

System G3C

August 1, 2021

Contents

1	Height preserving admissibility of weakening on the left	2
2	Height preserving admissibility of weakening on the right	4
3	Measure of derivations	5
4	Invertibility of Rules	6
4.1	Status of \rightarrow_R : Invertible	6
4.2	Status of \wedge_R (Left Premise): Invertible	7
4.3	Status of \wedge_R (Right Premise): Invertible	8
4.4	Status of \vee_R : Invertible	9
4.5	Status of \perp_R : Invertible	11
4.6	Status of \top_R : Invertible	12
4.7	Status of \rightarrow_L (Left Premise): Invertible	13
4.8	Status of \rightarrow_L (Right Premise): Invertible	14
4.9	Status of \wedge_L : Invertible	15
4.10	Status of \vee_L (Left Premise): Invertible	17
4.11	Status of \vee_L (Right Premise): Invertible	18
4.12	Status of \perp_L : Invertible	19
4.13	Status of I : Invertible	20
4.14	Status of \top_L : Invertible	21
5	Height preserving admissibility of contraction on the left	23
6	Height preserving admissibility of contraction on the Right	25
7	Identity-Expansion	27
8	Cut-Elimination	28
8.1	Status of \rightarrow_R : OK	28
8.2	Status of \wedge_R : OK	32
8.3	Status of \vee_R : OK	37
8.4	Status of \perp_R : OK	41
8.5	Status of \top_R : OK	44
8.6	Status of \rightarrow_L : OK	47
8.7	Status of \wedge_L : OK	50
8.8	Status of \vee_L : OK	53
8.9	Status of \perp_L : OK	56
8.10	Status of I : OK	58
8.11	Status of \top_L : OK	62

1 Height preserving admissibility of weakening on the left

- Case(s) rule \rightarrow_R

$$\frac{h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \rightarrow F_5} \rightarrow_R \rightarrow \frac{\frac{h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5}{\bullet h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5} \text{ ax}}{h_1 : \Delta_2, F_4, F_W \vdash \Delta_3, F_5} \text{ IH}}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_3, F_4 \rightarrow F_5} \rightarrow_R$$

- Case(s) rule \wedge_R

$$\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4 \quad h_1 : \Delta_2 \vdash \Delta_3, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \wedge F_5} \wedge_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_3, F_4} \text{ ax} \quad \frac{h_1 : \Delta_2 \vdash \Delta_3, F_5}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_3, F_5} \text{ ax}}{h_1 : \Delta_2, F_W \vdash \Delta_3, F_4 \wedge F_5} \text{ IH}}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_3, F_4 \wedge F_5} \wedge_R$$

- Case(s) rule \vee_R

$$\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \vee F_5} \vee_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_3, F_4, F_5} \text{ ax}}{h_1 : \Delta_2, F_W \vdash \Delta_3, F_4 \vee F_5} \text{ IH}}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_3, F_4 \vee F_5} \vee_R$$

- Case(s) rule \perp_R

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \perp_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_3} \text{ ax}}{h_1 : \Delta_2, F_W \vdash \perp, \Delta_3} \text{ IH}}{\bullet h_1 : \Delta_2, F_W \vdash \perp, \Delta_3} \perp_R$$

- Case(s) rule \top_R

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R \rightarrow \frac{}{\bullet h_1 : \Delta_2, F_W \vdash \top, \Delta_3} \top_R$$

- Case(s) rule \rightarrow_L

$$\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3 \quad h_1 : \Delta_2, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash \Delta_5} \rightarrow_L \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3}{h_1 : \Delta_2, F_W \vdash \Delta_5, F_3} \text{ ax} \quad \frac{h_1 : \Delta_2, F_4 \vdash \Delta_5}{h_1 : \Delta_2, F_4, F_W \vdash \Delta_5} \text{ ax}}{h_1 : \Delta_2, F_W, F_3 \rightarrow F_4 \vdash \Delta_5} \text{ IH}}{\bullet h_1 : \Delta_2, F_W, F_3 \rightarrow F_4 \vdash \Delta_5} \rightarrow_L$$

- Case(s) rule \wedge_L

$$\frac{h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash \Delta_5} \wedge_L \rightarrow \frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3, F_4, F_W \vdash \Delta_5} \text{ ax}}{h_1 : \Delta_2, F_W, F_3 \wedge F_4 \vdash \Delta_5} \text{ IH}}{\bullet h_1 : \Delta_2, F_W, F_3 \wedge F_4 \vdash \Delta_5} \wedge_L$$

- Case(s) rule \vee_L

$$\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5 \quad h_1 : \Delta_2, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5} \vee_L \rightarrow \frac{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5}{h_1 : \Delta_2, F_3, F_W \vdash \Delta_5} \text{ ax} \quad \frac{h_1 : \Delta_2, F_4 \vdash \Delta_5}{h_1 : \Delta_2, F_4, F_W \vdash \Delta_5} \text{ ax}}{h_1 : \Delta_2, F_W, F_3 \vee F_4 \vdash \Delta_5} \text{ IH}}{\bullet h_1 : \Delta_2, F_W, F_3 \vee F_4 \vdash \Delta_5} \vee_L$$

- Case(s) rule \perp_L

$$\frac{}{\bullet h_1 : \perp, \Delta_2 \vdash \Delta_3} \perp_L \rightarrow \frac{}{\bullet h_1 : \perp, \Delta_2, F_W \vdash \Delta_3} \perp_L$$

- Case(s) rule I

$$\frac{}{\bullet h_1 : \Delta_2, P_3 \vdash \Delta_4, P_3} I \quad \rightarrow \quad \frac{}{\bullet h_1 : \Delta_2, F_W, P_3 \vdash \Delta_4, P_3} I$$

- Case(s) rule \top_L

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \top, \Delta_2 \vdash \Delta_3} \top_L \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2, F_W \vdash \Delta_3} \text{IH}}{\bullet h_1 : \top, \Delta_2, F_W \vdash \Delta_3} \top_L \text{ax}$$

2 Height preserving admissibility of weakening on the right

- Case(s) rule \rightarrow_R

$$\frac{\frac{h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \rightarrow F_5} \rightarrow_R}{\frac{\frac{h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \rightarrow F_5} \text{ ax}}{\frac{h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5, F_W}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_W, F_4 \rightarrow F_5} \text{ IH}} \rightarrow_R} \rightarrow$$

- Case(s) rule \wedge_R

$$\frac{\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \wedge F_5} \wedge_R}{\frac{\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_W, F_4 \wedge F_5} \text{ ax}}{\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_W}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_W, F_4 \wedge F_5} \text{ IH}} \wedge_R} \rightarrow$$

- Case(s) rule \vee_R

$$\frac{\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \vee F_5} \vee_R}{\frac{\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_W, F_4 \vee F_5} \text{ ax}}{\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5, F_W}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_W, F_4 \vee F_5} \text{ IH}} \vee_R} \rightarrow$$

- Case(s) rule \perp_R

$$\frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \perp_R}{\frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3, F_W} \text{ ax}}{\frac{h_1 : \Delta_2 \vdash \Delta_3, F_W}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3, F_W} \text{ IH}} \perp_R} \rightarrow$$

- Case(s) rule \top_R

$$\frac{\frac{h_1 : \Delta_2 \vdash \top, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R}{\frac{h_1 : \Delta_2 \vdash \top, \Delta_3, F_W}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3, F_W} \top_R} \rightarrow$$

- Case(s) rule \rightarrow_L

$$\frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash \Delta_5} \rightarrow_L}{\frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3, F_W} \text{ ax}}{\frac{h_1 : \Delta_2, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash \Delta_5, F_W} \text{ IH}} \rightarrow_L} \rightarrow$$

- Case(s) rule \wedge_L

$$\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash \Delta_5} \wedge_L}{\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash \Delta_5, F_W} \text{ ax}}{\frac{h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5, F_W}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash \Delta_5, F_W} \text{ IH}} \wedge_L} \rightarrow$$

- Case(s) rule \vee_L

$$\frac{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5} \vee_L}{\frac{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5, F_W} \text{ ax}}{\frac{h_1 : \Delta_2, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5, F_W} \text{ IH}} \vee_L} \rightarrow$$

- Case(s) rule \perp_L

$$\frac{\frac{h_1 : \perp, \Delta_2 \vdash \Delta_3}{\bullet h_1 : \perp, \Delta_2 \vdash \Delta_3} \perp_L}{\frac{h_1 : \perp, \Delta_2 \vdash \Delta_3, F_W}{\bullet h_1 : \perp, \Delta_2 \vdash \Delta_3, F_W} \perp_L} \rightarrow$$

- Case(s) rule I

$$\frac{\frac{h_1 : \Delta_2, P_3 \vdash \Delta_4, P_3}{\bullet h_1 : \Delta_2, P_3 \vdash \Delta_4, P_3} I}{\frac{h_1 : \Delta_2, P_3 \vdash \Delta_4, F_W, P_3}{\bullet h_1 : \Delta_2, P_3 \vdash \Delta_4, F_W, P_3} I} \rightarrow$$

- Case(s) rule \top_L

$$\frac{\frac{h_1 : \Delta_2 \vdash \top, \Delta_3}{\bullet h_1 : \top, \Delta_2 \vdash \Delta_3} \top_L}{\frac{\frac{h_1 : \Delta_2 \vdash \top, \Delta_3}{\bullet h_1 : \top, \Delta_2 \vdash \Delta_3, F_W} \text{ ax}}{\frac{h_1 : \top, \Delta_2 \vdash \Delta_3, F_W}{\bullet h_1 : \top, \Delta_2 \vdash \Delta_3, F_W} \text{ IH}} \top_L} \rightarrow$$

3 Measure of derivations

- Case(s) rule \rightarrow_R

$$\frac{h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \rightarrow F_5} \rightarrow_R \rightarrow \frac{\frac{h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5}{\bullet h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5} \text{ ax}}{\bullet \bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \rightarrow F_5} \text{ IH} \rightarrow_R$$

- Case(s) rule \wedge_R

$$\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4 \quad h_1 : \Delta_2 \vdash \Delta_3, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \wedge F_5} \wedge_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4} \text{ ax} \quad \frac{h_1 : \Delta_2 \vdash \Delta_3, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_5} \text{ ax}}{\bullet \bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \wedge F_5} \text{ IH} \wedge_R$$

- Case(s) rule \vee_R

$$\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \vee F_5} \vee_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5} \text{ ax}}{\bullet \bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \vee F_5} \text{ IH} \vee_R$$

- Case(s) rule \perp_R

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \perp_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3} \text{ ax}}{\bullet \bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \text{ IH} \perp_R$$

- Case(s) rule \top_R

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R \rightarrow \frac{}{\bullet \bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R$$

- Case(s) rule \rightarrow_L

$$\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3 \quad h_1 : \Delta_2, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash \Delta_5} \rightarrow_L \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3} \text{ ax} \quad \frac{h_1 : \Delta_2, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_4 \vdash \Delta_5} \text{ ax}}{\bullet \bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash \Delta_5} \text{ IH} \rightarrow_L$$

- Case(s) rule \wedge_L

$$\frac{h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash \Delta_5} \wedge_L \rightarrow \frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5} \text{ ax}}{\bullet \bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash \Delta_5} \text{ IH} \wedge_L$$

- Case(s) rule \vee_L

$$\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5 \quad h_1 : \Delta_2, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5} \vee_L \rightarrow \frac{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \vdash \Delta_5} \text{ ax} \quad \frac{h_1 : \Delta_2, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_4 \vdash \Delta_5} \text{ ax}}{\bullet \bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5} \text{ IH} \vee_L$$

- Case(s) rule \perp_L

$$\frac{}{\bullet h_1 : \perp, \Delta_2 \vdash \Delta_3} \perp_L \rightarrow \frac{}{\bullet \bullet h_1 : \perp, \Delta_2 \vdash \Delta_3} \perp_L$$

- Case(s) rule I

$$\frac{}{\bullet h_1 : \Delta_2, P_3 \vdash \Delta_4, P_3} I \rightarrow \frac{}{\bullet \bullet h_1 : \Delta_2, P_3 \vdash \Delta_4, P_3} I$$

- Case(s) rule \top_L

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \top, \Delta_2 \vdash \Delta_3} \top_L \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3} \text{ ax}}{\bullet \bullet h_1 : \top, \Delta_2 \vdash \Delta_3} \text{ IH} \top_L$$

4 Invertibility of Rules

4.1 Status of \rightarrow_R : : Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : \Delta_4, F_5 \vdash \Delta_7, F_6, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \rightarrow F_2), F_5 \rightarrow F_6} \rightarrow_R \rightarrow \frac{}{\bullet h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5 \rightarrow F_6} \text{ax/ind}$$

$$\frac{h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \rightarrow F_5} \rightarrow_R \rightarrow \frac{}{\bullet h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5} \text{ax}$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_7, F_5, F_1 \rightarrow F_2 \quad h_3 : \Delta_4 \vdash \Delta_7, F_6, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \rightarrow F_2), F_5 \wedge F_6} \wedge_R \rightarrow \frac{}{h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5} \text{ax/ind} \frac{}{h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_6} \text{ax/ind} \wedge_R$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_7, F_5, F_6, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \rightarrow F_2), F_5 \vee F_6} \vee_R \rightarrow \frac{}{h_3 : \Delta_4, F_1 \vdash \Delta_7, F_2, F_5, F_6} \text{ax/ind} \vee_R$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_5, F_1 \rightarrow F_2}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1 \rightarrow F_2} \perp_R \rightarrow \frac{}{h_3 : \Delta_4, F_1 \vdash \Delta_5, F_2} \text{ax/ind} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1 \rightarrow F_2} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_4, F_1 \vdash \top, \Delta_5, F_2} \top_R$$

- Case rule \rightarrow_L

$$\frac{h_4 : \Delta_5 \vdash \Delta_1, F_6, F_2 \rightarrow F_3 \quad h_4 : \Delta_5, F_7 \vdash \Delta_1, F_2 \rightarrow F_3}{\bullet h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash \Delta_1, F_2 \rightarrow F_3} \rightarrow_L \rightarrow \frac{}{h_4 : \Delta_5, F_2 \vdash \Delta_1, F_3, F_6} \text{ax/ind} \frac{}{h_4 : \Delta_5, F_2, F_7 \vdash \Delta_1, F_3} \text{ax/ind} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_4 : \Delta_5, F_6, F_7 \vdash \Delta_1, F_2 \rightarrow F_3}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_2 \rightarrow F_3} \wedge_L \rightarrow \frac{}{h_4 : \Delta_5, F_2, F_6, F_7 \vdash \Delta_1, F_3} \text{ax/ind} \wedge_L$$

- Case rule \vee_L

$$\frac{h_4 : \Delta_5, F_6 \vdash \Delta_1, F_2 \rightarrow F_3 \quad h_4 : \Delta_5, F_7 \vdash \Delta_1, F_2 \rightarrow F_3}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_2 \rightarrow F_3} \vee_L \rightarrow \frac{}{h_4 : \Delta_5, F_2, F_6 \vdash \Delta_1, F_3} \text{ax/ind} \frac{}{h_4 : \Delta_5, F_2, F_7 \vdash \Delta_1, F_3} \text{ax/ind} \vee_L$$

- Case rule \perp_L

$$\frac{}{\bullet h_4 : \perp, \Delta_5 \vdash \Delta_1, F_2 \rightarrow F_3} \perp_L \rightarrow \frac{}{\bullet h_4 : \perp, \Delta_5, F_2 \vdash \Delta_1, F_3} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : P_5, \Delta_4 \vdash P_5, \Delta_6, F_1 \rightarrow F_2} I \rightarrow \frac{}{\bullet h_3 : \Delta_4, F_1, P_5 \vdash \Delta_6, F_2, P_5} I$$

- Case rule \top_L

$$\frac{h_4 : \Delta_5 \vdash \Delta_1, F_2 \rightarrow F_3}{\bullet h_4 : \top, \Delta_5 \vdash \Delta_1, F_2 \rightarrow F_3} \top_L \rightarrow \frac{h_4 : \Delta_5, F_2 \vdash \Delta_1, F_3}{\bullet h_4 : \top, \Delta_5, F_2 \vdash \Delta_1, F_3} \text{ax/ind} \top_L$$

4.2 Status of \wedge_R : (Left Premise): Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : \Delta_4, F_5 \vdash \Delta_7, F_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \rightarrow F_6} \rightarrow_R \rightarrow \frac{h_3 : \Delta_4, F_5 \vdash \Delta_7, F_1, F_6}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \rightarrow F_6} \text{ax/ind} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_7, F_5, F_1 \wedge F_2 \quad h_3 : \Delta_4 \vdash \Delta_7, F_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \wedge F_6} \wedge_R \rightarrow \frac{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \quad h_3 : \Delta_4 \vdash \Delta_7, F_1, F_6}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \wedge F_6} \text{ax/ind} \wedge_R$$

$$\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4 \quad h_1 : \Delta_2 \vdash \Delta_3, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \wedge F_5} \wedge_R \rightarrow \frac{h_1 : \Delta_2 \vdash \Delta_3, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4} \text{ax} \text{height}$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_7, F_5, F_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \vee F_6} \vee_R \rightarrow \frac{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5, F_6}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_5 \vee F_6} \text{ax/ind} \vee_R$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_5, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1 \wedge F_2} \perp_R \rightarrow \frac{h_3 : \Delta_4 \vdash \Delta_5, F_1}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1} \text{ax/ind} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1 \wedge F_2} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1} \top_R$$

- Case rule \rightarrow_L

$$\frac{h_4 : \Delta_5 \vdash \Delta_1, F_6, F_2 \wedge F_3 \quad h_4 : \Delta_5, F_7 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash \Delta_1, F_2 \wedge F_3} \rightarrow_L \rightarrow \frac{h_4 : \Delta_5 \vdash \Delta_1, F_2, F_6 \quad h_4 : \Delta_5, F_7 \vdash \Delta_1, F_2}{\bullet h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash \Delta_1, F_2} \text{ax/ind} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_4 : \Delta_5, F_6, F_7 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_2 \wedge F_3} \wedge_L \rightarrow \frac{\overline{h_4 : \Delta_5, F_6, F_7 \vdash \Delta_1, F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_2} \wedge_L$$

- Case rule \vee_L

$$\frac{h_4 : \Delta_5, F_6 \vdash \Delta_1, F_2 \wedge F_3 \quad h_4 : \Delta_5, F_7 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_2 \wedge F_3} \vee_L \rightarrow \frac{\overline{h_4 : \Delta_5, F_6 \vdash \Delta_1, F_2} \text{ ax/ind} \quad \overline{h_4 : \Delta_5, F_7 \vdash \Delta_1, F_2} \text{ ax/ind}}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_2} \vee_L$$

- Case rule \perp_L

$$\frac{}{\bullet h_4 : \perp, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3} \perp_L \rightarrow \frac{}{\bullet h_4 : \perp, \Delta_5 \vdash \Delta_1, F_2} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : P_5, \Delta_4 \vdash P_5, \Delta_6, F_1 \wedge F_2} I \rightarrow \frac{}{\bullet h_3 : \Delta_4, P_5 \vdash \Delta_6, F_1, P_5} I$$

- Case rule \top_L

$$\frac{h_4 : \Delta_5 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \top, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3} \top_L \rightarrow \frac{\overline{h_4 : \Delta_5 \vdash \Delta_1, F_2} \text{ ax/ind}}{\bullet h_4 : \top, \Delta_5 \vdash \Delta_1, F_2} \top_L$$

4.3 Status of \wedge_R (Right Premise): : Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : \Delta_4, F_5 \vdash \Delta_7, F_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \rightarrow F_6} \rightarrow_R \rightarrow \frac{\overline{h_3 : \Delta_4, F_5 \vdash \Delta_7, F_2, F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5 \rightarrow F_6} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_7, F_5, F_1 \wedge F_2 \quad h_3 : \Delta_4 \vdash \Delta_7, F_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \wedge F_6} \wedge_R \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5} \text{ ax/ind} \quad \overline{h_3 : \Delta_4 \vdash \Delta_7, F_2, F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5 \wedge F_6} \wedge_R$$

$$\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4 \quad h_1 : \Delta_2 \vdash \Delta_3, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \wedge F_5} \wedge_R \rightarrow \frac{\overline{h_1 : \Delta_2 \vdash \Delta_3, F_5} \text{ ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_5} \text{ height}$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_7, F_5, F_6, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \wedge F_2), F_5 \vee F_6} \vee_R \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5, F_6} \text{ ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_2, F_5 \vee F_6} \vee_R$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_5, F_1 \wedge F_2}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1 \wedge F_2} \perp_R \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash \Delta_5, F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_2} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1 \wedge F_2} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_2} \top_R$$

- Case rule \rightarrow_L

$$\frac{h_4 : \Delta_5 \vdash \Delta_1, F_6, F_2 \wedge F_3 \quad h_4 : \Delta_5, F_7 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash \Delta_1, F_2 \wedge F_3} \rightarrow_L \rightarrow \frac{\frac{h_4 : \Delta_5 \vdash \Delta_1, F_3, F_6}{\bullet h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash \Delta_1, F_3} \text{ax/ind} \quad \frac{h_4 : \Delta_5, F_7 \vdash \Delta_1, F_3}{\bullet h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash \Delta_1, F_3} \text{ax/ind}}{\bullet h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash \Delta_1, F_3} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_4 : \Delta_5, F_6, F_7 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_2 \wedge F_3} \wedge_L \rightarrow \frac{\frac{h_4 : \Delta_5, F_6, F_7 \vdash \Delta_1, F_3}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_3} \text{ax/ind}}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_3} \wedge_L$$

- Case rule \vee_L

$$\frac{h_4 : \Delta_5, F_6 \vdash \Delta_1, F_2 \wedge F_3 \quad h_4 : \Delta_5, F_7 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_2 \wedge F_3} \vee_L \rightarrow \frac{\frac{h_4 : \Delta_5, F_6 \vdash \Delta_1, F_3}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_3} \text{ax/ind} \quad \frac{h_4 : \Delta_5, F_7 \vdash \Delta_1, F_3}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_3} \text{ax/ind}}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_3} \vee_L$$

- Case rule \perp_L

$$\frac{}{\bullet h_4 : \perp, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3} \perp_L \rightarrow \frac{}{\bullet h_4 : \perp, \Delta_5 \vdash \Delta_1, F_3} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : P_5, \Delta_4 \vdash P_5, \Delta_6, F_1 \wedge F_2} I \rightarrow \frac{}{\bullet h_3 : \Delta_4, P_5 \vdash \Delta_6, F_2, P_5} I$$

- Case rule \top_L

$$\frac{h_4 : \Delta_5 \vdash \Delta_1, F_2 \wedge F_3}{\bullet h_4 : \top, \Delta_5 \vdash \Delta_1, F_2 \wedge F_3} \top_L \rightarrow \frac{\frac{h_4 : \Delta_5 \vdash \Delta_1, F_3}{\bullet h_4 : \top, \Delta_5 \vdash \Delta_1, F_3} \text{ax/ind}}{\bullet h_4 : \top, \Delta_5 \vdash \Delta_1, F_3} \top_L$$

4.4 Status of \vee_R : : Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : \Delta_4, F_5 \vdash \Delta_7, F_6, F_1 \vee F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \vee F_2), F_5 \rightarrow F_6} \rightarrow_R \rightarrow \frac{\frac{h_3 : \Delta_4, F_5 \vdash \Delta_7, F_1, F_2, F_6}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \rightarrow F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \rightarrow F_6} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_7, F_5, F_1 \vee F_2 \quad h_3 : \Delta_4 \vdash \Delta_7, F_6, F_1 \vee F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \vee F_2), F_5 \wedge F_6} \wedge_R \rightarrow \frac{\frac{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5} \text{ax/ind} \quad \frac{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_6}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \wedge F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \wedge F_6} \wedge_R$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_7, F_5, F_6, F_1 \vee F_2}{\bullet h_3 : \Delta_4 \vdash (\Delta_7, F_1 \vee F_2), F_5 \vee F_6} \vee_R \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5, F_6}}{\bullet h_3 : \Delta_4 \vdash \Delta_7, F_1, F_2, F_5 \vee F_6} \text{ax/ind} \vee_R$$

$$\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \vee F_5} \vee_R \rightarrow \frac{\overline{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5}}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5} \text{ax} \text{height}$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_4 \vdash \Delta_5, F_1 \vee F_2}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1 \vee F_2} \perp_R \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash \Delta_5, F_1, F_2}}{\bullet h_3 : \Delta_4 \vdash \perp, \Delta_5, F_1, F_2} \text{ax/ind} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1 \vee F_2} \top_R \rightarrow \frac{}{\bullet h_3 : \Delta_4 \vdash \top, \Delta_5, F_1, F_2} \top_R$$

- Case rule \rightarrow_L

$$\frac{h_4 : \Delta_5 \vdash \Delta_1, F_6, F_2 \vee F_3 \quad h_4 : \Delta_5, F_7 \vdash \Delta_1, F_2 \vee F_3}{\bullet h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash \Delta_1, F_2 \vee F_3} \rightarrow_L \rightarrow \frac{\overline{h_4 : \Delta_5 \vdash \Delta_1, F_2, F_3, F_6}}{\bullet h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash \Delta_1, F_2, F_3} \text{ax/ind} \frac{\overline{h_4 : \Delta_5, F_7 \vdash \Delta_1, F_2, F_3}}{\bullet h_4 : \Delta_5, F_6 \rightarrow F_7 \vdash \Delta_1, F_2, F_3} \text{ax/ind} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_4 : \Delta_5, F_6, F_7 \vdash \Delta_1, F_2 \vee F_3}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_2 \vee F_3} \wedge_L \rightarrow \frac{\overline{h_4 : \Delta_5, F_6, F_7 \vdash \Delta_1, F_2, F_3}}{\bullet h_4 : \Delta_5, F_6 \wedge F_7 \vdash \Delta_1, F_2, F_3} \text{ax/ind} \wedge_L$$

- Case rule \vee_L

$$\frac{h_4 : \Delta_5, F_6 \vdash \Delta_1, F_2 \vee F_3 \quad h_4 : \Delta_5, F_7 \vdash \Delta_1, F_2 \vee F_3}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_2 \vee F_3} \vee_L \rightarrow \frac{\overline{h_4 : \Delta_5, F_6 \vdash \Delta_1, F_2, F_3}}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_2, F_3} \text{ax/ind} \frac{\overline{h_4 : \Delta_5, F_7 \vdash \Delta_1, F_2, F_3}}{\bullet h_4 : \Delta_5, F_6 \vee F_7 \vdash \Delta_1, F_2, F_3} \text{ax/ind} \vee_L$$

- Case rule \perp_L

$$\frac{}{\bullet h_4 : \perp, \Delta_5 \vdash \Delta_1, F_2 \vee F_3} \perp_L \rightarrow \frac{}{\bullet h_4 : \perp, \Delta_5 \vdash \Delta_1, F_2, F_3} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_5, \Delta_4 \vdash p_5, \Delta_6, F_1 \vee F_2} I \rightarrow \frac{}{\bullet h_3 : \Delta_4, p_5 \vdash \Delta_6, F_1, F_2, p_5} I$$

- Case rule \top_L

$$\frac{}{\bullet h_4 : \top, \Delta_5 \vdash \Delta_1, F_2 \vee F_3} \top_L \rightarrow \frac{\overline{h_4 : \Delta_5 \vdash \Delta_1, F_2, F_3}}{\bullet h_4 : \top, \Delta_5 \vdash \Delta_1, F_2, F_3} \text{ax/ind} \top_L$$

4.5 Status of \perp_R : : Invertible

- Case rule \rightarrow_R

$$\frac{h_1 : \Delta_2, F_3 \vdash \perp, \Delta_5, F_4}{\bullet h_1 : \Delta_2 \vdash (\perp, \Delta_5), F_3 \rightarrow F_4} \rightarrow_R \rightarrow \frac{\overline{h_1 : \Delta_2, F_3 \vdash \Delta_5, F_4}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \rightarrow F_4} \text{ax/ind} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_1 : \Delta_2 \vdash \perp, \Delta_5, F_3 \quad h_1 : \Delta_2 \vdash \perp, \Delta_5, F_4}{\bullet h_1 : \Delta_2 \vdash (\perp, \Delta_5), F_3 \wedge F_4} \wedge_R \rightarrow \frac{\overline{h_1 : \Delta_2 \vdash \Delta_5, F_3} \text{ ax/ind} \quad \overline{h_1 : \Delta_2 \vdash \Delta_5, F_4} \text{ ax/ind}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \wedge F_4} \wedge_R$$

- Case rule \vee_R

$$\frac{h_1 : \Delta_2 \vdash \perp, \Delta_5, F_3, F_4}{\bullet h_1 : \Delta_2 \vdash (\perp, \Delta_5), F_3 \vee F_4} \vee_R \rightarrow \frac{\overline{h_1 : \Delta_2 \vdash \Delta_5, F_3, F_4} \text{ ax/ind}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3 \vee F_4} \vee_R$$

- Case rule \perp_R

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \perp_R \rightarrow \frac{\overline{h_1 : \Delta_2 \vdash \Delta_3} \text{ ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_3} \text{height}$$

- Case rule \top_R

$$\overline{\bullet h_1 : \Delta_2 \vdash \top, \perp, \Delta_3} \top_R \rightarrow \overline{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R$$

- Case rule \rightarrow_L

$$\frac{h_2 : \Delta_3 \vdash \perp, \Delta_1, F_4 \quad h_2 : \Delta_3, F_5 \vdash \perp, \Delta_1}{\bullet h_2 : \Delta_3, F_4 \rightarrow F_5 \vdash \perp, \Delta_1} \rightarrow_L \rightarrow \frac{\overline{h_2 : \Delta_3 \vdash \Delta_1, F_4} \text{ ax/ind} \quad \overline{h_2 : \Delta_3, F_5 \vdash \Delta_1} \text{ ax/ind}}{\bullet h_2 : \Delta_3, F_4 \rightarrow F_5 \vdash \Delta_1} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_2 : \Delta_3, F_4, F_5 \vdash \perp, \Delta_1}{\bullet h_2 : \Delta_3, F_4 \wedge F_5 \vdash \perp, \Delta_1} \wedge_L \rightarrow \frac{\overline{h_2 : \Delta_3, F_4, F_5 \vdash \Delta_1} \text{ ax/ind}}{\bullet h_2 : \Delta_3, F_4 \wedge F_5 \vdash \Delta_1} \wedge_L$$

- Case rule \vee_L

$$\frac{h_2 : \Delta_3, F_4 \vdash \perp, \Delta_1 \quad h_2 : \Delta_3, F_5 \vdash \perp, \Delta_1}{\bullet h_2 : \Delta_3, F_4 \vee F_5 \vdash \perp, \Delta_1} \vee_L \rightarrow \frac{\overline{h_2 : \Delta_3, F_4 \vdash \Delta_1} \text{ ax/ind} \quad \overline{h_2 : \Delta_3, F_5 \vdash \Delta_1} \text{ ax/ind}}{\bullet h_2 : \Delta_3, F_4 \vee F_5 \vdash \Delta_1} \vee_L$$

- Case rule \perp_L

$$\overline{\bullet h_2 : \perp, \Delta_3 \vdash \perp, \Delta_1} \perp_L \rightarrow \overline{\bullet h_2 : \perp, \Delta_3 \vdash \Delta_1} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_1 : p_3, \Delta_2 \vdash p_3, \perp, \Delta_4} I \rightarrow \frac{}{\bullet h_1 : \Delta_2, p_3 \vdash \Delta_4, p_3} I$$

- Case rule \top_L

$$\frac{h_2 : \Delta_3 \vdash \perp, \Delta_1}{\bullet h_2 : \top, \Delta_3 \vdash \perp, \Delta_1} \top_L \rightarrow \frac{}{h_2 : \Delta_3 \vdash \Delta_1} \text{ax/ind} \top_L$$

4.6 Status of \top_R : : Invertible

- Case rule \rightarrow_R

$$\frac{h_1 : \Delta_2, F_3 \vdash \top, \Delta_5, F_4}{\bullet h_1 : \Delta_2 \vdash (\top, \Delta_5), F_3 \rightarrow F_4} \rightarrow_R \rightarrow \text{trivial}$$

- Case rule \wedge_R

$$\frac{h_1 : \Delta_2 \vdash \top, \Delta_5, F_3 \quad h_1 : \Delta_2 \vdash \top, \Delta_5, F_4}{\bullet h_1 : \Delta_2 \vdash (\top, \Delta_5), F_3 \wedge F_4} \wedge_R \rightarrow \text{trivial}$$

- Case rule \vee_R

$$\frac{h_1 : \Delta_2 \vdash \top, \Delta_5, F_3, F_4}{\bullet h_1 : \Delta_2 \vdash (\top, \Delta_5), F_3 \vee F_4} \vee_R \rightarrow \text{trivial}$$

- Case rule \perp_R

$$\frac{h_1 : \Delta_2 \vdash \top, \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \top, \Delta_3} \perp_R \rightarrow \text{trivial}$$

- Case rule \top_R

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R \rightarrow \text{trivial}$$

- Case rule \rightarrow_L

$$\frac{h_2 : \Delta_3 \vdash \top, \Delta_1, F_4 \quad h_2 : \Delta_3, F_5 \vdash \top, \Delta_1}{\bullet h_2 : \Delta_3, F_4 \rightarrow F_5 \vdash \top, \Delta_1} \rightarrow_L \rightarrow \text{trivial}$$

- Case rule \wedge_L

$$\frac{h_2 : \Delta_3, F_4, F_5 \vdash \top, \Delta_1}{\bullet h_2 : \Delta_3, F_4 \wedge F_5 \vdash \top, \Delta_1} \wedge_L \rightarrow \text{trivial}$$

- Case rule \vee_L

$$\frac{h_2 : \Delta_3, F_4 \vdash \top, \Delta_1 \quad h_2 : \Delta_3, F_5 \vdash \top, \Delta_1}{\bullet h_2 : \Delta_3, F_4 \vee F_5 \vdash \top, \Delta_1} \vee_L \rightarrow \text{trivial}$$

- Case rule \perp_L

$$\frac{}{\bullet h_2 : \perp, \Delta_3 \vdash \top, \Delta_1} \perp_L \rightarrow \text{trivial}$$

- Case rule I

$$\frac{}{\bullet h_1 : p_3, \Delta_2 \vdash p_3, \top, \Delta_4} I \rightarrow \text{trivial}$$

- Case rule \top_L

$$\frac{h_2 : \Delta_3 \vdash \top, \Delta_1}{\bullet h_2 : \top, \Delta_3 \vdash \top, \Delta_1} \top_L \rightarrow \text{trivial}$$

4.7 Status of \rightarrow_L : (Left Premise): Invertible

- Case rule \rightarrow_R

$$\frac{h_4 : \Delta_1, F_6, F_2 \rightarrow F_3 \vdash \Delta_5, F_7}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \rightarrow_R \rightarrow \frac{h_4 : \Delta_1, F_6 \vdash \Delta_5, F_2, F_7}{\bullet h_4 : \Delta_1 \vdash \Delta_5, F_2, F_6 \rightarrow F_7} \text{ax/ind} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \Delta_5, F_6 \quad h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \Delta_5, F_7}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R \rightarrow \frac{\frac{h_4 : \Delta_1 \vdash \Delta_5, F_2, F_6}{\bullet h_4 : \Delta_1 \vdash \Delta_5, F_2, F_6 \wedge F_7} \text{ax/ind} \quad \frac{h_4 : \Delta_1 \vdash \Delta_5, F_2, F_7}{\bullet h_4 : \Delta_1 \vdash \Delta_5, F_2, F_7} \text{ax/ind}}{\bullet h_4 : \Delta_1 \vdash \Delta_5, F_2, F_6 \wedge F_7} \wedge_R$$

- Case rule \vee_R

$$\frac{h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \Delta_5, F_6, F_7}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \Delta_5, F_6 \vee F_7} \vee_R \rightarrow \frac{\frac{h_4 : \Delta_1 \vdash \Delta_5, F_2, F_6, F_7}{\bullet h_4 : \Delta_1 \vdash \Delta_5, F_2, F_6 \vee F_7} \text{ax/ind}}{\bullet h_4 : \Delta_1 \vdash \Delta_5, F_2, F_6 \vee F_7} \vee_R$$

- Case rule \perp_R

$$\frac{h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \Delta_5}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \perp, \Delta_5} \perp_R \rightarrow \frac{\frac{h_4 : \Delta_1 \vdash \Delta_5, F_2}{\bullet h_4 : \Delta_1 \vdash \perp, \Delta_5, F_2} \text{ax/ind}}{\bullet h_4 : \Delta_1 \vdash \perp, \Delta_5, F_2} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \top, \Delta_5} \top_R \rightarrow \frac{}{\bullet h_4 : \Delta_1 \vdash \top, \Delta_5, F_2} \top_R$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6, F_4 \quad h_3 : \Delta_7, F_5, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L \rightarrow \frac{\frac{h_3 : \Delta_7 \vdash \Delta_6, F_1, F_4}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_5 \vdash \Delta_6, F_1}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_4 \rightarrow F_5 \vdash \Delta_6, F_1} \rightarrow_L$$

$$\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3 \quad h_1 : \Delta_2, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash \Delta_5} \rightarrow_L \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3} \text{ax}}{\bullet h_1 : \Delta_2 \vdash \Delta_5, F_3} \text{height}$$

- Case rule \wedge_L

$$\frac{\frac{h_3 : \Delta_7, F_4, F_5, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \wedge F_5 \vdash \Delta_6} \wedge_L}{\frac{h_3 : \Delta_7, F_4, F_5 \vdash \Delta_6, F_1}{\bullet h_3 : \Delta_7, F_4 \wedge F_5 \vdash \Delta_6, F_1} \wedge_L} \text{ax/ind}$$

- Case rule \vee_L

$$\frac{\frac{h_3 : \Delta_7, F_4, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \vee F_5 \vdash \Delta_6} \vee_L \quad \frac{h_3 : \Delta_7, F_5, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \vee F_5 \vdash \Delta_6} \vee_L}{\frac{h_3 : \Delta_7, F_4 \vdash \Delta_6, F_1}{\bullet h_3 : \Delta_7, F_4 \vee F_5 \vdash \Delta_6, F_1} \vee_L} \text{ax/ind}$$

- Case rule \perp_L

$$\frac{\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4} \perp_L}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_4, F_1} \perp_L$$

- Case rule I

$$\frac{\frac{}{\bullet h_3 : p_4, \Delta_6, F_1 \rightarrow F_2 \vdash p_4, \Delta_5} I}{\bullet h_3 : \Delta_6, p_4 \vdash \Delta_5, F_1, p_4} I$$

- Case rule \top_L

$$\frac{\frac{h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4} \top_L}{\frac{h_3 : \Delta_5 \vdash \Delta_4, F_1}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_4, F_1} \top_L} \text{ax/ind}$$

4.8 Status of \rightarrow_L (Right Premise): : Invertible

- Case rule \rightarrow_R

$$\frac{\frac{h_4 : \Delta_1, F_6, F_2 \rightarrow F_3 \vdash \Delta_5, F_7}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \rightarrow_R}{\frac{h_4 : \Delta_1, F_3, F_6 \vdash \Delta_5, F_7}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \rightarrow_R} \text{ax/ind}$$

- Case rule \wedge_R

$$\frac{\frac{h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \Delta_5, F_6 \quad h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \Delta_5, F_7}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R}{\frac{h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R} \text{ax/ind}$$

- Case rule \vee_R

$$\frac{\frac{h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \Delta_5, F_6, F_7}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \Delta_5, F_6 \vee F_7} \vee_R}{\frac{h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6, F_7}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \vee F_7} \vee_R} \text{ax/ind}$$

