- Case rule  $\vee_2$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_6 \vee F_7} \vee_1 \quad \frac{h_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \vee F_{10}} \vee_2 \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \vee F_{10} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_6 \vee F_7 \quad \text{ax/W} \quad h_8 : \Delta_5, F_6 \vee F_7 \vdash F_{10} \quad \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_{10}} \text{hCut} \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \vee F_{10} \vee_2
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_1 \quad \frac{h_7 : \Delta_{11}, F_8 \rightarrow F_9, F_5 \vee F_6 \vdash F_8 \quad h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \rightarrow F_9), F_5 \vee F_6 \vdash F_{10}} \rightarrow_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \rightarrow F_9 \vdash F_{10} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6 \quad \text{ax/W} \quad h_7 : \Delta_{11}, F_8 \rightarrow F_9, F_5 \vee F_6 \vdash F_8 \quad \text{ax/W} \quad \bullet h_1 : \Delta_2 \vdash F_5 \vee F_6 \quad \text{ax/W} \quad h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10} \quad \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \rightarrow F_9 \vdash F_8} \text{hCut} \\
\hline
- : \Delta_{11}, \Delta_2, F_8 \rightarrow F_9 \vdash F_{10} \rightarrow_L
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_1 \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \wedge F_9), F_5 \vee F_6 \vdash F_{10}} \wedge_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \wedge F_9 \vdash F_{10} \\
\hline
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6 \quad \text{ax/W} \quad h_7 : \Delta_{11}, F_8, F_9, F_5 \vee F_6 \vdash F_{10} \quad \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8, F_9 \vdash F_{10}} \text{hCut} \\
\hline
- : \Delta_{11}, \Delta_2, F_8 \wedge F_9 \vdash F_{10} \wedge_L
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_1 \quad \frac{\frac{h_7 : \Delta_{11}, F_8, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_5 \vee F_6 \vdash F_{10}} \vee_L \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_5 \vee F_6 \vdash F_{10}} \vee_L}{- : \Delta_2, \Delta_{11}, F_8 \vee F_9 \vdash F_{10}} \text{Cut}}{\rightarrow} \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5 \vee F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_5 \vee F_6 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{hCut} \quad \frac{\frac{h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_5 \vee F_6 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \vee F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_5 \vee F_6 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, F_8 \vee F_9 \vdash F_{10}} \vee_L}{- : \Delta_{11}, \Delta_2, F_8 \vee F_9 \vdash F_{10}} \rightarrow} \\
\frac{\frac{h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_7 \vee F_8} \vee_1 \quad \frac{\frac{h_5 : \Delta_6, F_7 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \vee F_8 \vdash F_9} \vee_L \quad \frac{h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \vee F_8 \vdash F_9} \text{Cut}}{- : \Delta_2, \Delta_6 \vdash F_9} \text{Cut}}{\rightarrow} \\
\frac{\frac{- : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_7 \vee F_8} \text{ax/W} \quad \frac{- : \Delta_6, F_7 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \vee F_8 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_6 \vdash F_9} \text{sCut}
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_1 \quad \frac{\bullet h_7 : (\perp, \Delta_9), F_5 \vee F_6 \vdash F_8}{\bullet h_7 : (\perp, \Delta_9), F_5 \vee F_6 \vdash F_8} \perp_L}{- : \Delta_2, \perp, \Delta_9 \vdash F_8} \text{Cut}}{\rightarrow} \\
\frac{- : \perp, \Delta_2, \Delta_9 \vdash F_8}{\bullet h_7 : (\perp, \Delta_9), F_5 \vee F_6 \vdash F_8} \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_1 \quad \frac{\bullet h_7 : (\Delta_9, p_8), F_5 \vee F_6 \vdash p_8}{\bullet h_7 : (\Delta_9, p_8), F_5 \vee F_6 \vdash p_8} I}{- : \Delta_2, \Delta_9, p_8 \vdash p_8} \text{Cut}}{\rightarrow} \\
\frac{- : \Delta_2, \Delta_9, p_8 \vdash p_8}{\bullet h_7 : (\Delta_9, p_8), F_5 \vee F_6 \vdash p_8} I
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_1 \quad \frac{h_7 : \Delta_9, F_5 \vee F_6 \vdash F_8}{\bullet h_7 : (\top, \Delta_9), F_5 \vee F_6 \vdash F_8} \top_L}{- : \Delta_2, \top, \Delta_9 \vdash F_8} \text{Cut}}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \vee F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \text{ax/W} \quad \frac{h_7 : \top, \Delta_9, F_5 \vee F_6 \vdash F_8}{\bullet h_7 : (\top, \Delta_9), F_5 \vee F_6 \vdash F_8} \text{ax/W}}{- : \top, \Delta_2, \Delta_9 \vdash F_8} \text{hCut}
\end{array}$$

## 7.5 Status of $\vee_2$ : OK

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \vee F_7} \vee_2 \quad \frac{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash \top}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash \top} \top_R}{- : \Delta_2, \Delta_5 \vdash \top} \text{Cut}}{\rightarrow} \\
\frac{- : \Delta_2, \Delta_5 \vdash \top}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash \top} \top_R
\end{array}$$

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \vee F_7} \vee_2 \quad \frac{h_8 : \Delta_5, F_9, F_6 \vee F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \rightarrow F_{10}} \rightarrow_R}{- : \Delta_2, \Delta_5 \vdash F_9 \rightarrow F_{10}} \text{Cut}}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \vee F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \vee F_7} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_9, F_6 \vee F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \vee F_7 \vdash F_9 \rightarrow F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_5, F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_2, \Delta_5 \vdash F_9 \rightarrow F_{10}} \rightarrow_R
\end{array}$$





$$\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_8}{\bullet h_1 : \Delta_2 \vdash F_7 \vee F_8} \vee_2 \quad \frac{\frac{h_5 : \Delta_6, F_7 \vdash F_9 \quad h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \vee F_8 \vdash F_9} \vee_L}{- : \Delta_2, \Delta_6 \vdash F_9} \text{Cut}}{\rightarrow} \quad \frac{\frac{- : \Delta_2 \vdash F_8}{\text{ax/W}} \quad \frac{- : \Delta_6, F_8 \vdash F_9}{\text{ax/W}}}{- : \Delta_2, \Delta_6 \vdash F_9} \text{sCut}}$$

- Case rule  $\perp_L$

$$\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_2 \quad \frac{\bullet h_7 : (\perp, \Delta_9), F_5 \vee F_6 \vdash F_8}{- : \Delta_2, \perp, \Delta_9 \vdash F_8} \text{Cut}}{\rightarrow} \quad \frac{- : \perp, \Delta_2, \Delta_9 \vdash F_8}{\perp_L}}$$

- Case rule  $I$

$$\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_2 \quad \frac{\bullet h_7 : (\Delta_9, p_8), F_5 \vee F_6 \vdash p_8}{- : \Delta_2, \Delta_9, p_8 \vdash p_8} I}{\rightarrow} \quad \frac{- : \Delta_2, \Delta_9, p_8 \vdash p_8}{I} \text{Cut}$$

- Case rule  $\top_L$

$$\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6} \vee_2 \quad \frac{\frac{h_7 : \Delta_9, F_5 \vee F_6 \vdash F_8}{\bullet h_7 : (\top, \Delta_9), F_5 \vee F_6 \vdash F_8} \top_L}{- : \Delta_2, \top, \Delta_9 \vdash F_8} \text{Cut}}{\rightarrow} \quad \frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \vee F_6}{\text{ax/W}} \quad \frac{h_7 : \top, \Delta_9, F_5 \vee F_6 \vdash F_8}{\text{hCut}}}{- : \top, \Delta_2, \Delta_9 \vdash F_8} \text{ax/W}$$

## 7.6 Status of $\rightarrow_L$ : OK

- Case rule  $\top_R$

$$\frac{\frac{\frac{h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_7} \rightarrow_L \quad \frac{\bullet h_8 : \Delta_6, F_7 \vdash \top}{- : (\Delta_2, F_3 \rightarrow F_4), \Delta_6 \vdash \top} \top_R}{\rightarrow} \quad \frac{- : \Delta_2, \Delta_6, F_3 \rightarrow F_4 \vdash \top}{\top_R} \text{Cut}$$