- Case rule \perp_R

$$\frac{\frac{h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \Delta_5}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \perp, \Delta_5} \perp_R}{\frac{h_4 : \Delta_1, F_3 \vdash \Delta_5}{\bullet h_4 : \Delta_1, F_3 \vdash \perp, \Delta_5} \perp_R} \text{ax/ind}$$

- Case rule \top_R

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \rightarrow F_3 \vdash \top, \Delta_5} \top_R \rightarrow \frac{}{\bullet h_4 : \Delta_1, F_3 \vdash \top, \Delta_5} \top_R$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7, F_1 \rightarrow F_2 \vdash \Delta_6, F_4 \quad h_3 : \Delta_7, F_5, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L \rightarrow \frac{\frac{}{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4} \text{ax/ind} \quad \frac{}{h_3 : \Delta_7, F_2, F_5 \vdash \Delta_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L$$

$$\frac{h_1 : \Delta_2 \vdash \Delta_5, F_3 \quad h_1 : \Delta_2, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \rightarrow F_4 \vdash \Delta_5} \rightarrow_L \rightarrow \frac{\frac{}{h_1 : \Delta_2, F_4 \vdash \Delta_5} \text{ax}}{\bullet h_1 : \Delta_2, F_4 \vdash \Delta_5} \text{height}}$$

- Case rule \wedge_L

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \wedge F_5 \vdash \Delta_6} \wedge_L \rightarrow \frac{\frac{}{h_3 : \Delta_7, F_2, F_4, F_5 \vdash \Delta_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \wedge F_5 \vdash \Delta_6} \wedge_L$$

- Case rule \vee_L

$$\frac{h_3 : \Delta_7, F_4, F_1 \rightarrow F_2 \vdash \Delta_6 \quad h_3 : \Delta_7, F_5, F_1 \rightarrow F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \rightarrow F_2), F_4 \vee F_5 \vdash \Delta_6} \vee_L \rightarrow \frac{\frac{}{h_3 : \Delta_7, F_2, F_4 \vdash \Delta_6} \text{ax/ind} \quad \frac{}{h_3 : \Delta_7, F_2, F_5 \vdash \Delta_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \vee F_5 \vdash \Delta_6} \vee_L$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5, F_2 \vdash \Delta_4} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_4, \Delta_6, F_1 \rightarrow F_2 \vdash p_4, \Delta_5} I \rightarrow \frac{}{\bullet h_3 : \Delta_6, F_2, p_4 \vdash \Delta_5, p_4} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \rightarrow F_2 \vdash \Delta_4} \top_L \rightarrow \frac{\frac{}{h_3 : \Delta_5, F_2 \vdash \Delta_4} \text{ax/ind}}{\bullet h_3 : \top, \Delta_5, F_2 \vdash \Delta_4} \top_L$$

4.9 Status of \wedge_L : : Invertible

- Case rule \rightarrow_R

$$\frac{h_4 : \Delta_1, F_6, F_2 \wedge F_3 \vdash \Delta_5, F_7}{\bullet h_4 : \Delta_1, F_2 \wedge F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \rightarrow_R \rightarrow \frac{\frac{}{h_4 : \Delta_1, F_2, F_3, F_6 \vdash \Delta_5, F_7} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_4 : \Delta_1, F_2 \wedge F_3 \vdash \Delta_5, F_6 \quad h_4 : \Delta_1, F_2 \wedge F_3 \vdash \Delta_5, F_7}{\bullet h_4 : \Delta_1, F_2 \wedge F_3 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R \rightarrow \frac{\frac{}{h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_6} \text{ax/ind} \quad \frac{}{h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_7} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R$$

- Case rule \vee_R

$$\frac{h_4 : \Delta_1, F_2 \wedge F_3 \vdash \Delta_5, F_6, F_7}{\bullet h_4 : \Delta_1, F_2 \wedge F_3 \vdash \Delta_5, F_6 \vee F_7} \vee_R \rightarrow \frac{\frac{h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_6, F_7}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5, F_6 \vee F_7} \text{ax/ind}}{\vee_R}$$

- Case rule \perp_R

$$\frac{h_4 : \Delta_1, F_2 \wedge F_3 \vdash \Delta_5}{\bullet h_4 : \Delta_1, F_2 \wedge F_3 \vdash \perp, \Delta_5} \perp_R \rightarrow \frac{\frac{h_4 : \Delta_1, F_2, F_3 \vdash \Delta_5}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \perp, \Delta_5} \text{ax/ind}}{\perp_R}$$

- Case rule \top_R

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \wedge F_3 \vdash \top, \Delta_5} \top_R \rightarrow \frac{}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \top, \Delta_5} \top_R$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7, F_1 \wedge F_2 \vdash \Delta_6, F_4}{\bullet h_3 : (\Delta_7, F_1 \wedge F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_2 \vdash \Delta_6, F_4}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \rightarrow F_5 \vdash \Delta_6} \text{ax/ind}}{\rightarrow_L} \frac{\frac{h_3 : \Delta_7, F_1, F_2, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_5 \vdash \Delta_6} \text{ax/ind}}{\rightarrow_L}$$

- Case rule \wedge_L

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \wedge F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \wedge F_2), F_4 \wedge F_5 \vdash \Delta_6} \wedge_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_2, F_4, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \wedge F_5 \vdash \Delta_6} \text{ax/ind}}{\wedge_L}$$

$$\frac{h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \wedge F_4 \vdash \Delta_5} \wedge_L \rightarrow \frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3, F_4 \vdash \Delta_5} \text{ax}}{\text{height}}$$

- Case rule \vee_L

$$\frac{h_3 : \Delta_7, F_4, F_1 \wedge F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \wedge F_2), F_4 \vee F_5 \vdash \Delta_6} \vee_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_2, F_4 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \vee F_5 \vdash \Delta_6} \text{ax/ind}}{\vee_L} \frac{\frac{h_3 : \Delta_7, F_1, F_2, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_5 \vdash \Delta_6} \text{ax/ind}}{\vee_L}$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \wedge F_2 \vdash \Delta_4} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5, F_1, F_2 \vdash \Delta_4} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_4, \Delta_6, F_1 \wedge F_2 \vdash p_4, \Delta_5} I \rightarrow \frac{}{\bullet h_3 : \Delta_6, F_1, F_2, p_4 \vdash \Delta_5, p_4} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5, F_1 \wedge F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \wedge F_2 \vdash \Delta_4} \top_L \rightarrow \frac{\frac{h_3 : \Delta_5, F_1, F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1, F_2 \vdash \Delta_4} \text{ax/ind}}{\top_L}$$

4.10 Status of \vee_L : (Left Premise): Invertible

- Case rule \rightarrow_R

$$\frac{\frac{h_4 : \Delta_1, F_6, F_2 \vee F_3 \vdash \Delta_5, F_7}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \rightarrow_R}{\bullet h_4 : \Delta_1, F_2 \vdash \Delta_5, F_6 \rightarrow F_7} \rightarrow \frac{\frac{h_4 : \Delta_1, F_2, F_6 \vdash \Delta_5, F_7}{\bullet h_4 : \Delta_1, F_2 \vdash \Delta_5, F_6 \rightarrow F_7} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_2 \vdash \Delta_5, F_6 \rightarrow F_7} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{\frac{h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5, F_6 \quad h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5, F_7}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R}{\bullet h_4 : \Delta_1, F_2 \vdash \Delta_5, F_6 \wedge F_7} \rightarrow \frac{\frac{h_4 : \Delta_1, F_2 \vdash \Delta_5, F_6}{\bullet h_4 : \Delta_1, F_2 \vdash \Delta_5, F_6 \wedge F_7} \text{ax/ind} \quad \frac{h_4 : \Delta_1, F_2 \vdash \Delta_5, F_7}{\bullet h_4 : \Delta_1, F_2 \vdash \Delta_5, F_6 \wedge F_7} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_2 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R$$

- Case rule \vee_R

$$\frac{\frac{h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5, F_6, F_7}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5, F_6 \vee F_7} \vee_R}{\bullet h_4 : \Delta_1, F_2 \vdash \Delta_5, F_6 \vee F_7} \rightarrow \frac{\frac{h_4 : \Delta_1, F_2 \vdash \Delta_5, F_6, F_7}{\bullet h_4 : \Delta_1, F_2 \vdash \Delta_5, F_6 \vee F_7} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_2 \vdash \Delta_5, F_6 \vee F_7} \vee_R$$

- Case rule \perp_R

$$\frac{\frac{h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \perp, \Delta_5} \perp_R}{\bullet h_4 : \Delta_1, F_2 \vdash \perp, \Delta_5} \rightarrow \frac{\frac{h_4 : \Delta_1, F_2 \vdash \Delta_5}{\bullet h_4 : \Delta_1, F_2 \vdash \perp, \Delta_5} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_2 \vdash \perp, \Delta_5} \perp_R$$

- Case rule \top_R

$$\frac{\frac{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \top, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \vdash \top, \Delta_5} \top_R}{\bullet h_4 : \Delta_1, F_2 \vdash \top, \Delta_5} \rightarrow \frac{\frac{\bullet h_4 : \Delta_1, F_2 \vdash \top, \Delta_5}{\bullet h_4 : \Delta_1, F_2 \vdash \top, \Delta_5} \top_R}{\bullet h_4 : \Delta_1, F_2 \vdash \top, \Delta_5} \top_R$$

- Case rule \rightarrow_L

$$\frac{\frac{h_3 : \Delta_7, F_1 \vee F_2 \vdash \Delta_6, F_4 \quad h_3 : \Delta_7, F_5, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L}{\bullet h_3 : (\Delta_7, F_1, F_4 \rightarrow F_5 \vdash \Delta_6)} \rightarrow \frac{\frac{h_3 : \Delta_7, F_1 \vdash \Delta_6, F_4}{\bullet h_3 : \Delta_7, F_1, F_4 \rightarrow F_5 \vdash \Delta_6} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_1, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_4 \rightarrow F_5 \vdash \Delta_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{\frac{h_3 : \Delta_7, F_4, F_5, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \wedge F_5 \vdash \Delta_6} \wedge_L}{\bullet h_3 : \Delta_7, F_1, F_4 \wedge F_5 \vdash \Delta_6} \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_4, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_4 \wedge F_5 \vdash \Delta_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \wedge F_5 \vdash \Delta_6} \wedge_L$$

- Case rule \vee_L

$$\frac{\frac{h_3 : \Delta_7, F_4, F_1 \vee F_2 \vdash \Delta_6 \quad h_3 : \Delta_7, F_5, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \vee F_5 \vdash \Delta_6} \vee_L}{\bullet h_3 : \Delta_7, F_1, F_4 \vee F_5 \vdash \Delta_6} \rightarrow \frac{\frac{h_3 : \Delta_7, F_1, F_4 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_4 \vee F_5 \vdash \Delta_6} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_1, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_1, F_4 \vee F_5 \vdash \Delta_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \vee F_5 \vdash \Delta_6} \vee_L$$

$$\frac{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5 \quad h_1 : \Delta_2, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5} \vee_L}{\bullet h_1 : \Delta_2, F_3 \vdash \Delta_5} \rightarrow \frac{\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \vdash \Delta_5} \text{ax}}{\bullet h_1 : \Delta_2, F_3 \vdash \Delta_5} \text{height}$$

- Case rule \perp_L

$$\frac{\frac{\bullet h_3 : \perp, \Delta_5, F_1 \vee F_2 \vdash \Delta_4}{\bullet h_3 : \perp, \Delta_5, F_1 \vdash \Delta_4} \perp_L}{\bullet h_3 : \perp, \Delta_5, F_1 \vdash \Delta_4} \rightarrow \frac{\frac{\bullet h_3 : \perp, \Delta_5, F_1 \vdash \Delta_4}{\bullet h_3 : \perp, \Delta_5, F_1 \vdash \Delta_4} \perp_L}{\bullet h_3 : \perp, \Delta_5, F_1 \vdash \Delta_4} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_4, \Delta_6, F_1 \vee F_2 \vdash p_4, \Delta_5} I \rightarrow \frac{}{\bullet h_3 : \Delta_6, F_1, p_4 \vdash \Delta_5, p_4} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5, F_1 \vee F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \vee F_2 \vdash \Delta_4} \top_L \rightarrow \frac{\frac{}{h_3 : \Delta_5, F_1 \vdash \Delta_4} \text{ax/ind}}{\bullet h_3 : \top, \Delta_5, F_1 \vdash \Delta_4} \top_L$$

4.11 Status of \vee_L (Right Premise): : Invertible

- Case rule \rightarrow_R

$$\frac{h_4 : \Delta_1, F_6, F_2 \vee F_3 \vdash \Delta_5, F_7}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \rightarrow_R \rightarrow \frac{\frac{}{h_4 : \Delta_1, F_3 \vdash \Delta_5, F_7} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \rightarrow F_7} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5, F_6 \quad h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5, F_7}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R \rightarrow \frac{\frac{}{h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6} \text{ax/ind} \quad \frac{}{h_4 : \Delta_1, F_3 \vdash \Delta_5, F_7} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \wedge F_7} \wedge_R$$

- Case rule \vee_R

$$\frac{h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5, F_6, F_7}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5, F_6 \vee F_7} \vee_R \rightarrow \frac{\frac{}{h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6, F_7} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash \Delta_5, F_6 \vee F_7} \vee_R$$

- Case rule \perp_R

$$\frac{h_4 : \Delta_1, F_2 \vee F_3 \vdash \Delta_5}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \perp, \Delta_5} \perp_R \rightarrow \frac{\frac{}{h_4 : \Delta_1, F_3 \vdash \Delta_5} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash \perp, \Delta_5} \perp_R$$

- Case rule \top_R

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \vee F_3 \vdash \top, \Delta_5} \top_R \rightarrow \frac{}{\bullet h_4 : \Delta_1, F_3 \vdash \top, \Delta_5} \top_R$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_7, F_1 \vee F_2 \vdash \Delta_6, F_4 \quad h_3 : \Delta_7, F_5, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L \rightarrow \frac{\frac{}{h_3 : \Delta_7, F_2 \vdash \Delta_6, F_4} \text{ax/ind} \quad \frac{}{h_3 : \Delta_7, F_5 \vdash \Delta_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \rightarrow F_5 \vdash \Delta_6} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \wedge F_5 \vdash \Delta_6} \wedge_L \rightarrow \frac{\frac{}{h_3 : \Delta_7, F_2, F_4, F_5 \vdash \Delta_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_2, F_4 \wedge F_5 \vdash \Delta_6} \wedge_L$$

- Case rule \vee_L

$$\frac{h_3 : \Delta_7, F_4, F_1 \vee F_2 \vdash \Delta_6 \quad h_3 : \Delta_7, F_5, F_1 \vee F_2 \vdash \Delta_6}{\bullet h_3 : (\Delta_7, F_1 \vee F_2), F_4 \vee F_5 \vdash \Delta_6} \vee_L \rightarrow \frac{\frac{h_3 : \Delta_7, F_2, F_4 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_2, F_4 \vee F_5 \vdash \Delta_6} \text{ax/ind} \quad \frac{h_3 : \Delta_7, F_2, F_5 \vdash \Delta_6}{\bullet h_3 : \Delta_7, F_2, F_4 \vee F_5 \vdash \Delta_6} \text{ax/ind}}{\vee_L}$$

$$\frac{h_1 : \Delta_2, F_3 \vdash \Delta_5 \quad h_1 : \Delta_2, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_3 \vee F_4 \vdash \Delta_5} \vee_L \rightarrow \frac{h_1 : \Delta_2, F_4 \vdash \Delta_5}{\bullet h_1 : \Delta_2, F_4 \vdash \Delta_5} \text{ax} \text{ height}$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_5, F_1 \vee F_2 \vdash \Delta_4} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_5, F_2 \vdash \Delta_4} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_3 : p_4, \Delta_6, F_1 \vee F_2 \vdash p_4, \Delta_5} I \rightarrow \frac{}{\bullet h_3 : \Delta_6, F_2, p_4 \vdash \Delta_5, p_4} I$$

- Case rule \top_L

$$\frac{h_3 : \Delta_5, F_1 \vee F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_1 \vee F_2 \vdash \Delta_4} \top_L \rightarrow \frac{h_3 : \Delta_5, F_2 \vdash \Delta_4}{\bullet h_3 : \top, \Delta_5, F_2 \vdash \Delta_4} \text{ax/ind} \top_L$$

4.12 Status of \perp_L : : Invertible

- Case rule \rightarrow_R

$$\frac{h_2 : \perp, \Delta_1, F_4 \vdash \Delta_3, F_5}{\bullet h_2 : \perp, \Delta_1 \vdash \Delta_3, F_4 \rightarrow F_5} \rightarrow_R \rightarrow \text{trivial}$$

- Case rule \wedge_R

$$\frac{h_2 : \perp, \Delta_1 \vdash \Delta_3, F_4 \quad h_2 : \perp, \Delta_1 \vdash \Delta_3, F_5}{\bullet h_2 : \perp, \Delta_1 \vdash \Delta_3, F_4 \wedge F_5} \wedge_R \rightarrow \text{trivial}$$

- Case rule \vee_R

$$\frac{h_2 : \perp, \Delta_1 \vdash \Delta_3, F_4, F_5}{\bullet h_2 : \perp, \Delta_1 \vdash \Delta_3, F_4 \vee F_5} \vee_R \rightarrow \text{trivial}$$

- Case rule \perp_R

$$\frac{h_2 : \perp, \Delta_1 \vdash \Delta_3}{\bullet h_2 : \perp, \Delta_1 \vdash \perp, \Delta_3} \perp_R \rightarrow \text{trivial}$$

- Case rule \top_R

$$\frac{}{\bullet h_2 : \perp, \Delta_1 \vdash \top, \Delta_3} \top_R \rightarrow \text{trivial}$$

- Case rule \rightarrow_L

$$\frac{h_1 : \perp, \Delta_5 \vdash \Delta_4, F_2 \quad h_1 : \perp, \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : (\perp, \Delta_5), F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L \rightarrow \text{trivial}$$

- Case rule \wedge_L

$$\frac{h_1 : \perp, \Delta_5, F_2, F_3 \vdash \Delta_4}{\bullet h_1 : (\perp, \Delta_5), F_2 \wedge F_3 \vdash \Delta_4} \wedge_L \rightarrow \text{trivial}$$

- Case rule \vee_L

$$\frac{h_1 : \perp, \Delta_5, F_2 \vdash \Delta_4 \quad h_1 : \perp, \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : (\perp, \Delta_5), F_2 \vee F_3 \vdash \Delta_4} \vee_L \rightarrow \text{trivial}$$

- Case rule \perp_L

$$\frac{}{\bullet h_1 : \perp, \Delta_2 \vdash \Delta_3} \perp_L \rightarrow \text{trivial}$$

- Case rule I

$$\frac{}{\bullet h_1 : p_2, \perp, \Delta_4 \vdash p_2, \Delta_3} I \rightarrow \text{trivial}$$

- Case rule \top_L

$$\frac{h_1 : \perp, \Delta_3 \vdash \Delta_2}{\bullet h_1 : \top, \perp, \Delta_3 \vdash \Delta_2} \top_L \rightarrow \text{trivial}$$

4.13 Status of I : : Invertible

- Case rule \rightarrow_R

$$\frac{h_3 : \Delta_1, F_4, p_2 \vdash \Delta_6, F_5, p_2}{\bullet h_3 : \Delta_1, p_2 \vdash (\Delta_6, p_2), F_4 \rightarrow F_5} \rightarrow_R \rightarrow \text{trivial}$$

- Case rule \wedge_R

$$\frac{h_3 : \Delta_1, p_2 \vdash \Delta_6, F_4, p_2 \quad h_3 : \Delta_1, p_2 \vdash \Delta_6, F_5, p_2}{\bullet h_3 : \Delta_1, p_2 \vdash (\Delta_6, p_2), F_4 \wedge F_5} \wedge_R \rightarrow \text{trivial}$$

- Case rule \vee_R

$$\frac{h_3 : \Delta_1, p_2 \vdash \Delta_6, F_4, F_5, p_2}{\bullet h_3 : \Delta_1, p_2 \vdash (\Delta_6, p_2), F_4 \vee F_5} \vee_R \rightarrow \text{trivial}$$

- Case rule \perp_R

$$\frac{h_3 : \Delta_1, p_2 \vdash \Delta_4, p_2}{\bullet h_3 : \Delta_1, p_2 \vdash \perp, \Delta_4, p_2} \perp_R \rightarrow \text{trivial}$$

- Case rule \top_R

$$\frac{}{\bullet h_3 : \Delta_1, P_2 \vdash \top, \Delta_4, P_2} \top_R \rightarrow \text{trivial}$$

- Case rule \rightarrow_L

$$\frac{h_3 : \Delta_6, P_1 \vdash \Delta_2, F_4, P_1 \quad h_3 : \Delta_6, F_5, P_1 \vdash \Delta_2, P_1}{\bullet h_3 : (\Delta_6, P_1), F_4 \rightarrow F_5 \vdash \Delta_2, P_1} \rightarrow_L \rightarrow \text{trivial}$$

- Case rule \wedge_L

$$\frac{h_3 : \Delta_6, F_4, F_5, P_1 \vdash \Delta_2, P_1}{\bullet h_3 : (\Delta_6, P_1), F_4 \wedge F_5 \vdash \Delta_2, P_1} \wedge_L \rightarrow \text{trivial}$$

- Case rule \vee_L

$$\frac{h_3 : \Delta_6, F_4, P_1 \vdash \Delta_2, P_1 \quad h_3 : \Delta_6, F_5, P_1 \vdash \Delta_2, P_1}{\bullet h_3 : (\Delta_6, P_1), F_4 \vee F_5 \vdash \Delta_2, P_1} \vee_L \rightarrow \text{trivial}$$

- Case rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_4, P_1 \vdash \Delta_2, P_1} \perp_L \rightarrow \text{trivial}$$

- Case rule I

$$\frac{}{\bullet h_2 : P_3, \Delta_5, P_1 \vdash P_3, \Delta_4, P_1} I \rightarrow \text{trivial}$$

$$\frac{}{\bullet h_1 : P_3, \Delta_2 \vdash P_3, \Delta_4} I \rightarrow \text{trivial}$$

- Case rule \top_L

$$\frac{h_3 : \Delta_4, P_1 \vdash \Delta_2, P_1}{\bullet h_3 : \top, \Delta_4, P_1 \vdash \Delta_2, P_1} \top_L \rightarrow \text{trivial}$$

4.14 Status of \top_L : : Invertible

- Case rule \rightarrow_R

$$\frac{h_2 : \top, \Delta_1, F_4 \vdash \Delta_3, F_5}{\bullet h_2 : \top, \Delta_1 \vdash \Delta_3, F_4 \rightarrow F_5} \rightarrow_R \rightarrow \frac{}{\bullet h_2 : \Delta_1 \vdash \Delta_3, F_4 \rightarrow F_5} \xrightarrow{\text{ax/ind}} \frac{h_2 : \Delta_1, F_4 \vdash \Delta_3, F_5}{\bullet h_2 : \Delta_1 \vdash \Delta_3, F_4 \rightarrow F_5} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{h_2 : \top, \Delta_1 \vdash \Delta_3, F_4 \quad h_2 : \top, \Delta_1 \vdash \Delta_3, F_5}{\bullet h_2 : \top, \Delta_1 \vdash \Delta_3, F_4 \wedge F_5} \wedge_R \rightarrow \frac{}{\bullet h_2 : \Delta_1 \vdash \Delta_3, F_4 \wedge F_5} \xrightarrow{\text{ax/ind}} \frac{h_2 : \Delta_1 \vdash \Delta_3, F_4}{\bullet h_2 : \Delta_1 \vdash \Delta_3, F_4 \wedge F_5} \xrightarrow{\text{ax/ind}} \frac{h_2 : \Delta_1 \vdash \Delta_3, F_5}{\bullet h_2 : \Delta_1 \vdash \Delta_3, F_4 \wedge F_5} \wedge_R$$

- Case rule \vee_R

$$\frac{h_2 : \top, \Delta_1 \vdash \Delta_3, F_4, F_5}{\bullet h_2 : \top, \Delta_1 \vdash \Delta_3, F_4 \vee F_5} \vee_R \rightarrow \frac{\frac{h_2 : \Delta_1 \vdash \Delta_3, F_4, F_5}{\bullet h_2 : \Delta_1 \vdash \Delta_3, F_4 \vee F_5} \text{ ax/ind}}{\vee_R}$$

- Case rule \perp_R

$$\frac{h_2 : \top, \Delta_1 \vdash \Delta_3}{\bullet h_2 : \top, \Delta_1 \vdash \perp, \Delta_3} \perp_R \rightarrow \frac{\frac{h_2 : \Delta_1 \vdash \Delta_3}{\bullet h_2 : \Delta_1 \vdash \perp, \Delta_3} \text{ ax/ind}}{\perp_R}$$

- Case rule \top_R

$$\frac{}{\bullet h_2 : \top, \Delta_1 \vdash \top, \Delta_3} \top_R \rightarrow \frac{}{\bullet h_2 : \Delta_1 \vdash \top, \Delta_3} \top_R$$

- Case rule \rightarrow_L

$$\frac{h_1 : \top, \Delta_5 \vdash \Delta_4, F_2 \quad h_1 : \top, \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : (\top, \Delta_5), F_2 \rightarrow F_3 \vdash \Delta_4} \rightarrow_L \rightarrow \frac{\frac{h_1 : \Delta_5 \vdash \Delta_4, F_2}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \text{ ax/ind} \quad \frac{h_1 : \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \rightarrow F_3 \vdash \Delta_4} \text{ ax/ind}}{\rightarrow_L}$$

- Case rule \wedge_L

$$\frac{h_1 : \top, \Delta_5, F_2, F_3 \vdash \Delta_4}{\bullet h_1 : (\top, \Delta_5), F_2 \wedge F_3 \vdash \Delta_4} \wedge_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \wedge F_3 \vdash \Delta_4} \text{ ax/ind}}{\wedge_L}$$

- Case rule \vee_L

$$\frac{h_1 : \top, \Delta_5, F_2 \vdash \Delta_4 \quad h_1 : \top, \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : (\top, \Delta_5), F_2 \vee F_3 \vdash \Delta_4} \vee_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \text{ ax/ind} \quad \frac{h_1 : \Delta_5, F_3 \vdash \Delta_4}{\bullet h_1 : \Delta_5, F_2 \vee F_3 \vdash \Delta_4} \text{ ax/ind}}{\vee_L}$$

- Case rule \perp_L

$$\frac{}{\bullet h_1 : \perp, \top, \Delta_3 \vdash \Delta_2} \perp_L \rightarrow \frac{}{\bullet h_1 : \perp, \Delta_3 \vdash \Delta_2} \perp_L$$

- Case rule I

$$\frac{}{\bullet h_1 : p_2, \top, \Delta_4 \vdash p_2, \Delta_3} I \rightarrow \frac{}{\bullet h_1 : \Delta_4, p_2 \vdash \Delta_3, p_2} I$$

- Case rule \top_L

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \top, \Delta_2 \vdash \Delta_3} \top_L \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3} \text{ ax}}{\text{height}}$$

- Case(s) rule \perp_L

$$\frac{}{\bullet h_2 : \Delta_1, \perp, \perp \vdash \Delta_3} \perp_L \rightarrow \frac{}{\bullet h_2 : \perp, \Delta_1 \vdash \Delta_3} \perp_L$$

$$\frac{}{\bullet h_2 : (\perp, \Delta_4), F_1, F_1 \vdash \Delta_3} \perp_L \rightarrow \frac{}{\bullet h_2 : \perp, \Delta_4, F_1 \vdash \Delta_3} \perp_L$$

- Case(s) rule I

$$\frac{}{\bullet h_2 : \Delta_1, P_3, P_3 \vdash \Delta_4, P_3} I \rightarrow \frac{}{\bullet h_2 : \Delta_1, P_3 \vdash \Delta_4, P_3} I$$

$$\frac{}{\bullet h_2 : (\Delta_5, P_3), F_1, F_1 \vdash \Delta_4, P_3} I \rightarrow \frac{}{\bullet h_2 : \Delta_5, F_1, P_3 \vdash \Delta_4, P_3} I$$

- Case(s) rule \top_L

$$\frac{h_2 : \top, \Delta_1 \vdash \Delta_3}{\bullet h_2 : \Delta_1, \top, \top \vdash \Delta_3} \top_L \rightarrow \frac{h_2 : \Delta_1 \vdash \Delta_3}{\bullet h_2 : \top, \Delta_1 \vdash \Delta_3} \top_L \text{ inv-th/ax}$$

$$\frac{h_2 : \Delta_4, F_1, F_1 \vdash \Delta_3}{\bullet h_2 : (\top, \Delta_4), F_1, F_1 \vdash \Delta_3} \top_L \rightarrow \frac{\frac{h_2 : \Delta_4, F_1, F_1 \vdash \Delta_3}{h_2 : \Delta_4, F_1 \vdash \Delta_3} \text{IH}}{\bullet h_2 : \top, \Delta_4, F_1 \vdash \Delta_3} \top_L \text{ ax}$$

6 Height preserving admissibility of contraction on the Right

- Case(s) rule \rightarrow_R

$$\frac{\frac{h_2 : \Delta_3, F_4 \vdash \Delta_1, F_5, F_4 \rightarrow F_5}{\bullet h_2 : \Delta_3 \vdash \Delta_1, F_4 \rightarrow F_5, F_4 \rightarrow F_5} \rightarrow_R}{\frac{h_2 : \Delta_3, F_4 \vdash \Delta_6, F_1, F_1, F_5}{\bullet h_2 : \Delta_3 \vdash (\Delta_6, F_4 \rightarrow F_5), F_1, F_1} \rightarrow_R} \rightarrow \frac{\frac{\frac{h_2 : \Delta_3, F_4, F_4 \vdash \Delta_1, F_5, F_5}{\bullet h_2 : \Delta_3 \vdash \Delta_1, F_4 \rightarrow F_5} \text{inv-th/ax}}{\frac{h_2 : \Delta_3, F_4 \vdash \Delta_1, F_5, F_5}{\bullet h_2 : \Delta_3 \vdash \Delta_1, F_4 \rightarrow F_5} \text{IH-Mutual}} \rightarrow_R}{\frac{h_2 : \Delta_3, F_4 \vdash \Delta_6, F_1, F_1, F_5}{\bullet h_2 : \Delta_3 \vdash \Delta_6, F_1, F_4 \rightarrow F_5} \text{ax}}{\frac{h_2 : \Delta_3, F_4 \vdash \Delta_6, F_1, F_5}{\bullet h_2 : \Delta_3 \vdash \Delta_6, F_1, F_4 \rightarrow F_5} \text{IH}} \rightarrow_R} \rightarrow_R$$

- Case(s) rule \wedge_R

$$\frac{\frac{h_2 : \Delta_3 \vdash \Delta_1, F_4, F_4 \wedge F_5}{\bullet h_2 : \Delta_3 \vdash \Delta_1, F_4 \wedge F_5, F_4 \wedge F_5} \wedge_R}{\frac{h_2 : \Delta_3 \vdash \Delta_6, F_1, F_1, F_4}{\bullet h_2 : \Delta_3 \vdash (\Delta_6, F_4 \wedge F_5), F_1, F_1} \wedge_R} \rightarrow \frac{\frac{\frac{h_2 : \Delta_3 \vdash \Delta_1, F_4, F_4}{\bullet h_2 : \Delta_3 \vdash \Delta_1, F_4} \text{inv-th/ax}}{\frac{h_2 : \Delta_3 \vdash \Delta_1, F_5, F_5}{\bullet h_2 : \Delta_3 \vdash \Delta_1, F_5} \text{IH}} \wedge_R}{\frac{h_2 : \Delta_3 \vdash \Delta_1, F_4}{\bullet h_2 : \Delta_3 \vdash \Delta_1, F_4 \wedge F_5} \text{inv-th/ax}}{\frac{h_2 : \Delta_3 \vdash \Delta_1, F_5}{\bullet h_2 : \Delta_3 \vdash \Delta_1, F_5} \text{IH}} \wedge_R} \wedge_R$$

$$\frac{\frac{h_2 : \Delta_3 \vdash \Delta_6, F_1, F_1, F_4}{\bullet h_2 : \Delta_3 \vdash (\Delta_6, F_4 \wedge F_5), F_1, F_1} \wedge_R}{\frac{h_2 : \Delta_3 \vdash \Delta_6, F_1, F_1, F_5}{\bullet h_2 : \Delta_3 \vdash \Delta_6, F_1, F_4 \wedge F_5} \wedge_R} \rightarrow \frac{\frac{\frac{h_2 : \Delta_3 \vdash \Delta_6, F_1, F_1, F_4}{\bullet h_2 : \Delta_3 \vdash \Delta_6, F_1, F_4} \text{ax}}{\frac{h_2 : \Delta_3 \vdash \Delta_6, F_1, F_5}{\bullet h_2 : \Delta_3 \vdash \Delta_6, F_1, F_5} \text{IH}} \wedge_R}{\frac{h_2 : \Delta_3 \vdash \Delta_6, F_1, F_4}{\bullet h_2 : \Delta_3 \vdash \Delta_6, F_1, F_4 \wedge F_5} \text{ax}}{\frac{h_2 : \Delta_3 \vdash \Delta_6, F_1, F_5}{\bullet h_2 : \Delta_3 \vdash \Delta_6, F_1, F_5} \text{IH}} \wedge_R} \wedge_R$$

- Case(s) rule \vee_R

$$\frac{\frac{h_2 : \Delta_3 \vdash \Delta_1, F_4, F_5, F_4 \vee F_5}{\bullet h_2 : \Delta_3 \vdash \Delta_1, F_4 \vee F_5, F_4 \vee F_5} \vee_R}{\frac{h_2 : \Delta_3 \vdash \Delta_6, F_1, F_1, F_4}{\bullet h_2 : \Delta_3 \vdash (\Delta_6, F_4 \vee F_5), F_1, F_1} \vee_R} \rightarrow \frac{\frac{\frac{h_2 : \Delta_3 \vdash \Delta_1, F_4, F_4, F_5, F_5}{\bullet h_2 : \Delta_3 \vdash \Delta_1, F_4, F_4, F_5} \text{inv-th/ax}}{\frac{h_2 : \Delta_3 \vdash \Delta_1, F_4, F_5}{\bullet h_2 : \Delta_3 \vdash \Delta_1, F_4 \vee F_5} \text{IH}} \vee_R}{\frac{h_2 : \Delta_3 \vdash \Delta_1, F_4, F_5}{\bullet h_2 : \Delta_3 \vdash \Delta_1, F_4 \vee F_5} \text{IH}} \vee_R} \vee_R$$

$$\frac{\frac{h_2 : \Delta_3 \vdash \Delta_6, F_1, F_1, F_4}{\bullet h_2 : \Delta_3 \vdash (\Delta_6, F_4 \vee F_5), F_1, F_1} \vee_R}{\frac{h_2 : \Delta_3 \vdash \Delta_6, F_1, F_1, F_5}{\bullet h_2 : \Delta_3 \vdash \Delta_6, F_1, F_4 \vee F_5} \vee_R} \rightarrow \frac{\frac{\frac{h_2 : \Delta_3 \vdash \Delta_6, F_1, F_1, F_4}{\bullet h_2 : \Delta_3 \vdash \Delta_6, F_1, F_4} \text{ax}}{\frac{h_2 : \Delta_3 \vdash \Delta_6, F_1, F_5}{\bullet h_2 : \Delta_3 \vdash \Delta_6, F_1, F_5} \text{IH}} \vee_R}{\frac{h_2 : \Delta_3 \vdash \Delta_6, F_1, F_4}{\bullet h_2 : \Delta_3 \vdash \Delta_6, F_1, F_4 \vee F_5} \text{ax}}{\frac{h_2 : \Delta_3 \vdash \Delta_6, F_1, F_5}{\bullet h_2 : \Delta_3 \vdash \Delta_6, F_1, F_5} \text{IH}} \vee_R} \vee_R$$

- Case(s) rule \perp_R

$$\frac{\frac{h_2 : \Delta_3 \vdash \perp, \Delta_1}{\bullet h_2 : \Delta_3 \vdash \Delta_1, \perp, \perp} \perp_R}{\frac{h_2 : \Delta_3 \vdash \Delta_4, F_1, F_1}{\bullet h_2 : \Delta_3 \vdash (\perp, \Delta_4), F_1, F_1} \perp_R} \rightarrow \frac{\frac{h_2 : \Delta_3 \vdash \perp, \Delta_1}{\bullet h_2 : \Delta_3 \vdash \perp, \Delta_1} \text{inv-th/ax}}{\frac{h_2 : \Delta_3 \vdash \perp, \Delta_1}{\bullet h_2 : \Delta_3 \vdash \perp, \Delta_1} \perp_R} \perp_R$$

$$\frac{\frac{h_2 : \Delta_3 \vdash \Delta_4, F_1, F_1}{\bullet h_2 : \Delta_3 \vdash (\perp, \Delta_4), F_1, F_1} \perp_R}{\frac{h_2 : \Delta_3 \vdash \perp, \Delta_4, F_1}{\bullet h_2 : \Delta_3 \vdash \perp, \Delta_4, F_1} \perp_R} \rightarrow \frac{\frac{\frac{h_2 : \Delta_3 \vdash \Delta_4, F_1, F_1}{\bullet h_2 : \Delta_3 \vdash \Delta_4, F_1} \text{ax}}{\frac{h_2 : \Delta_3 \vdash \perp, \Delta_4, F_1}{\bullet h_2 : \Delta_3 \vdash \perp, \Delta_4, F_1} \text{IH}} \perp_R}{\frac{h_2 : \Delta_3 \vdash \perp, \Delta_4, F_1}{\bullet h_2 : \Delta_3 \vdash \perp, \Delta_4, F_1} \perp_R} \perp_R$$

- Case(s) rule \top_R

$$\frac{\frac{h_2 : \Delta_3 \vdash \Delta_1, \top, \top}{\bullet h_2 : \Delta_3 \vdash \Delta_1, \top, \top} \top_R}{\frac{h_2 : \Delta_3 \vdash (\top, \Delta_4), F_1, F_1}{\bullet h_2 : \Delta_3 \vdash (\top, \Delta_4), F_1, F_1} \top_R} \rightarrow \frac{\frac{h_2 : \Delta_3 \vdash \Delta_1, \top, \top}{\bullet h_2 : \Delta_3 \vdash \Delta_1, \top, \top} \top_R}{\frac{h_2 : \Delta_3 \vdash \top, \Delta_4, F_1}{\bullet h_2 : \Delta_3 \vdash \top, \Delta_4, F_1} \top_R} \top_R$$

- Case(s) rule \rightarrow_L

$$\frac{\frac{h_3 : \Delta_4 \vdash \Delta_1, F_2, F_2, F_5}{\bullet h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash \Delta_1, F_2, F_2} \rightarrow_L}{\frac{h_3 : \Delta_4 \vdash \Delta_1, F_2, F_2, F_5}{\bullet h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash \Delta_1, F_2, F_2} \rightarrow_L} \rightarrow \frac{\frac{\frac{h_3 : \Delta_4 \vdash \Delta_1, F_2, F_2, F_5}{\bullet h_3 : \Delta_4 \vdash \Delta_1, F_2, F_5} \text{ax}}{\frac{h_3 : \Delta_4, F_6 \vdash \Delta_1, F_2, F_2}{\bullet h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash \Delta_1, F_2} \text{IH}} \rightarrow_L}{\frac{h_3 : \Delta_4, F_6 \vdash \Delta_1, F_2, F_2}{\bullet h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash \Delta_1, F_2} \text{ax}}{\frac{h_3 : \Delta_4, F_6 \vdash \Delta_1, F_2, F_2}{\bullet h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash \Delta_1, F_2} \text{IH}} \rightarrow_L} \rightarrow_L$$

- Case(s) rule \wedge_L

$$\frac{\frac{h_3 : \Delta_4, F_5, F_6 \vdash \Delta_1, F_2, F_2}{\bullet h_3 : \Delta_4, F_5 \wedge F_6 \vdash \Delta_1, F_2, F_2}}{\wedge_L} \rightarrow \frac{\frac{\frac{h_3 : \Delta_4, F_5, F_6 \vdash \Delta_1, F_2, F_2}{\bullet h_3 : \Delta_4, F_5 \wedge F_6 \vdash \Delta_1, F_2, F_2}}{\wedge_L} \text{ IH} \quad \frac{h_3 : \Delta_4, F_5, F_6 \vdash \Delta_1, F_2, F_2}{\bullet h_3 : \Delta_4, F_5 \wedge F_6 \vdash \Delta_1, F_2, F_2}}{\wedge_L} \text{ ax}}{\wedge_L}$$

- Case(s) rule \vee_L

$$\frac{\frac{h_3 : \Delta_4, F_5 \vdash \Delta_1, F_2, F_2}{\bullet h_3 : \Delta_4, F_5 \vee F_6 \vdash \Delta_1, F_2, F_2} \vee_L \quad \frac{h_3 : \Delta_4, F_6 \vdash \Delta_1, F_2, F_2}{\bullet h_3 : \Delta_4, F_5 \vee F_6 \vdash \Delta_1, F_2, F_2} \vee_L}{\vee_L} \rightarrow \frac{\frac{\frac{h_3 : \Delta_4, F_5 \vdash \Delta_1, F_2, F_2}{\bullet h_3 : \Delta_4, F_5 \vee F_6 \vdash \Delta_1, F_2, F_2}}{\vee_L} \text{ ax} \quad \frac{h_3 : \Delta_4, F_6 \vdash \Delta_1, F_2, F_2}{\bullet h_3 : \Delta_4, F_5 \vee F_6 \vdash \Delta_1, F_2, F_2}}{\vee_L} \text{ IH}}{\vee_L}$$

- Case(s) rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_4 \vdash \Delta_1, F_2, F_2} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_4 \vdash \Delta_1, F_2} \perp_L$$

- Case(s) rule I

$$\frac{}{\bullet h_2 : \Delta_3, P_4 \vdash \Delta_1, P_4, P_4} I \rightarrow \frac{}{\bullet h_2 : \Delta_3, P_4 \vdash \Delta_1, P_4} I$$

$$\frac{}{\bullet h_2 : \Delta_3, P_4 \vdash (\Delta_5, P_4), F_1, F_1} I \rightarrow \frac{}{\bullet h_2 : \Delta_3, P_4 \vdash \Delta_5, F_1, P_4} I$$

- Case(s) rule \top_L

$$\frac{\frac{h_3 : \Delta_4 \vdash \Delta_1, F_2, F_2}{\bullet h_3 : \top, \Delta_4 \vdash \Delta_1, F_2, F_2}}{\top_L} \rightarrow \frac{\frac{\frac{h_3 : \Delta_4 \vdash \Delta_1, F_2, F_2}{\bullet h_3 : \top, \Delta_4 \vdash \Delta_1, F_2, F_2}}{\top_L} \text{ IH} \quad \frac{h_3 : \Delta_4 \vdash \Delta_1, F_2, F_2}{\bullet h_3 : \top, \Delta_4 \vdash \Delta_1, F_2, F_2}}{\top_L} \text{ ax}}{\top_L}$$

7 Identity-Expansion

$$\frac{\frac{\overline{- : F_0 \vdash F_0} \text{ IH}}{- : F_0 \vdash F_0, F_1} W \quad \frac{\overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_1 \vdash F_0, F_1} W}{- : F_0 \vee F_1 \vdash F_0, F_1} \vee_L}{- : F_0 \vee F_1 \vdash F_0 \vee F_1} \vee_R$$

$$\frac{\frac{\overline{- : F_0 \vdash F_0} \text{ IH}}{- : F_0, F_1 \vdash F_0} W \quad \frac{\overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_0, F_1 \vdash F_1} W}{- : F_0, F_1 \vdash F_0 \wedge F_1} \wedge_R}{- : F_0 \wedge F_1 \vdash F_0 \wedge F_1} \wedge_L$$

$$\frac{\frac{\overline{- : F_0 \vdash F_0} \text{ IH}}{- : F_0 \vdash F_0, F_1} W \quad \frac{\overline{- : F_1 \vdash F_1} \text{ IH}}{- : F_0, F_1 \vdash F_1} W}{- : F_0, F_0 \rightarrow F_1 \vdash F_1} \rightarrow_L}{- : F_0 \rightarrow F_1 \vdash F_0 \rightarrow F_1} \rightarrow_R$$

$$\frac{}{- : \top \vdash \top} \top_R$$

$$\frac{}{- : \perp \vdash \perp} \perp_L$$

8 Cut-Elimination

8.1 Status of \rightarrow_R : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_2 : \Delta_7, F_9 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), F_8, F_{10} \quad \rightarrow_R \quad h_{11} : \Delta_7, F_8, F_{12} \vdash \Delta_{14}, F_{13}, F_9 \rightarrow F_{10} \quad \rightarrow_R}{\bullet h_2 : \Delta_7 \vdash ((\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \rightarrow F_{10}), F_8 \quad \bullet h_{11} : \Delta_7, F_8 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \rightarrow F_{10}} \rightarrow_R \text{ Cut} \\
\frac{}{- : \Delta_7 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \rightarrow F_{10}} \rightarrow \\
\frac{\frac{h_2 : \Delta_7, F_{12}, F_9 \vdash \Delta_{14}, F_{10}, F_{13}, F_8 \quad \text{inv-th/ax}}{\bullet h_2 : \Delta_7, F_{12} \vdash \Delta_{14}, F_{13}, F_8, F_9 \rightarrow F_{10}} \rightarrow_R \quad \frac{h_{11} : \Delta_7, F_{12}, F_8 \vdash \Delta_{14}, F_{13}, F_9 \rightarrow F_{10}}{\text{ax/W}}}{- : \Delta_7, F_{12} \vdash \Delta_{14}, F_{13}, F_9 \rightarrow F_{10}} \rightarrow_R \text{ hCut}}{- : \Delta_7 \vdash \Delta_{14}, F_{12} \rightarrow F_{13}, F_9 \rightarrow F_{10}} \rightarrow_R \\
\frac{h_2 : \Delta_7, F_{11} \vdash \Delta_{10}, F_8, F_{12} \quad \rightarrow_R \quad h_9 : \Delta_7, F_8, F_{11} \vdash \Delta_{10}, F_{12} \quad \rightarrow_R}{\bullet h_2 : \Delta_7 \vdash (\Delta_{10}, F_{11} \rightarrow F_{12}), F_8 \quad \bullet h_9 : \Delta_7, F_8 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \rightarrow_R \text{ Cut} \\
\frac{}{- : \Delta_7 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \rightarrow \\
\frac{\frac{h_9 : \Delta_7, F_{11}, F_8 \vdash \Delta_{10}, F_{12} \quad \text{ax/W}}{\bullet h_9 : \Delta_7, F_{11}, F_8 \vdash \Delta_{10}, F_{12}} \text{ height}}{\frac{h_2 : \Delta_7, F_{11} \vdash \Delta_{10}, F_{12}, F_8 \quad \text{ax/W}}{- : \Delta_7, F_{11} \vdash \Delta_{10}, F_{12}} \rightarrow_R \text{ hCut}}{- : \Delta_7 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \rightarrow_R \\
\frac{}{- : \Delta_7 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \rightarrow \\
\frac{h_1 : \Delta_6, F_7 \vdash (\Delta_{10}, F_{11} \rightarrow F_{12}), F_8 \quad \rightarrow_R \quad h_9 : \Delta_6, F_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{12} \quad \rightarrow_R}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \rightarrow F_{12}), F_7 \rightarrow F_8 \quad \bullet h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \rightarrow_R \text{ Cut} \\
\frac{}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \rightarrow \\
\frac{\frac{h_1 : \Delta_6, F_{11}, F_7 \vdash \Delta_{10}, F_{12}, F_8 \quad \text{inv-th/ax}}{\bullet h_1 : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_7 \rightarrow F_8} \rightarrow_R \quad \frac{h_9 : \Delta_6, F_{11}, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{12}}{\text{ax/W}}}{- : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}} \rightarrow_R \text{ hCut}}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \rightarrow_R \\
\frac{}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \rightarrow
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{h_2 : \Delta_7, F_9 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_8, F_{10} \quad \rightarrow_R \quad h_{11} : \Delta_7, F_8 \vdash \Delta_{14}, F_{12}, F_9 \rightarrow F_{10} \quad h_{11} : \Delta_7, F_8 \vdash \Delta_{14}, F_{13}, F_9 \rightarrow F_{10}}{\bullet h_2 : \Delta_7 \vdash ((\Delta_{14}, F_{12} \wedge F_{13}), F_9 \rightarrow F_{10}), F_8 \quad \bullet h_{11} : \Delta_7, F_8 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_9 \rightarrow F_{10}} \rightarrow_R \text{ Cut} \\
\frac{}{- : \Delta_7 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_9 \rightarrow F_{10}} \rightarrow \\
\frac{\frac{h_{11} : \Delta_7, F_8, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \quad \text{inv-th/ax}}{\bullet h_{11} : \Delta_7, F_8, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \wedge F_{13}} \text{ hCut}}{\frac{h_2 : \Delta_7, F_9 \vdash \Delta_{14}, F_{10}, F_8, F_{12} \wedge F_{13} \quad \text{ax/W}}{- : \Delta_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \wedge F_{13}} \rightarrow_R \text{ hCut}}{- : \Delta_7 \vdash \Delta_{14}, F_9 \rightarrow F_{10}, F_{12} \wedge F_{13}} \rightarrow_R \\
\frac{}{- : \Delta_7 \vdash \Delta_{14}, F_9 \rightarrow F_{10}, F_{12} \wedge F_{13}} \rightarrow \\
\frac{h_1 : \Delta_6, F_7 \vdash (\Delta_{10}, F_{11} \wedge F_{12}), F_8 \quad \rightarrow_R \quad h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{11} \quad h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{12}}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \wedge F_{12}), F_7 \rightarrow F_8 \quad \bullet h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \rightarrow_R \text{ Cut} \\
\frac{}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \rightarrow \\
\frac{\frac{h_1 : \Delta_6, F_7 \vdash \Delta_{10}, F_{11}, F_8 \quad \text{inv-th/ax}}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_7 \rightarrow F_8} \rightarrow_R \quad \frac{h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{11}}{\text{ax/W}}}{- : \Delta_6 \vdash \Delta_{10}, F_{11}} \rightarrow_R \text{ hCut}}{\frac{\frac{h_1 : \Delta_6, F_7 \vdash \Delta_{10}, F_{12}, F_8 \quad \text{inv-th/ax}}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{12}, F_7 \rightarrow F_8} \rightarrow_R \quad \frac{h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{12}}{\text{ax/W}}}{- : \Delta_6 \vdash \Delta_{10}, F_{12}} \rightarrow_R \text{ hCut}}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R \\
\frac{}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_2 : \Delta_7, F_9 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_8, F_{10} \quad \rightarrow_R \quad \frac{h_{11} : \Delta_7, F_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \rightarrow F_{10}}{\bullet h_{11} : \Delta_7, F_8 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_9 \rightarrow F_{10}} \quad \vee_R}{\bullet h_2 : \Delta_7 \vdash ((\Delta_{14}, F_{12} \vee F_{13}), F_9 \rightarrow F_{10}), F_8} \quad \text{Cut}}{\vdash : \Delta_7 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_9 \rightarrow F_{10}} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_7, F_9 \vdash \Delta_{14}, F_{10}, F_8, F_{12} \vee F_{13}}{\vdash : \Delta_7, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \vee F_{13}} \quad \text{ax/W} \quad \frac{\frac{h_{11} : \Delta_7, F_8, F_9 \vdash \Delta_{14}, F_{10}, F_{12}, F_{13}}{\bullet h_{11} : \Delta_7, F_8, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \vee F_{13}} \quad \vee_R}{\vdash : \Delta_7, F_8, F_9 \vdash \Delta_{14}, F_{10}, F_{12} \vee F_{13}} \quad \text{hCut}}{\vdash : \Delta_7 \vdash \Delta_{14}, F_9 \rightarrow F_{10}, F_{12} \vee F_{13}} \quad \rightarrow_R}{\vdash : \Delta_7 \vdash \Delta_{14}, F_9 \rightarrow F_{10}, F_{12} \vee F_{13}} \quad \text{inv-th/ax} \\
\rightarrow \\
\frac{h_1 : \Delta_6, F_7 \vdash (\Delta_{10}, F_{11} \vee F_{12}), F_8 \quad \rightarrow_R \quad \frac{h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{11}, F_{12}}{\bullet h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{11} \vee F_{12}} \quad \vee_R}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \vee F_{12}), F_7 \rightarrow F_8} \quad \text{Cut}}{\vdash : \Delta_6 \vdash \Delta_{10}, F_{11} \vee F_{12}} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_6, F_7 \vdash \Delta_{10}, F_{11}, F_{12}, F_8}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_7 \rightarrow F_8} \quad \text{inv-th/ax} \quad \frac{h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{11}, F_{12}}{\vdash : \Delta_6, F_7 \rightarrow F_8 \vdash \Delta_{10}, F_{11}, F_{12}} \quad \text{ax/W}}{\vdash : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}} \quad \rightarrow_R}{\vdash : \Delta_6 \vdash \Delta_{10}, F_{11} \vee F_{12}} \quad \vee_R \quad \text{hCut}
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_2 : \Delta_7, F_9 \vdash (\perp, \Delta_{12}), F_8, F_{10} \quad \rightarrow_R \quad \frac{h_{11} : \Delta_7, F_8 \vdash \Delta_{12}, F_9 \rightarrow F_{10}}{\bullet h_{11} : \Delta_7, F_8 \vdash (\perp, \Delta_{12}), F_9 \rightarrow F_{10}} \quad \perp_R}{\bullet h_2 : \Delta_7 \vdash ((\perp, \Delta_{12}), F_9 \rightarrow F_{10}), F_8} \quad \text{Cut}}{\vdash : \Delta_7 \vdash (\perp, \Delta_{12}), F_9 \rightarrow F_{10}} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : \Delta_7 \vdash \perp, \Delta_{12}, F_8, F_9 \rightarrow F_{10}}{\vdash : \Delta_7 \vdash \perp, \Delta_{12}, F_9 \rightarrow F_{10}} \quad \text{ax/W} \quad \frac{h_{11} : \Delta_7, F_8 \vdash \perp, \Delta_{12}, F_9 \rightarrow F_{10}}{\vdash : \Delta_7, F_8 \vdash \perp, \Delta_{12}, F_9 \rightarrow F_{10}} \quad \text{hCut}}{\vdash : \Delta_7 \vdash \perp, \Delta_{12}, F_9 \rightarrow F_{10}} \quad \text{ax/W} \\
\rightarrow \\
\frac{h_1 : \Delta_6, F_7 \vdash (\perp, \Delta_{10}), F_8 \quad \rightarrow_R \quad \frac{h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \perp, \Delta_{10}} \quad \perp_R}{\bullet h_1 : \Delta_6 \vdash (\perp, \Delta_{10}), F_7 \rightarrow F_8} \quad \text{Cut}}{\vdash : \Delta_6 \vdash \perp, \Delta_{10}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_6 \vdash \perp, \Delta_{10}, F_7 \rightarrow F_8}{\vdash : \Delta_6 \vdash \perp, \Delta_{10}} \quad \text{ax/W} \quad \frac{h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \perp, \Delta_{10}}{\vdash : \Delta_6, F_7 \rightarrow F_8 \vdash \perp, \Delta_{10}} \quad \text{ax/W}}{\vdash : \Delta_6 \vdash \perp, \Delta_{10}} \quad \text{hCut}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_2 : \Delta_7, F_9 \vdash (\top, \Delta_{12}), F_8, F_{10} \quad \rightarrow_R \quad \frac{h_{11} : \Delta_7, F_8 \vdash (\top, \Delta_{12}), F_9 \rightarrow F_{10}}{\bullet h_{11} : \Delta_7, F_8 \vdash (\top, \Delta_{12}), F_9 \rightarrow F_{10}} \quad \top_R}{\bullet h_2 : \Delta_7 \vdash ((\top, \Delta_{12}), F_9 \rightarrow F_{10}), F_8} \quad \text{Cut}}{\vdash : \Delta_7 \vdash (\top, \Delta_{12}), F_9 \rightarrow F_{10}} \\
\rightarrow \\
\frac{\vdash : \Delta_7 \vdash \top, \Delta_{12}, F_9 \rightarrow F_{10}}{\vdash : \Delta_7 \vdash \top, \Delta_{12}, F_9 \rightarrow F_{10}} \quad \top_R \\
\rightarrow \\
\frac{h_1 : \Delta_6, F_7 \vdash (\top, \Delta_{10}), F_8 \quad \rightarrow_R \quad \frac{h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \top, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \rightarrow F_8 \vdash \top, \Delta_{10}} \quad \top_R}{\bullet h_1 : \Delta_6 \vdash (\top, \Delta_{10}), F_7 \rightarrow F_8} \quad \text{Cut}}{\vdash : \Delta_6 \vdash \top, \Delta_{10}} \\
\rightarrow \\
\frac{\vdash : \Delta_6 \vdash \top, \Delta_{10}}{\vdash : \Delta_6 \vdash \top, \Delta_{10}} \quad \top_R
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_2 : (\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \vdash \Delta_8, F_7, F_{10} \quad \rightarrow_R \quad \frac{h_{11} : \Delta_{14}, F_7 \vdash \Delta_8, F_{12}, F_9 \rightarrow F_{10} \quad h_{11} : \Delta_{14}, F_7, F_{13} \vdash \Delta_8, F_9 \rightarrow F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \rightarrow F_{13}), F_7 \vdash \Delta_8, F_9 \rightarrow F_{10}} \quad \text{Cut}}{\bullet h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash (\Delta_8, F_9 \rightarrow F_{10}), F_7} \quad \rightarrow_L}{\vdash : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9 \rightarrow F_{10}} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{14}, F_9, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_{10}, F_7}{\vdash : \Delta_{14}, F_9, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_{10}, F_7} \quad \text{ax/W} \quad \frac{\frac{h_{11} : \Delta_{14}, F_7, F_9 \vdash \Delta_8, F_{10}, F_{12}}{\bullet h_{11} : \Delta_{14}, F_7, F_9, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_{10}} \quad \text{inv-th/ax} \quad \frac{h_{11} : \Delta_{14}, F_{13}, F_7, F_9 \vdash \Delta_8, F_{10}}{\vdash : \Delta_{14}, F_{13}, F_7, F_9 \vdash \Delta_8, F_{10}} \quad \text{inv-th/ax}}{\vdash : \Delta_{14}, F_7, F_9, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_{10}} \quad \text{hCut}}{\vdash : \Delta_{14}, F_9, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_{10}} \quad \rightarrow_R}{\vdash : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9 \rightarrow F_{10}} \quad \rightarrow_R
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_2 : \Delta_{11}, F_8 \vdash \Delta_7, F_{12} \rightarrow F_{13}, F_9}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \rightarrow F_9), F_{12} \rightarrow F_{13}} \rightarrow_R \quad \frac{h_{10} : \Delta_{11} \vdash \Delta_7, F_{12}, F_8 \rightarrow F_9 \quad h_{10} : \Delta_{11}, F_{13} \vdash \Delta_7, F_8 \rightarrow F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \rightarrow F_{13} \vdash \Delta_7, F_8 \rightarrow F_9} \rightarrow_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{11}, F_8 \vdash \Delta_7, F_9, F_{12} \rightarrow F_{13}}{\text{ax/W}} \quad \frac{\frac{h_{10} : \Delta_{11}, F_8 \vdash \Delta_7, F_{12}, F_9}{\bullet h_{10} : \Delta_{11}, F_8, F_{12} \rightarrow F_{13} \vdash \Delta_7, F_9} \text{inv-th/ax} \quad \frac{h_{10} : \Delta_{11}, F_{13}, F_8 \vdash \Delta_7, F_9}{\bullet h_{10} : \Delta_{11}, F_{13}, F_8 \vdash \Delta_7, F_9} \text{inv-th/ax}}{- : \Delta_{11}, F_8 \vdash \Delta_7, F_9} \text{hCut}}{- : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9} \rightarrow_R \\
\rightarrow \\
\frac{\frac{h_1 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_9 \quad h_8 : \Delta_{12}, F_{10}, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \rightarrow F_7 \vdash \Delta_{11}} \rightarrow_L}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_6 \vdash \Delta_{11}, F_7, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_9, F_6 \rightarrow F_7} \text{inv-th/ax} \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_6 \rightarrow F_7 \vdash \Delta_{11}, F_9}{\text{ax/W}} \quad \frac{h_1 : \Delta_{12}, F_{10}, F_6 \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6 \rightarrow F_7} \text{inv-th/ax}}{- : \Delta_{12} \vdash \Delta_{11}, F_9} \text{hCut} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\text{ax/W}} \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_{10}, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\text{hCut}}}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \rightarrow_L \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_7, F_8 \vdash \Delta_{10}, F_9}{\bullet h_1 : \Delta_7 \vdash \Delta_{10}, F_8 \rightarrow F_9} \rightarrow_R \quad \frac{h_6 : \Delta_7 \vdash \Delta_{10}, F_8 \quad h_6 : \Delta_7, F_9 \vdash \Delta_{10}}{\bullet h_6 : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{10}} \rightarrow_L}{- : \Delta_7 \vdash \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_7, F_8 \vdash \Delta_{10}, F_9}{\text{ax/W}} \quad \frac{- : \Delta_7, F_8, F_9 \vdash \Delta_{10}}{\text{ax/W}}}{- : \Delta_7 \vdash \Delta_{10}, F_8} \text{sCut} \quad \frac{- : \Delta_7, F_8 \vdash \Delta_{10}}{\text{sCut}}}{- : \Delta_7 \vdash \Delta_{10}}
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{\frac{h_2 : (\Delta_{14}, F_{12} \wedge F_{13}), F_9 \vdash \Delta_8, F_7, F_{10}}{\bullet h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash (\Delta_8, F_9 \rightarrow F_{10}), F_7} \rightarrow_R \quad \frac{h_{11} : \Delta_{14}, F_7, F_{12}, F_{13} \vdash \Delta_8, F_9 \rightarrow F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \wedge F_{13}), F_7 \vdash \Delta_8, F_9 \rightarrow F_{10}} \wedge_L}{- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{14}, F_9, F_{12} \wedge F_{13} \vdash \Delta_8, F_{10}, F_7}{\text{ax/W}} \quad \frac{h_{11} : \Delta_{14}, F_{12}, F_{13}, F_7, F_9 \vdash \Delta_8, F_{10}}{\bullet h_{11} : \Delta_{14}, F_7, F_9, F_{12} \wedge F_{13} \vdash \Delta_8, F_{10}} \text{inv-th/ax}}{- : \Delta_{14}, F_9, F_{12} \wedge F_{13} \vdash \Delta_8, F_{10}} \wedge_L \text{hCut} \\
\rightarrow \\
\frac{- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_9 \rightarrow F_{10}}{- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_9 \rightarrow F_{10}} \rightarrow_R \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{11}, F_8 \vdash \Delta_7, F_{12} \wedge F_{13}, F_9}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \rightarrow F_9), F_{12} \wedge F_{13}} \rightarrow_R \quad \frac{h_{10} : \Delta_{11}, F_{12}, F_{13} \vdash \Delta_7, F_8 \rightarrow F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \wedge F_{13} \vdash \Delta_7, F_8 \rightarrow F_9} \wedge_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{11}, F_8 \vdash \Delta_7, F_9, F_{12} \wedge F_{13}}{\text{ax/W}} \quad \frac{h_{10} : \Delta_{11}, F_{12}, F_{13}, F_8 \vdash \Delta_7, F_9}{\bullet h_{10} : \Delta_{11}, F_8, F_{12} \wedge F_{13} \vdash \Delta_7, F_9} \text{inv-th/ax}}{- : \Delta_{11}, F_8 \vdash \Delta_7, F_9} \wedge_L \text{hCut} \\
\rightarrow \\
\frac{- : \Delta_{11}, F_8 \vdash \Delta_7, F_9}{- : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9} \rightarrow_R \\
\rightarrow \\
\frac{\frac{h_1 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_9, F_{10}, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \rightarrow F_7 \vdash \Delta_{11}} \wedge_L}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_{10}, F_6, F_9 \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6 \rightarrow F_7} \text{inv-th/ax} \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_{10}, F_9, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_{10}, F_9, F_6 \rightarrow F_7 \vdash \Delta_{11}} \text{ax/W}}{- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}} \wedge_L \text{hCut} \\
\rightarrow \\
\frac{- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \wedge_L
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\frac{h_2 : (\Delta_{14}, F_{12} \vee F_{13}), F_9 \vdash \Delta_8, F_7, F_{10}}{\bullet h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash (\Delta_8, F_9 \rightarrow F_{10}), F_7} \rightarrow_R \quad \frac{h_{11} : \Delta_{14}, F_7, F_{12} \vdash \Delta_8, F_9 \rightarrow F_{10} \quad h_{11} : \Delta_{14}, F_7, F_{13} \vdash \Delta_8, F_9 \rightarrow F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \vee F_{13}), F_7 \vdash \Delta_8, F_9 \rightarrow F_{10}} \vee_L}{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{Cut}} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{14}, F_9, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}, F_7}{- : \Delta_{14}, F_9, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}} \text{ax/W} \quad \frac{\frac{h_{11} : \Delta_{14}, F_{12}, F_7, F_9 \vdash \Delta_8, F_{10}}{\bullet h_{11} : \Delta_{14}, F_7, F_9, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}} \text{hCut} \quad \frac{h_{11} : \Delta_{14}, F_{13}, F_7, F_9 \vdash \Delta_8, F_{10}}{\bullet h_{11} : \Delta_{14}, F_7, F_9, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}} \text{hCut}}{- : \Delta_{14}, F_9, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}} \rightarrow_R}{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \rightarrow F_{10}} \rightarrow_R} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{11}, F_8 \vdash \Delta_7, F_{12} \vee F_{13}, F_9}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \rightarrow F_9), F_{12} \vee F_{13}} \rightarrow_R \quad \frac{h_{10} : \Delta_{11}, F_{12} \vdash \Delta_7, F_8 \rightarrow F_9 \quad h_{10} : \Delta_{11}, F_{13} \vdash \Delta_7, F_8 \rightarrow F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \vee F_{13} \vdash \Delta_7, F_8 \rightarrow F_9} \vee_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9} \text{Cut}} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{11}, F_8 \vdash \Delta_7, F_9, F_{12} \vee F_{13}}{\bullet h_2 : \Delta_{11}, F_8 \vdash \Delta_7, F_9, F_{12} \vee F_{13}} \text{ax/W} \quad \frac{h_{10} : \Delta_{11}, F_{12}, F_8 \vdash \Delta_7, F_9}{\bullet h_{10} : \Delta_{11}, F_8, F_{12} \vee F_{13} \vdash \Delta_7, F_9} \text{hCut} \quad \frac{h_{10} : \Delta_{11}, F_{13}, F_8 \vdash \Delta_7, F_9}{\bullet h_{10} : \Delta_{11}, F_8, F_{12} \vee F_{13} \vdash \Delta_7, F_9} \text{hCut}}{- : \Delta_{11}, F_8 \vdash \Delta_7, F_9} \rightarrow_R}{- : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9} \rightarrow_R} \\
\rightarrow \\
\frac{\frac{h_1 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \rightarrow F_7 \vdash \Delta_{11} \quad h_8 : \Delta_{12}, F_{10}, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \rightarrow F_7 \vdash \Delta_{11}} \vee_L}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \text{Cut}} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_6, F_9 \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_6 \rightarrow F_7} \text{inv-th/ax} \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \text{ax/W} \quad \frac{h_1 : \Delta_{12}, F_{10}, F_6 \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6 \rightarrow F_7} \text{inv-th/ax} \rightarrow_R \quad \frac{h_8 : \Delta_{12}, F_{10}, F_6 \rightarrow F_7 \vdash \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_{10}, F_6 \rightarrow F_7 \vdash \Delta_{11}} \text{hCut}}{- : \Delta_{12}, F_9 \vdash \Delta_{11}} \rightarrow_R}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \vee_L}
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{\frac{h_2 : \Delta_{11}, F_8 \vdash \Delta_7, \perp, F_9}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \rightarrow F_9), \perp} \rightarrow_R \quad \frac{h_{10} : \Delta_{11}, \perp \vdash \Delta_7, F_8 \rightarrow F_9}{\bullet h_{10} : \Delta_{11}, \perp \vdash \Delta_7, F_8 \rightarrow F_9} \perp_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9} \text{Cut}} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{11}, F_8 \vdash \perp, \Delta_7, F_9}{\bullet h_2 : \Delta_{11}, F_8 \vdash \perp, \Delta_7, F_9} \text{ax/W} \quad \frac{h_{10} : \perp, \Delta_{11}, F_8 \vdash \Delta_7, F_9}{\bullet h_{10} : \perp, \Delta_{11}, F_8 \vdash \Delta_7, F_9} \perp_L}{- : \Delta_{11}, F_8 \vdash \Delta_7, F_9} \text{hCut}}{- : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9} \rightarrow_R} \\
\rightarrow \\
\frac{\frac{h_2 : (\perp, \Delta_{12}), F_9 \vdash \Delta_8, F_7, F_{10}}{\bullet h_2 : \perp, \Delta_{12} \vdash (\Delta_8, F_9 \rightarrow F_{10}), F_7} \rightarrow_R \quad \frac{h_{11} : (\perp, \Delta_{12}), F_7 \vdash \Delta_8, F_9 \rightarrow F_{10}}{\bullet h_{11} : (\perp, \Delta_{12}), F_7 \vdash \Delta_8, F_9 \rightarrow F_{10}} \perp_L}{- : \perp, \Delta_{12} \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{Cut}} \\
\rightarrow \\
\frac{- : \perp, \Delta_{12} \vdash \Delta_8, F_9 \rightarrow F_{10}}{- : \perp, \Delta_{12} \vdash \Delta_8, F_9 \rightarrow F_{10}} \perp_L} \\
\rightarrow \\
\frac{\frac{h_1 : (\perp, \Delta_{10}), F_6 \vdash \Delta_9, F_7}{\bullet h_1 : \perp, \Delta_{10} \vdash \Delta_9, F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : (\perp, \Delta_{10}), F_6 \rightarrow F_7 \vdash \Delta_9}{\bullet h_8 : (\perp, \Delta_{10}), F_6 \rightarrow F_7 \vdash \Delta_9} \perp_L}{- : \perp, \Delta_{10} \vdash \Delta_9} \text{Cut}} \\
\rightarrow \\
\frac{- : \perp, \Delta_{10} \vdash \Delta_9}{- : \perp, \Delta_{10} \vdash \Delta_9} \perp_L}
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\frac{h_2 : \Delta_{10}, F_7 \vdash (\Delta_{12}, P_{11}), P_{11}, F_8}{\bullet h_2 : \Delta_{10} \vdash ((\Delta_{12}, P_{11}), F_7 \rightarrow F_8), P_{11}} \rightarrow_R \quad \frac{h_9 : \Delta_{10}, P_{11} \vdash (\Delta_{12}, P_{11}), F_7 \rightarrow F_8}{\bullet h_9 : \Delta_{10}, P_{11} \vdash (\Delta_{12}, P_{11}), F_7 \rightarrow F_8} I}{- : \Delta_{10} \vdash (\Delta_{12}, P_{11}), F_7 \rightarrow F_8} \text{Cut}} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{10}, F_7 \vdash \Delta_{12}, F_8, P_{11}, P_{11}}{\bullet h_2 : \Delta_{10}, F_7 \vdash \Delta_{12}, F_8, P_{11}, P_{11}} \text{ax/W} \quad \frac{h_9 : \Delta_{10}, F_7, P_{11} \vdash \Delta_{12}, F_8, P_{11}}{\bullet h_9 : \Delta_{10}, F_7, P_{11} \vdash \Delta_{12}, F_8, P_{11}} I}{- : \Delta_{10}, F_7 \vdash \Delta_{12}, F_8, P_{11}} \text{hCut}}{- : \Delta_{10} \vdash \Delta_{12}, P_{11}, F_7 \rightarrow F_8} \rightarrow_R}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : (\Delta_{13}, p_{11}), F_8 \vdash (\Delta_{12}, p_{11}), F_7, F_9}{\bullet h_2 : \Delta_{13}, p_{11} \vdash ((\Delta_{12}, p_{11}), F_8 \rightarrow F_9), F_7} \rightarrow_R \quad \frac{h_{10} : (\Delta_{13}, p_{11}), F_7 \vdash (\Delta_{12}, p_{11}), F_8 \rightarrow F_9}{\bullet h_{10} : (\Delta_{13}, p_{11}), F_8 \rightarrow F_9} I}{\frac{- : \Delta_{13}, p_{11} \vdash (\Delta_{12}, p_{11}), F_8 \rightarrow F_9}{\rightarrow} \quad \frac{- : \Delta_{13}, p_{11} \vdash \Delta_{12}, p_{11}, F_8 \rightarrow F_9}{I}}{I} \text{Cut} \\
\\
\frac{h_1 : (\Delta_{11}, p_9), F_6 \vdash (\Delta_{10}, p_9), F_7}{\bullet h_1 : \Delta_{11}, p_9 \vdash (\Delta_{10}, p_9), F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : (\Delta_{11}, p_9), F_6 \rightarrow F_7 \vdash \Delta_{10}, p_9}{\bullet h_8 : (\Delta_{11}, p_9), F_6 \rightarrow F_7 \vdash \Delta_{10}, p_9} I}{\frac{- : \Delta_{11}, p_9 \vdash \Delta_{10}, p_9}{\rightarrow} \quad \frac{- : \Delta_{11}, p_9 \vdash \Delta_{10}, p_9}{I}}{I} \text{Cut}
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{h_2 : \Delta_{11}, F_8 \vdash \Delta_7, \top, F_9}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \rightarrow F_9), \top} \rightarrow_R \quad \frac{h_{10} : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9}{\bullet h_{10} : \Delta_{11}, \top \vdash \Delta_7, F_8 \rightarrow F_9} \top_L}{\frac{- : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9}{\rightarrow} \quad \frac{- : \Delta_{11} \vdash \Delta_7, F_8 \rightarrow F_9}{\text{ax/W}}}{\text{Cut}} \\
\\
\frac{h_2 : (\top, \Delta_{12}), F_9 \vdash \Delta_8, F_7, F_{10}}{\bullet h_2 : \top, \Delta_{12} \vdash (\Delta_8, F_9 \rightarrow F_{10}), F_7} \rightarrow_R \quad \frac{h_{11} : \Delta_{12}, F_7 \vdash \Delta_8, F_9 \rightarrow F_{10}}{\bullet h_{11} : (\top, \Delta_{12}), F_7 \vdash \Delta_8, F_9 \rightarrow F_{10}} \top_L}{\frac{- : \top, \Delta_{12} \vdash \Delta_8, F_9 \rightarrow F_{10}}{\rightarrow} \quad \frac{- : \top, \Delta_{12} \vdash \Delta_8, F_9 \rightarrow F_{10}}{\text{ax/W}}}{\text{Cut}} \\
\\
\frac{\bullet h_2 : \top, \Delta_{12} \vdash \Delta_8, F_7, F_9 \rightarrow F_{10}}{\text{ax/W}} \quad \frac{h_{11} : \top, \Delta_{12}, F_7 \vdash \Delta_8, F_9 \rightarrow F_{10}}{h_{11} : \top, \Delta_{12}, F_7 \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{ax/W}}{\frac{- : \top, \Delta_{12} \vdash \Delta_8, F_9 \rightarrow F_{10}}{h\text{Cut}}} \\
\\
\frac{h_1 : (\top, \Delta_{10}), F_6 \vdash \Delta_9, F_7}{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \rightarrow F_7} \rightarrow_R \quad \frac{h_8 : \Delta_{10}, F_6 \rightarrow F_7 \vdash \Delta_9}{\bullet h_8 : (\top, \Delta_{10}), F_6 \rightarrow F_7 \vdash \Delta_9} \top_L}{\frac{- : \top, \Delta_{10} \vdash \Delta_9}{\rightarrow} \quad \frac{- : \top, \Delta_{10} \vdash \Delta_9}{\text{Cut}}} \\
\\
\frac{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \rightarrow F_7}{\text{ax/W}} \quad \frac{h_8 : \top, \Delta_{10}, F_6 \rightarrow F_7 \vdash \Delta_9}{h_8 : \top, \Delta_{10}, F_6 \rightarrow F_7 \vdash \Delta_9} \text{ax/W}}{\frac{- : \top, \Delta_{10} \vdash \Delta_9}{h\text{Cut}}}
\end{array}$$