- Case rule  $\rightarrow_R$

$$\frac{\frac{\frac{h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_7} \rightarrow_L \quad \frac{\frac{h_8 : \Delta_6, F_7, F_9 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \rightarrow F_{10}} \rightarrow_R}{- : (\Delta_2, F_3 \rightarrow F_4), \Delta_6 \vdash F_9 \rightarrow F_{10}} \text{Cut}}{\rightarrow} \quad \frac{\frac{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_7}{\text{ax/W}} \quad \frac{h_8 : \Delta_6, F_7, F_9 \vdash F_{10}}{\text{hCut}}}{- : \Delta_2, \Delta_6, F_9, F_3 \rightarrow F_4 \vdash F_{10}} \text{ax/W} \\ \frac{- : \Delta_2, \Delta_6, F_3 \rightarrow F_4 \vdash F_9 \rightarrow F_{10}}{\rightarrow_R} \rightarrow_R$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_7 \quad \rightarrow_L \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9 \quad h_8 : \Delta_6, F_7 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \wedge F_{10}} \wedge_R}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_7} \text{Cut} \\
\frac{}{- : (\Delta_2, F_3 \rightarrow F_4), \Delta_6 \vdash F_9 \wedge F_{10}}{\rightarrow} \\
\frac{}{- : \Delta_2, \Delta_6, F_3 \rightarrow F_4 \vdash F_3} \text{ax/W} \quad \frac{h_1 : \Delta_2, F_4 \vdash F_7 \quad \text{ax/W} \quad \bullet h_8 : \Delta_6, F_7 \vdash F_9 \wedge F_{10}}{- : \Delta_2, \Delta_6, F_4 \vdash F_9 \wedge F_{10}} \text{hCut} \\
\frac{}{- : \Delta_2, \Delta_6, F_3 \rightarrow F_4 \vdash F_9 \wedge F_{10}}{\rightarrow_L}
\end{array}$$

- Case rule  $\vee_1$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_7 \quad \rightarrow_L \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \vee F_{10}} \vee_1}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_7} \text{Cut} \\
\frac{}{- : (\Delta_2, F_3 \rightarrow F_4), \Delta_6 \vdash F_9 \vee F_{10}}{\rightarrow} \\
\frac{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_7 \quad \text{ax/W} \quad h_8 : \Delta_6, F_7 \vdash F_9 \quad \text{ax/W}}{- : \Delta_2, \Delta_6, F_3 \rightarrow F_4 \vdash F_9} \text{hCut} \\
\frac{}{- : \Delta_2, \Delta_6, F_3 \rightarrow F_4 \vdash F_9 \vee F_{10}}{\vee_1} \\
\frac{}{- : \Delta_2, \Delta_6, F_3 \rightarrow F_4 \vdash F_9 \vee F_{10}}{\rightarrow_L}
\end{array}$$

- Case rule  $\vee_2$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_7 \quad \rightarrow_L \quad \frac{h_8 : \Delta_6, F_7 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \vee F_{10}} \vee_2}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_7} \text{Cut} \\
\frac{}{- : (\Delta_2, F_3 \rightarrow F_4), \Delta_6 \vdash F_9 \vee F_{10}}{\rightarrow} \\
\frac{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_7 \quad \text{ax/W} \quad h_8 : \Delta_6, F_7 \vdash F_{10} \quad \text{ax/W}}{- : \Delta_2, \Delta_6, F_3 \rightarrow F_4 \vdash F_{10}} \text{hCut} \\
\frac{}{- : \Delta_2, \Delta_6, F_3 \rightarrow F_4 \vdash F_9 \vee F_{10}}{\vee_2} \\
\frac{}{- : \Delta_2, \Delta_6, F_3 \rightarrow F_4 \vdash F_9 \vee F_{10}}{\rightarrow_L}
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_6 \quad \rightarrow_L \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \rightarrow F_9 \vdash F_8 \quad h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \rightarrow F_9), F_6 \vdash F_{10}} \rightarrow_L}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_6} \text{Cut} \\
\frac{}{- : (\Delta_2, F_3 \rightarrow F_4), \Delta_{11}, F_8 \rightarrow F_9 \vdash F_{10}}{\rightarrow} \\
\frac{}{- : \Delta_{11}, \Delta_2, F_3 \rightarrow F_4, F_8 \rightarrow F_9 \vdash F_3} \text{ax/W} \quad \frac{h_1 : \Delta_2, F_4 \vdash F_6 \quad \text{ax/W} \quad \bullet h_7 : \Delta_{11}, F_6, F_8 \rightarrow F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_4, F_8 \rightarrow F_9 \vdash F_{10}} \text{hCut} \\
\frac{}{- : \Delta_{11}, \Delta_2, F_3 \rightarrow F_4, F_8 \rightarrow F_9 \vdash F_{10}}{\rightarrow_L} \\
\frac{h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_8 \rightarrow F_9 \quad \rightarrow_L \quad \frac{h_6 : \Delta_7, F_8 \rightarrow F_9 \vdash F_8 \quad h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \rightarrow F_9 \vdash F_{10}} \rightarrow_L}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_8 \rightarrow F_9} \text{Cut} \\
\frac{}{- : (\Delta_2, F_3 \rightarrow F_4), \Delta_7 \vdash F_{10}}{\rightarrow} \\
\frac{}{- : \Delta_2, \Delta_7, F_3 \rightarrow F_4 \vdash F_3} \text{ax/W} \quad \frac{h_1 : \Delta_2, F_4 \vdash F_8 \rightarrow F_9 \quad \text{ax/W} \quad \bullet h_6 : \Delta_7, F_8 \rightarrow F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_4 \vdash F_{10}} \text{hCut} \\
\frac{}{- : \Delta_2, \Delta_7, F_3 \rightarrow F_4 \vdash F_{10}}{\rightarrow_L}
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_6 \quad \rightarrow_L \quad \frac{h_7 : \Delta_{11}, F_6, F_8, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \wedge F_9), F_6 \vdash F_{10}} \wedge_L}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_6} \text{Cut} \\
\frac{}{- : (\Delta_2, F_3 \rightarrow F_4), \Delta_{11}, F_8 \wedge F_9 \vdash F_{10}}{\rightarrow} \\
\frac{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_6 \quad \text{ax/W} \quad h_7 : \Delta_{11}, F_6, F_8, F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8, F_9, F_3 \rightarrow F_4 \vdash F_{10}} \text{hCut} \\
\frac{}{- : \Delta_{11}, \Delta_2, F_3 \rightarrow F_4, F_8 \wedge F_9 \vdash F_{10}}{\wedge_L}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_8 \wedge F_9 \rightarrow_L \quad \frac{h_6 : \Delta_7, F_8, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \wedge F_9 \vdash F_{10}} \wedge_L}{\frac{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_8 \wedge F_9}{- : (\Delta_2, F_3 \rightarrow F_4), \Delta_7 \vdash F_{10}} \rightarrow_L \quad \text{Cut}}{\rightarrow} \\
\frac{\frac{- : \Delta_2, \Delta_7, F_3 \rightarrow F_4 \vdash F_3}{- : \Delta_2, \Delta_7, F_3 \rightarrow F_4 \vdash F_{10}} \text{ax/W} \quad \frac{\frac{h_1 : \Delta_2, F_4 \vdash F_8 \wedge F_9}{- : \Delta_2, \Delta_7, F_4 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \wedge F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_4 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_7, F_3 \rightarrow F_4 \vdash F_{10}} \text{hCut}}{\rightarrow_L}
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_6 \rightarrow_L \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \vdash F_{10} \quad h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_6 \vdash F_{10}} \vee_L}{\frac{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_6}{- : (\Delta_2, F_3 \rightarrow F_4), \Delta_{11}, F_8 \vee F_9 \vdash F_{10}} \text{Cut}}{\rightarrow} \\
\frac{\frac{- : \Delta_{11}, \Delta_2, F_3 \rightarrow F_4, F_8 \vee F_9 \vdash F_3}{- : \Delta_{11}, \Delta_2, F_3 \rightarrow F_4, F_8 \vee F_9 \vdash F_{10}} \text{ax/W} \quad \frac{\frac{h_1 : \Delta_2, F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_4, F_8 \vee F_9 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_{11}, F_6, F_8 \vee F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_4, F_8 \vee F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_3 \rightarrow F_4, F_8 \vee F_9 \vdash F_{10}} \text{hCut}}{\rightarrow_L} \\
\frac{h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_8 \vee F_9 \rightarrow_L \quad \frac{h_6 : \Delta_7, F_8 \vdash F_{10} \quad h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \vee F_9 \vdash F_{10}} \vee_L}{\frac{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_8 \vee F_9}{- : (\Delta_2, F_3 \rightarrow F_4), \Delta_7 \vdash F_{10}} \text{Cut}}{\rightarrow} \\
\frac{\frac{- : \Delta_2, \Delta_7, F_3 \rightarrow F_4 \vdash F_3}{- : \Delta_2, \Delta_7, F_3 \rightarrow F_4 \vdash F_{10}} \text{ax/W} \quad \frac{\frac{h_1 : \Delta_2, F_4 \vdash F_8 \vee F_9}{- : \Delta_2, \Delta_7, F_4 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \vee F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_4 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_7, F_3 \rightarrow F_4 \vdash F_{10}} \text{hCut}}{\rightarrow_L}
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash \perp \rightarrow_L \quad \frac{\bullet h_6 : \Delta_7, \perp \vdash F_8}{\bullet h_6 : \Delta_7, \perp \vdash F_8} \perp_L}{\frac{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash \perp}{- : (\Delta_2, F_3 \rightarrow F_4), \Delta_7 \vdash F_8} \text{Cut}}{\rightarrow} \\
\frac{\frac{- : \Delta_2, \Delta_7, F_3 \rightarrow F_4 \vdash F_3}{- : \Delta_2, \Delta_7, F_3 \rightarrow F_4 \vdash F_8} \text{ax/W} \quad \frac{\frac{h_1 : \Delta_2, F_4 \vdash \perp}{- : \Delta_2, \Delta_7, F_4 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_6 : \perp, \Delta_7 \vdash F_8}{- : \Delta_2, \Delta_7, F_4 \vdash F_8} \perp_L}{- : \Delta_2, \Delta_7, F_3 \rightarrow F_4 \vdash F_8} \text{hCut}}{\rightarrow_L} \\
\frac{h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_6 \rightarrow_L \quad \frac{\bullet h_7 : (\perp, \Delta_9), F_6 \vdash F_8}{\bullet h_7 : (\perp, \Delta_9), F_6 \vdash F_8} \perp_L}{\frac{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_6}{- : (\Delta_2, F_3 \rightarrow F_4), \perp, \Delta_9 \vdash F_8} \text{Cut}}{\rightarrow} \\
\frac{- : \perp, \Delta_2, \Delta_9, F_3 \rightarrow F_4 \vdash F_8}{- : \perp, \Delta_2, \Delta_9, F_3 \rightarrow F_4 \vdash F_8} \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_6 \rightarrow_L \quad \frac{\bullet h_7 : (\Delta_9, p_8), F_6 \vdash p_8}{\bullet h_7 : (\Delta_9, p_8), F_6 \vdash p_8} I}{\frac{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_6}{- : (\Delta_2, F_3 \rightarrow F_4), \Delta_9, p_8 \vdash p_8} \text{Cut}}{\rightarrow} \\
\frac{- : \Delta_2, \Delta_9, p_8, F_3 \rightarrow F_4 \vdash p_8}{- : \Delta_2, \Delta_9, p_8, F_3 \rightarrow F_4 \vdash p_8} I \\
\frac{h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash p_8 \rightarrow_L \quad \frac{\bullet h_6 : \Delta_7, p_8 \vdash p_8}{\bullet h_6 : \Delta_7, p_8 \vdash p_8} I}{\frac{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash p_8}{- : (\Delta_2, F_3 \rightarrow F_4), \Delta_7 \vdash p_8} \text{Cut}}{\rightarrow} \\
\frac{- : \Delta_2, \Delta_7, F_3 \rightarrow F_4 \vdash p_8}{- : \Delta_2, \Delta_7, F_3 \rightarrow F_4 \vdash p_8} \text{ax/W}
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash \top}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash \top} \rightarrow_L \quad \frac{h_6 : \Delta_7 \vdash F_8}{\bullet h_6 : \Delta_7, \top \vdash F_8} \top_L}{\frac{- : (\Delta_2, F_3 \rightarrow F_4), \Delta_7 \vdash F_8}{\rightarrow} \text{Cut}} \\
\frac{}{- : \Delta_2, \Delta_7, F_3 \rightarrow F_4 \vdash F_8} \text{ax/W} \\
\frac{\frac{h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_6} \rightarrow_L \quad \frac{h_7 : \Delta_9, F_6 \vdash F_8}{\bullet h_7 : (\top, \Delta_9), F_6 \vdash F_8} \top_L}{\frac{- : (\Delta_2, F_3 \rightarrow F_4), \top, \Delta_9 \vdash F_8}{\rightarrow} \text{Cut}} \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash F_6}{\text{ax/W}} \quad \frac{h_7 : \top, \Delta_9, F_6 \vdash F_8}{\text{hCut}}}{- : \top, \Delta_2, \Delta_9, F_3 \rightarrow F_4 \vdash F_8} \text{hCut}
\end{array}$$