8.2 Status of \wedge_R : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_2 : \Delta_7 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), F_8, F_9 \quad h_2 : \Delta_7 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), F_8, F_{10}}{\bullet h_2 : \Delta_7 \vdash ((\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \wedge F_{10}), F_8} \wedge_R \quad \frac{h_{11} : \Delta_7, F_8, F_{12} \vdash \Delta_{14}, F_{13}, F_9 \wedge F_{10}}{\bullet h_{11} : \Delta_7, F_8 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \wedge F_{10}} \rightarrow_R}{\frac{- : \Delta_7 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \wedge F_{10}}{\rightarrow} \quad \frac{- : \Delta_7 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \wedge F_{10}}{\text{Cut}}} \\
\\
\frac{h_2 : \Delta_7, F_{12} \vdash \Delta_{14}, F_{13}, F_8, F_9}{\bullet h_2 : \Delta_7, F_{12} \vdash \Delta_{14}, F_{13}, F_8, F_9 \wedge F_{10}} \text{inv-th/ax} \quad \frac{h_2 : \Delta_7, F_{12} \vdash \Delta_{14}, F_{10}, F_{13}, F_8}{\bullet h_2 : \Delta_7, F_{12} \vdash \Delta_{14}, F_{10}, F_{13}, F_8} \text{inv-th/ax}}{\frac{- : \Delta_7, F_{12} \vdash \Delta_{14}, F_{13}, F_9 \wedge F_{10}}{\rightarrow} \quad \frac{- : \Delta_7, F_{12} \vdash \Delta_{14}, F_{13}, F_9 \wedge F_{10}}{\text{hCut}}} \\
\\
\frac{h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \rightarrow F_{12}), F_7 \quad h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \rightarrow F_{12}), F_8}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \rightarrow F_{12}), F_7 \wedge F_8} \wedge_R \quad \frac{h_9 : \Delta_6, F_{11}, F_7 \wedge F_8 \vdash \Delta_{10}, F_{12}}{\bullet h_9 : \Delta_6, F_7 \wedge F_8 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \rightarrow_R}{\frac{- : \Delta_6 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}}{\rightarrow} \quad \frac{- : \Delta_6 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}}{\text{Cut}}} \\
\\
\frac{h_1 : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_7}{\bullet h_1 : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_7 \wedge F_8} \text{inv-th/ax} \quad \frac{h_1 : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_8}{\bullet h_1 : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_8} \text{inv-th/ax}}{\frac{- : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_7 \wedge F_8}{\rightarrow} \quad \frac{- : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_7 \wedge F_8}{\text{hCut}}} \\
\\
\frac{- : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}}{\rightarrow} \quad \frac{- : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}}{\rightarrow_R}}{\frac{- : \Delta_6 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}}{\rightarrow_R}}
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{h_1 : \Delta_6 \vdash (\perp, \Delta_{10}), F_7 \quad h_1 : \Delta_6 \vdash (\perp, \Delta_{10}), F_8}{\bullet h_1 : \Delta_6 \vdash (\perp, \Delta_{10}), F_7 \wedge F_8} \wedge_R \quad \frac{h_9 : \Delta_6, F_7 \wedge F_8 \vdash \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \wedge F_8 \vdash \perp, \Delta_{10}} \perp_R \\
\hline
- : \Delta_6 \vdash \perp, \Delta_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_6 \vdash \perp, \Delta_{10}, F_7 \wedge F_8}{- : \Delta_6 \vdash \perp, \Delta_{10}} \text{ ax/W} \quad \frac{h_9 : \Delta_6, F_7 \wedge F_8 \vdash \perp, \Delta_{10}}{- : \Delta_6 \vdash \perp, \Delta_{10}} \text{ ax/W}}{- : \Delta_6 \vdash \perp, \Delta_{10}} \text{ hCut}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_2 : \Delta_7 \vdash (\top, \Delta_{12}), F_8, F_9 \quad h_2 : \Delta_7 \vdash (\top, \Delta_{12}), F_8, F_{10}}{\bullet h_2 : \Delta_7 \vdash ((\top, \Delta_{12}), F_9 \wedge F_{10}), F_8} \wedge_R \quad \frac{}{\bullet h_{11} : \Delta_7, F_8 \vdash (\top, \Delta_{12}), F_9 \wedge F_{10}} \top_R \\
\hline
- : \Delta_7 \vdash (\top, \Delta_{12}), F_9 \wedge F_{10} \\
\rightarrow \\
- : \Delta_7 \vdash \top, \Delta_{12}, F_9 \wedge F_{10} \quad \top_R \\
\hline
\frac{h_1 : \Delta_6 \vdash (\top, \Delta_{10}), F_7 \quad h_1 : \Delta_6 \vdash (\top, \Delta_{10}), F_8}{\bullet h_1 : \Delta_6 \vdash (\top, \Delta_{10}), F_7 \wedge F_8} \wedge_R \quad \frac{}{\bullet h_9 : \Delta_6, F_7 \wedge F_8 \vdash \top, \Delta_{10}} \top_R \\
\hline
- : \Delta_6 \vdash \top, \Delta_{10} \\
\rightarrow \\
- : \Delta_6 \vdash \top, \Delta_{10} \quad \top_R
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_7, F_9 \quad h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_7, F_{10}}{\bullet h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash (\Delta_8, F_9 \wedge F_{10}), F_7} \wedge_R \quad \frac{h_{11} : \Delta_{14}, F_7 \vdash \Delta_8, F_{12}, F_9 \wedge F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \rightarrow F_{13}) \vdash \Delta_8, F_9 \wedge F_{10}} \\
\hline
- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9 \wedge F_{10} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_7, F_9}{- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_7, F_9} \text{ ax/W} \quad \frac{\frac{h_{11} : \Delta_{14}, F_7 \vdash \Delta_8, F_{12}, F_9}{\bullet h_{11} : \Delta_{14}, F_7, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9} \text{ inv-th/ax}}{- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9} \text{ hCut}}{- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9 \wedge F_{10}} \rightarrow_L \quad \frac{h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9 \wedge F_{10}}{- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9 \wedge F_{10}} \\
\hline
\frac{h_2 : \Delta_{11} \vdash \Delta_7, F_{12} \rightarrow F_{13}, F_8 \quad h_2 : \Delta_{11} \vdash \Delta_7, F_{12} \rightarrow F_{13}, F_9}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \wedge F_9), F_{12} \rightarrow F_{13}} \wedge_R \quad \frac{h_{10} : \Delta_{11} \vdash \Delta_7, F_{12}, F_8 \wedge F_9 \quad h_{10} : \Delta_{11}, F_{13} \vdash \Delta_7, F_8 \wedge F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \rightarrow F_{13} \vdash \Delta_7, F_8 \wedge F_9} \rightarrow_L \\
\hline
- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9 \\
\rightarrow \\
\frac{\frac{- : \Delta_{11}, F_{12} \vdash \Delta_7, F_{13}, F_8}{- : \Delta_{11}, F_{12} \vdash \Delta_7, F_{13}, F_8 \wedge F_9} \text{ inv-th/ax}}{- : \Delta_{11}, F_{12} \vdash \Delta_7, F_8 \wedge F_9} \wedge_R \quad \frac{- : \Delta_{11}, F_{12} \vdash \Delta_7, F_{13}, F_9}{- : \Delta_{11}, F_{12} \vdash \Delta_7, F_8 \wedge F_9} \text{ inv-th/ax}}{- : \Delta_{11}, F_{12} \vdash \Delta_7, F_8 \wedge F_9} \text{ sCut} \\
\hline
- : \Delta_{11} \vdash \Delta_7, F_{12}, F_8 \wedge F_9 \quad \text{ ax/W} \\
\hline
- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9 \\
\hline
\frac{h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6 \quad h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_{12}, F_6 \wedge F_7 \vdash \Delta_{11}, F_9 \quad h_8 : \Delta_{12}, F_{10}, F_6 \wedge F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \wedge F_7 \vdash \Delta_{11}} \rightarrow_L \\
\hline
- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11} \\
\rightarrow \\
\frac{\frac{- : \Delta_{12}, F_6, F_7 \vdash \Delta_{11}, F_9}{- : \Delta_{12}, F_6, F_7, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{ inv-th/ax}}{- : \Delta_{12}, F_6, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{ ax/W} \quad \frac{- : \Delta_{12}, F_{10}, F_6, F_7 \vdash \Delta_{11}}{- : \Delta_{12}, F_6, F_7, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{ inv-th/ax}}{- : \Delta_{12}, F_6, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{ sCut} \\
\hline
- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6 \quad \text{ ax/W} \\
\hline
- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11} \\
\hline
- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11} \quad \text{ sCut}
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_7, F_9 \quad h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_7, F_{10}}{\bullet h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash (\Delta_8, F_9 \wedge F_{10}), F_7} \wedge_R \quad \frac{h_{11} : \Delta_{14}, F_7, F_{12}, F_{13} \vdash \Delta_8, F_9 \wedge F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \wedge F_{13}), F_7 \vdash \Delta_8, F_9 \wedge F_{10}} \wedge_L \\
\hline
- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_9 \wedge F_{10} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{14}, F_{12}, F_{13} \vdash \Delta_8, F_7, F_9}{\bullet h_2 : \Delta_{14}, F_{12}, F_{13} \vdash \Delta_8, F_7, F_9 \wedge F_{10}} \text{ inv-th/ax}}{- : \Delta_{14}, F_{12}, F_{13} \vdash \Delta_8, F_7, F_9 \wedge F_{10}} \wedge_R \quad \frac{h_{11} : \Delta_{14}, F_{12}, F_{13}, F_7 \vdash \Delta_8, F_9 \wedge F_{10}}{- : \Delta_{14}, F_{12}, F_{13} \vdash \Delta_8, F_9 \wedge F_{10}} \text{ ax/W}}{- : \Delta_{14}, F_{12}, F_{13} \vdash \Delta_8, F_9 \wedge F_{10}} \text{ hCut} \\
\hline
- : \Delta_{14}, F_{12}, F_{13} \vdash \Delta_8, F_9 \wedge F_{10} \\
\rightarrow \\
- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_9 \wedge F_{10} \quad \wedge_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_2 : \Delta_{11} \vdash \Delta_7, F_{12} \wedge F_{13}, F_8 \quad h_2 : \Delta_{11} \vdash \Delta_7, F_{12} \wedge F_{13}, F_9}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \wedge F_9), F_{12} \wedge F_{13}} \wedge_R \quad \frac{h_{10} : \Delta_{11}, F_{12}, F_{13} \vdash \Delta_7, F_8 \wedge F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \wedge F_{13} \vdash \Delta_7, F_8 \wedge F_9} \wedge_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{11} \vdash \Delta_7, F_8, F_{12} \wedge F_{13}}{\bullet h_2 : \Delta_{11} \vdash \Delta_7, F_8} \text{ax/W} \quad \frac{\frac{h_{10} : \Delta_{11}, F_{12}, F_{13} \vdash \Delta_7, F_8}{\bullet h_{10} : \Delta_{11}, F_{12} \wedge F_{13} \vdash \Delta_7, F_8} \wedge_L \quad \frac{h_2 : \Delta_{11} \vdash \Delta_7, F_9, F_{12} \wedge F_{13}}{\bullet h_2 : \Delta_{11} \vdash \Delta_7, F_9} \text{ax/W}}{- : \Delta_{11} \vdash \Delta_7, F_9} \wedge_R}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \text{hCut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6 \quad h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_{12}, F_9, F_{10}, F_6 \wedge F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \wedge F_7 \vdash \Delta_{11}} \wedge_L}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6}{\bullet h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6 \wedge F_7} \text{inv-th/ax} \quad \frac{h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_7} \text{inv-th/ax}}{- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}} \wedge_R \\
\frac{h_8 : \Delta_{12}, F_{10}, F_9, F_6 \wedge F_7 \vdash \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_{10}, F_9, F_6 \wedge F_7 \vdash \Delta_{11}} \text{ax/W}}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \text{hCut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_7 \vdash \Delta_{10}, F_8 \quad h_1 : \Delta_7 \vdash \Delta_{10}, F_9}{\bullet h_1 : \Delta_7 \vdash \Delta_{10}, F_8 \wedge F_9} \wedge_R \quad \frac{h_6 : \Delta_7, F_8, F_9 \vdash \Delta_{10}}{\bullet h_6 : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{10}} \wedge_L}{- : \Delta_7 \vdash \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_7 \vdash \Delta_{10}, F_8}{\bullet - : \Delta_7 \vdash \Delta_{10}, F_8} \text{ax/W} \quad \frac{- : \Delta_7, F_8 \vdash \Delta_{10}, F_9}{\bullet - : \Delta_7, F_8 \vdash \Delta_{10}, F_9} \text{ax/W} \quad \frac{- : \Delta_7, F_8, F_9 \vdash \Delta_{10}}{\bullet - : \Delta_7, F_8, F_9 \vdash \Delta_{10}} \text{ax/W}}{- : \Delta_7 \vdash \Delta_{10}} \text{sCut} \\
\rightarrow \\
\frac{- : \Delta_7 \vdash \Delta_{10}, F_8}{\bullet - : \Delta_7 \vdash \Delta_{10}, F_8} \text{ax/W} \quad \frac{- : \Delta_7, F_8 \vdash \Delta_{10}}{\bullet - : \Delta_7, F_8 \vdash \Delta_{10}} \text{sCut}}{- : \Delta_7 \vdash \Delta_{10}} \text{sCut}
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\frac{h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_7, F_9 \quad h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_7, F_{10}}{\bullet h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash (\Delta_8, F_9 \wedge F_{10}), F_7} \wedge_R \quad \frac{h_{11} : \Delta_{14}, F_7, F_{12} \vdash \Delta_8, F_9 \wedge F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \vee F_{13}), F_7 \vdash \Delta_8, F_9 \wedge F_{10}} \wedge_L}{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \wedge F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_7, F_9}{\bullet h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_7, F_9} \text{ax/W} \quad \frac{\frac{h_{11} : \Delta_{14}, F_{12}, F_7 \vdash \Delta_8, F_9}{\bullet h_{11} : \Delta_{14}, F_7, F_{12} \vee F_{13} \vdash \Delta_8, F_9} \text{inv-th/ax} \quad \frac{h_{11} : \Delta_{14}, F_{13}, F_7 \vdash \Delta_8, F_9}{\bullet h_{11} : \Delta_{14}, F_{13}, F_7 \vdash \Delta_8, F_9} \text{inv-th/ax}}{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9} \vee_L}{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9} \text{hCut} \\
\rightarrow \\
\frac{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9}{\bullet - : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9} \text{ax/W} \quad \frac{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \wedge F_{10}}{\bullet - : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \wedge F_{10}} \text{hCut}}{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \wedge F_{10}} \text{ax/W} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{11} \vdash \Delta_7, F_{12} \vee F_{13}, F_8 \quad h_2 : \Delta_{11} \vdash \Delta_7, F_{12} \vee F_{13}, F_9}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \wedge F_9), F_{12} \vee F_{13}} \wedge_R \quad \frac{h_{10} : \Delta_{11}, F_{12} \vdash \Delta_7, F_8 \wedge F_9 \quad h_{10} : \Delta_{11}, F_{13} \vdash \Delta_7, F_8 \wedge F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \vee F_{13} \vdash \Delta_7, F_8 \wedge F_9} \wedge_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_{11} \vdash \Delta_7, F_{12}, F_{13}, F_8}{\bullet - : \Delta_{11} \vdash \Delta_7, F_{12}, F_{13}, F_8} \text{inv-th/ax} \quad \frac{- : \Delta_{11} \vdash \Delta_7, F_{12}, F_{13}, F_9}{\bullet - : \Delta_{11} \vdash \Delta_7, F_{12}, F_{13}, F_9} \text{inv-th/ax}}{- : \Delta_{11} \vdash \Delta_7, F_{12}, F_{13}, F_8 \wedge F_9} \wedge_R \\
\frac{- : \Delta_{11}, F_{13} \vdash \Delta_7, F_{12}, F_8 \wedge F_9}{\bullet - : \Delta_{11}, F_{13} \vdash \Delta_7, F_{12}, F_8 \wedge F_9} \text{ax/W}}{- : \Delta_{11} \vdash \Delta_7, F_{12}, F_8 \wedge F_9} \text{sCut} \\
\rightarrow \\
\frac{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9}{\bullet - : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \text{ax/W} \quad \frac{- : \Delta_{11}, F_{12} \vdash \Delta_7, F_8 \wedge F_9}{\bullet - : \Delta_{11}, F_{12} \vdash \Delta_7, F_8 \wedge F_9} \text{sCut}}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \text{sCut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6 \quad h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_7}{\bullet h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \wedge F_7 \vdash \Delta_{11} \quad h_8 : \Delta_{12}, F_{10}, F_6 \wedge F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \wedge F_7 \vdash \Delta_{11}} \wedge_L}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_{12}, F_6, F_9 \vee F_{10} \vdash \Delta_{11}, F_7}{\bullet - : \Delta_{12}, F_6, F_9 \vee F_{10} \vdash \Delta_{11}, F_7} \text{ax/W} \quad \frac{- : \Delta_{12}, F_6, F_7, F_9 \vdash \Delta_{11}}{\bullet - : \Delta_{12}, F_6, F_7, F_9 \vee F_{10} \vdash \Delta_{11}} \text{inv-th/ax} \quad \frac{- : \Delta_{12}, F_{10}, F_6, F_7 \vdash \Delta_{11}}{\bullet - : \Delta_{12}, F_{10}, F_6, F_7 \vdash \Delta_{11}} \text{inv-th/ax}}{- : \Delta_{12}, F_6, F_9 \vee F_{10} \vdash \Delta_{11}} \vee_L \\
\frac{- : \Delta_{12}, F_6, F_9 \vee F_{10} \vdash \Delta_{11}, F_7}{\bullet - : \Delta_{12}, F_6, F_9 \vee F_{10} \vdash \Delta_{11}, F_7} \text{ax/W} \quad \frac{- : \Delta_{12}, F_6, F_9 \vee F_{10} \vdash \Delta_{11}}{\bullet - : \Delta_{12}, F_6, F_9 \vee F_{10} \vdash \Delta_{11}} \text{sCut}}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \text{sCut}
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{h_2 : \Delta_{11} \vdash \Delta_7, \perp, F_8 \quad h_2 : \Delta_{11} \vdash \Delta_7, \perp, F_9}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \wedge F_9), \perp} \wedge_R \quad \frac{}{\bullet h_{10} : \Delta_{11}, \perp \vdash \Delta_7, F_8 \wedge F_9} \perp_L \\
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{11} \vdash \perp, \Delta_7, F_8}{- : \Delta_{11} \vdash \Delta_7, F_8} \text{ax/W} \quad \frac{\bullet h_{10} : \perp, \Delta_{11} \vdash \Delta_7, F_8}{- : \Delta_{11} \vdash \Delta_7, F_8} \perp_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \text{hCut} \quad \frac{\frac{h_2 : \Delta_{11} \vdash \perp, \Delta_7, F_9}{- : \Delta_{11} \vdash \Delta_7, F_9} \text{ax/W} \quad \frac{\bullet h_{10} : \perp, \Delta_{11} \vdash \Delta_7, F_9}{- : \Delta_{11} \vdash \Delta_7, F_9} \perp_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \text{hCut} \\
\wedge_R
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \perp, \Delta_{12} \vdash \Delta_8, F_7, F_9 \quad h_2 : \perp, \Delta_{12} \vdash \Delta_8, F_7, F_{10}}{\bullet h_2 : \perp, \Delta_{12} \vdash (\Delta_8, F_9 \wedge F_{10}), F_7} \wedge_R \quad \frac{}{\bullet h_{11} : (\perp, \Delta_{12}), F_7 \vdash \Delta_8, F_9 \wedge F_{10}} \perp_L \\
\frac{}{- : \perp, \Delta_{12} \vdash \Delta_8, F_9 \wedge F_{10}} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_{12} \vdash \Delta_8, F_9 \wedge F_{10}} \perp_L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \perp, \Delta_{10} \vdash \Delta_9, F_6 \quad h_1 : \perp, \Delta_{10} \vdash \Delta_9, F_7}{\bullet h_1 : \perp, \Delta_{10} \vdash \Delta_9, F_6 \wedge F_7} \wedge_R \quad \frac{}{\bullet h_8 : (\perp, \Delta_{10}), F_6 \wedge F_7 \vdash \Delta_9} \perp_L \\
\frac{}{- : \perp, \Delta_{10} \vdash \Delta_9} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_{10} \vdash \Delta_9} \perp_L
\end{array}$$

• Case rule I

$$\begin{array}{c}
\frac{h_2 : \Delta_{10} \vdash (\Delta_{12}, p_{11}), p_{11}, F_7 \quad h_2 : \Delta_{10} \vdash (\Delta_{12}, p_{11}), p_{11}, F_8}{\bullet h_2 : \Delta_{10} \vdash ((\Delta_{12}, p_{11}), F_7 \wedge F_8), p_{11}} \wedge_R \quad \frac{}{\bullet h_9 : \Delta_{10}, p_{11} \vdash (\Delta_{12}, p_{11}), F_7 \wedge F_8} I \\
\frac{}{- : \Delta_{10} \vdash (\Delta_{12}, p_{11}), F_7 \wedge F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{10} \vdash \Delta_{12}, F_7, p_{11}, p_{11}}{- : \Delta_{10} \vdash \Delta_{12}, F_7, p_{11}} \text{ax/W} \quad \frac{\bullet h_9 : \Delta_{10}, p_{11} \vdash \Delta_{12}, F_7, p_{11}}{- : \Delta_{10} \vdash \Delta_{12}, F_7, p_{11}} I}{- : \Delta_{10} \vdash \Delta_{12}, F_7 \wedge F_8} \text{hCut} \quad \frac{\frac{h_2 : \Delta_{10} \vdash \Delta_{12}, F_8, p_{11}, p_{11}}{- : \Delta_{10} \vdash \Delta_{12}, F_8, p_{11}} \text{ax/W} \quad \frac{\bullet h_9 : \Delta_{10}, p_{11} \vdash \Delta_{12}, F_8, p_{11}}{- : \Delta_{10} \vdash \Delta_{12}, F_8, p_{11}} I}{- : \Delta_{10} \vdash \Delta_{12}, F_7 \wedge F_8} \text{hCut} \\
\wedge_R
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Delta_{13}, p_{11} \vdash (\Delta_{12}, p_{11}), F_7, F_8 \quad h_2 : \Delta_{13}, p_{11} \vdash (\Delta_{12}, p_{11}), F_7, F_9}{\bullet h_2 : \Delta_{13}, p_{11} \vdash ((\Delta_{12}, p_{11}), F_8 \wedge F_9), F_7} \wedge_R \quad \frac{}{\bullet h_{10} : (\Delta_{13}, p_{11}), F_7 \vdash (\Delta_{12}, p_{11}), F_8 \wedge F_9} I \\
\frac{}{- : \Delta_{13}, p_{11} \vdash (\Delta_{12}, p_{11}), F_8 \wedge F_9} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{13}, p_{11} \vdash \Delta_{12}, p_{11}, F_8 \wedge F_9} I
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_{11}, p_9 \vdash (\Delta_{10}, p_9), F_6 \quad h_1 : \Delta_{11}, p_9 \vdash (\Delta_{10}, p_9), F_7}{\bullet h_1 : \Delta_{11}, p_9 \vdash (\Delta_{10}, p_9), F_6 \wedge F_7} \wedge_R \quad \frac{}{\bullet h_8 : (\Delta_{11}, p_9), F_6 \wedge F_7 \vdash \Delta_{10}, p_9} I \\
\frac{}{- : \Delta_{11}, p_9 \vdash \Delta_{10}, p_9} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{11}, p_9 \vdash \Delta_{10}, p_9} I
\end{array}$$

• Case rule \top_L

$$\begin{array}{c}
\frac{h_2 : \Delta_{11} \vdash \Delta_7, \top, F_8 \quad h_2 : \Delta_{11} \vdash \Delta_7, \top, F_9}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \wedge F_9), \top} \wedge_R \quad \frac{h_{10} : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9}{\bullet h_{10} : \Delta_{11}, \top \vdash \Delta_7, F_8 \wedge F_9} \top_L \\
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9} \text{ax/W}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \top, \Delta_{12} \vdash \Delta_8, F_7, F_9 \quad h_2 : \top, \Delta_{12} \vdash \Delta_8, F_7, F_{10}}{\bullet h_2 : \top, \Delta_{12} \vdash (\Delta_8, F_9 \wedge F_{10}), F_7} \wedge_R \quad \frac{h_{11} : \Delta_{12}, F_7 \vdash \Delta_8, F_9 \wedge F_{10}}{\bullet h_{11} : (\top, \Delta_{12}), F_7 \vdash \Delta_8, F_9 \wedge F_{10}} \top_L \\
\frac{}{- : \top, \Delta_{12} \vdash \Delta_8, F_9 \wedge F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : \top, \Delta_{12} \vdash \Delta_8, F_7, F_9 \wedge F_{10}}{- : \top, \Delta_{12} \vdash \Delta_8, F_9 \wedge F_{10}} \text{ax/W} \quad \frac{h_{11} : \top, \Delta_{12}, F_7 \vdash \Delta_8, F_9 \wedge F_{10}}{- : \top, \Delta_{12} \vdash \Delta_8, F_9 \wedge F_{10}} \text{ax/W}}{- : \top, \Delta_{12} \vdash \Delta_8, F_9 \wedge F_{10}} \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \quad h_1 : \top, \Delta_{10} \vdash \Delta_9, F_7}{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \wedge F_7} \wedge_R \quad \frac{h_8 : \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_9}{\bullet h_8 : (\top, \Delta_{10}), F_6 \wedge F_7 \vdash \Delta_9} \top_L \\
\frac{}{- : \top, \Delta_{10} \vdash \Delta_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \wedge F_7}{- : \top, \Delta_{10} \vdash \Delta_9} \text{ax/W} \quad \frac{h_8 : \top, \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_9}{- : \top, \Delta_{10} \vdash \Delta_9} \text{ax/W}}{- : \top, \Delta_{10} \vdash \Delta_9} \text{hCut}
\end{array}$$

8.3 Status of \vee_R : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_2 : \Delta_7 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), F_8, F_9, F_{10}}{\bullet h_2 : \Delta_7 \vdash ((\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \vee F_{10}), F_8} \vee_R \quad \frac{h_{11} : \Delta_7, F_8, F_{12} \vdash \Delta_{14}, F_{13}, F_9 \vee F_{10}}{\bullet h_{11} : \Delta_7, F_8 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \vee F_{10}} \rightarrow_R}{- : \Delta_7 \vdash (\Delta_{14}, F_{12} \rightarrow F_{13}), F_9 \vee F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_7, F_{12} \vdash \Delta_{14}, F_{10}, F_{13}, F_8, F_9}{\bullet h_2 : \Delta_7, F_{12} \vdash \Delta_{14}, F_{13}, F_8, F_9 \vee F_{10}} \text{inv-th/ax}}{\vee_R} \quad \frac{h_{11} : \Delta_7, F_{12}, F_8 \vdash \Delta_{14}, F_{13}, F_9 \vee F_{10}}{\text{ax/W}}}{- : \Delta_7, F_{12} \vdash \Delta_{14}, F_{13}, F_9 \vee F_{10}} \text{hCut} \\
\rightarrow_R \\
- : \Delta_7 \vdash \Delta_{14}, F_{12} \rightarrow F_{13}, F_9 \vee F_{10} \\
\rightarrow \\
\frac{h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \rightarrow F_{12}), F_7, F_8}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \rightarrow F_{12}), F_7 \vee F_8} \vee_R \quad \frac{h_9 : \Delta_6, F_{11}, F_7 \vee F_8 \vdash \Delta_{10}, F_{12}}{\bullet h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \rightarrow_R}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}} \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_7, F_8}{\bullet h_1 : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_7 \vee F_8} \text{inv-th/ax}}{\vee_R} \quad \frac{h_9 : \Delta_6, F_{11}, F_7 \vee F_8 \vdash \Delta_{10}, F_{12}}{\text{ax/W}}}{- : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}} \text{hCut} \\
\rightarrow_R \\
- : \Delta_6 \vdash \Delta_{10}, F_{11} \rightarrow F_{12}
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{h_2 : \Delta_7 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_8, F_9, F_{10}}{\bullet h_2 : \Delta_7 \vdash ((\Delta_{14}, F_{12} \wedge F_{13}), F_9 \vee F_{10}), F_8} \vee_R \quad \frac{h_{11} : \Delta_7, F_8 \vdash \Delta_{14}, F_{12}, F_9 \vee F_{10} \quad h_{11} : \Delta_7, F_8 \vdash \Delta_{14}, F_{13}, F_9 \vee F_{10}}{\bullet h_{11} : \Delta_7, F_8 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_9 \vee F_{10}} \wedge_R}{- : \Delta_7 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_9 \vee F_{10}} \text{Cut} \\
\rightarrow \\
\frac{h_2 : \Delta_7 \vdash \Delta_{14}, F_{10}, F_8, F_9, F_{12} \wedge F_{13}}{\text{ax/W}} \quad \frac{\frac{h_{11} : \Delta_7, F_8 \vdash \Delta_{14}, F_{10}, F_{12}, F_9}{\bullet h_{11} : \Delta_7, F_8 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \wedge F_{13}} \text{inv-th/ax}}{\wedge_R} \quad \frac{h_{11} : \Delta_7, F_8 \vdash \Delta_{14}, F_{10}, F_{13}, F_9}{\text{inv-th/ax}}}{- : \Delta_7 \vdash \Delta_{14}, F_{10}, F_9, F_{12} \wedge F_{13}} \text{hCut} \\
\vee_R \\
- : \Delta_7 \vdash \Delta_{14}, F_{12} \wedge F_{13}, F_9 \vee F_{10} \\
\rightarrow \\
\frac{h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \wedge F_{12}), F_7, F_8}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \wedge F_{12}), F_7 \vee F_8} \vee_R \quad \frac{h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11} \quad h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{12}}{\bullet h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \wedge_R}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_7, F_8}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_7 \vee F_8} \text{inv-th/ax}}{\vee_R} \quad \frac{h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11}}{\text{ax/W}} \quad \frac{h_1 : \Delta_6 \vdash \Delta_{10}, F_{12}, F_7, F_8}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{12}, F_7 \vee F_8} \text{inv-th/ax}}{\vee_R} \quad \frac{h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{12}}{\text{ax/W}}}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \wedge F_{12}} \text{hCut} \\
\wedge_R \\
- : \Delta_6 \vdash \Delta_{10}, F_{11} \wedge F_{12}
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_2 : \Delta_7 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_8, F_9, F_{10}}{\bullet h_2 : \Delta_7 \vdash ((\Delta_{14}, F_{12} \vee F_{13}), F_9 \vee F_{10}), F_8} \vee_R \quad \frac{h_{11} : \Delta_7, F_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \vee F_{10}}{\bullet h_{11} : \Delta_7, F_8 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_9 \vee F_{10}} \vee_R}{- : \Delta_7 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_9 \vee F_{10}} \text{Cut} \\
\rightarrow \\
\frac{h_2 : \Delta_7 \vdash \Delta_{14}, F_{10}, F_{12}, F_{13}, F_8, F_9}{\bullet h_2 : \Delta_7 \vdash \Delta_{14}, F_{12}, F_{13}, F_8, F_9 \vee F_{10}} \text{inv-th/ax}}{\vee_R} \quad \frac{h_{11} : \Delta_7, F_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \vee F_{10}}{\text{ax/W}}}{- : \Delta_7 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \vee F_{10}} \text{hCut} \\
\vee_R \\
- : \Delta_7 \vdash \Delta_{14}, F_{12} \vee F_{13}, F_9 \vee F_{10}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Delta_7 \vdash \Delta_{10}, F_8, F_{11}, F_{12}}{\bullet h_2 : \Delta_7 \vdash (\Delta_{10}, F_{11} \vee F_{12}), F_8} \vee_R \quad \frac{h_9 : \Delta_7, F_8 \vdash \Delta_{10}, F_{11}, F_{12}}{\bullet h_9 : \Delta_7, F_8 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R}{- : \Delta_7 \vdash \Delta_{10}, F_{11} \vee F_{12}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_7 \vdash \Delta_{10}, F_{11}, F_{12}, F_8}{\bullet h_2 : \Delta_7 \vdash \Delta_{10}, F_{11}, F_{12}, F_8} \text{ax/W} \quad \frac{h_9 : \Delta_7, F_8 \vdash \Delta_{10}, F_{11}, F_{12}}{\bullet h_9 : \Delta_7, F_8 \vdash \Delta_{10}, F_{11}, F_{12}} \text{ax/W}}{- : \Delta_7 \vdash \Delta_{10}, F_{11}, F_{12}} \vee_R \quad \frac{- : \Delta_7 \vdash \Delta_{10}, F_{11} \vee F_{12}}{- : \Delta_7 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R}{- : \Delta_7 \vdash \Delta_{10}, F_{11} \vee F_{12}} \text{hCut} \\
\frac{h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \vee F_{12}), F_7, F_8}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \vee F_{12}), F_7 \vee F_8} \vee_R \quad \frac{h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11}, F_{12}}{\bullet h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \vee F_{12}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_7, F_8}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_7 \vee F_8} \text{inv-th/ax} \quad \frac{h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11}, F_{12}}{\bullet h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11}, F_{12}} \text{ax/W}}{- : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}} \vee_R \quad \frac{- : \Delta_6 \vdash \Delta_{10}, F_{11} \vee F_{12}}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \vee F_{12}} \vee_R}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \vee F_{12}} \text{hCut}
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_2 : \Delta_7 \vdash (\perp, \Delta_{12}), F_8, F_9, F_{10}}{\bullet h_2 : \Delta_7 \vdash ((\perp, \Delta_{12}), F_9 \vee F_{10}), F_8} \vee_R \quad \frac{h_{11} : \Delta_7, F_8 \vdash \Delta_{12}, F_9 \vee F_{10}}{\bullet h_{11} : \Delta_7, F_8 \vdash (\perp, \Delta_{12}), F_9 \vee F_{10}} \perp_R}{- : \Delta_7 \vdash (\perp, \Delta_{12}), F_9 \vee F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_7 \vdash \perp, \Delta_{12}, F_8, F_9 \vee F_{10}}{\bullet h_2 : \Delta_7 \vdash \perp, \Delta_{12}, F_8, F_9 \vee F_{10}} \text{ax/W} \quad \frac{h_{11} : \Delta_7, F_8 \vdash \perp, \Delta_{12}, F_9 \vee F_{10}}{\bullet h_{11} : \Delta_7, F_8 \vdash \perp, \Delta_{12}, F_9 \vee F_{10}} \text{ax/W}}{- : \Delta_7 \vdash \perp, \Delta_{12}, F_9 \vee F_{10}} \text{hCut} \\
\frac{h_1 : \Delta_6 \vdash (\perp, \Delta_{10}), F_7, F_8}{\bullet h_1 : \Delta_6 \vdash (\perp, \Delta_{10}), F_7 \vee F_8} \vee_R \quad \frac{h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \vee F_8 \vdash \perp, \Delta_{10}} \perp_R}{- : \Delta_6 \vdash \perp, \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_6 \vdash \perp, \Delta_{10}, F_7 \vee F_8}{\bullet h_1 : \Delta_6 \vdash \perp, \Delta_{10}, F_7 \vee F_8} \text{ax/W} \quad \frac{h_9 : \Delta_6, F_7 \vee F_8 \vdash \perp, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \vee F_8 \vdash \perp, \Delta_{10}} \text{ax/W}}{- : \Delta_6 \vdash \perp, \Delta_{10}} \text{hCut}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_2 : \Delta_7 \vdash (\top, \Delta_{12}), F_8, F_9, F_{10}}{\bullet h_2 : \Delta_7 \vdash ((\top, \Delta_{12}), F_9 \vee F_{10}), F_8} \vee_R \quad \frac{h_{11} : \Delta_7, F_8 \vdash (\top, \Delta_{12}), F_9 \vee F_{10}}{\bullet h_{11} : \Delta_7, F_8 \vdash (\top, \Delta_{12}), F_9 \vee F_{10}} \top_R}{- : \Delta_7 \vdash (\top, \Delta_{12}), F_9 \vee F_{10}} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_7 \vdash \top, \Delta_{12}, F_9 \vee F_{10}}{- : \Delta_7 \vdash \top, \Delta_{12}, F_9 \vee F_{10}} \top_R \\
\frac{h_1 : \Delta_6 \vdash (\top, \Delta_{10}), F_7, F_8}{\bullet h_1 : \Delta_6 \vdash (\top, \Delta_{10}), F_7 \vee F_8} \vee_R \quad \frac{h_9 : \Delta_6, F_7 \vee F_8 \vdash \top, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \vee F_8 \vdash \top, \Delta_{10}} \top_R}{- : \Delta_6 \vdash \top, \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_6 \vdash \top, \Delta_{10}}{- : \Delta_6 \vdash \top, \Delta_{10}} \top_R
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_7, F_9, F_{10}}{\bullet h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash (\Delta_8, F_9 \vee F_{10}), F_7} \vee_R \quad \frac{h_{11} : \Delta_{14}, F_7 \vdash \Delta_8, F_{12}, F_9 \vee F_{10} \quad h_{11} : \Delta_{14}, F_7, F_{13} \vdash \Delta_8, F_9 \vee F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \rightarrow F_{13}), F_7 \vdash \Delta_8, F_9 \vee F_{10}} \rightarrow_L}{- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9 \vee F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_{10}, F_7, F_9}{\bullet h_2 : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_{10}, F_7, F_9} \text{ax/W} \quad \frac{h_{11} : \Delta_{14}, F_7 \vdash \Delta_8, F_{10}, F_{12}, F_9}{\bullet h_{11} : \Delta_{14}, F_7, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_{10}, F_9} \text{inv-th/ax}}{- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_{10}, F_9} \vee_R \quad \frac{h_{11} : \Delta_{14}, F_{13}, F_7 \vdash \Delta_8, F_{10}, F_9}{\bullet h_{11} : \Delta_{14}, F_{13}, F_7 \vdash \Delta_8, F_{10}, F_9} \text{inv-th/ax}}{- : \Delta_{14}, F_{12} \rightarrow F_{13} \vdash \Delta_8, F_9 \vee F_{10}} \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \Delta_{11} \vdash \Delta_7, F_{12} \rightarrow F_{13}, F_8, F_9}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \vee F_9), F_{12} \rightarrow F_{13}} \vee_R \quad \frac{h_{10} : \Delta_{11} \vdash \Delta_7, F_{12}, F_8 \vee F_9 \quad h_{10} : \Delta_{11}, F_{13} \vdash \Delta_7, F_8 \vee F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \rightarrow F_{13} \vdash \Delta_7, F_8 \vee F_9} \rightarrow_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{11} \vdash \Delta_7, F_8, F_9, F_{12} \rightarrow F_{13}}{- : \Delta_{11} \vdash \Delta_7, F_8, F_9} \text{ax/W} \quad \frac{h_{10} : \Delta_{11} \vdash \Delta_7, F_{12}, F_8, F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \rightarrow F_{13} \vdash \Delta_7, F_8, F_9} \text{inv-th/ax} \quad \frac{h_{10} : \Delta_{11}, F_{13} \vdash \Delta_7, F_8, F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \rightarrow F_{13} \vdash \Delta_7, F_8, F_9} \text{inv-th/ax}}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \vee_R}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \text{hCut} \\
\rightarrow \\
\frac{h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \vee_R \quad \frac{h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_9 \quad h_8 : \Delta_{12}, F_{10}, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \rightarrow F_{10}), F_6 \vee F_7 \vdash \Delta_{11}} \rightarrow_L}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12} \vdash \Delta_{11}, F_6, F_7, F_9}{\bullet h_1 : \Delta_{12} \vdash \Delta_{11}, F_9, F_6 \vee F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_6 \vee F_7 \vdash \Delta_{11}, F_9}{\bullet h_8 : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{ax/W}}{- : \Delta_{12} \vdash \Delta_{11}, F_9} \vee_R \quad \frac{\frac{h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_{10}, F_6 \vee F_7 \vdash \Delta_{11}} \text{ax/W}}{- : \Delta_{12}, F_{10} \vdash \Delta_{11}} \vee_R}{- : \Delta_{12}, F_9 \rightarrow F_{10} \vdash \Delta_{11}} \text{hCut} \rightarrow_L
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_7, F_9, F_{10}}{\bullet h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash (\Delta_8, F_9 \vee F_{10}), F_7} \vee_R \quad \frac{h_{11} : \Delta_{14}, F_7, F_{12}, F_{13} \vdash \Delta_8, F_9 \vee F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \wedge F_{13}), F_7 \vdash \Delta_8, F_9 \vee F_{10}} \wedge_L}{- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_9 \vee F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_{10}, F_7, F_9}{\bullet h_2 : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_{10}, F_7, F_9} \text{ax/W} \quad \frac{h_{11} : \Delta_{14}, F_{12}, F_{13}, F_7 \vdash \Delta_8, F_{10}, F_9}{\bullet h_{11} : \Delta_{14}, F_7, F_{12} \wedge F_{13} \vdash \Delta_8, F_{10}, F_9} \text{inv-th/ax}}{- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_{10}, F_9} \wedge_L}{- : \Delta_{14}, F_{12} \wedge F_{13} \vdash \Delta_8, F_9 \vee F_{10}} \vee_R \text{hCut} \\
\rightarrow \\
\frac{h_2 : \Delta_{11} \vdash \Delta_7, F_{12} \wedge F_{13}, F_8, F_9}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \vee F_9), F_{12} \wedge F_{13}} \vee_R \quad \frac{h_{10} : \Delta_{11}, F_{12}, F_{13} \vdash \Delta_7, F_8 \vee F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \wedge F_{13} \vdash \Delta_7, F_8 \vee F_9} \wedge_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{11} \vdash \Delta_7, F_8, F_9, F_{12} \wedge F_{13}}{\bullet h_2 : \Delta_{11} \vdash \Delta_7, F_8, F_9} \text{ax/W} \quad \frac{h_{10} : \Delta_{11}, F_{12}, F_{13} \vdash \Delta_7, F_8, F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \wedge F_{13} \vdash \Delta_7, F_8, F_9} \text{inv-th/ax}}{- : \Delta_{11} \vdash \Delta_7, F_8, F_9} \wedge_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \vee_R \text{hCut} \\
\rightarrow \\
\frac{h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \vee_R \quad \frac{h_8 : \Delta_{12}, F_9, F_{10}, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \wedge F_{10}), F_6 \vee F_7 \vdash \Delta_{11}} \wedge_L}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}, F_6 \vee F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_9, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : \Delta_{12}, F_{10}, F_9, F_6 \vee F_7 \vdash \Delta_{11}} \text{ax/W}}{- : \Delta_{12}, F_{10}, F_9 \vdash \Delta_{11}} \wedge_L}{- : \Delta_{12}, F_9 \wedge F_{10} \vdash \Delta_{11}} \vee_R \text{hCut}
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_7, F_9, F_{10}}{\bullet h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash (\Delta_8, F_9 \vee F_{10}), F_7} \vee_R \quad \frac{h_{11} : \Delta_{14}, F_7, F_{12} \vdash \Delta_8, F_9 \vee F_{10} \quad h_{11} : \Delta_{14}, F_7, F_{13} \vdash \Delta_8, F_9 \vee F_{10}}{\bullet h_{11} : (\Delta_{14}, F_{12} \vee F_{13}), F_7 \vdash \Delta_8, F_9 \vee F_{10}} \vee_L}{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \vee F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}, F_7, F_9}{\bullet h_2 : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}, F_7, F_9} \text{ax/W} \quad \frac{h_{11} : \Delta_{14}, F_{12}, F_7 \vdash \Delta_8, F_{10}, F_9}{\bullet h_{11} : \Delta_{14}, F_7, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}, F_9} \text{inv-th/ax}}{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_{10}, F_9} \vee_L}{- : \Delta_{14}, F_{12} \vee F_{13} \vdash \Delta_8, F_9 \vee F_{10}} \vee_R \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_2 : \Delta_{11} \vdash \Delta_7, F_{12} \vee F_{13}, F_8, F_9}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \vee F_9), F_{12} \vee F_{13}} \vee_R \quad \frac{h_{10} : \Delta_{11}, F_{12} \vdash \Delta_7, F_8 \vee F_9 \quad h_{10} : \Delta_{11}, F_{13} \vdash \Delta_7, F_8 \vee F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \vee F_{13} \vdash \Delta_7, F_8 \vee F_9} \vee_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{11} \vdash \Delta_7, F_8, F_9, F_{12} \vee F_{13}}{\bullet h_2 : \Delta_{11} \vdash \Delta_7, F_8, F_9, F_{12} \vee F_{13}} \text{ax/W} \quad \frac{\frac{h_{10} : \Delta_{11}, F_{12} \vdash \Delta_7, F_8, F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \vee F_{13} \vdash \Delta_7, F_8, F_9} \text{inv-th/ax} \quad \frac{h_{10} : \Delta_{11}, F_{13} \vdash \Delta_7, F_8, F_9}{\bullet h_{10} : \Delta_{11}, F_{12} \vee F_{13} \vdash \Delta_7, F_8, F_9} \text{inv-th/ax}}{- : \Delta_{11} \vdash \Delta_7, F_8, F_9} \vee_R}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \text{hCut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \vee_R \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \vee F_7 \vdash \Delta_{11} \quad h_8 : \Delta_{12}, F_{10}, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \vee F_7 \vdash \Delta_{11}} \vee_L}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_9 \vdash \Delta_{11}, F_6 \vee F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \vee F_{10}), F_6 \vee F_7 \vdash \Delta_{11}} \text{ax/W}}{- : \Delta_{12}, F_9 \vdash \Delta_{11}} \vee_R \quad \frac{\frac{h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6, F_7}{\bullet h_1 : \Delta_{12}, F_{10} \vdash \Delta_{11}, F_6 \vee F_7} \text{inv-th/ax} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_6 \vee F_7 \vdash \Delta_{11}}{\bullet h_8 : (\Delta_{12}, F_{10} \vee F_7), F_6 \vee F_7 \vdash \Delta_{11}} \text{ax/W}}{- : \Delta_{12}, F_{10} \vdash \Delta_{11}} \vee_R}{- : \Delta_{12}, F_9 \vee F_{10} \vdash \Delta_{11}} \text{hCut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_7 \vdash \Delta_{10}, F_8, F_9}{\bullet h_1 : \Delta_7 \vdash \Delta_{10}, F_8 \vee F_9} \vee_R \quad \frac{h_6 : \Delta_7, F_8 \vdash \Delta_{10} \quad h_6 : \Delta_7, F_9 \vdash \Delta_{10}}{\bullet h_6 : \Delta_7, F_8 \vee F_9 \vdash \Delta_{10}} \vee_L}{- : \Delta_7 \vdash \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_7 \vdash \Delta_{10}, F_8, F_9 \quad \text{ax/W} \quad - : \Delta_7, F_9 \vdash \Delta_{10}, F_8 \quad \text{ax/W}}{- : \Delta_7 \vdash \Delta_{10}, F_8} \text{sCut} \quad \frac{- : \Delta_7, F_8 \vdash \Delta_{10} \quad \text{ax/W}}{- : \Delta_7 \vdash \Delta_{10}} \text{sCut} \\
\rightarrow \\
- : \Delta_7 \vdash \Delta_{10}
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{\frac{h_2 : \Delta_{11} \vdash \Delta_7, \perp, F_8, F_9}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \vee F_9), \perp} \vee_R \quad \frac{}{\bullet h_{10} : \Delta_{11}, \perp \vdash \Delta_7, F_8 \vee F_9} \perp_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{11} \vdash \perp, \Delta_7, F_8, F_9}{\bullet h_2 : \Delta_{11} \vdash \perp, \Delta_7, F_8, F_9} \text{ax/W} \quad \frac{}{\bullet h_{10} : \perp, \Delta_{11} \vdash \Delta_7, F_8, F_9} \perp_L}{- : \Delta_{11} \vdash \Delta_7, F_8, F_9} \text{hCut} \\
\rightarrow \\
\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \vee_R \\
\rightarrow \\
\frac{h_2 : \perp, \Delta_{12} \vdash \Delta_8, F_7, F_9, F_{10}}{\bullet h_2 : \perp, \Delta_{12} \vdash (\Delta_8, F_9 \vee F_{10}), F_7} \vee_R \quad \frac{}{\bullet h_{11} : (\perp, \Delta_{12}), F_7 \vdash \Delta_8, F_9 \vee F_{10}} \perp_L}{- : \perp, \Delta_{12} \vdash \Delta_8, F_9 \vee F_{10}} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_{12} \vdash \Delta_8, F_9 \vee F_{10}} \perp_L \\
\rightarrow \\
\frac{h_1 : \perp, \Delta_{10} \vdash \Delta_9, F_6, F_7}{\bullet h_1 : \perp, \Delta_{10} \vdash \Delta_9, F_6 \vee F_7} \vee_R \quad \frac{}{\bullet h_8 : (\perp, \Delta_{10}), F_6 \vee F_7 \vdash \Delta_9} \perp_L}{- : \perp, \Delta_{10} \vdash \Delta_9} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_{10} \vdash \Delta_9} \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\frac{h_2 : \Delta_{10} \vdash (\Delta_{12}, p_{11}), p_{11}, F_7, F_8}{\bullet h_2 : \Delta_{10} \vdash ((\Delta_{12}, p_{11}), F_7 \vee F_8), p_{11}} \vee_R \quad \frac{}{\bullet h_9 : \Delta_{10}, p_{11} \vdash (\Delta_{12}, p_{11}), F_7 \vee F_8} I}{- : \Delta_{10} \vdash (\Delta_{12}, p_{11}), F_7 \vee F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{10} \vdash \Delta_{12}, F_7, F_8, p_{11}, p_{11}}{\bullet h_2 : \Delta_{10} \vdash \Delta_{12}, F_7, F_8, p_{11}, p_{11}} \text{ax/W} \quad \frac{}{\bullet h_9 : \Delta_{10}, p_{11} \vdash \Delta_{12}, F_7, F_8, p_{11}} I}{- : \Delta_{10} \vdash \Delta_{12}, F_7, F_8, p_{11}} \text{hCut} \\
\rightarrow \\
\frac{}{- : \Delta_{10} \vdash \Delta_{12}, p_{11}, F_7 \vee F_8} \vee_R \\
\rightarrow \\
\frac{h_2 : \Delta_{13}, p_{11} \vdash (\Delta_{12}, p_{11}), F_7, F_8, F_9}{\bullet h_2 : \Delta_{13}, p_{11} \vdash ((\Delta_{12}, p_{11}), F_8 \vee F_9), F_7} \vee_R \quad \frac{}{\bullet h_{10} : (\Delta_{13}, p_{11}), F_7 \vdash (\Delta_{12}, p_{11}), F_8 \vee F_9} I}{- : \Delta_{13}, p_{11} \vdash (\Delta_{12}, p_{11}), F_8 \vee F_9} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{13}, p_{11} \vdash \Delta_{12}, p_{11}, F_8 \vee F_9} I
\end{array}$$

$$\frac{\frac{h_1 : \Delta_{11}, p_9 \vdash (\Delta_{10}, p_9), F_6, F_7}{\bullet h_1 : \Delta_{11}, p_9 \vdash (\Delta_{10}, p_9), F_6 \vee F_7} \vee_R \quad \frac{h_8 : (\Delta_{11}, p_9), F_6 \vee F_7 \vdash \Delta_{10}, p_9}{\text{Cut}} \quad I}{\frac{- : \Delta_{11}, p_9 \vdash \Delta_{10}, p_9}{\rightarrow} \quad I} \text{Cut}$$

$$\frac{}{- : \Delta_{11}, p_9 \vdash \Delta_{10}, p_9} \text{I}$$

- Case rule \top_L

$$\frac{\frac{h_2 : \Delta_{11} \vdash \Delta_7, \top, F_8, F_9}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \vee F_9), \top} \vee_R \quad \frac{h_{10} : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9}{\bullet h_{10} : \Delta_{11}, \top \vdash \Delta_7, F_8 \vee F_9} \top_L}{\frac{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9}{\rightarrow} \quad \text{Cut}} \text{Cut}$$

$$\frac{}{- : \Delta_{11} \vdash \Delta_7, F_8 \vee F_9} \text{ax/W}$$

$$\frac{\frac{h_2 : \top, \Delta_{12} \vdash \Delta_8, F_7, F_9, F_{10}}{\bullet h_2 : \top, \Delta_{12} \vdash (\Delta_8, F_9 \vee F_{10}), F_7} \vee_R \quad \frac{h_{11} : \Delta_{12}, F_7 \vdash \Delta_8, F_9 \vee F_{10}}{\bullet h_{11} : (\top, \Delta_{12}), F_7 \vdash \Delta_8, F_9 \vee F_{10}} \top_L}{\frac{- : \top, \Delta_{12} \vdash \Delta_8, F_9 \vee F_{10}}{\rightarrow} \quad \text{Cut}} \text{Cut}$$

$$\frac{\frac{\bullet h_2 : \top, \Delta_{12} \vdash \Delta_8, F_7, F_9 \vee F_{10}}{\rightarrow} \text{ax/W} \quad \frac{h_{11} : \top, \Delta_{12}, F_7 \vdash \Delta_8, F_9 \vee F_{10}}{\bullet h_{11} : \top, \Delta_{12}, F_7 \vdash \Delta_8, F_9 \vee F_{10}} \text{ax/W}}{- : \top, \Delta_{12} \vdash \Delta_8, F_9 \vee F_{10}} \text{hCut}$$

$$\frac{\frac{h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6, F_7}{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \vee F_7} \vee_R \quad \frac{h_8 : \Delta_{10}, F_6 \vee F_7 \vdash \Delta_9}{\bullet h_8 : (\top, \Delta_{10}), F_6 \vee F_7 \vdash \Delta_9} \top_L}{\frac{- : \top, \Delta_{10} \vdash \Delta_9}{\rightarrow} \quad \text{Cut}} \text{Cut}$$

$$\frac{\frac{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \vee F_7}{\rightarrow} \text{ax/W} \quad \frac{h_8 : \top, \Delta_{10}, F_6 \vee F_7 \vdash \Delta_9}{\bullet h_8 : \top, \Delta_{10}, F_6 \vee F_7 \vdash \Delta_9} \text{ax/W}}{- : \top, \Delta_{10} \vdash \Delta_9} \text{hCut}$$

8.4 Status of \perp_R : OK

- Case rule \rightarrow_R

$$\frac{\frac{h_1 : \Delta_4 \vdash \Delta_6, F_7 \rightarrow F_8}{\bullet h_1 : \Delta_4 \vdash (\Delta_6, F_7 \rightarrow F_8), \perp} \perp_R \quad \frac{h_5 : \perp, \Delta_4, F_7 \vdash \Delta_6, F_8}{\bullet h_5 : \Delta_4, \perp \vdash \Delta_6, F_7 \rightarrow F_8} \rightarrow_R}{\frac{- : \Delta_4 \vdash \Delta_6, F_7 \rightarrow F_8}{\rightarrow} \quad \text{Cut}} \text{Cut}$$

$$\frac{}{- : \Delta_4 \vdash \Delta_6, F_7 \rightarrow F_8} \text{ax/W}$$

$$\frac{\frac{h_2 : \Delta_5 \vdash (\Delta_{10}, F_8 \rightarrow F_9), F_6}{\bullet h_2 : \Delta_5 \vdash (\perp, \Delta_{10}, F_8 \rightarrow F_9), F_6} \perp_R \quad \frac{h_7 : \Delta_5, F_6, F_8 \vdash \perp, \Delta_{10}, F_9}{\bullet h_7 : \Delta_5, F_6 \vdash \perp, \Delta_{10}, F_8 \rightarrow F_9} \rightarrow_R}{\frac{- : \Delta_5 \vdash \perp, \Delta_{10}, F_8 \rightarrow F_9}{\rightarrow} \quad \text{Cut}} \text{Cut}$$

$$\frac{\frac{h_2 : \Delta_5 \vdash \perp, \Delta_{10}, F_6, F_8 \rightarrow F_9}{\rightarrow} \text{ax/W} \quad \frac{h_7 : \Delta_5, F_6 \vdash \perp, \Delta_{10}, F_8 \rightarrow F_9}{\bullet h_7 : \Delta_5, F_6 \vdash \perp, \Delta_{10}, F_8 \rightarrow F_9} \text{ax/W}}{- : \Delta_5 \vdash \perp, \Delta_{10}, F_8 \rightarrow F_9} \text{hCut}$$

- Case rule \wedge_R

$$\frac{\frac{h_1 : \Delta_4 \vdash \Delta_6, F_7 \wedge F_8}{\bullet h_1 : \Delta_4 \vdash (\Delta_6, F_7 \wedge F_8), \perp} \perp_R \quad \frac{h_5 : \perp, \Delta_4 \vdash \Delta_6, F_7 \quad h_5 : \perp, \Delta_4 \vdash \Delta_6, F_8}{\bullet h_5 : \Delta_4, \perp \vdash \Delta_6, F_7 \wedge F_8} \wedge_R}{\frac{- : \Delta_4 \vdash \Delta_6, F_7 \wedge F_8}{\rightarrow} \quad \text{Cut}} \text{Cut}$$

$$\frac{}{- : \Delta_4 \vdash \Delta_6, F_7 \wedge F_8} \text{ax/W}$$

$$\frac{\frac{h_2 : \Delta_5 \vdash (\Delta_{10}, F_8 \wedge F_9), F_6}{\bullet h_2 : \Delta_5 \vdash (\perp, \Delta_{10}, F_8 \wedge F_9), F_6} \perp_R \quad \frac{h_7 : \Delta_5, F_6 \vdash \perp, \Delta_{10}, F_8 \quad h_7 : \Delta_5, F_6 \vdash \perp, \Delta_{10}, F_9}{\bullet h_7 : \Delta_5, F_6 \vdash \perp, \Delta_{10}, F_8 \wedge F_9} \wedge_R}{\frac{- : \Delta_5 \vdash \perp, \Delta_{10}, F_8 \wedge F_9}{\rightarrow} \quad \text{Cut}} \text{Cut}$$

$$\frac{\frac{h_2 : \Delta_5 \vdash \perp, \Delta_{10}, F_6, F_8 \wedge F_9}{\rightarrow} \text{ax/W} \quad \frac{h_7 : \Delta_5, F_6 \vdash \perp, \Delta_{10}, F_8 \wedge F_9}{\bullet h_7 : \Delta_5, F_6 \vdash \perp, \Delta_{10}, F_8 \wedge F_9} \text{ax/W}}{- : \Delta_5 \vdash \perp, \Delta_{10}, F_8 \wedge F_9} \text{hCut}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_1 : \Delta_4 \vdash \Delta_6, F_7 \vee F_8}{\bullet h_1 : \Delta_4 \vdash (\Delta_6, F_7 \vee F_8), \perp} \perp_R \quad \frac{h_5 : \perp, \Delta_4 \vdash \Delta_6, F_7, F_8}{\bullet h_5 : \Delta_4, \perp \vdash \Delta_6, F_7 \vee F_8} \vee_R}{\frac{}{- : \Delta_4 \vdash \Delta_6, F_7 \vee F_8} \rightarrow} \text{Cut} \\
\frac{}{- : \Delta_4 \vdash \Delta_6, F_7 \vee F_8} \text{ax/W} \\
\frac{h_2 : \Delta_5 \vdash (\Delta_{10}, F_8 \vee F_9), F_6}{\bullet h_2 : \Delta_5 \vdash (\perp, \Delta_{10}, F_8 \vee F_9), F_6} \perp_R \quad \frac{h_7 : \Delta_5, F_6 \vdash \perp, \Delta_{10}, F_8, F_9}{\bullet h_7 : \Delta_5, F_6 \vdash \perp, \Delta_{10}, F_8 \vee F_9} \vee_R}{\frac{}{- : \Delta_5 \vdash \perp, \Delta_{10}, F_8 \vee F_9} \rightarrow} \text{Cut} \\
\frac{}{- : \Delta_5 \vdash \perp, \Delta_{10}, F_8 \vee F_9} \text{ax/W} \quad \frac{}{- : \Delta_5 \vdash \perp, \Delta_{10}, F_8 \vee F_9} \text{ax/W} \\
\frac{}{- : \Delta_5 \vdash \perp, \Delta_{10}, F_8 \vee F_9} \text{hCut}
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_1 : \Delta_4 \vdash \perp, \Delta_6}{\bullet h_1 : \Delta_4 \vdash (\perp, \Delta_6), \perp} \perp_R \quad \frac{h_5 : \perp, \Delta_4 \vdash \Delta_6}{\bullet h_5 : \Delta_4, \perp \vdash \perp, \Delta_6} \perp_R}{\frac{}{- : \Delta_4 \vdash \perp, \Delta_6} \rightarrow} \text{Cut} \\
\frac{}{- : \Delta_4 \vdash \perp, \Delta_6} \text{ax/W} \\
\frac{h_2 : \Delta_5 \vdash \Delta_8, F_6}{\bullet h_2 : \Delta_5 \vdash (\perp, \Delta_8), F_6} \perp_R \quad \frac{h_7 : \Delta_5, F_6 \vdash \Delta_8}{\bullet h_7 : \Delta_5, F_6 \vdash \perp, \Delta_8} \perp_R}{\frac{}{- : \Delta_5 \vdash \perp, \Delta_8} \rightarrow} \text{Cut} \\
\frac{}{- : \Delta_5 \vdash \perp, \Delta_8} \text{ax/W} \quad \frac{}{- : \Delta_5 \vdash \perp, \Delta_8} \text{ax/W} \\
\frac{}{- : \Delta_5 \vdash \perp, \Delta_8} \text{hCut}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_1 : \Delta_4 \vdash \top, \Delta_6}{\bullet h_1 : \Delta_4 \vdash (\top, \Delta_6), \perp} \perp_R \quad \frac{}{\bullet h_5 : \Delta_4, \perp \vdash \top, \Delta_6} \top_R}{\frac{}{- : \Delta_4 \vdash \top, \Delta_6} \rightarrow} \text{Cut} \\
\frac{}{- : \Delta_4 \vdash \top, \Delta_6} \top_R \\
\frac{h_2 : \Delta_5 \vdash (\top, \Delta_8), F_6}{\bullet h_2 : \Delta_5 \vdash (\perp, \top, \Delta_8), F_6} \perp_R \quad \frac{}{\bullet h_7 : \Delta_5, F_6 \vdash \perp, \top, \Delta_8} \top_R}{\frac{}{- : \Delta_5 \vdash \perp, \top, \Delta_8} \rightarrow} \text{Cut} \\
\frac{}{- : \Delta_5 \vdash \perp, \top, \Delta_8} \top_R
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_1 : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7}{\bullet h_1 : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7, \perp} \perp_R \quad \frac{h_4 : \perp, \Delta_8 \vdash \Delta_7, F_5 \quad h_4 : \perp, \Delta_8, F_6 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \rightarrow F_6), \perp \vdash \Delta_7} \rightarrow_L}{\frac{}{- : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7} \rightarrow} \text{Cut} \\
\frac{}{- : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7} \text{ax/W} \\
\frac{h_2 : \Delta_{10}, F_8 \rightarrow F_9 \vdash \Delta_6, F_5}{\bullet h_2 : \Delta_{10}, F_8 \rightarrow F_9 \vdash (\perp, \Delta_6), F_5} \perp_R \quad \frac{h_7 : \Delta_{10}, F_5 \vdash \perp, \Delta_6, F_8 \quad h_7 : \Delta_{10}, F_5, F_9 \vdash \perp, \Delta_6}{\bullet h_7 : (\Delta_{10}, F_8 \rightarrow F_9), F_5 \vdash \perp, \Delta_6} \rightarrow_L}{\frac{}{- : \Delta_{10}, F_8 \rightarrow F_9 \vdash \perp, \Delta_6} \rightarrow} \text{Cut} \\
\frac{}{- : \Delta_{10}, F_8 \rightarrow F_9 \vdash \perp, \Delta_6} \text{ax/W} \quad \frac{}{- : \Delta_{10}, F_8 \rightarrow F_9 \vdash \perp, \Delta_6} \text{ax/W} \\
\frac{}{- : \Delta_{10}, F_8 \rightarrow F_9 \vdash \perp, \Delta_6} \text{hCut} \\
\frac{h_2 : \Delta_7 \vdash \Delta_5, F_8 \rightarrow F_9}{\bullet h_2 : \Delta_7 \vdash (\perp, \Delta_5), F_8 \rightarrow F_9} \perp_R \quad \frac{h_6 : \Delta_7 \vdash \perp, \Delta_5, F_8 \quad h_6 : \Delta_7, F_9 \vdash \perp, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \rightarrow F_9 \vdash \perp, \Delta_5} \rightarrow_L}{\frac{}{- : \Delta_7 \vdash \perp, \Delta_5} \rightarrow} \text{Cut} \\
\frac{}{- : \Delta_7 \vdash \perp, \Delta_5} \text{ax/W} \quad \frac{}{- : \Delta_7 \vdash \perp, \Delta_5} \text{ax/W} \\
\frac{}{- : \Delta_7 \vdash \perp, \Delta_5} \text{hCut}
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7}{\bullet h_1 : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7, \perp} \perp_R \quad \frac{h_4 : \perp, \Delta_8, F_5, F_6 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \wedge F_6), \perp \vdash \Delta_7} \wedge_L}{- : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7} \text{ax/W} \\
\frac{\frac{h_2 : \Delta_{10}, F_8 \wedge F_9 \vdash \Delta_6, F_5}{\bullet h_2 : \Delta_{10}, F_8 \wedge F_9 \vdash (\perp, \Delta_6), F_5} \perp_R \quad \frac{h_7 : \Delta_{10}, F_5, F_8, F_9 \vdash \perp, \Delta_6}{\bullet h_7 : (\Delta_{10}, F_8 \wedge F_9), F_5 \vdash \perp, \Delta_6} \wedge_L}{- : \Delta_{10}, F_8 \wedge F_9 \vdash \perp, \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{10}, F_8 \wedge F_9 \vdash \perp, \Delta_6, F_5}{- : \Delta_{10}, F_8 \wedge F_9 \vdash \perp, \Delta_6} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5, F_8 \wedge F_9 \vdash \perp, \Delta_6}{\bullet h_7 : \Delta_{10}, F_5, F_8 \wedge F_9 \vdash \perp, \Delta_6} \text{ax/W}}{- : \Delta_{10}, F_8 \wedge F_9 \vdash \perp, \Delta_6} \text{hCut} \\
\frac{\frac{h_2 : \Delta_7 \vdash \Delta_5, F_8 \wedge F_9}{\bullet h_2 : \Delta_7 \vdash (\perp, \Delta_5), F_8 \wedge F_9} \perp_R \quad \frac{h_6 : \Delta_7, F_8, F_9 \vdash \perp, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \wedge F_9 \vdash \perp, \Delta_5} \wedge_L}{- : \Delta_7 \vdash \perp, \Delta_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_7 \vdash \perp, \Delta_5, F_8 \wedge F_9}{- : \Delta_7 \vdash \perp, \Delta_5} \text{ax/W} \quad \frac{h_6 : \Delta_7, F_8 \wedge F_9 \vdash \perp, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \wedge F_9 \vdash \perp, \Delta_5} \text{ax/W}}{- : \Delta_7 \vdash \perp, \Delta_5} \text{hCut}
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_8, F_5 \vee F_6 \vdash \Delta_7}{\bullet h_1 : \Delta_8, F_5 \vee F_6 \vdash \Delta_7, \perp} \perp_R \quad \frac{h_4 : \perp, \Delta_8, F_5 \vdash \Delta_7 \quad h_4 : \perp, \Delta_8, F_6 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \vee F_6), \perp \vdash \Delta_7} \vee_L}{- : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \text{ax/W} \\
\frac{\frac{h_2 : \Delta_{10}, F_8 \vee F_9 \vdash \Delta_6, F_5}{\bullet h_2 : \Delta_{10}, F_8 \vee F_9 \vdash (\perp, \Delta_6), F_5} \perp_R \quad \frac{h_7 : \Delta_{10}, F_5, F_8 \vdash \perp, \Delta_6 \quad h_7 : \Delta_{10}, F_5, F_9 \vdash \perp, \Delta_6}{\bullet h_7 : (\Delta_{10}, F_8 \vee F_9), F_5 \vdash \perp, \Delta_6} \vee_L}{- : \Delta_{10}, F_8 \vee F_9 \vdash \perp, \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_{10}, F_8 \vee F_9 \vdash \perp, \Delta_6, F_5}{- : \Delta_{10}, F_8 \vee F_9 \vdash \perp, \Delta_6} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_5, F_8 \vee F_9 \vdash \perp, \Delta_6}{\bullet h_7 : \Delta_{10}, F_5, F_8 \vee F_9 \vdash \perp, \Delta_6} \text{ax/W}}{- : \Delta_{10}, F_8 \vee F_9 \vdash \perp, \Delta_6} \text{hCut} \\
\frac{\frac{h_2 : \Delta_7 \vdash \Delta_5, F_8 \vee F_9}{\bullet h_2 : \Delta_7 \vdash (\perp, \Delta_5), F_8 \vee F_9} \perp_R \quad \frac{h_6 : \Delta_7, F_8 \vdash \perp, \Delta_5 \quad h_6 : \Delta_7, F_9 \vdash \perp, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \vee F_9 \vdash \perp, \Delta_5} \vee_L}{- : \Delta_7 \vdash \perp, \Delta_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \Delta_7 \vdash \perp, \Delta_5, F_8 \vee F_9}{- : \Delta_7 \vdash \perp, \Delta_5} \text{ax/W} \quad \frac{h_6 : \Delta_7, F_8 \vee F_9 \vdash \perp, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \vee F_9 \vdash \perp, \Delta_5} \text{ax/W}}{- : \Delta_7 \vdash \perp, \Delta_5} \text{hCut}
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_5 \vdash \Delta_6}{\bullet h_1 : \Delta_5 \vdash \Delta_6, \perp} \perp_R \quad \frac{}{\bullet h_4 : \Delta_5, \perp \vdash \Delta_6} \perp_L}{- : \Delta_5 \vdash \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_5 \vdash \Delta_6} \text{ax/W} \\
\frac{\frac{h_2 : \Delta_7 \vdash \Delta_5, \perp}{\bullet h_2 : \Delta_7 \vdash (\perp, \Delta_5), \perp} \perp_R \quad \frac{}{\bullet h_6 : \Delta_7, \perp \vdash \perp, \Delta_5} \perp_L}{- : \Delta_7 \vdash \perp, \Delta_5} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_7 \vdash \perp, \Delta_5} \text{ax/W} \\
\frac{\frac{h_2 : \perp, \Delta_8 \vdash \Delta_6, F_5}{\bullet h_2 : \perp, \Delta_8 \vdash (\perp, \Delta_6), F_5} \perp_R \quad \frac{}{\bullet h_7 : (\perp, \Delta_8), F_5 \vdash \perp, \Delta_6} \perp_L}{- : \perp, \Delta_8 \vdash \perp, \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_8 \vdash \perp, \Delta_6} \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_1 : \Delta_7, P_5 \vdash \Delta_6, P_5}{\bullet h_1 : \Delta_7, P_5 \vdash (\Delta_6, P_5), \perp} \perp_R \quad \frac{}{\bullet h_4 : (\Delta_7, P_5), \perp \vdash \Delta_6, P_5} I \\
\hline
- : \Delta_7, P_5 \vdash \Delta_6, P_5 \\
\rightarrow \\
\frac{}{- : \Delta_7, P_5 \vdash \Delta_6, P_5} I \\
\hline
\frac{h_2 : \Delta_6 \vdash (\Delta_8, P_7), P_7}{\bullet h_2 : \Delta_6 \vdash (\perp, \Delta_8, P_7), P_7} \perp_R \quad \frac{}{\bullet h_5 : \Delta_6, P_7 \vdash \perp, \Delta_8, P_7} I \\
\hline
- : \Delta_6 \vdash \perp, \Delta_8, P_7 \\
\rightarrow \\
\frac{h_2 : \Delta_6 \vdash \perp, \Delta_8, P_7, P_7}{\bullet h_2 : \Delta_6 \vdash \perp, \Delta_8, P_7, P_7} \text{ax/W} \quad \frac{}{\bullet h_5 : \Delta_6, P_7 \vdash \perp, \Delta_8, P_7} I \\
\hline
- : \Delta_6 \vdash \perp, \Delta_8, P_7 \quad \text{hCut} \\
\hline
\frac{h_2 : \Delta_9, P_7 \vdash (\Delta_8, P_7), F_5}{\bullet h_2 : \Delta_9, P_7 \vdash (\perp, \Delta_8, P_7), F_5} \perp_R \quad \frac{}{\bullet h_6 : (\Delta_9, P_7), F_5 \vdash \perp, \Delta_8, P_7} I \\
\hline
- : \Delta_9, P_7 \vdash \perp, \Delta_8, P_7 \\
\rightarrow \\
\frac{}{- : \Delta_9, P_7 \vdash \perp, \Delta_8, P_7} I \\
\hline
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{h_1 : \top, \Delta_6 \vdash \Delta_5}{\bullet h_1 : \top, \Delta_6 \vdash \Delta_5, \perp} \perp_R \quad \frac{h_4 : \perp, \Delta_6 \vdash \Delta_5}{\bullet h_4 : (\top, \Delta_6), \perp \vdash \Delta_5} \top_L \\
\hline
- : \top, \Delta_6 \vdash \Delta_5 \\
\rightarrow \\
\frac{}{- : \top, \Delta_6 \vdash \Delta_5} \text{ax/W} \\
\hline
\frac{h_2 : \Delta_7 \vdash \Delta_5, \top}{\bullet h_2 : \Delta_7 \vdash (\perp, \Delta_5), \top} \perp_R \quad \frac{h_6 : \Delta_7 \vdash \perp, \Delta_5}{\bullet h_6 : \Delta_7, \top \vdash \perp, \Delta_5} \top_L \\
\hline
- : \Delta_7 \vdash \perp, \Delta_5 \\
\rightarrow \\
\frac{}{- : \Delta_7 \vdash \perp, \Delta_5} \text{ax/W} \\
\hline
\frac{h_2 : \top, \Delta_8 \vdash \Delta_6, F_5}{\bullet h_2 : \top, \Delta_8 \vdash (\perp, \Delta_6), F_5} \perp_R \quad \frac{h_7 : \Delta_8, F_5 \vdash \perp, \Delta_6}{\bullet h_7 : (\top, \Delta_8), F_5 \vdash \perp, \Delta_6} \top_L \\
\hline
- : \top, \Delta_8 \vdash \perp, \Delta_6 \\
\rightarrow \\
\frac{h_2 : \top, \Delta_8 \vdash \perp, \Delta_6, F_5}{\bullet h_2 : \top, \Delta_8 \vdash \perp, \Delta_6, F_5} \text{ax/W} \quad \frac{h_7 : \top, \Delta_8, F_5 \vdash \perp, \Delta_6}{\bullet h_7 : \top, \Delta_8, F_5 \vdash \perp, \Delta_6} \text{ax/W} \\
\hline
- : \top, \Delta_8 \vdash \perp, \Delta_6 \quad \text{hCut} \\
\hline
\end{array}$$