## 7.7 Status of $\wedge_L$ : OK

- Case rule  $\top_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash F_7} \wedge_L \quad \frac{}{\bullet h_8 : \Delta_6, F_7 \vdash \top} \top_R}{\frac{- : (\Delta_2, F_3 \wedge F_4), \Delta_6 \vdash \top}{\rightarrow} \text{Cut}} \\
\frac{}{- : \Delta_2, \Delta_6, F_3 \wedge F_4 \vdash \top} \top_R
\end{array}$$

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash F_7} \wedge_L \quad \frac{h_8 : \Delta_6, F_7, F_9 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \rightarrow F_{10}} \rightarrow_R}{\frac{- : (\Delta_2, F_3 \wedge F_4), \Delta_6 \vdash F_9 \rightarrow F_{10}}{\rightarrow} \text{Cut}} \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash F_7}{\text{ax/W}} \quad \frac{h_8 : \Delta_6, F_7, F_9 \vdash F_{10}}{\text{hCut}}}{\frac{- : \Delta_2, \Delta_6, F_9, F_3 \wedge F_4 \vdash F_{10}}{\rightarrow_R}} \text{hCut} \\
\frac{}{- : \Delta_2, \Delta_6, F_3 \wedge F_4 \vdash F_9 \rightarrow F_{10}} \rightarrow_R
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash F_7} \wedge_L \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9 \quad h_8 : \Delta_6, F_7 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \wedge F_{10}} \wedge_R}{\frac{- : (\Delta_2, F_3 \wedge F_4), \Delta_6 \vdash F_9 \wedge F_{10}}{\rightarrow} \text{Cut}} \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_7}{\text{ax/W}} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9 \wedge F_{10}}{\text{hCut}}}{\frac{- : \Delta_2, \Delta_6, F_3, F_4 \vdash F_9 \wedge F_{10}}{\wedge_L}} \wedge_L \\
\frac{}{- : \Delta_2, \Delta_6, F_3 \wedge F_4 \vdash F_9 \wedge F_{10}} \wedge_L
\end{array}$$

- Case rule  $\vee_1$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash F_7} \wedge_L \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \vee F_{10}} \vee_1}{\frac{- : (\Delta_2, F_3 \wedge F_4), \Delta_6 \vdash F_9 \vee F_{10}}{\rightarrow} \text{Cut}} \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash F_7}{\text{ax/W}} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9}{\text{hCut}}}{\frac{- : \Delta_2, \Delta_6, F_3 \wedge F_4 \vdash F_9}{\vee_1}} \vee_1 \\
\frac{}{- : \Delta_2, \Delta_6, F_3 \wedge F_4 \vdash F_9 \vee F_{10}} \vee_1
\end{array}$$