8.5 Status of \top_R : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{}{\bullet h_1 : \Delta_4 \vdash (\Delta_6, F_7 \rightarrow F_8), \top} \top_R \quad \frac{h_5 : \top, \Delta_4, F_7 \vdash \Delta_6, F_8}{\bullet h_5 : \Delta_4, \top \vdash \Delta_6, F_7 \rightarrow F_8} \rightarrow_R \\
\hline
- : \Delta_4 \vdash \Delta_6, F_7 \rightarrow F_8 \\
\rightarrow \\
\frac{}{\bullet h_1 : \Delta_4, F_7 \vdash \top, \Delta_6, F_8} \top_R \quad \frac{h_5 : \top, \Delta_4, F_7 \vdash \Delta_6, F_8}{\bullet h_5 : \top, \Delta_4, F_7 \vdash \Delta_6, F_8} \text{ax/W} \\
\hline
- : \Delta_4, F_7 \vdash \Delta_6, F_8 \quad \rightarrow_R \\
\hline
- : \Delta_4 \vdash \Delta_6, F_7 \rightarrow F_8 \\
\hline
\frac{}{\bullet h_2 : \Delta_5 \vdash (\top, \Delta_{10}, F_8 \rightarrow F_9), F_6} \top_R \quad \frac{h_7 : \Delta_5, F_6, F_8 \vdash \top, \Delta_{10}, F_9}{\bullet h_7 : \Delta_5, F_6 \vdash \top, \Delta_{10}, F_8 \rightarrow F_9} \rightarrow_R \\
\hline
- : \Delta_5 \vdash \top, \Delta_{10}, F_8 \rightarrow F_9 \\
\rightarrow \\
\frac{}{- : \Delta_5 \vdash \top, \Delta_{10}, F_8 \rightarrow F_9} \top_R \\
\hline
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_4 \vdash (\Delta_6, F_7 \wedge F_8), \top}{\vdash : \Delta_4 \vdash \Delta_6, F_7 \wedge F_8} \top_R \quad \frac{\frac{h_5 : \top, \Delta_4 \vdash \Delta_6, F_7 \quad h_5 : \top, \Delta_4 \vdash \Delta_6, F_8}{\bullet h_5 : \Delta_4, \top \vdash \Delta_6, F_7 \wedge F_8} \wedge_R}{\vdash : \Delta_4 \vdash \Delta_6, F_7 \wedge F_8} \text{Cut}}{\vdash : \Delta_4 \vdash \Delta_6, F_7 \wedge F_8} \rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_4 \vdash \top, \Delta_6, F_7}{\vdash : \Delta_4 \vdash \Delta_6, F_7} \top_R \quad \frac{h_5 : \top, \Delta_4 \vdash \Delta_6, F_7}{\bullet h_5 : \Delta_4, \top \vdash \Delta_6, F_7} \text{ax/W} \quad \frac{\frac{\bullet h_1 : \Delta_4 \vdash \top, \Delta_6, F_8}{\vdash : \Delta_4 \vdash \Delta_6, F_8} \top_R \quad \frac{h_5 : \top, \Delta_4 \vdash \Delta_6, F_8}{\bullet h_5 : \Delta_4, \top \vdash \Delta_6, F_8} \text{ax/W}}{\vdash : \Delta_4 \vdash \Delta_6, F_8} \text{hCut}}{\vdash : \Delta_4 \vdash \Delta_6, F_7 \wedge F_8} \wedge_R}{\vdash : \Delta_4 \vdash \Delta_6, F_7 \wedge F_8} \text{hCut} \\
\frac{\frac{\bullet h_2 : \Delta_5 \vdash (\top, \Delta_{10}, F_8 \wedge F_9), F_6}{\vdash : \Delta_5 \vdash \top, \Delta_{10}, F_8 \wedge F_9} \top_R \quad \frac{\frac{h_7 : \Delta_5, F_6 \vdash \top, \Delta_{10}, F_8 \quad h_7 : \Delta_5, F_6 \vdash \top, \Delta_{10}, F_9}{\bullet h_7 : \Delta_5, F_6 \vdash \top, \Delta_{10}, F_8 \wedge F_9} \wedge_R}{\vdash : \Delta_5 \vdash \top, \Delta_{10}, F_8 \wedge F_9} \text{Cut}}{\vdash : \Delta_5 \vdash \top, \Delta_{10}, F_8 \wedge F_9} \rightarrow \\
\frac{\vdash : \Delta_5 \vdash \top, \Delta_{10}, F_8 \wedge F_9}{\vdash : \Delta_5 \vdash \top, \Delta_{10}, F_8 \wedge F_9} \top_R
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_4 \vdash (\Delta_6, F_7 \vee F_8), \top}{\vdash : \Delta_4 \vdash \Delta_6, F_7 \vee F_8} \top_R \quad \frac{\frac{h_5 : \top, \Delta_4 \vdash \Delta_6, F_7, F_8}{\bullet h_5 : \Delta_4, \top \vdash \Delta_6, F_7 \vee F_8} \vee_R}{\vdash : \Delta_4 \vdash \Delta_6, F_7 \vee F_8} \text{Cut}}{\vdash : \Delta_4 \vdash \Delta_6, F_7 \vee F_8} \rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_4 \vdash \top, \Delta_6, F_7, F_8}{\vdash : \Delta_4 \vdash \Delta_6, F_7, F_8} \top_R \quad \frac{h_5 : \top, \Delta_4 \vdash \Delta_6, F_7, F_8}{\bullet h_5 : \Delta_4, \top \vdash \Delta_6, F_7, F_8} \text{ax/W}}{\vdash : \Delta_4 \vdash \Delta_6, F_7, F_8} \text{hCut}}{\vdash : \Delta_4 \vdash \Delta_6, F_7 \vee F_8} \vee_R \\
\frac{\frac{\bullet h_2 : \Delta_5 \vdash (\top, \Delta_{10}, F_8 \vee F_9), F_6}{\vdash : \Delta_5 \vdash \top, \Delta_{10}, F_8 \vee F_9} \top_R \quad \frac{\frac{h_7 : \Delta_5, F_6 \vdash \top, \Delta_{10}, F_8, F_9}{\bullet h_7 : \Delta_5, F_6 \vdash \top, \Delta_{10}, F_8 \vee F_9} \vee_R}{\vdash : \Delta_5 \vdash \top, \Delta_{10}, F_8 \vee F_9} \text{Cut}}{\vdash : \Delta_5 \vdash \top, \Delta_{10}, F_8 \vee F_9} \rightarrow \\
\frac{\vdash : \Delta_5 \vdash \top, \Delta_{10}, F_8 \vee F_9}{\vdash : \Delta_5 \vdash \top, \Delta_{10}, F_8 \vee F_9} \top_R
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_4 \vdash (\perp, \Delta_6), \top}{\vdash : \Delta_4 \vdash \perp, \Delta_6} \top_R \quad \frac{h_5 : \top, \Delta_4 \vdash \Delta_6}{\bullet h_5 : \Delta_4, \top \vdash \perp, \Delta_6} \perp_R}{\vdash : \Delta_4 \vdash \perp, \Delta_6} \text{Cut}}{\vdash : \Delta_4 \vdash \perp, \Delta_6} \rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_4 \vdash \perp, \top, \Delta_6}{\vdash : \Delta_4 \vdash \perp, \Delta_6} \text{ax/W} \quad \frac{h_5 : \top, \Delta_4 \vdash \perp, \Delta_6}{\bullet h_5 : \Delta_4, \top \vdash \perp, \Delta_6} \text{ax/W}}{\vdash : \Delta_4 \vdash \perp, \Delta_6} \text{hCut}}{\vdash : \Delta_4 \vdash \perp, \Delta_6} \\
\frac{\frac{\bullet h_2 : \Delta_5 \vdash (\top, \perp, \Delta_8), F_6}{\vdash : \Delta_5 \vdash \top, \perp, \Delta_8} \top_R \quad \frac{\frac{h_7 : \Delta_5, F_6 \vdash \top, \Delta_8}{\bullet h_7 : \Delta_5, F_6 \vdash \top, \perp, \Delta_8} \perp_R}{\vdash : \Delta_5 \vdash \top, \perp, \Delta_8} \text{Cut}}{\vdash : \Delta_5 \vdash \top, \perp, \Delta_8} \rightarrow \\
\frac{\vdash : \Delta_5 \vdash \top, \perp, \Delta_8}{\vdash : \Delta_5 \vdash \perp, \top, \Delta_8} \top_R
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_4 \vdash (\top, \Delta_6), \top}{\vdash : \Delta_4 \vdash \top, \Delta_6} \top_R \quad \frac{h_5 : \top, \Delta_4 \vdash \Delta_6}{\bullet h_5 : \Delta_4, \top \vdash \top, \Delta_6} \top_R}{\vdash : \Delta_4 \vdash \top, \Delta_6} \text{Cut}}{\vdash : \Delta_4 \vdash \top, \Delta_6} \rightarrow \\
\frac{\vdash : \Delta_4 \vdash \top, \Delta_6}{\vdash : \Delta_4 \vdash \top, \Delta_6} \top_R \\
\frac{\frac{\bullet h_2 : \Delta_5 \vdash (\top, \Delta_8), F_6}{\vdash : \Delta_5 \vdash \top, \Delta_8} \top_R \quad \frac{h_7 : \Delta_5, F_6 \vdash \top, \Delta_8}{\bullet h_7 : \Delta_5, F_6 \vdash \top, \Delta_8} \top_R}{\vdash : \Delta_5 \vdash \top, \Delta_8} \text{Cut}}{\vdash : \Delta_5 \vdash \top, \Delta_8} \rightarrow \\
\frac{\vdash : \Delta_5 \vdash \top, \Delta_8}{\vdash : \Delta_5 \vdash \top, \Delta_8} \top_R
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_1 : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7, \top} \top_R \quad \frac{h_4 : \top, \Delta_8 \vdash \Delta_7, F_5 \quad h_4 : \top, \Delta_8, F_6 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \rightarrow F_6), \top \vdash \Delta_7} \rightarrow_L}{- : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7} \text{Cut}}{\rightarrow} \\
\frac{\frac{\frac{\frac{}{\bullet h_1 : \Delta_8 \vdash \top, \Delta_7, F_5} \top_R \quad \frac{h_4 : \top, \Delta_8 \vdash \Delta_7, F_5}{hCut} \quad \frac{\frac{ax/W}{\bullet h_1 : \Delta_8, F_6 \vdash \top, \Delta_7} \top_R \quad \frac{h_4 : \top, \Delta_8, F_6 \vdash \Delta_7}{hCut}}{- : \Delta_8, F_6 \vdash \Delta_7} \rightarrow_L}{- : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7} \text{hCut}}{- : \Delta_8, F_5 \rightarrow F_6 \vdash \Delta_7} \rightarrow_L} \\
\frac{\frac{\frac{}{\bullet h_2 : \Delta_{10}, F_8 \rightarrow F_9 \vdash (\top, \Delta_6), F_5} \top_R \quad \frac{h_7 : \Delta_{10}, F_5 \vdash \top, \Delta_6, F_8 \quad h_7 : \Delta_{10}, F_5, F_9 \vdash \top, \Delta_6}{\bullet h_7 : (\Delta_{10}, F_8 \rightarrow F_9), F_5 \vdash \top, \Delta_6} \rightarrow_L}{- : \Delta_{10}, F_8 \rightarrow F_9 \vdash \top, \Delta_6} \text{Cut}}{\rightarrow} \\
\frac{\frac{}{- : \Delta_{10}, F_8 \rightarrow F_9 \vdash \top, \Delta_6} \top_R}{- : \Delta_{10}, F_8 \rightarrow F_9 \vdash \top, \Delta_6} \rightarrow_L} \\
\frac{\frac{\frac{}{\bullet h_2 : \Delta_7 \vdash (\top, \Delta_5), F_8 \rightarrow F_9} \top_R \quad \frac{h_6 : \Delta_7 \vdash \top, \Delta_5, F_8 \quad h_6 : \Delta_7, F_9 \vdash \top, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \rightarrow F_9 \vdash \top, \Delta_5} \rightarrow_L}{- : \Delta_7 \vdash \top, \Delta_5} \text{Cut}}{\rightarrow} \\
\frac{\frac{}{- : \Delta_7 \vdash \top, \Delta_5} \top_R}{- : \Delta_7 \vdash \top, \Delta_5} \rightarrow_L}
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_1 : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7, \top} \top_R \quad \frac{h_4 : \top, \Delta_8, F_5, F_6 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \wedge F_6), \top \vdash \Delta_7} \wedge_L}{- : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7} \text{Cut}}{\rightarrow} \\
\frac{\frac{\frac{\frac{}{\bullet h_1 : \Delta_8, F_5, F_6 \vdash \top, \Delta_7} \top_R \quad \frac{h_4 : \top, \Delta_8, F_5, F_6 \vdash \Delta_7}{hCut} \quad \frac{ax/W}{\bullet h_1 : \Delta_8, F_5 \wedge F_6 \vdash \top, \Delta_7} \top_R}{- : \Delta_8, F_5, F_6 \vdash \Delta_7} \wedge_L}{- : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7} \text{hCut}}{- : \Delta_8, F_5 \wedge F_6 \vdash \Delta_7} \wedge_L} \\
\frac{\frac{\frac{}{\bullet h_2 : \Delta_{10}, F_8 \wedge F_9 \vdash (\top, \Delta_6), F_5} \top_R \quad \frac{h_7 : \Delta_{10}, F_5, F_8, F_9 \vdash \top, \Delta_6}{\bullet h_7 : (\Delta_{10}, F_8 \wedge F_9), F_5 \vdash \top, \Delta_6} \wedge_L}{- : \Delta_{10}, F_8 \wedge F_9 \vdash \top, \Delta_6} \text{Cut}}{\rightarrow} \\
\frac{\frac{}{- : \Delta_{10}, F_8 \wedge F_9 \vdash \top, \Delta_6} \top_R}{- : \Delta_{10}, F_8 \wedge F_9 \vdash \top, \Delta_6} \rightarrow_L} \\
\frac{\frac{\frac{}{\bullet h_2 : \Delta_7 \vdash (\top, \Delta_5), F_8 \wedge F_9} \top_R \quad \frac{h_6 : \Delta_7, F_8, F_9 \vdash \top, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \wedge F_9 \vdash \top, \Delta_5} \wedge_L}{- : \Delta_7 \vdash \top, \Delta_5} \text{Cut}}{\rightarrow} \\
\frac{\frac{}{- : \Delta_7 \vdash \top, \Delta_5} \top_R}{- : \Delta_7 \vdash \top, \Delta_5} \rightarrow_L}
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_1 : \Delta_8, F_5 \vee F_6 \vdash \Delta_7, \top} \top_R \quad \frac{h_4 : \top, \Delta_8, F_5 \vdash \Delta_7 \quad h_4 : \top, \Delta_8, F_6 \vdash \Delta_7}{\bullet h_4 : (\Delta_8, F_5 \vee F_6), \top \vdash \Delta_7} \vee_L}{- : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \text{Cut}}{\rightarrow} \\
\frac{\frac{\frac{\frac{}{\bullet h_1 : \Delta_8, F_5 \vdash \top, \Delta_7} \top_R \quad \frac{h_4 : \top, \Delta_8, F_5 \vdash \Delta_7}{hCut} \quad \frac{ax/W}{\bullet h_1 : \Delta_8, F_6 \vdash \top, \Delta_7} \top_R \quad \frac{h_4 : \top, \Delta_8, F_6 \vdash \Delta_7}{hCut}}{- : \Delta_8, F_6 \vdash \Delta_7} \vee_L}{- : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \text{hCut}}{- : \Delta_8, F_5 \vee F_6 \vdash \Delta_7} \vee_L} \\
\frac{\frac{\frac{}{\bullet h_2 : \Delta_{10}, F_8 \vee F_9 \vdash (\top, \Delta_6), F_5} \top_R \quad \frac{h_7 : \Delta_{10}, F_5, F_8 \vdash \top, \Delta_6 \quad h_7 : \Delta_{10}, F_5, F_9 \vdash \top, \Delta_6}{\bullet h_7 : (\Delta_{10}, F_8 \vee F_9), F_5 \vdash \top, \Delta_6} \vee_L}{- : \Delta_{10}, F_8 \vee F_9 \vdash \top, \Delta_6} \text{Cut}}{\rightarrow} \\
\frac{\frac{}{- : \Delta_{10}, F_8 \vee F_9 \vdash \top, \Delta_6} \top_R}{- : \Delta_{10}, F_8 \vee F_9 \vdash \top, \Delta_6} \rightarrow_L} \\
\frac{\frac{\frac{}{\bullet h_2 : \Delta_7 \vdash (\top, \Delta_5), F_8 \vee F_9} \top_R \quad \frac{h_6 : \Delta_7, F_8 \vdash \top, \Delta_5 \quad h_6 : \Delta_7, F_9 \vdash \top, \Delta_5}{\bullet h_6 : \Delta_7, F_8 \vee F_9 \vdash \top, \Delta_5} \vee_L}{- : \Delta_7 \vdash \top, \Delta_5} \text{Cut}}{\rightarrow} \\
\frac{\frac{}{- : \Delta_7 \vdash \top, \Delta_5} \top_R}{- : \Delta_7 \vdash \top, \Delta_5} \rightarrow_L}
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \perp, \Delta_6 \vdash \Delta_5, \top}{- : \perp, \Delta_6 \vdash \Delta_5} \top_R \quad \frac{\bullet h_4 : (\perp, \Delta_6), \top \vdash \Delta_5}{\text{Cut}} \perp_L}{\rightarrow} \\
\frac{\perp_L}{- : \perp, \Delta_6 \vdash \Delta_5} \\
\frac{\bullet h_2 : \Delta_7 \vdash (\top, \Delta_5), \perp}{- : \Delta_7 \vdash \top, \Delta_5} \top_R \quad \frac{\bullet h_6 : \Delta_7, \perp \vdash \top, \Delta_5}{\text{Cut}} \perp_L}{\rightarrow} \\
\frac{\top_R}{- : \Delta_7 \vdash \top, \Delta_5} \\
\frac{\bullet h_2 : \perp, \Delta_8 \vdash (\top, \Delta_6), F_5}{- : \perp, \Delta_8 \vdash \top, \Delta_6} \top_R \quad \frac{\bullet h_7 : (\perp, \Delta_8), F_5 \vdash \top, \Delta_6}{\text{Cut}} \perp_L}{\rightarrow} \\
\frac{\top_R}{- : \perp, \Delta_8 \vdash \top, \Delta_6}
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_7, p_5 \vdash (\Delta_6, p_5), \top}{- : \Delta_7, p_5 \vdash \Delta_6, p_5} \top_R \quad \frac{\bullet h_4 : (\Delta_7, p_5), \top \vdash \Delta_6, p_5}{\text{Cut}} I}{\rightarrow} \\
\frac{I}{- : \Delta_7, p_5 \vdash \Delta_6, p_5} \\
\frac{\bullet h_2 : \Delta_6 \vdash (\top, \Delta_8, p_7), p_7}{- : \Delta_6 \vdash \top, \Delta_8, p_7} \top_R \quad \frac{\bullet h_5 : \Delta_6, p_7 \vdash \top, \Delta_8, p_7}{\text{Cut}} I}{\rightarrow} \\
\frac{I}{- : \Delta_6 \vdash \top, \Delta_8, p_7} \\
\frac{\bullet h_2 : \Delta_9, p_7 \vdash (\top, \Delta_8, p_7), F_5}{- : \Delta_9, p_7 \vdash \top, \Delta_8, p_7} \top_R \quad \frac{\bullet h_6 : (\Delta_9, p_7), F_5 \vdash \top, \Delta_8, p_7}{\text{Cut}} I}{\rightarrow} \\
\frac{I}{- : \Delta_9, p_7 \vdash \top, \Delta_8, p_7}
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{\frac{\bullet h_1 : \Delta_5 \vdash \Delta_6, \top}{- : \Delta_5 \vdash \Delta_6} \top_R \quad \frac{h_4 : \Delta_5 \vdash \Delta_6}{\bullet h_4 : \Delta_5, \top \vdash \Delta_6} \top_L}{\rightarrow} \\
\frac{\text{ax/w}}{- : \Delta_5 \vdash \Delta_6} \\
\frac{\bullet h_2 : \Delta_7 \vdash (\top, \Delta_5), \top}{- : \Delta_7 \vdash \top, \Delta_5} \top_R \quad \frac{h_6 : \Delta_7 \vdash \top, \Delta_5}{\bullet h_6 : \Delta_7, \top \vdash \top, \Delta_5} \top_L}{\rightarrow} \\
\frac{\top_L}{- : \Delta_7 \vdash \top, \Delta_5} \\
\frac{\bullet h_2 : \top, \Delta_8 \vdash (\top, \Delta_6), F_5}{- : \top, \Delta_8 \vdash \top, \Delta_6} \top_R \quad \frac{h_7 : \Delta_8, F_5 \vdash \top, \Delta_6}{\bullet h_7 : (\top, \Delta_8), F_5 \vdash \top, \Delta_6} \top_L}{\rightarrow} \\
\frac{\top_L}{- : \top, \Delta_8 \vdash \top, \Delta_6}
\end{array}$$

8.6 Status of \rightarrow_L : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{h_3 : \Delta_7 \vdash \Delta_{13}, F_{11} \rightarrow F_{12}, F_8 \quad h_3 : \Delta_7, F_9 \vdash \Delta_{13}, F_{11} \rightarrow F_{12} \quad \rightarrow_L \quad \frac{h_{10} : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11} \quad h_{10} : \Delta_7, F_{12}, F_8 \rightarrow F_9 \vdash \Delta_{13} \quad \rightarrow_L}{\bullet h_3 : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11} \rightarrow F_{12}} \quad \text{Cut}}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}} \\
\rightarrow \\
\frac{\frac{\frac{- : \Delta_7, F_{11} \vdash \Delta_{13}, F_{12}, F_8 \quad \text{inv-th/ax}}{- : \Delta_7, F_{11}, F_9 \vdash \Delta_{13}, F_{12}} \quad \text{inv-th/ax}}{- : \Delta_7, F_{11}, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{12}} \quad \rightarrow_L \quad \frac{- : \Delta_7, F_{11}, F_{12}, F_8 \rightarrow F_9 \vdash \Delta_{13}}{- : \Delta_7, F_{11}, F_8 \rightarrow F_9 \vdash \Delta_{13}} \quad \text{sCut}}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11}} \quad \text{ax/W}}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}} \\
\rightarrow \\
\frac{h_3 : \Delta_7 \vdash \Delta_{12}, F_8, F_{10} \quad h_3 : \Delta_7, F_{11} \vdash \Delta_{12}, F_8 \quad \rightarrow_L \quad \frac{h_9 : \Delta_7, F_8 \vdash \Delta_{12}, F_{10} \quad h_9 : \Delta_7, F_8, F_{11} \vdash \Delta_{12} \quad \rightarrow_L}{\bullet h_9 : (\Delta_7, F_{10} \rightarrow F_{11}), F_8 \vdash \Delta_{12}} \quad \text{Cut}}{- : \Delta_7, F_{10} \rightarrow F_{11} \vdash \Delta_{12}} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_7 \vdash \Delta_{12}, F_{10}, F_8 \quad \text{ax/W} \quad \frac{h_9 : \Delta_7, F_8 \vdash \Delta_{12}, F_{10}}{\bullet h_9 : \Delta_7, F_8 \vdash \Delta_{12}, F_{10}} \quad \text{ax/W} \quad \text{height}}{- : \Delta_7 \vdash \Delta_{12}, F_{10}} \quad \text{hCut} \quad \frac{h_3 : \Delta_7, F_{11} \vdash \Delta_{12}, F_8 \quad \text{ax/W} \quad \frac{h_9 : \Delta_7, F_{11}, F_8 \vdash \Delta_{12}}{\bullet h_9 : \Delta_7, F_{11}, F_8 \vdash \Delta_{12}} \quad \text{ax/W} \quad \text{height}}{- : \Delta_7, F_{11} \vdash \Delta_{12}} \quad \text{hCut}}{- : \Delta_7, F_{10} \rightarrow F_{11} \vdash \Delta_{12}} \quad \rightarrow_L
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_3 : \Delta_{14}, F_{11} \wedge F_{12} \vdash \Delta_{13}, F_7, F_8 \quad h_3 : (\Delta_{14}, F_{11} \wedge F_{12}), F_9 \vdash \Delta_{13}, F_7 \quad \rightarrow_L \quad \frac{h_{10} : \Delta_{14}, F_7, F_{11}, F_{12}, F_8 \rightarrow F_9 \vdash \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \wedge F_{12}), F_8 \rightarrow F_9), F_7 \vdash \Delta_{13}} \quad \text{Cut}}{- : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \rightarrow F_9 \vdash \Delta_{13}} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_{14}, F_{11}, F_{12} \vdash \Delta_{13}, F_7, F_8 \quad \text{inv-th/ax}}{\bullet h_3 : \Delta_{14}, F_{11}, F_{12}, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_7} \quad \text{inv-th/ax}}{- : \Delta_{14}, F_{11}, F_{12}, F_8 \rightarrow F_9 \vdash \Delta_{13}} \quad \text{ax/W} \quad \frac{h_{10} : \Delta_{14}, F_{11}, F_{12}, F_7, F_8 \rightarrow F_9 \vdash \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \wedge F_{12}), F_8 \rightarrow F_9), F_7 \vdash \Delta_{13}} \quad \text{hCut}}{- : \Delta_{14}, F_8 \rightarrow F_9, F_{11} \wedge F_{12} \vdash \Delta_{13}} \quad \wedge_L \\
\rightarrow \\
\frac{h_3 : \Delta_7 \vdash \Delta_{13}, F_{11} \wedge F_{12}, F_8 \quad h_3 : \Delta_7, F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12} \quad \rightarrow_L \quad \frac{h_{10} : \Delta_7, F_{11}, F_{12}, F_8 \rightarrow F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \rightarrow F_9), F_{11} \wedge F_{12} \vdash \Delta_{13}} \quad \text{Cut}}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_7 \vdash \Delta_{13}, F_8, F_{11} \wedge F_{12} \quad \text{ax/W} \quad \frac{h_{10} : \Delta_7, F_{11}, F_{12} \vdash \Delta_{13}, F_8}{\bullet h_{10} : \Delta_7, F_{11} \wedge F_{12} \vdash \Delta_{13}, F_8} \quad \text{inv-th/ax}}{- : \Delta_7 \vdash \Delta_{13}, F_8} \quad \wedge_L \quad \text{hCut} \quad \frac{h_3 : \Delta_7, F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12} \quad \text{ax/W} \quad \frac{h_{10} : \Delta_7, F_{11}, F_{12}, F_9 \vdash \Delta_{13}}{\bullet h_{10} : \Delta_7, F_9, F_{11} \wedge F_{12} \vdash \Delta_{13}} \quad \text{inv-th/ax}}{- : \Delta_7, F_9 \vdash \Delta_{13}} \quad \wedge_L \quad \text{hCut}}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}} \quad \rightarrow_L
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{h_3 : \Delta_{14}, F_{11} \vee F_{12} \vdash \Delta_{13}, F_7, F_8 \quad h_3 : (\Delta_{14}, F_{11} \vee F_{12}), F_9 \vdash \Delta_{13}, F_7 \quad \rightarrow_L \quad \frac{h_{10} : \Delta_{14}, F_7, F_{11}, F_8 \rightarrow F_9 \vdash \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \vee F_{12}), F_8 \rightarrow F_9), F_7 \vdash \Delta_{13}} \quad \text{Cut}}{- : (\Delta_{14}, F_{11} \vee F_{12}), F_8 \rightarrow F_9 \vdash \Delta_{13}} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_{14}, F_{11} \vee F_{12} \vdash \Delta_{13}, F_7, F_8 \quad \text{ax/W} \quad \frac{h_{10} : \Delta_{14}, F_{11}, F_7 \vdash \Delta_{13}, F_8}{\bullet h_{10} : \Delta_{14}, F_7, F_{11} \vee F_{12} \vdash \Delta_{13}, F_8} \quad \text{inv-th/ax}}{- : \Delta_{14}, F_{11} \vee F_{12} \vdash \Delta_{13}, F_8} \quad \text{hCut} \quad \frac{h_{10} : \Delta_{14}, F_{12}, F_7 \vdash \Delta_{13}, F_8}{\bullet h_{10} : \Delta_{14}, F_7, F_{11} \vee F_{12} \vdash \Delta_{13}, F_8} \quad \text{inv-th/ax}}{- : \Delta_{14}, F_8 \rightarrow F_9, F_{11} \vee F_{12} \vdash \Delta_{13}} \quad \vee_L \\
\rightarrow \\
\frac{h_3 : \Delta_7 \vdash \Delta_{13}, F_{11} \vee F_{12}, F_8 \quad h_3 : \Delta_7, F_9 \vdash \Delta_{13}, F_{11} \vee F_{12} \quad \rightarrow_L \quad \frac{h_{10} : \Delta_7, F_{11}, F_8 \rightarrow F_9 \vdash \Delta_{13} \quad h_{10} : \Delta_7, F_{12}, F_8 \rightarrow F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \rightarrow F_9), F_{11} \vee F_{12} \vdash \Delta_{13}} \quad \text{Cut}}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}} \\
\rightarrow \\
\frac{\frac{- : \Delta_7 \vdash \Delta_{13}, F_{11}, F_{12}, F_8 \quad \text{inv-th/ax}}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11}, F_{12}} \quad \text{inv-th/ax}}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11}, F_{12}} \quad \rightarrow_L \quad \frac{- : \Delta_7, F_{12}, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11}}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11}} \quad \text{ax/W} \quad \text{sCut}}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}, F_{11}} \quad \vee_L \quad \frac{- : \Delta_7, F_{11}, F_8 \rightarrow F_9 \vdash \Delta_{13}}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}} \quad \text{ax/sCut}}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{13}}
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_7 \vdash \Delta_{11}, \perp, F_8 \quad h_3 : \Delta_7, F_9 \vdash \Delta_{11}, \perp}{\bullet h_3 : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{11}, \perp} \rightarrow_L \quad \frac{\bullet h_{10} : (\Delta_7, F_8 \rightarrow F_9), \perp \vdash \Delta_{11}}{\perp} \text{Cut}}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{11}} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_7 \vdash \perp, \Delta_{11}, F_8}{- : \Delta_7 \vdash \Delta_{11}, F_8} \text{ax/W} \quad \frac{\bullet h_{10} : \perp, \Delta_7 \vdash \Delta_{11}, F_8}{- : \Delta_7, F_9 \vdash \Delta_{11}} \text{hCut} \quad \frac{h_3 : \Delta_7, F_9 \vdash \perp, \Delta_{11}}{- : \Delta_7, F_9 \vdash \Delta_{11}} \text{ax/W} \quad \frac{\bullet h_{10} : \perp, \Delta_7, F_9 \vdash \Delta_{11}}{- : \Delta_7, F_9 \vdash \Delta_{11}} \text{hCut}}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{11}} \rightarrow_L \\
\rightarrow \\
\frac{h_3 : \perp, \Delta_{12} \vdash \Delta_{11}, F_7, F_8 \quad h_3 : (\perp, \Delta_{12}), F_9 \vdash \Delta_{11}, F_7}{\bullet h_3 : (\perp, \Delta_{12}), F_8 \rightarrow F_9 \vdash \Delta_{11}, F_7} \rightarrow_L \quad \frac{\bullet h_{10} : ((\perp, \Delta_{12}), F_8 \rightarrow F_9), F_7 \vdash \Delta_{11}}{- : (\perp, \Delta_{12}), F_8 \rightarrow F_9 \vdash \Delta_{11}} \text{Cut}}{- : \perp, \Delta_{12}, F_8 \rightarrow F_9 \vdash \Delta_{11}} \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_{12}, F_8 \rightarrow F_9 \vdash \Delta_{11}} \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_7 \vdash (\Delta_{12}, P_{11}), P_{11}, F_8 \quad h_3 : \Delta_7, F_9 \vdash (\Delta_{12}, P_{11}), P_{11}}{\bullet h_3 : \Delta_7, F_8 \rightarrow F_9 \vdash (\Delta_{12}, P_{11}), P_{11}} \rightarrow_L \quad \frac{\bullet h_{10} : (\Delta_7, F_8 \rightarrow F_9), P_{11} \vdash \Delta_{12}, P_{11}}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{12}, P_{11}} \text{Cut}}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{12}, P_{11}} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_7 \vdash \Delta_{12}, F_8, P_{11}, P_{11}}{- : \Delta_7 \vdash \Delta_{12}, F_8, P_{11}} \text{ax/W} \quad \frac{\bullet h_{10} : \Delta_7, P_{11} \vdash \Delta_{12}, F_8, P_{11}}{- : \Delta_7, F_9 \vdash \Delta_{12}, P_{11}} \text{I} \quad \frac{h_3 : \Delta_7, F_9 \vdash \Delta_{12}, P_{11}, P_{11}}{- : \Delta_7, F_9 \vdash \Delta_{12}, P_{11}} \text{ax/W} \quad \frac{\bullet h_{10} : \Delta_7, F_9, P_{11} \vdash \Delta_{12}, P_{11}}{- : \Delta_7, F_9 \vdash \Delta_{12}, P_{11}} \text{I}}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{12}, P_{11}} \text{hCut} \rightarrow_L \\
\rightarrow \\
\frac{h_3 : \Delta_{13}, P_{11} \vdash (\Delta_{12}, P_{11}), F_7, F_8 \quad h_3 : (\Delta_{13}, P_{11}), F_9 \vdash (\Delta_{12}, P_{11}), F_7}{\bullet h_3 : (\Delta_{13}, P_{11}), F_8 \rightarrow F_9 \vdash (\Delta_{12}, P_{11}), F_7} \rightarrow_L \quad \frac{\bullet h_{10} : ((\Delta_{13}, P_{11}), F_8 \rightarrow F_9), F_7 \vdash \Delta_{12}, P_{11}}{- : (\Delta_{13}, P_{11}), F_8 \rightarrow F_9 \vdash \Delta_{12}, P_{11}} \text{I} \text{Cut}}{- : \Delta_{13}, P_{11}, F_8 \rightarrow F_9 \vdash \Delta_{12}, P_{11}} \text{I}
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_7 \vdash \Delta_{11}, \top, F_8 \quad h_3 : \Delta_7, F_9 \vdash \Delta_{11}, \top}{\bullet h_3 : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{11}, \top} \rightarrow_L \quad \frac{h_{10} : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{11}}{\bullet h_{10} : (\Delta_7, F_8 \rightarrow F_9), \top \vdash \Delta_{11}} \top_L \text{Cut}}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{11}} \\
\rightarrow \\
\frac{}{- : \Delta_7, F_8 \rightarrow F_9 \vdash \Delta_{11}} \text{ax/W} \\
\rightarrow \\
\frac{h_3 : \top, \Delta_{12} \vdash \Delta_{11}, F_7, F_8 \quad h_3 : (\top, \Delta_{12}), F_9 \vdash \Delta_{11}, F_7}{\bullet h_3 : (\top, \Delta_{12}), F_8 \rightarrow F_9 \vdash \Delta_{11}, F_7} \rightarrow_L \quad \frac{h_{10} : \Delta_{12}, F_7, F_8 \rightarrow F_9 \vdash \Delta_{11}}{\bullet h_{10} : ((\top, \Delta_{12}), F_8 \rightarrow F_9), F_7 \vdash \Delta_{11}} \top_L \text{Cut}}{- : (\top, \Delta_{12}), F_8 \rightarrow F_9 \vdash \Delta_{11}} \\
\rightarrow \\
\frac{\frac{\bullet h_3 : \top, \Delta_{12}, F_8 \rightarrow F_9 \vdash \Delta_{11}, F_7}{- : \top, \Delta_{12}, F_8 \rightarrow F_9 \vdash \Delta_{11}} \text{ax/W} \quad \frac{h_{10} : \top, \Delta_{12}, F_7, F_8 \rightarrow F_9 \vdash \Delta_{11}}{- : \top, \Delta_{12}, F_8 \rightarrow F_9 \vdash \Delta_{11}} \text{hCut}}{- : \top, \Delta_{12}, F_8 \rightarrow F_9 \vdash \Delta_{11}} \text{hCut}
\end{array}$$

8.7 Status of \wedge_L : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_7, F_9, F_{10} \vdash (\Delta_{12}, F_{13} \rightarrow F_{14}), F_8}{\bullet h_3 : \Delta_7, F_9 \wedge F_{10} \vdash (\Delta_{12}, F_{13} \rightarrow F_{14}), F_8} \wedge_L \quad \frac{h_{11} : \Delta_7, F_8, F_{13}, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{14}}{\bullet h_{11} : (\Delta_7, F_9 \wedge F_{10}), F_8 \vdash \Delta_{12}, F_{13} \rightarrow F_{14}} \rightarrow_R \text{Cut}}{- : \Delta_7, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13} \rightarrow F_{14}} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_7, F_{10}, F_{13}, F_9 \vdash \Delta_{12}, F_{14}, F_8}{\bullet h_3 : \Delta_7, F_{13}, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{14}, F_8} \text{inv-th/ax} \quad \frac{h_{11} : \Delta_7, F_{13}, F_8, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{14}}{- : \Delta_7, F_{13}, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{14}} \text{ax/W} \text{hCut}}{- : \Delta_7, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13} \rightarrow F_{14}} \rightarrow_R
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{h_3 : \Delta_7, F_9, F_{10} \vdash (\Delta_{12}, F_{13} \wedge F_{14}), F_8}{\bullet h_3 : \Delta_7, F_9 \wedge F_{10} \vdash (\Delta_{12}, F_{13} \wedge F_{14}), F_8} \wedge_L \quad \frac{h_{11} : \Delta_7, F_8, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13} \quad h_{11} : \Delta_7, F_8, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{14}}{\bullet h_{11} : (\Delta_7, F_9 \wedge F_{10}), F_8 \vdash \Delta_{12}, F_{13} \wedge F_{14}} \wedge_R \\
\hline
- : \Delta_7, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13} \wedge F_{14} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_7, F_{10}, F_9 \vdash \Delta_{12}, F_8, F_{13} \wedge F_{14}}{\bullet h_3 : \Delta_7, F_{10}, F_9 \vdash \Delta_{12}, F_8, F_{13} \wedge F_{14}} \text{ax/W} \quad \frac{h_{11} : \Delta_7, F_{10}, F_8, F_9 \vdash \Delta_{12}, F_{13} \quad \text{inv-th/ax} \quad h_{11} : \Delta_7, F_{10}, F_8, F_9 \vdash \Delta_{12}, F_{14}}{\bullet h_{11} : \Delta_7, F_{10}, F_8, F_9 \vdash \Delta_{12}, F_{13} \wedge F_{14}} \text{inv-th/ax}}{- : \Delta_7, F_{10}, F_9 \vdash \Delta_{12}, F_8, F_{13} \wedge F_{14}} \wedge_L \quad \frac{- : \Delta_7, F_{10}, F_9 \vdash \Delta_{12}, F_{13} \wedge F_{14} \quad h_{11} : \Delta_7, F_8, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13}, F_{14}}{- : \Delta_7, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13} \wedge F_{14}} \wedge_R}{- : \Delta_7, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13} \wedge F_{14}} \text{hCut}
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_3 : \Delta_7, F_9, F_{10} \vdash (\Delta_{12}, F_{13} \vee F_{14}), F_8}{\bullet h_3 : \Delta_7, F_9 \wedge F_{10} \vdash (\Delta_{12}, F_{13} \vee F_{14}), F_8} \wedge_L \quad \frac{h_{11} : \Delta_7, F_8, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13}, F_{14}}{\bullet h_{11} : (\Delta_7, F_9 \wedge F_{10}), F_8 \vdash \Delta_{12}, F_{13} \vee F_{14}} \vee_R \\
\hline
- : \Delta_7, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13} \vee F_{14} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_7, F_{10}, F_9 \vdash \Delta_{12}, F_{13}, F_{14}, F_8}{\bullet h_3 : \Delta_7, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13}, F_{14}, F_8} \text{inv-th/ax} \quad \frac{h_{11} : \Delta_7, F_8, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13}, F_{14}}{\bullet h_{11} : (\Delta_7, F_9 \wedge F_{10}), F_8 \vdash \Delta_{12}, F_{13}, F_{14}} \text{ax/W}}{- : \Delta_7, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13}, F_{14}} \wedge_L \quad \frac{- : \Delta_7, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13}, F_{14} \quad h_{11} : \Delta_7, F_8, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13}, F_{14}}{- : \Delta_7, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13} \vee F_{14}} \vee_R}{- : \Delta_7, F_9 \wedge F_{10} \vdash \Delta_{12}, F_{13} \vee F_{14}} \text{hCut}
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_3 : \Delta_7, F_9, F_{10} \vdash (\perp, \Delta_{12}), F_8}{\bullet h_3 : \Delta_7, F_9 \wedge F_{10} \vdash (\perp, \Delta_{12}), F_8} \wedge_L \quad \frac{h_{11} : \Delta_7, F_8, F_9 \wedge F_{10} \vdash \perp, \Delta_{12}}{\bullet h_{11} : (\Delta_7, F_9 \wedge F_{10}), F_8 \vdash \perp, \Delta_{12}} \perp_R \\
\hline
- : \Delta_7, F_9 \wedge F_{10} \vdash \perp, \Delta_{12} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_7, F_9 \wedge F_{10} \vdash \perp, \Delta_{12}, F_8}{\bullet h_3 : \Delta_7, F_9 \wedge F_{10} \vdash \perp, \Delta_{12}, F_8} \text{ax/W} \quad \frac{h_{11} : \Delta_7, F_8, F_9 \wedge F_{10} \vdash \perp, \Delta_{12}}{\bullet h_{11} : (\Delta_7, F_9 \wedge F_{10}), F_8 \vdash \perp, \Delta_{12}} \text{ax/W}}{- : \Delta_7, F_9 \wedge F_{10} \vdash \perp, \Delta_{12}} \wedge_L \quad \frac{- : \Delta_7, F_9 \wedge F_{10} \vdash \perp, \Delta_{12} \quad h_{11} : \Delta_7, F_8, F_9 \wedge F_{10} \vdash \perp, \Delta_{12}}{- : \Delta_7, F_9 \wedge F_{10} \vdash \perp, \Delta_{12}} \text{hCut}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_3 : \Delta_7, F_9, F_{10} \vdash (\top, \Delta_{12}), F_8}{\bullet h_3 : \Delta_7, F_9 \wedge F_{10} \vdash (\top, \Delta_{12}), F_8} \wedge_L \quad \frac{h_{11} : (\Delta_7, F_9 \wedge F_{10}), F_8 \vdash \top, \Delta_{12}}{\bullet h_{11} : (\Delta_7, F_9 \wedge F_{10}), F_8 \vdash \top, \Delta_{12}} \top_R \\
\hline
- : \Delta_7, F_9 \wedge F_{10} \vdash \top, \Delta_{12} \quad \text{Cut} \\
\rightarrow \\
- : \Delta_7, F_9 \wedge F_{10} \vdash \top, \Delta_{12} \quad \top_R
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_3 : (\Delta_{14}, F_{11} \rightarrow F_{12}), F_8, F_9 \vdash \Delta_{13}, F_7}{\bullet h_3 : (\Delta_{14}, F_{11} \rightarrow F_{12}), F_8 \wedge F_9 \vdash \Delta_{13}, F_7} \wedge_L \quad \frac{h_{10} : \Delta_{14}, F_7, F_8 \wedge F_9 \vdash \Delta_{13}, F_{11} \quad h_{10} : \Delta_{14}, F_7, F_{12}, F_8 \wedge F_9 \vdash \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \rightarrow F_{12}), F_8 \wedge F_9), F_7 \vdash \Delta_{13}} \rightarrow_L \\
\hline
- : (\Delta_{14}, F_{11} \rightarrow F_{12}), F_8 \wedge F_9 \vdash \Delta_{13} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_{14}, F_8, F_9, F_{11} \rightarrow F_{12} \vdash \Delta_{13}, F_7}{\bullet h_3 : \Delta_{14}, F_8, F_9, F_{11} \rightarrow F_{12} \vdash \Delta_{13}, F_7} \text{ax/W} \quad \frac{h_{10} : \Delta_{14}, F_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \quad \text{inv-th/ax} \quad h_{10} : \Delta_{14}, F_{12}, F_7, F_8, F_9 \vdash \Delta_{13}}{\bullet h_{10} : \Delta_{14}, F_7, F_8, F_9, F_{11} \rightarrow F_{12} \vdash \Delta_{13}} \text{inv-th/ax}}{- : \Delta_{14}, F_8, F_9, F_{11} \rightarrow F_{12} \vdash \Delta_{13}} \wedge_L \quad \frac{- : \Delta_{14}, F_8, F_9, F_{11} \rightarrow F_{12} \vdash \Delta_{13} \quad h_{10} : \Delta_{14}, F_7, F_8, F_9 \wedge F_{10} \vdash \Delta_{13}}{- : \Delta_{14}, F_{11} \rightarrow F_{12}, F_8 \wedge F_9 \vdash \Delta_{13}} \rightarrow_L}{- : \Delta_{14}, F_{11} \rightarrow F_{12}, F_8 \wedge F_9 \vdash \Delta_{13}} \text{hCut} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \rightarrow F_{12}}{\bullet h_3 : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}, F_{11} \rightarrow F_{12}} \wedge_L \quad \frac{h_{10} : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}, F_{11} \quad h_{10} : \Delta_7, F_{12}, F_8 \wedge F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), F_{11} \rightarrow F_{12} \vdash \Delta_{13}} \rightarrow_L}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \rightarrow F_{12}}{\bullet h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \rightarrow F_{12}} \text{ax/W} \quad \frac{h_{10} : \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \quad \text{inv-th/ax} \quad h_{10} : \Delta_7, F_{12}, F_8, F_9 \vdash \Delta_{13}}{\bullet h_{10} : \Delta_7, F_8, F_9, F_{11} \rightarrow F_{12} \vdash \Delta_{13}} \text{inv-th/ax}}{- : \Delta_7, F_8, F_9 \vdash \Delta_{13}} \wedge_L \quad \frac{- : \Delta_7, F_8, F_9 \vdash \Delta_{13} \quad h_{10} : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}} \rightarrow_L}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}} \text{hCut}
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_3 : (\Delta_{14}, F_{11} \wedge F_{12}), F_8, F_9 \vdash \Delta_{13}, F_7}{\bullet h_3 : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \wedge F_9 \vdash \Delta_{13}, F_7} \wedge_L \quad \frac{h_{10} : \Delta_{14}, F_7, F_{11}, F_{12}, F_8 \wedge F_9 \vdash \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \wedge F_{12}), F_8 \wedge F_9), F_7 \vdash \Delta_{13}} \wedge_L}{- : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \wedge F_9 \vdash \Delta_{13}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_{14}, F_{11}, F_{12}, F_8, F_9 \vdash \Delta_{13}, F_7}{\bullet h_3 : \Delta_{14}, F_{11}, F_{12}, F_8 \wedge F_9 \vdash \Delta_{13}, F_7} \text{inv-th/ax}}{- : \Delta_{14}, F_{11}, F_{12}, F_8 \wedge F_9 \vdash \Delta_{13}} \wedge_L \quad \frac{h_{10} : \Delta_{14}, F_{11}, F_{12}, F_7, F_8 \wedge F_9 \vdash \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11}, F_{12}), F_7, F_8 \wedge F_9) \vdash \Delta_{13}} \text{ax/W}}{- : \Delta_{14}, F_{11} \wedge F_{12}, F_8 \wedge F_9 \vdash \Delta_{13}} \wedge_L}{- : \Delta_{14}, F_{11} \wedge F_{12}, F_8 \wedge F_9 \vdash \Delta_{13}} \text{hCut} \\
\rightarrow \\
\frac{h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}}{\bullet h_3 : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}} \wedge_L \quad \frac{h_{10} : \Delta_7, F_{11}, F_{12}, F_8 \wedge F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), F_{11} \wedge F_{12} \vdash \Delta_{13}} \wedge_L}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}}{\bullet h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}} \text{ax/W}}{- : \Delta_7, F_8, F_9 \vdash \Delta_{13}} \wedge_L \quad \frac{h_{10} : \Delta_7, F_{11}, F_{12}, F_8, F_9 \vdash \Delta_{13}}{\bullet h_{10} : \Delta_7, F_8, F_9, F_{11} \wedge F_{12} \vdash \Delta_{13}} \text{inv-th/ax}}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}} \wedge_L}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}} \text{hCut} \\
\rightarrow \\
\frac{h_3 : \Delta_7, F_{10}, F_{11} \vdash \Delta_{12}, F_8}{\bullet h_3 : \Delta_7, F_{10} \wedge F_{11} \vdash \Delta_{12}, F_8} \wedge_L \quad \frac{h_9 : \Delta_7, F_8, F_{10}, F_{11} \vdash \Delta_{12}}{\bullet h_9 : (\Delta_7, F_{10} \wedge F_{11}), F_8 \vdash \Delta_{12}} \wedge_L}{- : \Delta_7, F_{10} \wedge F_{11} \vdash \Delta_{12}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_7, F_{10}, F_{11} \vdash \Delta_{12}, F_8}{\bullet h_3 : \Delta_7, F_{10}, F_{11} \vdash \Delta_{12}, F_8} \text{ax/W}}{- : \Delta_7, F_{10}, F_{11} \vdash \Delta_{12}} \wedge_L \quad \frac{h_9 : \Delta_7, F_{10}, F_{11}, F_8 \vdash \Delta_{12}}{\bullet h_9 : \Delta_7, F_{10}, F_{11}, F_8 \vdash \Delta_{12}} \text{ax/W}}{- : \Delta_7, F_{10}, F_{11}, F_8 \vdash \Delta_{12}} \text{height}}{- : \Delta_7, F_{10} \wedge F_{11} \vdash \Delta_{12}} \wedge_L}{- : \Delta_7, F_{10} \wedge F_{11} \vdash \Delta_{12}} \text{hCut}
\end{array}$$