- Case rule  $\vee_2$



- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash \perp}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash \perp} \wedge_L \quad \frac{}{\bullet h_6 : \Delta_7, \perp \vdash F_8} \perp_L}{- : (\Delta_2, F_3 \wedge F_4), \Delta_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash \perp}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash \perp} \text{ax/W} \quad \frac{\bullet h_6 : \perp, \Delta_7 \vdash F_8}{- : \Delta_2, \Delta_7, F_3, F_4 \vdash F_8} \perp_L}{- : \Delta_2, \Delta_7, F_3 \wedge F_4 \vdash F_8} \wedge_L \text{hCut} \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash F_6} \wedge_L \quad \frac{}{\bullet h_7 : (\perp, \Delta_9), F_6 \vdash F_8} \perp_L}{- : (\Delta_2, F_3 \wedge F_4), \perp, \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_2, \Delta_9, F_3 \wedge F_4 \vdash F_8} \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash F_6} \wedge_L \quad \frac{}{\bullet h_7 : (\Delta_9, p_8), F_6 \vdash p_8} I}{- : (\Delta_2, F_3 \wedge F_4), \Delta_9, p_8 \vdash p_8} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_9, p_8, F_3 \wedge F_4 \vdash p_8} I \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash p_8}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash p_8} \wedge_L \quad \frac{}{\bullet h_6 : \Delta_7, p_8 \vdash p_8} I}{- : (\Delta_2, F_3 \wedge F_4), \Delta_7 \vdash p_8} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_7, F_3 \wedge F_4 \vdash p_8} \text{ax/W}
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash \top}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash \top} \wedge_L \quad \frac{h_6 : \Delta_7 \vdash F_8}{\bullet h_6 : \Delta_7, \top \vdash F_8} \top_L}{- : (\Delta_2, F_3 \wedge F_4), \Delta_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_7, F_3 \wedge F_4 \vdash F_8} \text{ax/W} \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash F_6} \wedge_L \quad \frac{h_7 : \Delta_9, F_6 \vdash F_8}{\bullet h_7 : (\top, \Delta_9), F_6 \vdash F_8} \top_L}{- : (\Delta_2, F_3 \wedge F_4), \top, \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash F_6} \text{ax/W} \quad \frac{h_7 : \top, \Delta_9, F_6 \vdash F_8}{h_7 : \top, \Delta_9, F_6 \vdash F_8} \text{ax/W}}{- : \top, \Delta_2, \Delta_9, F_3 \wedge F_4 \vdash F_8} \text{hCut}
\end{array}$$

## 7.8 Status of $\vee_L$ : OK

- Case rule  $\top_R$

$$\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_7} \vee_L \quad \frac{}{\bullet h_8 : \Delta_6, F_7 \vdash \top} \top_R}{- : (\Delta_2, F_3 \vee F_4), \Delta_6 \vdash \top} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_6, F_3 \vee F_4 \vdash \top} \top_R$$

- Case rule  $\rightarrow_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_7 \quad h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_7} \vee_L \quad \frac{h_8 : \Delta_6, F_7, F_9 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \rightarrow F_{10}} \rightarrow_R \\
\frac{}{- : (\Delta_2, F_3 \vee F_4), \Delta_6 \vdash F_9 \rightarrow F_{10}}{\text{Cut}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_7}{- : \Delta_2, \Delta_6, F_9, F_3 \vee F_4 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7, F_9 \vdash F_{10}}{h_8 : \Delta_6, F_7 \vdash F_9 \rightarrow F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_6, F_3 \vee F_4 \vdash F_9 \rightarrow F_{10}} \text{hCut} \\
\rightarrow_R \\
- : \Delta_2, \Delta_6, F_3 \vee F_4 \vdash F_9 \rightarrow F_{10}
\end{array}$$

- Case rule  $\wedge_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_7 \quad h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_7} \vee_L \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9 \quad h_8 : \Delta_6, F_7 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \wedge F_{10}} \wedge_R \\
\frac{}{- : (\Delta_2, F_3 \vee F_4), \Delta_6 \vdash F_9 \wedge F_{10}}{\text{Cut}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_7}{- : \Delta_2, \Delta_6, F_3 \vee F_4 \vdash F_9} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9}{h_8 : \Delta_6, F_7 \vdash F_9 \wedge F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_6, F_3 \vee F_4 \vdash F_9} \text{hCut} \quad \frac{\frac{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_7}{- : \Delta_2, \Delta_6, F_3 \vee F_4 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_{10}}{h_8 : \Delta_6, F_7 \vdash F_9 \wedge F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_6, F_3 \vee F_4 \vdash F_9 \wedge F_{10}} \text{hCut} \\
\wedge_R \\
- : \Delta_2, \Delta_6, F_3 \vee F_4 \vdash F_9 \wedge F_{10}
\end{array}$$

- Case rule  $\vee_1$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_7 \quad h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_7} \vee_L \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \vee F_{10}} \vee_1 \\
\frac{}{- : (\Delta_2, F_3 \vee F_4), \Delta_6 \vdash F_9 \vee F_{10}}{\text{Cut}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_7}{- : \Delta_2, \Delta_6, F_3 \vee F_4 \vdash F_9} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9}{h_8 : \Delta_6, F_7 \vdash F_9 \vee F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_6, F_3 \vee F_4 \vdash F_9} \text{hCut} \\
\vee_1 \\
- : \Delta_2, \Delta_6, F_3 \vee F_4 \vdash F_9 \vee F_{10}
\end{array}$$

- Case rule  $\vee_2$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_7 \quad h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_7} \vee_L \quad \frac{h_8 : \Delta_6, F_7 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \vee F_{10}} \vee_2 \\
\frac{}{- : (\Delta_2, F_3 \vee F_4), \Delta_6 \vdash F_9 \vee F_{10}}{\text{Cut}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_7}{- : \Delta_2, \Delta_6, F_3 \vee F_4 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_{10}}{h_8 : \Delta_6, F_7 \vdash F_9 \vee F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_6, F_3 \vee F_4 \vdash F_{10}} \text{hCut} \\
\vee_2 \\
- : \Delta_2, \Delta_6, F_3 \vee F_4 \vdash F_9 \vee F_{10}
\end{array}$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_6} \vee_L \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \rightarrow F_9 \vdash F_8 \quad h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \rightarrow F_9), F_6 \vdash F_{10}} \rightarrow_L \\
\frac{}{- : (\Delta_2, F_3 \vee F_4), \Delta_{11}, F_8 \rightarrow F_9 \vdash F_{10}}{\text{Cut}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_8 \rightarrow F_9, F_3 \vee F_4 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \rightarrow F_9 \vdash F_8}{h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \rightarrow F_9, F_3 \vee F_4 \vdash F_8} \text{hCut} \quad \frac{\frac{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_9, F_3 \vee F_4 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \rightarrow F_9, F_3 \vee F_4 \vdash F_{10}} \text{hCut} \\
\rightarrow_L \\
- : \Delta_{11}, \Delta_2, F_8 \rightarrow F_9, F_3 \vee F_4 \vdash F_{10} \\
\frac{h_1 : \Delta_2, F_3 \vdash F_8 \rightarrow F_9 \quad h_1 : \Delta_2, F_4 \vdash F_8 \rightarrow F_9}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_8 \rightarrow F_9} \vee_L \quad \frac{h_6 : \Delta_7, F_8 \rightarrow F_9 \vdash F_8 \quad h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \rightarrow F_9 \vdash F_{10}} \rightarrow_L \\
\frac{}{- : (\Delta_2, F_3 \vee F_4), \Delta_7 \vdash F_{10}}{\text{Cut}} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_8 \rightarrow F_9}{- : \Delta_2, \Delta_7, F_3 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \rightarrow F_9 \vdash F_{10}}{h_6 : \Delta_7, F_8 \rightarrow F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_7, F_3 \vdash F_{10}} \text{hCut} \quad \frac{\frac{h_1 : \Delta_2, F_4 \vdash F_8 \rightarrow F_9}{- : \Delta_2, \Delta_7, F_4 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \rightarrow F_9 \vdash F_{10}}{h_6 : \Delta_7, F_8 \rightarrow F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_7, F_4 \vdash F_{10}} \text{hCut} \\
\vee_L \\
- : \Delta_2, \Delta_7, F_3 \vee F_4 \vdash F_{10}
\end{array}$$

- Case rule  $\wedge_L$



$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_6} \vee_L \quad \frac{h_7 : \Delta_{11}, F_6, F_8, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \wedge F_9), F_6 \vdash F_{10}} \wedge_L}{- : (\Delta_2, F_3 \vee F_4), \Delta_{11}, F_8 \wedge F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vee F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_8, F_9, F_3 \vee F_4 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_6, F_8, F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \wedge F_9, F_3 \vee F_4 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \wedge F_9, F_3 \vee F_4 \vdash F_{10}} \text{hCut} \wedge_L \\
\frac{h_1 : \Delta_2, F_3 \vdash F_8 \wedge F_9 \quad h_1 : \Delta_2, F_4 \vdash F_8 \wedge F_9}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_8 \wedge F_9} \vee_L \quad \frac{h_6 : \Delta_7, F_8, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \wedge F_9 \vdash F_{10}} \wedge_L}{- : (\Delta_2, F_3 \vee F_4), \Delta_7 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_2, F_3 \vdash F_8 \wedge F_9}{- : \Delta_2, \Delta_7, F_3 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \wedge F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_3 \vee F_4 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_7, F_3 \vee F_4 \vdash F_{10}} \text{hCut} \wedge_L \\
\frac{h_1 : \Delta_2, F_4 \vdash F_8 \wedge F_9}{- : \Delta_2, \Delta_7, F_4 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \wedge F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_3 \vee F_4 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_7, F_3 \vee F_4 \vdash F_{10}} \text{hCut} \wedge_L
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_6} \vee_L \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \vdash F_{10} \quad h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \vee F_9), F_6 \vdash F_{10}} \vee_L}{- : (\Delta_2, F_3 \vee F_4), \Delta_{11}, F_8 \vee F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_2, F_3 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_3, F_8 \vee F_9 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_{11}, F_6, F_8 \vee F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_3 \vee F_4, F_8 \vee F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_3 \vee F_4, F_8 \vee F_9 \vdash F_{10}} \text{hCut} \wedge_L \\
\frac{h_1 : \Delta_2, F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_4, F_8 \vee F_9 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_{11}, F_6, F_8 \vee F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_3 \vee F_4, F_8 \vee F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_3 \vee F_4, F_8 \vee F_9 \vdash F_{10}} \text{hCut} \wedge_L \\
\frac{h_1 : \Delta_2, F_3 \vdash F_8 \vee F_9 \quad h_1 : \Delta_2, F_4 \vdash F_8 \vee F_9}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_8 \vee F_9} \vee_L \quad \frac{h_6 : \Delta_7, F_8 \vdash F_{10} \quad h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \vee F_9 \vdash F_{10}} \vee_L}{- : (\Delta_2, F_3 \vee F_4), \Delta_7 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_2, F_3 \vdash F_8 \vee F_9}{- : \Delta_2, \Delta_7, F_3 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \vee F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_3 \vee F_4 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_7, F_3 \vee F_4 \vdash F_{10}} \text{hCut} \wedge_L \\
\frac{h_1 : \Delta_2, F_4 \vdash F_8 \vee F_9}{- : \Delta_2, \Delta_7, F_4 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \vee F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_3 \vee F_4 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_7, F_3 \vee F_4 \vdash F_{10}} \text{hCut} \wedge_L
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash \perp \quad h_1 : \Delta_2, F_4 \vdash \perp}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \perp} \vee_L \quad \frac{}{\bullet h_6 : \Delta_7, \perp \vdash F_8} \perp_L}{- : (\Delta_2, F_3 \vee F_4), \Delta_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_2, F_3 \vdash \perp}{- : \Delta_2, \Delta_7, F_3 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_6 : \perp, \Delta_7 \vdash F_8}{- : \Delta_2, \Delta_7, F_3 \vee F_4 \vdash F_8} \text{ax/W}}{- : \Delta_2, \Delta_7, F_3 \vee F_4 \vdash F_8} \text{hCut} \perp_L \\
\frac{h_1 : \Delta_2, F_4 \vdash \perp}{- : \Delta_2, \Delta_7, F_4 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_6 : \perp, \Delta_7 \vdash F_8}{- : \Delta_2, \Delta_7, F_3 \vee F_4 \vdash F_8} \text{ax/W}}{- : \Delta_2, \Delta_7, F_3 \vee F_4 \vdash F_8} \text{hCut} \perp_L \\
\frac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_6} \vee_L \quad \frac{}{\bullet h_7 : (\perp, \Delta_9), F_6 \vdash F_8} \perp_L}{- : (\Delta_2, F_3 \vee F_4), \perp, \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
- : \perp, \Delta_2, \Delta_9, F_3 \vee F_4 \vdash F_8 \quad \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_6} \vee_L \quad \frac{}{\bullet h_7 : (\Delta_9, P_8), F_6 \vdash P_8} I}{- : (\Delta_2, F_3 \vee F_4), \Delta_9, P_8 \vdash P_8} \text{Cut} \\
\rightarrow \\
- : \Delta_2, \Delta_9, P_8, F_3 \vee F_4 \vdash P_8 \quad I \\
\frac{h_1 : \Delta_2, F_3 \vdash P_8 \quad h_1 : \Delta_2, F_4 \vdash P_8}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash P_8} \vee_L \quad \frac{}{\bullet h_6 : \Delta_7, P_8 \vdash P_8} I}{- : (\Delta_2, F_3 \vee F_4), \Delta_7 \vdash P_8} \text{Cut} \\
\rightarrow \\
- : \Delta_2, \Delta_7, P_8, F_3 \vee F_4 \vdash P_8 \quad \text{ax/W}
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash \top \quad h_1 : \Delta_2, F_4 \vdash \top}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \top} \vee_L \quad \frac{h_6 : \Delta_7 \vdash F_8}{\bullet h_6 : \Delta_7, \top \vdash F_8} \top_L}{\frac{- : (\Delta_2, F_3 \vee F_4), \Delta_7 \vdash F_8}{\rightarrow} \text{Cut}} \\
\frac{}{- : \Delta_2, \Delta_7, F_3 \vee F_4 \vdash F_8} \text{ax/W} \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_6} \vee_L \quad \frac{h_7 : \Delta_9, F_6 \vdash F_8}{\bullet h_7 : (\top, \Delta_9), F_6 \vdash F_8} \top_L}{\frac{- : (\Delta_2, F_3 \vee F_4), \top, \Delta_9 \vdash F_8}{\rightarrow} \text{Cut}} \\
\frac{\frac{}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash F_6} \text{ax/W} \quad \frac{h_7 : \top, \Delta_9, F_6 \vdash F_8}{h_7 : \top, \Delta_9, F_6 \vdash F_8} \text{ax/W}}{- : \top, \Delta_2, \Delta_9, F_3 \vee F_4 \vdash F_8} \text{hCut}
\end{array}$$

## 7.9 Status of $\perp_L$ : OK

- Case rule  $\top_R$

$$\frac{\frac{}{\bullet h_1 : \perp, \Delta_2 \vdash F_5} \perp_L \quad \frac{}{\bullet h_6 : \Delta_4, F_5 \vdash \top} \top_R}{\frac{- : (\perp, \Delta_2), \Delta_4 \vdash \top}{\rightarrow} \text{Cut}} \\
\frac{}{- : \perp, \Delta_2, \Delta_4 \vdash \top} \top_R$$

- Case rule  $\rightarrow_R$

$$\frac{\frac{}{\bullet h_1 : \perp, \Delta_2 \vdash F_5} \perp_L \quad \frac{h_6 : \Delta_4, F_5, F_7 \vdash F_8}{\bullet h_6 : \Delta_4, F_5 \vdash F_7 \rightarrow F_8} \rightarrow_R}{\frac{- : (\perp, \Delta_2), \Delta_4 \vdash F_7 \rightarrow F_8}{\rightarrow} \text{Cut}} \\
\frac{}{- : \perp, \Delta_2, \Delta_4 \vdash F_7 \rightarrow F_8} \perp_L$$