• Case rule \vee_L

$$\begin{array}{c}
\frac{h_3 : (\Delta_{14}, F_{11} \vee F_{12}), F_8, F_9 \vdash \Delta_{13}, F_7}{\bullet h_3 : (\Delta_{14}, F_{11} \vee F_{12}), F_8 \wedge F_9 \vdash \Delta_{13}, F_7} \wedge_L \quad \frac{h_{10} : \Delta_{14}, F_7, F_{11}, F_8 \wedge F_9 \vdash \Delta_{13} \quad h_{10} : \Delta_{14}, F_7, F_{12}, F_8 \wedge F_9 \vdash \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \vee F_{12}), F_8 \wedge F_9), F_7 \vdash \Delta_{13}} \wedge_L}{- : (\Delta_{14}, F_{11} \vee F_{12}), F_8 \wedge F_9 \vdash \Delta_{13}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_{14}, F_8, F_9, F_{11} \vee F_{12} \vdash \Delta_{13}, F_7}{\bullet h_3 : \Delta_{14}, F_8, F_9, F_{11} \vee F_{12} \vdash \Delta_{13}, F_7} \text{ax/W}}{- : \Delta_{14}, F_8, F_9, F_{11} \vee F_{12} \vdash \Delta_{13}} \wedge_L \quad \frac{h_{10} : \Delta_{14}, F_{11}, F_7, F_8, F_9 \vdash \Delta_{13}}{\bullet h_{10} : \Delta_{14}, F_7, F_8, F_9, F_{11} \vee F_{12} \vdash \Delta_{13}} \text{inv-th/ax}}{- : \Delta_{14}, F_8 \wedge F_9, F_{11} \vee F_{12} \vdash \Delta_{13}} \wedge_L}{- : \Delta_{14}, F_8 \wedge F_9, F_{11} \vee F_{12} \vdash \Delta_{13}} \text{hCut}}{- : \Delta_{14}, F_8 \wedge F_9, F_{11} \vee F_{12} \vdash \Delta_{13}} \text{inv-th/ax} \\
\rightarrow \\
\frac{h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \vee F_{12}}{\bullet h_3 : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}, F_{11} \vee F_{12}} \wedge_L \quad \frac{h_{10} : \Delta_7, F_{11}, F_8 \wedge F_9 \vdash \Delta_{13} \quad h_{10} : \Delta_7, F_{12}, F_8 \wedge F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), F_{11} \vee F_{12} \vdash \Delta_{13}} \wedge_L}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \vee F_{12}}{\bullet h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{13}, F_{11} \vee F_{12}} \text{ax/W}}{- : \Delta_7, F_8, F_9 \vdash \Delta_{13}} \wedge_L \quad \frac{h_{10} : \Delta_7, F_{11}, F_8, F_9 \vdash \Delta_{13}}{\bullet h_{10} : \Delta_7, F_8, F_9, F_{11} \vee F_{12} \vdash \Delta_{13}} \text{inv-th/ax}}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}} \wedge_L}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}} \text{hCut}}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{13}} \text{inv-th/ax}
\end{array}$$

• Case rule \perp_L

$$\begin{array}{c}
\frac{h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{11}, \perp}{\bullet h_3 : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}, \perp} \wedge_L \quad \frac{h_{10} : (\Delta_7, F_8 \wedge F_9), \perp \vdash \Delta_{11}}{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), \perp \vdash \Delta_{11}} \wedge_L}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_7, F_8, F_9 \vdash \perp, \Delta_{11}}{\bullet h_3 : \Delta_7, F_8, F_9 \vdash \perp, \Delta_{11}} \text{ax/W}}{- : \Delta_7, F_8, F_9 \vdash \perp, \Delta_{11}} \wedge_L \quad \frac{h_{10} : \perp, \Delta_7, F_8, F_9 \vdash \Delta_{11}}{\bullet h_{10} : \perp, \Delta_7, F_8, F_9 \vdash \Delta_{11}} \wedge_L}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}} \wedge_L}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}} \text{hCut} \\
\rightarrow \\
\frac{h_3 : (\perp, \Delta_{12}), F_8, F_9 \vdash \Delta_{11}, F_7}{\bullet h_3 : (\perp, \Delta_{12}), F_8 \wedge F_9 \vdash \Delta_{11}, F_7} \wedge_L \quad \frac{h_{10} : ((\perp, \Delta_{12}), F_8 \wedge F_9), F_7 \vdash \Delta_{11}}{\bullet h_{10} : ((\perp, \Delta_{12}), F_8 \wedge F_9), F_7 \vdash \Delta_{11}} \wedge_L}{- : (\perp, \Delta_{12}), F_8 \wedge F_9 \vdash \Delta_{11}} \text{Cut} \\
\rightarrow \\
\frac{- : \perp, \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{11}}{- : \perp, \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{11}} \wedge_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_7, F_8, F_9 \vdash (\Delta_{12}, P_{11}), P_{11}}{\bullet h_3 : \Delta_7, F_8 \wedge F_9 \vdash (\Delta_{12}, P_{11}), P_{11}} \wedge L \quad \frac{}{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), P_{11} \vdash \Delta_{12}, P_{11}} I}{\frac{}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{12}, P_{11}} \text{Cut}} \rightarrow \\
\frac{\frac{h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{12}, P_{11}, P_{11}}{\bullet h_3 : \Delta_7, F_8, F_9, P_{11} \vdash \Delta_{12}, P_{11}} \text{ax/W} \quad \frac{}{\bullet h_{10} : \Delta_7, F_8, F_9, P_{11} \vdash \Delta_{12}, P_{11}} I}{\frac{}{- : \Delta_7, F_8, F_9 \vdash \Delta_{12}, P_{11}} \wedge L} \text{hCut} \\
\frac{}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{12}, P_{11}} \wedge L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_3 : (\Delta_{13}, P_{11}), F_8, F_9 \vdash (\Delta_{12}, P_{11}), F_7}{\bullet h_3 : (\Delta_{13}, P_{11}), F_8 \wedge F_9 \vdash (\Delta_{12}, P_{11}), F_7} \wedge L \quad \frac{}{\bullet h_{10} : ((\Delta_{13}, P_{11}), F_8 \wedge F_9), F_7 \vdash \Delta_{12}, P_{11}} I}{\frac{}{- : (\Delta_{13}, P_{11}), F_8 \wedge F_9 \vdash \Delta_{12}, P_{11}} \text{Cut}} \rightarrow \\
\frac{}{- : \Delta_{13}, P_{11}, F_8 \wedge F_9 \vdash \Delta_{12}, P_{11}} I
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_7, F_8, F_9 \vdash \Delta_{11}, \top}{\bullet h_3 : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}, \top} \wedge L \quad \frac{h_{10} : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}}{\bullet h_{10} : (\Delta_7, F_8 \wedge F_9), \top \vdash \Delta_{11}} \top_L}{\frac{}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}} \text{Cut}} \rightarrow \\
\frac{}{- : \Delta_7, F_8 \wedge F_9 \vdash \Delta_{11}} \text{ax/W}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_3 : (\top, \Delta_{12}), F_8, F_9 \vdash \Delta_{11}, F_7}{\bullet h_3 : (\top, \Delta_{12}), F_8 \wedge F_9 \vdash \Delta_{11}, F_7} \wedge L \quad \frac{h_{10} : \Delta_{12}, F_7, F_8 \wedge F_9 \vdash \Delta_{11}}{\bullet h_{10} : ((\top, \Delta_{12}), F_8 \wedge F_9), F_7 \vdash \Delta_{11}} \top_L}{\frac{}{- : (\top, \Delta_{12}), F_8 \wedge F_9 \vdash \Delta_{11}} \text{Cut}} \rightarrow \\
\frac{\frac{h_3 : \top, \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{11}, F_7}{\bullet h_3 : \top, \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{11}, F_7} \text{ax/W} \quad \frac{h_{10} : \top, \Delta_{12}, F_7, F_8 \wedge F_9 \vdash \Delta_{11}}{\bullet h_{10} : \top, \Delta_{12}, F_7, F_8 \wedge F_9 \vdash \Delta_{11}} \text{ax/W}}{\frac{}{- : \top, \Delta_{12}, F_8 \wedge F_9 \vdash \Delta_{11}} \text{hCut}} \rightarrow_R
\end{array}$$

8.8 Status of \vee_L : OK

- Case rule \rightarrow_R

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_7, F_9 \vdash (\Delta_{12}, F_{13} \rightarrow F_{14}), F_8}{\bullet h_3 : \Delta_7, F_9 \vee F_{10} \vdash (\Delta_{12}, F_{13} \rightarrow F_{14}), F_8} \vee_L \quad \frac{h_3 : \Delta_7, F_{10} \vdash (\Delta_{12}, F_{13} \rightarrow F_{14}), F_8}{\bullet h_{11} : (\Delta_7, F_9 \vee F_{10}), F_8 \vdash \Delta_{12}, F_{13} \rightarrow F_{14}} \rightarrow_R}{\frac{}{- : \Delta_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \rightarrow F_{14}} \text{Cut}} \rightarrow \\
\frac{\frac{h_3 : \Delta_7, F_{13}, F_9 \vdash \Delta_{12}, F_{14}, F_8}{\bullet h_3 : \Delta_7, F_{13}, F_9 \vee F_{10} \vdash \Delta_{12}, F_{14}, F_8} \text{inv-th/ax} \quad \frac{h_3 : \Delta_7, F_{10}, F_{13} \vdash \Delta_{12}, F_{14}, F_8}{\bullet h_{11} : \Delta_7, F_{13}, F_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{14}} \text{inv-th/ax}}{\frac{}{- : \Delta_7, F_{13}, F_9 \vee F_{10} \vdash \Delta_{12}, F_{14}} \text{hCut}} \rightarrow_R \\
\frac{}{- : \Delta_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \rightarrow F_{14}} \rightarrow_R
\end{array}$$

- Case rule \wedge_R

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_7, F_9 \vdash (\Delta_{12}, F_{13} \wedge F_{14}), F_8}{\bullet h_3 : \Delta_7, F_9 \vee F_{10} \vdash (\Delta_{12}, F_{13} \wedge F_{14}), F_8} \wedge_L \quad \frac{h_3 : \Delta_7, F_{10} \vdash (\Delta_{12}, F_{13} \wedge F_{14}), F_8}{\bullet h_{11} : (\Delta_7, F_9 \vee F_{10}) \vdash \Delta_{12}, F_{13} \wedge F_{14}} \wedge_R}{\frac{}{- : \Delta_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \wedge F_{14}} \text{Cut}} \rightarrow \\
\frac{\frac{h_3 : \Delta_7, F_9 \vdash \Delta_{12}, F_{13}, F_8}{\bullet h_3 : \Delta_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}, F_8} \text{inv-th/ax} \quad \frac{h_3 : \Delta_7, F_{10} \vdash \Delta_{12}, F_{13}, F_8}{\bullet h_{11} : \Delta_7, F_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}} \text{inv-th/ax}}{\frac{}{- : \Delta_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}} \text{hCut}} \rightarrow_R \\
\frac{}{- : \Delta_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \wedge F_{14}} \rightarrow_R
\end{array}$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{h_3 : \Delta_7, F_9 \vdash (\Delta_{12}, F_{13} \vee F_{14}), F_8 \quad h_3 : \Delta_7, F_{10} \vdash (\Delta_{12}, F_{13} \vee F_{14}), F_8}{\bullet h_3 : \Delta_7, F_9 \vee F_{10} \vdash (\Delta_{12}, F_{13} \vee F_{14}), F_8} \vee_L \quad \frac{h_{11} : \Delta_7, F_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}, F_{14}}{\bullet h_{11} : (\Delta_7, F_9 \vee F_{10}), F_8 \vdash \Delta_{12}, F_{13} \vee F_{14}} \vee_R \\
\frac{}{- : \Delta_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \vee F_{14}}{\rightarrow} \text{Cut} \\
\frac{\frac{h_3 : \Delta_7, F_9 \vdash \Delta_{12}, F_{13}, F_{14}, F_8}{\bullet h_3 : \Delta_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}, F_{14}, F_8} \text{inv-th/ax} \quad \frac{h_3 : \Delta_7, F_{10} \vdash \Delta_{12}, F_{13}, F_{14}, F_8}{\bullet h_3 : \Delta_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}, F_{14}, F_8} \text{inv-th/ax}}{\frac{}{- : \Delta_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}, F_{14}} \vee_L} \vee_L \quad \frac{h_{11} : \Delta_7, F_8, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13}, F_{14}}{\bullet h_{11} : (\Delta_7, F_9 \vee F_{10}), F_8 \vdash \Delta_{12}, F_{13}, F_{14}} \text{ax/W}}{\frac{}{- : \Delta_7, F_9 \vee F_{10} \vdash \Delta_{12}, F_{13} \vee F_{14}} \vee_R} \text{hCut}
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{h_3 : \Delta_7, F_9 \vdash (\perp, \Delta_{12}), F_8 \quad h_3 : \Delta_7, F_{10} \vdash (\perp, \Delta_{12}), F_8}{\bullet h_3 : \Delta_7, F_9 \vee F_{10} \vdash (\perp, \Delta_{12}), F_8} \vee_L \quad \frac{h_{11} : \Delta_7, F_8, F_9 \vee F_{10} \vdash \Delta_{12}}{\bullet h_{11} : (\Delta_7, F_9 \vee F_{10}), F_8 \vdash \perp, \Delta_{12}} \perp_R \\
\frac{}{- : \Delta_7, F_9 \vee F_{10} \vdash \perp, \Delta_{12}}{\rightarrow} \text{Cut} \\
\frac{\frac{h_3 : \Delta_7, F_9 \vee F_{10} \vdash \perp, \Delta_{12}, F_8}{\bullet h_3 : \Delta_7, F_9 \vee F_{10} \vdash \perp, \Delta_{12}, F_8} \text{ax/W} \quad \frac{h_{11} : \Delta_7, F_8, F_9 \vee F_{10} \vdash \perp, \Delta_{12}}{\bullet h_{11} : (\Delta_7, F_9 \vee F_{10}), F_8 \vdash \perp, \Delta_{12}} \text{ax/W}}{\frac{}{- : \Delta_7, F_9 \vee F_{10} \vdash \perp, \Delta_{12}} \text{hCut}}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{h_3 : \Delta_7, F_9 \vdash (\top, \Delta_{12}), F_8 \quad h_3 : \Delta_7, F_{10} \vdash (\top, \Delta_{12}), F_8}{\bullet h_3 : \Delta_7, F_9 \vee F_{10} \vdash (\top, \Delta_{12}), F_8} \vee_L \quad \frac{h_{11} : (\Delta_7, F_9 \vee F_{10}), F_8 \vdash \top, \Delta_{12}}{\bullet h_{11} : (\Delta_7, F_9 \vee F_{10}), F_8 \vdash \top, \Delta_{12}} \top_R \\
\frac{}{- : \Delta_7, F_9 \vee F_{10} \vdash \top, \Delta_{12}}{\rightarrow} \text{Cut} \\
\frac{}{- : \Delta_7, F_9 \vee F_{10} \vdash \top, \Delta_{12}}{\top_R}
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{h_3 : (\Delta_{14}, F_{11} \rightarrow F_{12}), F_8 \vdash \Delta_{13}, F_7 \quad h_3 : (\Delta_{14}, F_{11} \rightarrow F_{12}), F_9 \vdash \Delta_{13}, F_7}{\bullet h_3 : (\Delta_{14}, F_{11} \rightarrow F_{12}), F_8 \vee F_9 \vdash \Delta_{13}, F_7} \vee_L \quad \frac{h_{10} : \Delta_{14}, F_7, F_8 \vee F_9 \vdash \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \rightarrow F_{12}), F_8 \vee F_9) \vdash \Delta_{13}} \rightarrow_L \\
\frac{}{- : (\Delta_{14}, F_{11} \rightarrow F_{12}), F_8 \vee F_9 \vdash \Delta_{13}}{\rightarrow} \text{Cut} \\
\frac{\frac{h_3 : \Delta_{14}, F_8 \vdash \Delta_{13}, F_{11}, F_7}{\bullet h_3 : \Delta_{14}, F_8 \vee F_9 \vdash \Delta_{13}, F_{11}, F_7} \text{inv-th/ax} \quad \frac{h_3 : \Delta_{14}, F_9 \vdash \Delta_{13}, F_{11}, F_7}{\bullet h_3 : \Delta_{14}, F_8 \vee F_9 \vdash \Delta_{13}, F_{11}, F_7} \text{inv-th/ax}}{\frac{}{- : \Delta_{14}, F_8 \vee F_9 \vdash \Delta_{13}, F_{11}} \vee_L} \vee_L \quad \frac{h_{10} : \Delta_{14}, F_7, F_8 \vee F_9 \vdash \Delta_{13}, F_{11}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \rightarrow F_{12}), F_8 \vee F_9) \vdash \Delta_{13}, F_{11}} \text{ax/W}}{\frac{}{- : \Delta_{14}, F_{11} \rightarrow F_{12}, F_8 \vee F_9 \vdash \Delta_{13}} \text{hCut}} \text{hCut} \\
\frac{}{- : \Delta_{14}, F_{11} \rightarrow F_{12}, F_8 \vee F_9 \vdash \Delta_{13}} \\
\frac{h_3 : \Delta_7, F_8 \vdash \Delta_{13}, F_{11} \rightarrow F_{12} \quad h_3 : \Delta_7, F_9 \vdash \Delta_{13}, F_{11} \rightarrow F_{12}}{\bullet h_3 : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}, F_{11} \rightarrow F_{12}} \vee_L \quad \frac{h_{10} : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}, F_{11} \quad h_{10} : \Delta_7, F_{12}, F_8 \vee F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \vee F_9), F_{11} \rightarrow F_{12} \vdash \Delta_{13}} \rightarrow_L \\
\frac{}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}}{\rightarrow} \text{Cut} \\
\frac{\frac{}{- : \Delta_7, F_{11}, F_8 \vdash \Delta_{13}, F_{12}} \text{inv-th/ax} \quad \frac{}{- : \Delta_7, F_{11}, F_9 \vdash \Delta_{13}, F_{12}} \text{inv-th/ax}}{\frac{}{- : \Delta_7, F_{11}, F_8 \vee F_9 \vdash \Delta_{13}, F_{12}} \vee_L} \vee_L \quad \frac{}{- : \Delta_7, F_{11}, F_{12}, F_8 \vee F_9 \vdash \Delta_{13}} \text{ax/W}}{\frac{}{- : \Delta_7, F_{11}, F_8 \vee F_9 \vdash \Delta_{13}} \text{sCut}} \text{sCut} \\
\frac{}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}}
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_3 : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \vdash \Delta_{13}, F_7 \quad h_3 : (\Delta_{14}, F_{11} \wedge F_{12}), F_9 \vdash \Delta_{13}, F_7}{\bullet h_3 : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \vee F_9 \vdash \Delta_{13}, F_7} \vee_L \quad \frac{h_{10} : \Delta_{14}, F_7, F_{11}, F_{12}, F_8 \vee F_9 \vdash \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \wedge F_{12}), F_8 \vee F_9), F_7 \vdash \Delta_{13}} \wedge_L \\
\frac{}{- : (\Delta_{14}, F_{11} \wedge F_{12}), F_8 \vee F_9 \vdash \Delta_{13}}{\rightarrow} \text{Cut} \\
\frac{\frac{h_3 : \Delta_{14}, F_{11}, F_{12}, F_8 \vdash \Delta_{13}, F_7}{\bullet h_3 : \Delta_{14}, F_{11}, F_{12}, F_8 \vee F_9 \vdash \Delta_{13}, F_7} \text{inv-th/ax} \quad \frac{h_3 : \Delta_{14}, F_{11}, F_{12}, F_9 \vdash \Delta_{13}, F_7}{\bullet h_3 : \Delta_{14}, F_{11}, F_{12}, F_8 \vee F_9 \vdash \Delta_{13}, F_7} \text{inv-th/ax}}{\frac{}{- : \Delta_{14}, F_{11}, F_{12}, F_8 \vee F_9 \vdash \Delta_{13}} \vee_L} \vee_L \quad \frac{h_{10} : \Delta_{14}, F_{11}, F_{12}, F_7, F_8 \vee F_9 \vdash \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \wedge F_{12}), F_8 \vee F_9), F_7 \vdash \Delta_{13}} \text{ax/W}}{\frac{}{- : \Delta_{14}, F_{11} \wedge F_{12}, F_8 \vee F_9 \vdash \Delta_{13}} \wedge_L} \text{hCut} \\
\frac{}{- : \Delta_{14}, F_{11} \wedge F_{12}, F_8 \vee F_9 \vdash \Delta_{13}}
\end{array}$$

$$\begin{array}{c}
\frac{h_3 : \Delta_7, F_8 \vdash \Delta_{13}, F_{11} \wedge F_{12} \quad h_3 : \Delta_7, F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}}{\bullet h_3 : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}} \vee_L \quad \frac{h_{10} : \Delta_7, F_{11}, F_{12}, F_8 \vee F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \vee F_9), F_{11} \wedge F_{12} \vdash \Delta_{13}} \wedge_L \\
\hline
- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_7, F_8 \vdash \Delta_{13}, F_{11} \wedge F_{12}}{- : \Delta_7, F_8 \vdash \Delta_{13}} \text{ax/W} \quad \frac{h_{10} : \Delta_7, F_{11}, F_{12}, F_8 \vdash \Delta_{13}}{\bullet h_{10} : \Delta_7, F_8, F_{11} \wedge F_{12} \vdash \Delta_{13}} \wedge_L}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}} \text{hCut} \quad \frac{h_3 : \Delta_7, F_9 \vdash \Delta_{13}, F_{11} \wedge F_{12}}{- : \Delta_7, F_9 \vdash \Delta_{13}} \text{ax/W} \quad \frac{h_{10} : \Delta_7, F_{11}, F_{12}, F_9 \vdash \Delta_{13}}{\bullet h_{10} : \Delta_7, F_9, F_{11} \wedge F_{12} \vdash \Delta_{13}} \wedge_L}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}} \text{hCut} \quad \text{inv-t}
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{h_3 : (\Delta_{14}, F_{11} \vee F_{12}), F_8 \vdash \Delta_{13}, F_7 \quad h_3 : (\Delta_{14}, F_{11} \vee F_{12}), F_9 \vdash \Delta_{13}, F_7}{\bullet h_3 : (\Delta_{14}, F_{11} \vee F_{12}), F_8 \vee F_9 \vdash \Delta_{13}, F_7} \vee_L \quad \frac{h_{10} : \Delta_{14}, F_7, F_{11}, F_8 \vee F_9 \vdash \Delta_{13}}{\bullet h_{10} : ((\Delta_{14}, F_{11} \vee F_{12}), F_8 \vee F_9) \vdash \Delta_{13}} \wedge_L \\
\hline
- : (\Delta_{14}, F_{11} \vee F_{12}), F_8 \vee F_9 \vdash \Delta_{13} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_{14}, F_{11}, F_8 \vdash \Delta_{13}, F_7}{\bullet h_3 : \Delta_{14}, F_{11}, F_8 \vee F_9 \vdash \Delta_{13}, F_7} \text{inv-th/ax} \quad \frac{h_3 : \Delta_{14}, F_{11}, F_9 \vdash \Delta_{13}, F_7}{- : \Delta_{14}, F_{11}, F_8 \vee F_9 \vdash \Delta_{13}} \text{inv-th/ax}}{- : \Delta_{14}, F_{11}, F_8 \vee F_9 \vdash \Delta_{13}} \vee_L \quad \frac{h_{10} : \Delta_{14}, F_{11}, F_7, F_8 \vee F_9 \vdash \Delta_{13}}{- : \Delta_{14}, F_{11} \vee F_{12}, F_8 \vee F_9 \vdash \Delta_{13}} \text{ax/W}}{- : \Delta_{14}, F_{11} \vee F_{12}, F_8 \vee F_9 \vdash \Delta_{13}} \text{hCut} \quad \frac{h_3 : \Delta_{14}, F_{12}, F_8 \vdash \Delta_{13}, F_7}{\bullet h_3 : \Delta_{14}, F_{12}, F_8 \vee F_9 \vdash \Delta_{13}, F_7} \text{inv-t} \\
\hline
\frac{h_3 : \Delta_7, F_8 \vdash \Delta_{13}, F_{11} \vee F_{12} \quad h_3 : \Delta_7, F_9 \vdash \Delta_{13}, F_{11} \vee F_{12}}{\bullet h_3 : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}, F_{11} \vee F_{12}} \vee_L \quad \frac{h_{10} : \Delta_7, F_{11}, F_8 \vee F_9 \vdash \Delta_{13} \quad h_{10} : \Delta_7, F_{12}, F_8 \vee F_9 \vdash \Delta_{13}}{\bullet h_{10} : (\Delta_7, F_8 \vee F_9), F_{11} \vee F_{12} \vdash \Delta_{13}} \vee_L \\
\hline
- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_7, F_8 \vdash \Delta_{13}, F_{11}, F_{12}}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}, F_{11}, F_{12}} \text{inv-th/ax} \quad \frac{- : \Delta_7, F_9 \vdash \Delta_{13}, F_{11}, F_{12}}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}, F_{11}} \text{inv-th/ax}}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}, F_{11}} \vee_L \quad \frac{- : \Delta_7, F_{12}, F_8 \vee F_9 \vdash \Delta_{13}, F_{11}}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}} \text{ax/W}}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}, F_{11}} \text{sCut} \quad \frac{- : \Delta_7, F_{11}, F_8 \vee F_9 \vdash \Delta_{13}}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}} \text{ax/W}}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{13}} \text{sCut} \\
\hline
\frac{h_3 : \Delta_7, F_{10} \vdash \Delta_{12}, F_8 \quad h_3 : \Delta_7, F_{11} \vdash \Delta_{12}, F_8}{\bullet h_3 : \Delta_7, F_{10} \vee F_{11} \vdash \Delta_{12}, F_8} \vee_L \quad \frac{h_9 : \Delta_7, F_8, F_{10} \vdash \Delta_{12} \quad h_9 : \Delta_7, F_8, F_{11} \vdash \Delta_{12}}{\bullet h_9 : (\Delta_7, F_{10} \vee F_{11}), F_8 \vdash \Delta_{12}} \vee_L \\
\hline
- : \Delta_7, F_{10} \vee F_{11} \vdash \Delta_{12} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_7, F_{10} \vdash \Delta_{12}, F_8}{- : \Delta_7, F_{10} \vdash \Delta_{12}} \text{ax/W} \quad \frac{h_9 : \Delta_7, F_{10}, F_8 \vdash \Delta_{12}}{\bullet h_9 : \Delta_7, F_{10}, F_8 \vdash \Delta_{12}} \text{height}}{- : \Delta_7, F_{10} \vee F_{11} \vdash \Delta_{12}} \text{hCut} \quad \frac{h_3 : \Delta_7, F_{11} \vdash \Delta_{12}, F_8}{- : \Delta_7, F_{11} \vdash \Delta_{12}} \text{ax/W} \quad \frac{h_9 : \Delta_7, F_{11}, F_8 \vdash \Delta_{12}}{\bullet h_9 : \Delta_7, F_{11}, F_8 \vdash \Delta_{12}} \text{height}}{- : \Delta_7, F_{10} \vee F_{11} \vdash \Delta_{12}} \vee_L \text{hCut}
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{h_3 : \Delta_7, F_8 \vdash \Delta_{11}, \perp \quad h_3 : \Delta_7, F_9 \vdash \Delta_{11}, \perp}{\bullet h_3 : \Delta_7, F_8 \vee F_9 \vdash \Delta_{11}, \perp} \vee_L \quad \frac{h_{10} : (\Delta_7, F_8 \vee F_9), \perp \vdash \Delta_{11}}{\bullet h_{10} : (\Delta_7, F_8 \vee F_9), \perp \vdash \Delta_{11}} \perp_L \\
\hline
- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{11} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 : \Delta_7, F_8 \vdash \perp, \Delta_{11}}{- : \Delta_7, F_8 \vdash \Delta_{11}} \text{ax/W} \quad \frac{h_{10} : \perp, \Delta_7, F_8 \vdash \Delta_{11}}{\bullet h_{10} : \perp, \Delta_7, F_8 \vdash \Delta_{11}} \perp_L}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{11}} \text{hCut} \quad \frac{h_3 : \Delta_7, F_9 \vdash \perp, \Delta_{11}}{- : \Delta_7, F_9 \vdash \Delta_{11}} \text{ax/W} \quad \frac{h_{10} : \perp, \Delta_7, F_9 \vdash \Delta_{11}}{\bullet h_{10} : \perp, \Delta_7, F_9 \vdash \Delta_{11}} \perp_L}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{11}} \vee_L \text{hCut} \\
\hline
\frac{h_3 : (\perp, \Delta_{12}), F_8 \vdash \Delta_{11}, F_7 \quad h_3 : (\perp, \Delta_{12}), F_9 \vdash \Delta_{11}, F_7}{\bullet h_3 : (\perp, \Delta_{12}), F_8 \vee F_9 \vdash \Delta_{11}, F_7} \vee_L \quad \frac{h_{10} : ((\perp, \Delta_{12}), F_8 \vee F_9), F_7 \vdash \Delta_{11}}{\bullet h_{10} : ((\perp, \Delta_{12}), F_8 \vee F_9), F_7 \vdash \Delta_{11}} \perp_L \\
\hline
- : (\perp, \Delta_{12}), F_8 \vee F_9 \vdash \Delta_{11} \quad \text{Cut} \\
\rightarrow \\
- : \perp, \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{11} \quad \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_7, F_8 \vdash (\Delta_{12}, P_{11}), P_{11} \quad h_3 : \Delta_7, F_9 \vdash (\Delta_{12}, P_{11}), P_{11}}{\bullet h_3 : \Delta_7, F_8 \vee F_9 \vdash (\Delta_{12}, P_{11}), P_{11}} \vee_L \quad \frac{\bullet h_{10} : (\Delta_7, F_8 \vee F_9), P_{11} \vdash \Delta_{12}, P_{11}}{I} \text{Cut}}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{12}, P_{11}} \rightarrow \\
\frac{\frac{h_3 : \Delta_7, F_8 \vdash \Delta_{12}, P_{11}, P_{11}}{\text{ax/W}} \quad \frac{\bullet h_{10} : \Delta_7, F_8, P_{11} \vdash \Delta_{12}, P_{11}}{I} \text{hCut}}{- : \Delta_7, F_8 \vdash \Delta_{12}, P_{11}} \rightarrow \frac{\frac{h_3 : \Delta_7, F_9 \vdash \Delta_{12}, P_{11}, P_{11}}{\text{ax/W}} \quad \bullet h_{10} : \Delta_7, F_9, P_{11} \vdash \Delta_{12}, P_{11}}{- : \Delta_7, F_9 \vdash \Delta_{12}, P_{11}} \vee_L \quad \frac{I}{\text{hCut}}}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{12}, P_{11}} \\
\\
\frac{\frac{h_3 : (\Delta_{13}, P_{11}), F_8 \vdash (\Delta_{12}, P_{11}), F_7 \quad h_3 : (\Delta_{13}, P_{11}), F_9 \vdash (\Delta_{12}, P_{11}), F_7}{\bullet h_3 : (\Delta_{13}, P_{11}), F_8 \vee F_9 \vdash (\Delta_{12}, P_{11}), F_7} \vee_L \quad \frac{\bullet h_{10} : ((\Delta_{13}, P_{11}), F_8 \vee F_9), F_7 \vdash \Delta_{12}, P_{11}}{I} \text{Cut}}{- : (\Delta_{13}, P_{11}), F_8 \vee F_9 \vdash \Delta_{12}, P_{11}} \rightarrow \\
\frac{I}{- : \Delta_{13}, P_{11}, F_8 \vee F_9 \vdash \Delta_{12}, P_{11}}
\end{array}$$

- Case rule \top_L

$$\begin{array}{c}
\frac{\frac{h_3 : \Delta_7, F_8 \vdash \Delta_{11}, \top \quad h_3 : \Delta_7, F_9 \vdash \Delta_{11}, \top}{\bullet h_3 : \Delta_7, F_8 \vee F_9 \vdash \Delta_{11}, \top} \vee_L \quad \frac{h_{10} : \Delta_7, F_8 \vee F_9 \vdash \Delta_{11}}{\bullet h_{10} : (\Delta_7, F_8 \vee F_9), \top \vdash \Delta_{11}} \top_L \text{Cut}}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{11}} \rightarrow \\
\frac{\text{ax/W}}{- : \Delta_7, F_8 \vee F_9 \vdash \Delta_{11}} \\
\\
\frac{\frac{h_3 : (\top, \Delta_{12}), F_8 \vdash \Delta_{11}, F_7 \quad h_3 : (\top, \Delta_{12}), F_9 \vdash \Delta_{11}, F_7}{\bullet h_3 : (\top, \Delta_{12}), F_8 \vee F_9 \vdash \Delta_{11}, F_7} \vee_L \quad \frac{h_{10} : \Delta_{12}, F_7, F_8 \vee F_9 \vdash \Delta_{11}}{\bullet h_{10} : ((\top, \Delta_{12}), F_8 \vee F_9), F_7 \vdash \Delta_{11}} \top_L \text{Cut}}{- : (\top, \Delta_{12}), F_8 \vee F_9 \vdash \Delta_{11}} \rightarrow \\
\frac{\frac{\bullet h_3 : \top, \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{11}, F_7}{\text{ax/W}} \quad \frac{h_{10} : \top, \Delta_{12}, F_7, F_8 \vee F_9 \vdash \Delta_{11}}{\text{hCut}}}{- : \top, \Delta_{12}, F_8 \vee F_9 \vdash \Delta_{11}}
\end{array}$$

8.9 Status of \perp_L : OK

- Case rule \rightarrow_R

$$\frac{\frac{\bullet h_3 : \perp, \Delta_5 \vdash (\Delta_8, F_9 \rightarrow F_{10}), F_6}{\perp_L} \quad \frac{h_7 : \perp, \Delta_5, F_6, F_9 \vdash \Delta_8, F_{10}}{\bullet h_7 : (\perp, \Delta_5), F_6 \vdash \Delta_8, F_9 \rightarrow F_{10}} \rightarrow_R \text{Cut}}{- : \perp, \Delta_5 \vdash \Delta_8, F_9 \rightarrow F_{10}} \rightarrow \\
\frac{\perp_L}{- : \perp, \Delta_5 \vdash \Delta_8, F_9 \rightarrow F_{10}}$$

- Case rule \wedge_R

$$\frac{\frac{\bullet h_3 : \perp, \Delta_5 \vdash (\Delta_8, F_9 \wedge F_{10}), F_6}{\perp_L} \quad \frac{h_7 : \perp, \Delta_5, F_6 \vdash \Delta_8, F_9 \quad h_7 : \perp, \Delta_5, F_6 \vdash \Delta_8, F_{10}}{\bullet h_7 : (\perp, \Delta_5), F_6 \vdash \Delta_8, F_9 \wedge F_{10}} \wedge_R \text{Cut}}{- : \perp, \Delta_5 \vdash \Delta_8, F_9 \wedge F_{10}} \rightarrow \\
\frac{\perp_L}{- : \perp, \Delta_5 \vdash \Delta_8, F_9 \wedge F_{10}}$$

- Case rule \vee_R

$$\frac{\frac{\bullet h_3 : \perp, \Delta_5 \vdash (\Delta_8, F_9 \vee F_{10}), F_6}{\perp_L} \quad \frac{h_7 : \perp, \Delta_5, F_6 \vdash \Delta_8, F_9, F_{10}}{\bullet h_7 : (\perp, \Delta_5), F_6 \vdash \Delta_8, F_9 \vee F_{10}} \vee_R \text{Cut}}{- : \perp, \Delta_5 \vdash \Delta_8, F_9 \vee F_{10}} \rightarrow \\
\frac{\perp_L}{- : \perp, \Delta_5 \vdash \Delta_8, F_9 \vee F_{10}}$$

- Case rule \perp_R

$$\frac{\frac{\frac{}{\bullet h_3 : \perp, \Delta_5 \vdash (\perp, \Delta_8), F_6} \perp_L \quad \frac{h_7 : \perp, \Delta_5, F_6 \vdash \Delta_8}{\bullet h_7 : (\perp, \Delta_5), F_6 \vdash \perp, \Delta_8} \perp_R}{\perp : \perp, \Delta_5 \vdash \perp, \Delta_8} \text{Cut}}{\perp : \perp, \Delta_5 \vdash \perp, \Delta_8} \perp_L}{\perp : \perp, \Delta_5 \vdash \perp, \Delta_8} \perp_L$$