- Case rule  $\wedge_R$

$$\frac{\frac{}{\bullet h_1 : \perp, \Delta_2 \vdash F_5} \perp_L \quad \frac{h_6 : \Delta_4, F_5 \vdash F_7 \quad h_6 : \Delta_4, F_5 \vdash F_8}{\bullet h_6 : \Delta_4, F_5 \vdash F_7 \wedge F_8} \wedge_R}{\frac{- : (\perp, \Delta_2), \Delta_4 \vdash F_7 \wedge F_8}{\rightarrow} \text{Cut}} \\
\frac{}{- : \perp, \Delta_2, \Delta_4 \vdash F_7 \wedge F_8} \perp_L$$

- Case rule  $\vee_1$

$$\frac{\frac{}{\bullet h_1 : \perp, \Delta_2 \vdash F_5} \perp_L \quad \frac{h_6 : \Delta_4, F_5 \vdash F_7}{\bullet h_6 : \Delta_4, F_5 \vdash F_7 \vee F_8} \vee_1}{\frac{- : (\perp, \Delta_2), \Delta_4 \vdash F_7 \vee F_8}{\rightarrow} \text{Cut}} \\
\frac{}{- : \perp, \Delta_2, \Delta_4 \vdash F_7 \vee F_8} \perp_L$$

- Case rule  $\vee_2$

$$\frac{\frac{}{\bullet h_1 : \perp, \Delta_2 \vdash F_5} \perp_L \quad \frac{h_6 : \Delta_4, F_5 \vdash F_8}{\bullet h_6 : \Delta_4, F_5 \vdash F_7 \vee F_8} \vee_2}{\frac{- : (\perp, \Delta_2), \Delta_4 \vdash F_7 \vee F_8}{\rightarrow} \text{Cut}} \\
\frac{}{- : \perp, \Delta_2, \Delta_4 \vdash F_7 \vee F_8} \perp_L$$

- Case rule  $\rightarrow_L$

$$\begin{array}{c}
\frac{\bullet h_1 : \perp, \Delta_2 \vdash F_4}{\perp_L} \quad \frac{h_5 : \Delta_9, F_4, F_6 \rightarrow F_7 \vdash F_6 \quad h_5 : \Delta_9, F_4, F_7 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \rightarrow F_7), F_4 \vdash F_8} \rightarrow_L \\
\hline
\frac{}{- : (\perp, \Delta_2), \Delta_9, F_6 \rightarrow F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_2, \Delta_9, F_6 \rightarrow F_7 \vdash F_8} \perp_L \\
\hline
\frac{\bullet h_1 : \perp, \Delta_2 \vdash F_6 \rightarrow F_7}{\perp_L} \quad \frac{h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash F_6 \quad h_4 : \Delta_5, F_7 \vdash F_8}{\bullet h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash F_8} \rightarrow_L \\
\hline
\frac{}{- : (\perp, \Delta_2), \Delta_5 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_2, \Delta_5 \vdash F_8} \perp_L
\end{array}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{\bullet h_1 : \perp, \Delta_2 \vdash F_4}{\perp_L} \quad \frac{h_5 : \Delta_9, F_4, F_6, F_7 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \wedge F_7), F_4 \vdash F_8} \wedge_L \\
\hline
\frac{}{- : (\perp, \Delta_2), \Delta_9, F_6 \wedge F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_2, \Delta_9, F_6 \wedge F_7 \vdash F_8} \perp_L \\
\hline
\frac{\bullet h_1 : \perp, \Delta_2 \vdash F_6 \wedge F_7}{\perp_L} \quad \frac{h_4 : \Delta_5, F_6, F_7 \vdash F_8}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash F_8} \wedge_L \\
\hline
\frac{}{- : (\perp, \Delta_2), \Delta_5 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_2, \Delta_5 \vdash F_8} \perp_L
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{\bullet h_1 : \perp, \Delta_2 \vdash F_4}{\perp_L} \quad \frac{h_5 : \Delta_9, F_4, F_6 \vdash F_8 \quad h_5 : \Delta_9, F_4, F_7 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \vee F_7), F_4 \vdash F_8} \vee_L \\
\hline
\frac{}{- : (\perp, \Delta_2), \Delta_9, F_6 \vee F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_2, \Delta_9, F_6 \vee F_7 \vdash F_8} \perp_L \\
\hline
\frac{\bullet h_1 : \perp, \Delta_2 \vdash F_6 \vee F_7}{\perp_L} \quad \frac{h_4 : \Delta_5, F_6 \vdash F_8 \quad h_4 : \Delta_5, F_7 \vdash F_8}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash F_8} \vee_L \\
\hline
\frac{}{- : (\perp, \Delta_2), \Delta_5 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_2, \Delta_5 \vdash F_8} \perp_L
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{\bullet h_1 : \perp, \Delta_2 \vdash \perp}{\perp_L} \quad \frac{\bullet h_4 : \Delta_5, \perp \vdash F_6}{\perp_L} \\
\hline
\frac{}{- : (\perp, \Delta_2), \Delta_5 \vdash F_6} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_2, \Delta_5 \vdash F_6} \perp_L \\
\hline
\frac{\bullet h_1 : \perp, \Delta_2 \vdash F_4}{\perp_L} \quad \frac{\bullet h_5 : (\perp, \Delta_7), F_4 \vdash F_6}{\perp_L} \\
\hline
\frac{}{- : (\perp, \Delta_2), \perp, \Delta_7 \vdash F_6} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \perp, \Delta_2, \Delta_7 \vdash F_6} \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{\bullet h_1 : \perp, \Delta_2 \vdash F_4}{\perp_L} \quad \frac{\bullet h_5 : (\Delta_7, P_6), F_4 \vdash P_6}{I} \\
\hline
\frac{}{- : (\perp, \Delta_2), \Delta_7, P_6 \vdash P_6} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_2, \Delta_7, P_6 \vdash P_6} \perp_L
\end{array}$$

$$\frac{\frac{\frac{}{\bullet h_1 : \perp, \Delta_2 \vdash p_6} \perp_L}{\rightarrow} \quad \frac{\frac{}{\bullet h_4 : \Delta_5, p_6 \vdash p_6} I}{\text{Cut}}}{\rightarrow} \text{Cut}}{\rightarrow} \perp_L$$

- Case rule  $\top_L$

$$\frac{\frac{\frac{}{\bullet h_1 : \perp, \Delta_2 \vdash \top} \perp_L}{\rightarrow} \quad \frac{\frac{\frac{}{\bullet h_4 : \Delta_5, \top \vdash F_6} \top_L}{\text{Cut}}}{\rightarrow} \text{Cut}}{\rightarrow} \perp_L}{\rightarrow} \perp_L$$

$$\frac{\frac{\frac{}{\bullet h_1 : \perp, \Delta_2 \vdash F_4} \perp_L}{\rightarrow} \quad \frac{\frac{\frac{}{\bullet h_5 : (\top, \Delta_7), F_4 \vdash F_6} \top_L}{\text{Cut}}}{\rightarrow} \text{Cut}}{\rightarrow} \perp_L}{\rightarrow} \perp_L$$

## 7.10 Status of $I$ : OK

- Case rule  $\top_R$

$$\frac{\frac{\frac{}{\bullet h_1 : \Delta_2, p_5 \vdash p_5} I}{\rightarrow} \quad \frac{\frac{}{\bullet h_6 : \Delta_4, p_5 \vdash \top} \top_R}{\text{Cut}}}{\rightarrow} \text{Cut}}{\rightarrow} \top_R$$

- Case rule  $\rightarrow_R$

$$\frac{\frac{\frac{}{\bullet h_1 : \Delta_2, p_5 \vdash p_5} I}{\rightarrow} \quad \frac{\frac{\frac{}{\bullet h_6 : \Delta_4, p_5 \vdash F_7 \rightarrow F_8} \rightarrow_R}{\text{Cut}}}{\rightarrow} \text{Cut}}{\rightarrow} \text{Cut}}{\rightarrow} \text{ax/W}$$

- Case rule  $\wedge_R$

$$\frac{\frac{\frac{}{\bullet h_1 : \Delta_2, p_5 \vdash p_5} I}{\rightarrow} \quad \frac{\frac{\frac{}{\bullet h_6 : \Delta_4, p_5 \vdash F_7} \quad \frac{}{\bullet h_6 : \Delta_4, p_5 \vdash F_8}}{\wedge_R}}{\text{Cut}}}{\rightarrow} \text{Cut}}{\rightarrow} \text{ax/W}$$

- Case rule  $\vee_1$

$$\frac{\frac{\frac{}{\bullet h_1 : \Delta_2, p_5 \vdash p_5} I}{\rightarrow} \quad \frac{\frac{\frac{}{\bullet h_6 : \Delta_4, p_5 \vdash F_7} \quad \frac{}{\bullet h_6 : \Delta_4, p_5 \vdash F_7 \vee F_8}}{\vee_1}}{\text{Cut}}}{\rightarrow} \text{Cut}}{\rightarrow} \text{ax/W}$$

- Case rule  $\vee_2$

$$\frac{\frac{\frac{}{\bullet h_1 : \Delta_2, p_5 \vdash p_5} I}{\rightarrow} \quad \frac{\frac{\frac{}{\bullet h_6 : \Delta_4, p_5 \vdash F_8} \quad \frac{}{\bullet h_6 : \Delta_4, p_5 \vdash F_7 \vee F_8}}{\vee_2}}{\text{Cut}}}{\rightarrow} \text{Cut}}{\rightarrow} \text{ax/W}$$

- Case rule  $\rightarrow_L$

$$\frac{\frac{\frac{}{\bullet h_1 : \Delta_2, p_4 \vdash p_4} I \quad \frac{h_5 : \Delta_9, p_4, F_6 \rightarrow F_7 \vdash F_6 \quad h_5 : \Delta_9, F_7, p_4 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \rightarrow F_7), p_4 \vdash F_8} \rightarrow_L}{- : (\Delta_2, p_4), \Delta_9, F_6 \rightarrow F_7 \vdash F_8} \text{Cut}}{\rightarrow} \text{ax/w}}{- : \Delta_2, \Delta_9, p_4, F_6 \rightarrow F_7 \vdash F_8}$$

- Case rule  $\wedge_L$

$$\frac{\frac{\frac{}{\bullet h_1 : \Delta_2, p_4 \vdash p_4} I \quad \frac{h_5 : \Delta_9, F_6, F_7, p_4 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \wedge F_7), p_4 \vdash F_8} \wedge_L}{- : (\Delta_2, p_4), \Delta_9, F_6 \wedge F_7 \vdash F_8} \text{Cut}}{\rightarrow} \text{ax/w}}{- : \Delta_2, \Delta_9, p_4, F_6 \wedge F_7 \vdash F_8}$$

- Case rule  $\vee_L$

$$\frac{\frac{\frac{}{\bullet h_1 : \Delta_2, p_4 \vdash p_4} I \quad \frac{h_5 : \Delta_9, F_6, p_4 \vdash F_8 \quad h_5 : \Delta_9, F_7, p_4 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \vee F_7), p_4 \vdash F_8} \vee_L}{- : (\Delta_2, p_4), \Delta_9, F_6 \vee F_7 \vdash F_8} \text{Cut}}{\rightarrow} \text{ax/w}}{- : \Delta_2, \Delta_9, p_4, F_6 \vee F_7 \vdash F_8}$$

- Case rule  $\perp_L$

$$\frac{\frac{\frac{}{\bullet h_1 : \Delta_2, p_4 \vdash p_4} I \quad \frac{}{\bullet h_5 : (\perp, \Delta_7), p_4 \vdash F_6} \perp_L}{- : (\Delta_2, p_4), \perp, \Delta_7 \vdash F_6} \text{Cut}}{\rightarrow} \perp_L}{- : \perp, \Delta_2, \Delta_7, p_4 \vdash F_6}$$

- Case rule  $I$

$$\frac{\frac{\frac{}{\bullet h_1 : \Delta_2, p_4 \vdash p_4} I \quad \frac{}{\bullet h_5 : (\Delta_7, p_6), p_4 \vdash p_6} I}{- : (\Delta_2, p_4), \Delta_7, p_6 \vdash p_6} \text{Cut}}{\rightarrow} I}{- : \Delta_2, \Delta_7, p_4, p_6 \vdash p_6} I$$

$$\frac{\frac{\frac{}{\bullet h_1 : \Delta_2, p_6 \vdash p_6} I \quad \frac{}{\bullet h_4 : \Delta_5, p_6 \vdash p_6} I}{- : (\Delta_2, p_6), \Delta_5 \vdash p_6} \text{Cut}}{\rightarrow} I}{- : \Delta_2, \Delta_5, p_6 \vdash p_6} I$$

- Case rule  $\top_L$

$$\frac{\frac{\frac{}{\bullet h_1 : \Delta_2, p_4 \vdash p_4} I \quad \frac{h_5 : \Delta_7, p_4 \vdash F_6}{\bullet h_5 : (\top, \Delta_7), p_4 \vdash F_6} \top_L}{- : (\Delta_2, p_4), \top, \Delta_7 \vdash F_6} \text{Cut}}{\rightarrow} \text{ax/w}}{- : \top, \Delta_2, \Delta_7, p_4 \vdash F_6}$$

## 7.11 Status of $\top_L$ : OK

- Case rule  $\top_R$

$$\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \top, \Delta_2 \vdash F_5} \top_L \quad \frac{}{\bullet h_6 : \Delta_4, F_5 \vdash \top} \top_R}{- : (\top, \Delta_2), \Delta_4 \vdash \top} \text{Cut} \rightarrow \frac{}{- : \top, \Delta_2, \Delta_4 \vdash \top} \top_R$$

- Case rule  $\rightarrow_R$

$$\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \top, \Delta_2 \vdash F_5} \top_L \quad \frac{h_6 : \Delta_4, F_5, F_7 \vdash F_8}{\bullet h_6 : \Delta_4, F_5 \vdash F_7 \rightarrow F_8} \rightarrow_R}{- : (\top, \Delta_2), \Delta_4 \vdash F_7 \rightarrow F_8} \text{Cut} \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \top, \Delta_2 \vdash F_5} \text{ax/W} \quad \frac{h_6 : \Delta_4, F_5, F_7 \vdash F_8}{\bullet h_6 : \top, \Delta_4, F_5 \vdash F_7 \rightarrow F_8} \text{ax/W}}{- : \top, \Delta_2, \Delta_4 \vdash F_7 \rightarrow F_8} \text{hCut}$$

- Case rule  $\wedge_R$

$$\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \top, \Delta_2 \vdash F_5} \top_L \quad \frac{h_6 : \Delta_4, F_5 \vdash F_7 \quad h_6 : \Delta_4, F_5 \vdash F_8}{\bullet h_6 : \Delta_4, F_5 \vdash F_7 \wedge F_8} \wedge_R}{- : (\top, \Delta_2), \Delta_4 \vdash F_7 \wedge F_8} \text{Cut} \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \top, \Delta_2 \vdash F_5} \text{ax/W} \quad \frac{h_6 : \Delta_4, F_5 \vdash F_7 \wedge F_8}{\bullet h_6 : \top, \Delta_4, F_5 \vdash F_7 \wedge F_8} \text{ax/W}}{- : \top, \Delta_2, \Delta_4 \vdash F_7 \wedge F_8} \text{hCut}$$

- Case rule  $\vee_1$

$$\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \top, \Delta_2 \vdash F_5} \top_L \quad \frac{h_6 : \Delta_4, F_5 \vdash F_7}{\bullet h_6 : \Delta_4, F_5 \vdash F_7 \vee F_8} \vee_1}{- : (\top, \Delta_2), \Delta_4 \vdash F_7 \vee F_8} \text{Cut} \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \top, \Delta_2 \vdash F_5} \text{ax/W} \quad \frac{h_6 : \Delta_4, F_5 \vdash F_7 \vee F_8}{\bullet h_6 : \top, \Delta_4, F_5 \vdash F_7 \vee F_8} \text{ax/W}}{- : \top, \Delta_2, \Delta_4 \vdash F_7 \vee F_8} \text{hCut}$$

- Case rule  $\vee_2$

$$\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \top, \Delta_2 \vdash F_5} \top_L \quad \frac{h_6 : \Delta_4, F_5 \vdash F_8}{\bullet h_6 : \Delta_4, F_5 \vdash F_7 \vee F_8} \vee_2}{- : (\top, \Delta_2), \Delta_4 \vdash F_7 \vee F_8} \text{Cut} \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \top, \Delta_2 \vdash F_5} \text{ax/W} \quad \frac{h_6 : \Delta_4, F_5 \vdash F_7 \vee F_8}{\bullet h_6 : \top, \Delta_4, F_5 \vdash F_7 \vee F_8} \text{ax/W}}{- : \top, \Delta_2, \Delta_4 \vdash F_7 \vee F_8} \text{hCut}$$

- Case rule  $\rightarrow_L$

$$\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \top, \Delta_2 \vdash F_4} \top_L \quad \frac{h_5 : \Delta_9, F_4, F_6 \rightarrow F_7 \vdash F_6 \quad h_5 : \Delta_9, F_4, F_7 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \rightarrow F_7), F_4 \vdash F_8} \rightarrow_L}{- : (\top, \Delta_2), \Delta_9, F_6 \rightarrow F_7 \vdash F_8} \text{Cut} \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \top, \Delta_2 \vdash F_4} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_4, F_6 \rightarrow F_7 \vdash F_8}{\bullet h_5 : \top, \Delta_9, F_4, F_6 \rightarrow F_7 \vdash F_8} \text{ax/W}}{- : \top, \Delta_2, \Delta_9, F_6 \rightarrow F_7 \vdash F_8} \text{hCut}$$

$$\frac{\frac{h_1 : \Delta_2 \vdash F_6 \rightarrow F_7}{\bullet h_1 : \top, \Delta_2 \vdash F_6 \rightarrow F_7} \top_L \quad \frac{h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash F_6 \quad h_4 : \Delta_5, F_7 \vdash F_8}{\bullet h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash F_8} \rightarrow_L}{- : (\top, \Delta_2), \Delta_5 \vdash F_8} \text{Cut} \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_6 \rightarrow F_7}{\bullet h_1 : \top, \Delta_2 \vdash F_6 \rightarrow F_7} \text{ax/W} \quad \frac{h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash F_8}{\bullet h_4 : \top, \Delta_5, F_6 \rightarrow F_7 \vdash F_8} \text{ax/W}}{- : \top, \Delta_2, \Delta_5 \vdash F_8} \text{hCut}$$

- Case rule  $\wedge_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \top, \Delta_2 \vdash F_4} \top_L \quad \frac{h_5 : \Delta_9, F_4, F_6, F_7 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \wedge F_7), F_4 \vdash F_8} \wedge_L}{- : (\top, \Delta_2), \Delta_9, F_6 \wedge F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \top, \Delta_2 \vdash F_4} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_4, F_6 \wedge F_7 \vdash F_8}{\bullet h_5 : \top, \Delta_9, F_6 \wedge F_7 \vdash F_8} \text{ax/W}}{- : \top, \Delta_2, \Delta_9, F_6 \wedge F_7 \vdash F_8} \text{hCut} \\
\frac{h_1 : \Delta_2 \vdash F_6 \wedge F_7}{\bullet h_1 : \top, \Delta_2 \vdash F_6 \wedge F_7} \top_L \quad \frac{h_4 : \Delta_5, F_6, F_7 \vdash F_8}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash F_8} \wedge_L}{- : (\top, \Delta_2), \Delta_5 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \wedge F_7}{\bullet h_1 : \top, \Delta_2 \vdash F_6 \wedge F_7} \text{ax/W} \quad \frac{h_4 : \Delta_5, F_6 \wedge F_7 \vdash F_8}{\bullet h_4 : \top, \Delta_5, F_6 \wedge F_7 \vdash F_8} \text{ax/W}}{- : \top, \Delta_2, \Delta_5 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule  $\vee_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \top, \Delta_2 \vdash F_4} \top_L \quad \frac{h_5 : \Delta_9, F_4, F_6 \vdash F_8 \quad h_5 : \Delta_9, F_4, F_7 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \vee F_7), F_4 \vdash F_8} \vee_L}{- : (\top, \Delta_2), \Delta_9, F_6 \vee F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \top, \Delta_2 \vdash F_4} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_4, F_6 \vee F_7 \vdash F_8}{\bullet h_5 : \top, \Delta_9, F_4, F_6 \vee F_7 \vdash F_8} \text{ax/W}}{- : \top, \Delta_2, \Delta_9, F_6 \vee F_7 \vdash F_8} \text{hCut} \\
\frac{h_1 : \Delta_2 \vdash F_6 \vee F_7}{\bullet h_1 : \top, \Delta_2 \vdash F_6 \vee F_7} \top_L \quad \frac{h_4 : \Delta_5, F_6 \vdash F_8 \quad h_4 : \Delta_5, F_7 \vdash F_8}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash F_8} \vee_L}{- : (\top, \Delta_2), \Delta_5 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \vee F_7}{\bullet h_1 : \top, \Delta_2 \vdash F_6 \vee F_7} \text{ax/W} \quad \frac{h_4 : \Delta_5, F_6 \vee F_7 \vdash F_8}{\bullet h_4 : \top, \Delta_5, F_6 \vee F_7 \vdash F_8} \text{ax/W}}{- : \top, \Delta_2, \Delta_5 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule  $\perp_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash \perp}{\bullet h_1 : \top, \Delta_2 \vdash \perp} \top_L \quad \frac{}{\bullet h_4 : \Delta_5, \perp \vdash F_6} \perp_L}{- : (\top, \Delta_2), \Delta_5 \vdash F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash \perp}{\bullet h_1 : \top, \Delta_2 \vdash \perp} \text{ax/W} \quad \frac{}{\bullet h_4 : \perp, \top, \Delta_5 \vdash F_6} \perp_L}{- : \top, \Delta_2, \Delta_5 \vdash F_6} \text{hCut} \\
\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \top, \Delta_2 \vdash F_4} \top_L \quad \frac{}{\bullet h_5 : (\perp, \Delta_7), F_4 \vdash F_6} \perp_L}{- : (\top, \Delta_2), \perp, \Delta_7 \vdash F_6} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \top, \Delta_2, \Delta_7 \vdash F_6} \perp_L
\end{array}$$

- Case rule  $I$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \top, \Delta_2 \vdash F_4} \top_L \quad \frac{}{\bullet h_5 : (\Delta_7, p_6), F_4 \vdash p_6} I}{- : (\top, \Delta_2), \Delta_7, p_6 \vdash p_6} \text{Cut} \\
\rightarrow \\
\frac{}{- : \top, \Delta_2, \Delta_7, p_6 \vdash p_6} I \\
\frac{h_1 : \Delta_2 \vdash p_6}{\bullet h_1 : \top, \Delta_2 \vdash p_6} \top_L \quad \frac{}{\bullet h_4 : \Delta_5, p_6 \vdash p_6} I}{- : (\top, \Delta_2), \Delta_5 \vdash p_6} \text{Cut} \\
\rightarrow \\
\frac{}{- : \top, \Delta_2, \Delta_5 \vdash p_6} \text{ax/W}
\end{array}$$

- Case rule  $\top_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash \top}{\bullet h_1 : \top, \Delta_2 \vdash \top} \top_L \quad \frac{h_4 : \Delta_5 \vdash F_6}{\bullet h_4 : \Delta_5, \top \vdash F_6} \top_L}{- : (\top, \Delta_2), \Delta_5 \vdash F_6} \text{Cut} \\
\rightarrow \\
\frac{}{- : \top, \Delta_2, \Delta_5 \vdash F_6} \text{ax/W} \\
\\
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \top, \Delta_2 \vdash F_4} \top_L \quad \frac{h_5 : \Delta_7, F_4 \vdash F_6}{\bullet h_5 : (\top, \Delta_7), F_4 \vdash F_6} \top_L}{- : (\top, \Delta_2), \top, \Delta_7 \vdash F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \top, \Delta_2 \vdash F_4} \text{ax/W} \quad \frac{h_5 : \top, \top, \Delta_7, F_4 \vdash F_6}{\bullet h_5 : \top, \top, \Delta_7, F_4 \vdash F_6} \text{ax/W}}{- : \top, \top, \Delta_2, \Delta_7 \vdash F_6} \text{hCut}
\end{array}$$