- Case rule \top_R

$$\frac{\frac{\frac{}{\bullet h_3 : \perp, \Delta_5 \vdash (\top, \Delta_8), F_6} \perp_L \quad \frac{}{\bullet h_7 : (\perp, \Delta_5), F_6 \vdash \top, \Delta_8} \top_R}{\perp : \perp, \Delta_5 \vdash \top, \Delta_8} \text{Cut}}{\perp : \perp, \Delta_5 \vdash \top, \Delta_8} \perp_L}{\perp : \perp, \Delta_5 \vdash \top, \Delta_8} \top_R$$

- Case rule \rightarrow_L

$$\frac{\frac{\frac{}{\bullet h_3 : \perp, \Delta_{10}, F_7 \rightarrow F_8 \vdash \Delta_9, F_5} \perp_L \quad \frac{h_6 : \perp, \Delta_{10}, F_5 \vdash \Delta_9, F_7 \quad h_6 : \perp, \Delta_{10}, F_5, F_8 \vdash \Delta_9}{\bullet h_6 : (\perp, \Delta_{10}, F_7 \rightarrow F_8), F_5 \vdash \Delta_9} \rightarrow_L}{\perp : \perp, \Delta_{10}, F_7 \rightarrow F_8 \vdash \Delta_9} \text{Cut}}{\perp : \perp, \Delta_{10}, F_7 \rightarrow F_8 \vdash \Delta_9} \perp_L}{\perp : \perp, \Delta_{10}, F_7 \rightarrow F_8 \vdash \Delta_9} \rightarrow_L$$

$$\frac{\frac{\frac{}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_9, F_7 \rightarrow F_8} \perp_L \quad \frac{h_6 : \perp, \Delta_5 \vdash \Delta_9, F_7 \quad h_6 : \perp, \Delta_5, F_8 \vdash \Delta_9}{\bullet h_6 : (\perp, \Delta_5), F_7 \rightarrow F_8 \vdash \Delta_9} \rightarrow_L}{\perp : \perp, \Delta_5 \vdash \Delta_9} \text{Cut}}{\perp : \perp, \Delta_5 \vdash \Delta_9} \perp_L}{\perp : \perp, \Delta_5 \vdash \Delta_9} \rightarrow_L$$

- Case rule \wedge_L

$$\frac{\frac{\frac{}{\bullet h_3 : \perp, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9, F_5} \perp_L \quad \frac{h_6 : \perp, \Delta_{10}, F_5, F_7, F_8 \vdash \Delta_9}{\bullet h_6 : (\perp, \Delta_{10}, F_7 \wedge F_8), F_5 \vdash \Delta_9} \wedge_L}{\perp : \perp, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9} \text{Cut}}{\perp : \perp, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9} \perp_L}{\perp : \perp, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9} \wedge_L$$

$$\frac{\frac{\frac{}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_9, F_7 \wedge F_8} \perp_L \quad \frac{h_6 : \perp, \Delta_5, F_7, F_8 \vdash \Delta_9}{\bullet h_6 : (\perp, \Delta_5), F_7 \wedge F_8 \vdash \Delta_9} \wedge_L}{\perp : \perp, \Delta_5 \vdash \Delta_9} \text{Cut}}{\perp : \perp, \Delta_5 \vdash \Delta_9} \perp_L}{\perp : \perp, \Delta_5 \vdash \Delta_9} \wedge_L$$

- Case rule \vee_L

$$\frac{\frac{\frac{}{\bullet h_3 : \perp, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9, F_5} \perp_L \quad \frac{h_6 : \perp, \Delta_{10}, F_5, F_7 \vdash \Delta_9 \quad h_6 : \perp, \Delta_{10}, F_5, F_8 \vdash \Delta_9}{\bullet h_6 : (\perp, \Delta_{10}, F_7 \vee F_8), F_5 \vdash \Delta_9} \vee_L}{\perp : \perp, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9} \text{Cut}}{\perp : \perp, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9} \perp_L}{\perp : \perp, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9} \vee_L$$

$$\frac{\frac{\frac{}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_9, F_7 \vee F_8} \perp_L \quad \frac{h_6 : \perp, \Delta_5, F_7 \vdash \Delta_9 \quad h_6 : \perp, \Delta_5, F_8 \vdash \Delta_9}{\bullet h_6 : (\perp, \Delta_5), F_7 \vee F_8 \vdash \Delta_9} \vee_L}{\perp : \perp, \Delta_5 \vdash \Delta_9} \text{Cut}}{\perp : \perp, \Delta_5 \vdash \Delta_9} \perp_L}{\perp : \perp, \Delta_5 \vdash \Delta_9} \vee_L$$

- Case rule \perp_L

$$\frac{\frac{\frac{}{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_8, F_6} \perp_L \quad \frac{}{\bullet h_7 : (\perp, \Delta_5), F_6 \vdash \Delta_8} \perp_L}{\perp : \perp, \Delta_5 \vdash \Delta_8} \text{Cut}}{\perp : \perp, \Delta_5 \vdash \Delta_8} \perp_L}{\perp : \perp, \Delta_5 \vdash \Delta_8} \perp_L$$

- Case rule I

$$\frac{\frac{\frac{\bullet h_3 : \perp, \Delta_5 \vdash (\Delta_8, p_7), p_7}{\vdash : \perp, \Delta_5 \vdash \Delta_8, p_7} \perp_L \quad \frac{\bullet h_6 : (\perp, \Delta_5), p_7 \vdash \Delta_8, p_7}{\vdash : \perp, \Delta_5 \vdash \Delta_8, p_7} I}{\vdash : \perp, \Delta_5 \vdash \Delta_8, p_7} \text{Cut}}{\vdash : \perp, \Delta_5 \vdash \Delta_8, p_7} \perp_L$$

$$\frac{\frac{\frac{\bullet h_3 : \perp, \Delta_9, p_7 \vdash (\Delta_8, p_7), F_5}{\vdash : \perp, \Delta_9, p_7 \vdash \Delta_8, p_7} \perp_L \quad \frac{\bullet h_6 : (\perp, \Delta_9, p_7), F_5 \vdash \Delta_8, p_7}{\vdash : \perp, \Delta_9, p_7 \vdash \Delta_8, p_7} I}{\vdash : \perp, \Delta_9, p_7 \vdash \Delta_8, p_7} \text{Cut}}{\vdash : \perp, \Delta_9, p_7 \vdash \Delta_8, p_7} \perp_L$$

- Case rule \top_L

$$\frac{\frac{\frac{\bullet h_3 : \perp, \Delta_5 \vdash \Delta_7, \top}{\vdash : \perp, \Delta_5 \vdash \Delta_7} \perp_L \quad \frac{\frac{h_6 : \perp, \Delta_5 \vdash \Delta_7}{\bullet h_6 : (\perp, \Delta_5), \top \vdash \Delta_7} \top_L}{\vdash : \perp, \Delta_5 \vdash \Delta_7} \text{Cut}}{\vdash : \perp, \Delta_5 \vdash \Delta_7} \perp_L$$

$$\frac{\frac{\frac{\bullet h_3 : \perp, \top, \Delta_8 \vdash \Delta_7, F_5}{\vdash : \perp, \top, \Delta_8 \vdash \Delta_7} \perp_L \quad \frac{\frac{h_6 : \perp, \Delta_8, F_5 \vdash \Delta_7}{\bullet h_6 : (\perp, \top, \Delta_8), F_5 \vdash \Delta_7} \top_L}{\vdash : \perp, \top, \Delta_8 \vdash \Delta_7} \text{Cut}}{\vdash : \perp, \top, \Delta_8 \vdash \Delta_7} \perp_L$$

8.10 Status of I : OK

- Case rule \rightarrow_R

$$\frac{\frac{\frac{\bullet h_2 : \Delta_6, p_8 \vdash ((\Delta_{12}, F_{10} \rightarrow F_{11}), p_8), F_7}{\vdash : \Delta_6, p_8 \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), p_8} I \quad \frac{\frac{h_9 : \Delta_6, F_7, F_{10}, p_8 \vdash \Delta_{12}, F_{11}, p_8}{\bullet h_9 : (\Delta_6, p_8), F_7 \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), p_8} \rightarrow_R}{\vdash : \Delta_6, p_8 \vdash (\Delta_{12}, F_{10} \rightarrow F_{11}), p_8} \text{Cut}}{\vdash : \Delta_6, p_8 \vdash \Delta_{12}, p_8, F_{10} \rightarrow F_{11}} I$$

$$\frac{\frac{\frac{\bullet h_1 : \Delta_5, p_6 \vdash (\Delta_8, F_9 \rightarrow F_{10}), p_6}{\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \rightarrow F_{10}} I \quad \frac{\frac{h_7 : \Delta_5, F_9, p_6, p_6 \vdash \Delta_8, F_{10}}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \Delta_8, F_9 \rightarrow F_{10}} \rightarrow_R}{\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{Cut}}{\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \rightarrow F_{10}} \rightarrow_R$$

$$\frac{\frac{\frac{\bullet h_1 : \Delta_5, F_9, p_6 \vdash \Delta_8, F_{10}, p_6}{\vdash : \Delta_5, F_9, p_6 \vdash \Delta_8, F_{10}} I \quad \frac{h_7 : \Delta_5, F_9, p_6, p_6 \vdash \Delta_8, F_{10}}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \Delta_8, F_{10}} \text{ax/w}}{\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{hCut} \rightarrow_R$$

- Case rule \wedge_R

$$\frac{\frac{\frac{\bullet h_2 : \Delta_6, p_8 \vdash ((\Delta_{12}, F_{10} \wedge F_{11}), p_8), F_7}{\vdash : \Delta_6, p_8 \vdash (\Delta_{12}, F_{10} \wedge F_{11}), p_8} I \quad \frac{\frac{h_9 : \Delta_6, F_7, p_8 \vdash \Delta_{12}, F_{10}, p_8 \quad h_9 : \Delta_6, F_7, p_8 \vdash \Delta_{12}, F_{11}, p_8}{\bullet h_9 : (\Delta_6, p_8), F_7 \vdash (\Delta_{12}, F_{10} \wedge F_{11}), p_8} \wedge_R}{\vdash : \Delta_6, p_8 \vdash (\Delta_{12}, F_{10} \wedge F_{11}), p_8} \text{Cut}}{\vdash : \Delta_6, p_8 \vdash \Delta_{12}, p_8, F_{10} \wedge F_{11}} I$$

$$\frac{\frac{\frac{\bullet h_1 : \Delta_5, p_6 \vdash (\Delta_8, F_9 \wedge F_{10}), p_6}{\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \wedge F_{10}} I \quad \frac{\frac{h_7 : \Delta_5, p_6, p_6 \vdash \Delta_8, F_9 \quad h_7 : \Delta_5, p_6, p_6 \vdash \Delta_8, F_{10}}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \Delta_8, F_9 \wedge F_{10}} \wedge_R}{\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \wedge F_{10}} \text{Cut}}{\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \wedge F_{10}} \rightarrow_R$$

$$\frac{\frac{\frac{\bullet h_1 : \Delta_5, p_6 \vdash \Delta_8, F_9, p_6}{\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9} I \quad \frac{h_7 : \Delta_5, p_6, p_6 \vdash \Delta_8, F_9}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \Delta_8, F_9} \text{ax/w}}{\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9} \text{hCut} \quad \frac{\frac{\frac{\bullet h_1 : \Delta_5, p_6 \vdash \Delta_8, F_{10}, p_6}{\vdash : \Delta_5, p_6 \vdash \Delta_8, F_{10}} I \quad \frac{h_7 : \Delta_5, p_6, p_6 \vdash \Delta_8, F_{10}}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \Delta_8, F_{10}} \text{ax/w}}{\vdash : \Delta_5, p_6 \vdash \Delta_8, F_{10}} \text{hCut}}{\vdash : \Delta_5, p_6 \vdash \Delta_8, F_9 \wedge F_{10}} \wedge_R$$

- Case rule \vee_R

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_2 : \Delta_6, p_8 \vdash ((\Delta_{12}, F_{10} \vee F_{11}), p_8), F_7} I \quad \frac{h_9 : \Delta_6, F_7, p_8 \vdash \Delta_{12}, F_{10}, F_{11}, p_8}{\bullet h_9 : (\Delta_6, p_8), F_7 \vdash (\Delta_{12}, F_{10} \vee F_{11}), p_8} \vee_R}{- : \Delta_6, p_8 \vdash (\Delta_{12}, F_{10} \vee F_{11}), p_8} \text{Cut}}{\rightarrow} \\
\frac{}{- : \Delta_6, p_8 \vdash \Delta_{12}, p_8, F_{10} \vee F_{11}} I \\
\frac{\frac{\frac{}{\bullet h_1 : \Delta_5, p_6 \vdash (\Delta_8, F_9 \vee F_{10}), p_6} I \quad \frac{h_7 : \Delta_5, p_6, p_6 \vdash \Delta_8, F_9, F_{10}}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \Delta_8, F_9 \vee F_{10}} \vee_R}{- : \Delta_5, p_6 \vdash \Delta_8, F_9 \vee F_{10}} \text{Cut}}{\rightarrow} \\
\frac{\frac{\frac{}{\bullet h_1 : \Delta_5, p_6 \vdash \Delta_8, F_{10}, F_9, p_6} I \quad \frac{h_7 : \Delta_5, p_6, p_6 \vdash \Delta_8, F_{10}, F_9}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \Delta_8, F_{10}, F_9} \text{ax/W}}{- : \Delta_5, p_6 \vdash \Delta_8, F_{10}, F_9} \text{hCut}}{- : \Delta_5, p_6 \vdash \Delta_8, F_9 \vee F_{10}} \vee_R
\end{array}$$

- Case rule \perp_R

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_2 : \Delta_6, p_8 \vdash ((\perp, \Delta_{10}), p_8), F_7} I \quad \frac{h_9 : \Delta_6, F_7, p_8 \vdash \Delta_{10}, p_8}{\bullet h_9 : (\Delta_6, p_8), F_7 \vdash (\perp, \Delta_{10}), p_8} \perp_R}{- : \Delta_6, p_8 \vdash (\perp, \Delta_{10}), p_8} \text{Cut}}{\rightarrow} \\
\frac{}{- : \Delta_6, p_8 \vdash \perp, \Delta_{10}, p_8} I \\
\frac{\frac{\frac{}{\bullet h_1 : \Delta_5, p_6 \vdash (\perp, \Delta_8), p_6} I \quad \frac{h_7 : \Delta_5, p_6, p_6 \vdash \Delta_8}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \perp, \Delta_8} \perp_R}{- : \Delta_5, p_6 \vdash \perp, \Delta_8} \text{Cut}}{\rightarrow} \\
\frac{\frac{\frac{}{\bullet h_1 : \Delta_5, p_6 \vdash \perp, \Delta_8, p_6} I \quad \frac{h_7 : \Delta_5, p_6, p_6 \vdash \perp, \Delta_8}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \perp, \Delta_8} \text{ax/W}}{- : \Delta_5, p_6 \vdash \perp, \Delta_8} \text{hCut}}{- : \Delta_5, p_6 \vdash \perp, \Delta_8}
\end{array}$$

- Case rule \top_R

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_2 : \Delta_6, p_8 \vdash ((\top, \Delta_{10}), p_8), F_7} I \quad \frac{h_9 : (\Delta_6, p_8), F_7 \vdash (\top, \Delta_{10}), p_8}{\bullet h_9 : (\Delta_6, p_8), F_7 \vdash (\top, \Delta_{10}), p_8} \top_R}{- : \Delta_6, p_8 \vdash (\top, \Delta_{10}), p_8} \text{Cut}}{\rightarrow} \\
\frac{}{- : \Delta_6, p_8 \vdash \top, \Delta_{10}, p_8} \top_R \\
\frac{\frac{\frac{}{\bullet h_1 : \Delta_5, p_6 \vdash (\top, \Delta_8), p_6} I \quad \frac{h_7 : (\Delta_5, p_6), p_6 \vdash \top, \Delta_8}{\bullet h_7 : (\Delta_5, p_6), p_6 \vdash \top, \Delta_8} \top_R}{- : \Delta_5, p_6 \vdash \top, \Delta_8} \text{Cut}}{\rightarrow} \\
\frac{}{- : \Delta_5, p_6 \vdash \top, \Delta_8} \top_R
\end{array}$$

- Case rule \rightarrow_L

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_2 : (\Delta_{12}, F_{10} \rightarrow F_{11}), p_7 \vdash (\Delta_8, p_7), F_6} I \quad \frac{h_9 : \Delta_{12}, F_6, p_7 \vdash \Delta_8, F_{10}, p_7 \quad h_9 : \Delta_{12}, F_6, F_{11}, p_7 \vdash \Delta_8, p_7}{\bullet h_9 : ((\Delta_{12}, F_{10} \rightarrow F_{11}), p_7), F_6 \vdash \Delta_8, p_7} \rightarrow_L}{- : (\Delta_{12}, F_{10} \rightarrow F_{11}), p_7 \vdash \Delta_8, p_7} \text{Cut}}{\rightarrow} \\
\frac{}{- : \Delta_{12}, p_7, F_{10} \rightarrow F_{11} \vdash \Delta_8, p_7} I \\
\frac{\frac{\frac{}{\bullet h_2 : \Delta_6, p_7 \vdash (\Delta_8, p_7), F_{10} \rightarrow F_{11}} I \quad \frac{h_9 : \Delta_6, p_7 \vdash \Delta_8, F_{10}, p_7 \quad h_9 : \Delta_6, F_{11}, p_7 \vdash \Delta_8, p_7}{\bullet h_9 : (\Delta_6, p_7), F_{10} \rightarrow F_{11} \vdash \Delta_8, p_7} \rightarrow_L}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} \text{Cut}}{\rightarrow} \\
\frac{}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} I
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_1 : (\Delta_{10}, F_7 \rightarrow F_8), P_5 \vdash \Delta_9, P_5}{- : (\Delta_{10}, F_7 \rightarrow F_8), P_5 \vdash \Delta_9} I \quad \frac{h_6 : \Delta_{10}, P_5, P_5 \vdash \Delta_9, F_7 \quad h_6 : \Delta_{10}, F_8, P_5, P_5 \vdash \Delta_9}{\bullet h_6 : ((\Delta_{10}, F_7 \rightarrow F_8), P_5), P_5 \vdash \Delta_9} \rightarrow_L \\
\hline
- : (\Delta_{10}, F_7 \rightarrow F_8), P_5 \vdash \Delta_9 \quad \text{Cut} \\
\hline
\frac{\bullet h_1 : \Delta_{10}, P_5 \vdash \Delta_9, F_7, P_5 \quad h_6 : \Delta_{10}, P_5, P_5 \vdash \Delta_9, F_7}{- : \Delta_{10}, P_5 \vdash \Delta_9, F_7} I \quad \frac{\text{ax/W}}{h\text{Cut}} \quad \frac{\bullet h_1 : \Delta_{10}, F_8, P_5 \vdash \Delta_9, P_5 \quad h_6 : \Delta_{10}, F_8, P_5, P_5 \vdash \Delta_9}{- : \Delta_{10}, F_8, P_5 \vdash \Delta_9} I \quad \frac{\text{ax/W}}{h\text{Cut}} \\
\hline
- : \Delta_{10}, P_5, F_7 \rightarrow F_8 \vdash \Delta_9 \quad \rightarrow_L
\end{array}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{\bullet h_2 : (\Delta_{12}, F_{10} \wedge F_{11}), P_7 \vdash (\Delta_8, P_7), F_6}{- : (\Delta_{12}, F_{10} \wedge F_{11}), P_7 \vdash \Delta_8, P_7} I \quad \frac{h_9 : \Delta_{12}, F_6, F_{10}, F_{11}, P_7 \vdash \Delta_8, P_7}{\bullet h_9 : ((\Delta_{12}, F_{10} \wedge F_{11}), P_7), F_6 \vdash \Delta_8, P_7} \wedge_L \\
\hline
- : (\Delta_{12}, F_{10} \wedge F_{11}), P_7 \vdash \Delta_8, P_7 \quad \text{Cut} \\
\hline
\frac{- : \Delta_{12}, P_7, F_{10} \wedge F_{11} \vdash \Delta_8, P_7}{- : \Delta_{12}, P_7, F_{10} \wedge F_{11} \vdash \Delta_8, P_7} I \\
\hline
\frac{\bullet h_2 : \Delta_6, P_7 \vdash (\Delta_8, P_7), F_{10} \wedge F_{11}}{- : \Delta_6, P_7 \vdash \Delta_8, P_7} I \quad \frac{h_9 : \Delta_6, F_{10}, F_{11}, P_7 \vdash \Delta_8, P_7}{\bullet h_9 : (\Delta_6, P_7), F_{10} \wedge F_{11} \vdash \Delta_8, P_7} \wedge_L \\
\hline
- : \Delta_6, P_7 \vdash \Delta_8, P_7 \quad \text{Cut} \\
\hline
\frac{- : \Delta_6, P_7 \vdash \Delta_8, P_7}{- : \Delta_6, P_7 \vdash \Delta_8, P_7} I \\
\hline
\frac{\bullet h_1 : (\Delta_{10}, F_7 \wedge F_8), P_5 \vdash \Delta_9, P_5}{- : (\Delta_{10}, F_7 \wedge F_8), P_5 \vdash \Delta_9} I \quad \frac{h_6 : \Delta_{10}, F_7, F_8, P_5, P_5 \vdash \Delta_9}{\bullet h_6 : ((\Delta_{10}, F_7 \wedge F_8), P_5), P_5 \vdash \Delta_9} \wedge_L \\
\hline
- : (\Delta_{10}, F_7 \wedge F_8), P_5 \vdash \Delta_9 \quad \text{Cut} \\
\hline
\frac{\bullet h_1 : \Delta_{10}, F_7, F_8, P_5 \vdash \Delta_9, P_5 \quad h_6 : \Delta_{10}, F_7, F_8, P_5, P_5 \vdash \Delta_9}{- : \Delta_{10}, F_7, F_8, P_5 \vdash \Delta_9} I \quad \frac{\text{ax/W}}{h\text{Cut}} \\
\hline
- : \Delta_{10}, P_5, F_7 \wedge F_8 \vdash \Delta_9 \quad \wedge_L
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{\bullet h_2 : (\Delta_{12}, F_{10} \vee F_{11}), P_7 \vdash (\Delta_8, P_7), F_6}{- : (\Delta_{12}, F_{10} \vee F_{11}), P_7 \vdash \Delta_8, P_7} I \quad \frac{h_9 : \Delta_{12}, F_6, F_{10}, P_7 \vdash \Delta_8, P_7 \quad h_9 : \Delta_{12}, F_6, F_{11}, P_7 \vdash \Delta_8, P_7}{\bullet h_9 : ((\Delta_{12}, F_{10} \vee F_{11}), P_7), F_6 \vdash \Delta_8, P_7} \vee_L \\
\hline
- : (\Delta_{12}, F_{10} \vee F_{11}), P_7 \vdash \Delta_8, P_7 \quad \text{Cut} \\
\hline
\frac{- : \Delta_{12}, P_7, F_{10} \vee F_{11} \vdash \Delta_8, P_7}{- : \Delta_{12}, P_7, F_{10} \vee F_{11} \vdash \Delta_8, P_7} I \\
\hline
\frac{\bullet h_2 : \Delta_6, P_7 \vdash (\Delta_8, P_7), F_{10} \vee F_{11}}{- : \Delta_6, P_7 \vdash \Delta_8, P_7} I \quad \frac{h_9 : \Delta_6, F_{10}, P_7 \vdash \Delta_8, P_7 \quad h_9 : \Delta_6, F_{11}, P_7 \vdash \Delta_8, P_7}{\bullet h_9 : (\Delta_6, P_7), F_{10} \vee F_{11} \vdash \Delta_8, P_7} \vee_L \\
\hline
- : \Delta_6, P_7 \vdash \Delta_8, P_7 \quad \text{Cut} \\
\hline
\frac{- : \Delta_6, P_7 \vdash \Delta_8, P_7}{- : \Delta_6, P_7 \vdash \Delta_8, P_7} I \\
\hline
\frac{\bullet h_1 : (\Delta_{10}, F_7 \vee F_8), P_5 \vdash \Delta_9, P_5}{- : (\Delta_{10}, F_7 \vee F_8), P_5 \vdash \Delta_9} I \quad \frac{h_6 : \Delta_{10}, F_7, P_5, P_5 \vdash \Delta_9 \quad h_6 : \Delta_{10}, F_8, P_5, P_5 \vdash \Delta_9}{\bullet h_6 : ((\Delta_{10}, F_7 \vee F_8), P_5), P_5 \vdash \Delta_9} \vee_L \\
\hline
- : (\Delta_{10}, F_7 \vee F_8), P_5 \vdash \Delta_9 \quad \text{Cut} \\
\hline
\frac{\bullet h_1 : \Delta_{10}, F_7, P_5 \vdash \Delta_9, P_5 \quad h_6 : \Delta_{10}, F_7, P_5, P_5 \vdash \Delta_9}{- : \Delta_{10}, F_7, P_5 \vdash \Delta_9} I \quad \frac{\text{ax/W}}{h\text{Cut}} \quad \frac{\bullet h_1 : \Delta_{10}, F_8, P_5 \vdash \Delta_9, P_5 \quad h_6 : \Delta_{10}, F_8, P_5, P_5 \vdash \Delta_9}{- : \Delta_{10}, F_8, P_5 \vdash \Delta_9} I \quad \frac{\text{ax/W}}{h\text{Cut}} \\
\hline
- : \Delta_{10}, P_5, F_7 \vee F_8 \vdash \Delta_9 \quad \vee_L
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{\bullet h_2 : \Delta_6, P_7 \vdash (\Delta_8, P_7), \perp}{- : \Delta_6, P_7 \vdash \Delta_8, P_7} I \quad \frac{\bullet h_9 : (\Delta_6, P_7), \perp \vdash \Delta_8, P_7}{- : \Delta_6, P_7 \vdash \Delta_8, P_7} \perp_L \\
\hline
- : \Delta_6, P_7 \vdash \Delta_8, P_7 \quad \text{Cut} \\
\hline
\frac{- : \Delta_6, P_7 \vdash \Delta_8, P_7}{- : \Delta_6, P_7 \vdash \Delta_8, P_7} I
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_2 : (\perp, \Delta_{10}), p_7 \vdash (\Delta_8, p_7), F_6 \quad I \quad \bullet h_9 : ((\perp, \Delta_{10}), p_7), F_6 \vdash \Delta_8, p_7}{- : (\perp, \Delta_{10}), p_7 \vdash \Delta_8, p_7} \text{Cut} \quad \perp_L \\
\rightarrow \\
\frac{}{- : \perp, \Delta_{10}, p_7 \vdash \Delta_8, p_7} \perp_L \\
\bullet h_1 : (\perp, \Delta_8), p_5 \vdash \Delta_7, p_5 \quad I \quad \bullet h_6 : ((\perp, \Delta_8), p_5), p_5 \vdash \Delta_7 \quad I \\
- : (\perp, \Delta_8), p_5 \vdash \Delta_7 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \perp, \Delta_8, p_5 \vdash \Delta_7} \perp_L
\end{array}$$

• Case rule I

$$\begin{array}{c}
\frac{\bullet h_2 : \Delta_6, p_7 \vdash ((\Delta_{10}, p_9), p_7), p_9 \quad I \quad \bullet h_8 : (\Delta_6, p_7), p_9 \vdash (\Delta_{10}, p_9), p_7 \quad I}{- : \Delta_6, p_7 \vdash (\Delta_{10}, p_9), p_7} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_6, p_7 \vdash \Delta_{10}, p_7, p_9} I \\
\bullet h_2 : (\Delta_{11}, p_9), p_7 \vdash ((\Delta_{10}, p_9), p_7), F_6 \quad I \quad \bullet h_8 : ((\Delta_{11}, p_9), p_7), F_6 \vdash (\Delta_{10}, p_9), p_7 \quad I \\
- : (\Delta_{11}, p_9), p_7 \vdash (\Delta_{10}, p_9), p_7 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_{11}, p_7, p_9 \vdash \Delta_{10}, p_7, p_9} I \\
\bullet h_2 : \Delta_6, p_9 \vdash (\Delta_{10}, p_9), F_7 \quad I \quad \bullet h_8 : (\Delta_6, p_9), F_7 \vdash \Delta_{10}, p_9 \quad I \\
- : \Delta_6, p_9 \vdash \Delta_{10}, p_9 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_6, p_9 \vdash \Delta_{10}, p_9} I \\
\bullet h_1 : \Delta_5, p_7 \vdash (\Delta_8, p_7), p_7 \quad I \quad \bullet h_6 : (\Delta_5, p_7), p_7 \vdash \Delta_8, p_7 \quad I \\
- : \Delta_5, p_7 \vdash \Delta_8, p_7 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_5, p_7 \vdash \Delta_8, p_7} I \\
\bullet h_1 : (\Delta_9, p_7), p_5 \vdash (\Delta_8, p_7), p_5 \quad I \quad \bullet h_6 : ((\Delta_9, p_7), p_5), p_5 \vdash \Delta_8, p_7 \quad I \\
- : (\Delta_9, p_7), p_5 \vdash \Delta_8, p_7 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_9, p_5, p_7 \vdash \Delta_8, p_7} I
\end{array}$$

• Case rule \top_L

$$\begin{array}{c}
\frac{\bullet h_2 : \Delta_6, p_7 \vdash (\Delta_8, p_7), \top \quad I \quad h_9 : \Delta_6, p_7 \vdash \Delta_8, p_7}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} \top_L \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_6, p_7 \vdash \Delta_8, p_7} I \\
\bullet h_2 : (\top, \Delta_{10}), p_7 \vdash (\Delta_8, p_7), F_6 \quad I \quad h_9 : \Delta_{10}, F_6, p_7 \vdash \Delta_8, p_7 \quad \top_L \\
- : (\top, \Delta_{10}), p_7 \vdash \Delta_8, p_7 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \top, \Delta_{10}, p_7 \vdash \Delta_8, p_7} I \\
\bullet h_1 : (\top, \Delta_8), p_5 \vdash \Delta_7, p_5 \quad I \quad h_6 : \Delta_8, p_5, p_5 \vdash \Delta_7 \quad \top_L \\
- : (\top, \Delta_8), p_5 \vdash \Delta_7 \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 : \top, \Delta_8, p_5 \vdash \Delta_7, p_5 \quad I \quad h_6 : \top, \Delta_8, p_5, p_5 \vdash \Delta_7 \quad \text{ax/W}}{- : \top, \Delta_8, p_5 \vdash \Delta_7} \text{hCut}
\end{array}$$

8.11 Status of \top_L : OK

- Case rule \rightarrow_R

$$\frac{\frac{\frac{h_3 : \Delta_5 \vdash (\Delta_8, F_9 \rightarrow F_{10}), F_6}{\bullet h_3 : \top, \Delta_5 \vdash (\Delta_8, F_9 \rightarrow F_{10}), F_6} \top_L \quad \frac{h_7 : \top, \Delta_5, F_6, F_9 \vdash \Delta_8, F_{10}}{\bullet h_7 : (\top, \Delta_5), F_6 \vdash \Delta_8, F_9 \rightarrow F_{10}} \rightarrow_R}{- : \top, \Delta_5 \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{Cut}}{\frac{\frac{h_3 : \top, \Delta_5 \vdash \Delta_8, F_6, F_9 \rightarrow F_{10}}{- : \top, \Delta_5 \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{ax/W} \quad \frac{h_7 : \top, \Delta_5, F_6 \vdash \Delta_8, F_9 \rightarrow F_{10}}{- : \top, \Delta_5 \vdash \Delta_8, F_9 \rightarrow F_{10}} \text{hCut}}{\rightarrow} \text{ax/W}}{\rightarrow} \text{hCut}}$$

- Case rule \wedge_R

$$\frac{\frac{\frac{h_3 : \Delta_5 \vdash (\Delta_8, F_9 \wedge F_{10}), F_6}{\bullet h_3 : \top, \Delta_5 \vdash (\Delta_8, F_9 \wedge F_{10}), F_6} \top_L \quad \frac{h_7 : \top, \Delta_5, F_6 \vdash \Delta_8, F_9 \quad h_7 : \top, \Delta_5, F_6 \vdash \Delta_8, F_{10}}{\bullet h_7 : (\top, \Delta_5), F_6 \vdash \Delta_8, F_9 \wedge F_{10}} \wedge_R}{- : \top, \Delta_5 \vdash \Delta_8, F_9 \wedge F_{10}} \text{Cut}}{\frac{\frac{h_3 : \top, \Delta_5 \vdash \Delta_8, F_6, F_9 \wedge F_{10}}{- : \top, \Delta_5 \vdash \Delta_8, F_9 \wedge F_{10}} \text{ax/W} \quad \frac{h_7 : \top, \Delta_5, F_6 \vdash \Delta_8, F_9 \wedge F_{10}}{- : \top, \Delta_5 \vdash \Delta_8, F_9 \wedge F_{10}} \text{hCut}}{\rightarrow} \text{ax/W}}{\rightarrow} \text{hCut}}$$

- Case rule \vee_R

$$\frac{\frac{\frac{h_3 : \Delta_5 \vdash (\Delta_8, F_9 \vee F_{10}), F_6}{\bullet h_3 : \top, \Delta_5 \vdash (\Delta_8, F_9 \vee F_{10}), F_6} \top_L \quad \frac{h_7 : \top, \Delta_5, F_6 \vdash \Delta_8, F_9, F_{10}}{\bullet h_7 : (\top, \Delta_5), F_6 \vdash \Delta_8, F_9 \vee F_{10}} \vee_R}{- : \top, \Delta_5 \vdash \Delta_8, F_9 \vee F_{10}} \text{Cut}}{\frac{\frac{h_3 : \top, \Delta_5 \vdash \Delta_8, F_6, F_9 \vee F_{10}}{- : \top, \Delta_5 \vdash \Delta_8, F_9 \vee F_{10}} \text{ax/W} \quad \frac{h_7 : \top, \Delta_5, F_6 \vdash \Delta_8, F_9 \vee F_{10}}{- : \top, \Delta_5 \vdash \Delta_8, F_9 \vee F_{10}} \text{hCut}}{\rightarrow} \text{ax/W}}{\rightarrow} \text{hCut}}$$

- Case rule \perp_R

$$\frac{\frac{\frac{h_3 : \Delta_5 \vdash (\perp, \Delta_8), F_6}{\bullet h_3 : \top, \Delta_5 \vdash (\perp, \Delta_8), F_6} \top_L \quad \frac{h_7 : \top, \Delta_5, F_6 \vdash \Delta_8}{\bullet h_7 : (\top, \Delta_5), F_6 \vdash \perp, \Delta_8} \perp_R}{- : \top, \Delta_5 \vdash \perp, \Delta_8} \text{Cut}}{\frac{\frac{h_3 : \top, \Delta_5 \vdash \perp, \Delta_8, F_6}{- : \top, \Delta_5 \vdash \perp, \Delta_8} \text{ax/W} \quad \frac{h_7 : \top, \Delta_5, F_6 \vdash \perp, \Delta_8}{- : \top, \Delta_5 \vdash \perp, \Delta_8} \text{hCut}}{\rightarrow} \text{ax/W}}{\rightarrow} \text{hCut}}$$

- Case rule \top_R

$$\frac{\frac{\frac{h_3 : \Delta_5 \vdash (\top, \Delta_8), F_6}{\bullet h_3 : \top, \Delta_5 \vdash (\top, \Delta_8), F_6} \top_L \quad \frac{h_7 : \top, \Delta_5, F_6 \vdash \top, \Delta_8}{\bullet h_7 : (\top, \Delta_5), F_6 \vdash \top, \Delta_8} \top_R}{- : \top, \Delta_5 \vdash \top, \Delta_8} \text{Cut}}{\frac{\frac{h_3 : \top, \Delta_5 \vdash \top, \Delta_8}{- : \top, \Delta_5 \vdash \top, \Delta_8} \top_R}}{\rightarrow} \text{Cut}}$$

- Case rule \rightarrow_L

$$\frac{\frac{\frac{\frac{h_3 : \Delta_{10}, F_7 \rightarrow F_8 \vdash \Delta_9, F_5}{\bullet h_3 : \top, \Delta_{10}, F_7 \rightarrow F_8 \vdash \Delta_9, F_5} \top_L \quad \frac{h_6 : \top, \Delta_{10}, F_5 \vdash \Delta_9, F_7 \quad h_6 : \top, \Delta_{10}, F_5, F_8 \vdash \Delta_9}{\bullet h_6 : (\top, \Delta_{10}, F_7 \rightarrow F_8), F_5 \vdash \Delta_9} \rightarrow_L}{- : \top, \Delta_{10}, F_7 \rightarrow F_8 \vdash \Delta_9} \text{Cut}}{\frac{\frac{h_3 : \top, \Delta_{10}, F_7 \rightarrow F_8 \vdash \Delta_9, F_5}{- : \top, \Delta_{10}, F_7 \rightarrow F_8 \vdash \Delta_9} \text{ax/W} \quad \frac{h_6 : \top, \Delta_{10}, F_5, F_7 \rightarrow F_8 \vdash \Delta_9}{- : \top, \Delta_{10}, F_7 \rightarrow F_8 \vdash \Delta_9} \text{hCut}}{\rightarrow} \text{ax/W}}{\rightarrow} \text{hCut}}$$

$$\frac{\frac{\frac{h_3 : \Delta_5 \vdash \Delta_9, F_7 \rightarrow F_8}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_9, F_7 \rightarrow F_8} \top_L \quad \frac{h_6 : \top, \Delta_5 \vdash \Delta_9, F_7 \quad h_6 : \top, \Delta_5, F_8 \vdash \Delta_9}{\bullet h_6 : (\top, \Delta_5), F_7 \rightarrow F_8 \vdash \Delta_9} \rightarrow_L}{- : \top, \Delta_5 \vdash \Delta_9} \text{Cut}}{\frac{\frac{h_3 : \top, \Delta_5 \vdash \Delta_9, F_7 \rightarrow F_8}{- : \top, \Delta_5 \vdash \Delta_9} \text{ax/W} \quad \frac{h_6 : \top, \Delta_5, F_7 \rightarrow F_8 \vdash \Delta_9}{- : \top, \Delta_5 \vdash \Delta_9} \text{hCut}}{\rightarrow} \text{ax/W}}{\rightarrow} \text{hCut}}$$

- Case rule \wedge_L

$$\begin{array}{c}
\frac{h_3 : \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9, F_5}{\bullet h_3 : \top, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9, F_5} \top_L \quad \frac{h_6 : \top, \Delta_{10}, F_5, F_7, F_8 \vdash \Delta_9}{\bullet h_6 : (\top, \Delta_{10}, F_7 \wedge F_8), F_5 \vdash \Delta_9} \wedge_L}{\vdash : \top, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9} \text{Cut} \\
\rightarrow \\
\frac{h_3 : \top, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9, F_5}{\vdash : \top, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9} \text{ax/W} \quad \frac{\bullet h_6 : \top, \Delta_{10}, F_5, F_7 \wedge F_8 \vdash \Delta_9}{\vdash : \top, \Delta_{10}, F_7 \wedge F_8 \vdash \Delta_9} \text{hCut} \\
\\
\frac{h_3 : \Delta_5 \vdash \Delta_9, F_7 \wedge F_8}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_9, F_7 \wedge F_8} \top_L \quad \frac{h_6 : \top, \Delta_5, F_7, F_8 \vdash \Delta_9}{\bullet h_6 : (\top, \Delta_5), F_7 \wedge F_8 \vdash \Delta_9} \wedge_L}{\vdash : \top, \Delta_5 \vdash \Delta_9} \text{Cut} \\
\rightarrow \\
\frac{h_3 : \top, \Delta_5 \vdash \Delta_9, F_7 \wedge F_8}{\vdash : \top, \Delta_5 \vdash \Delta_9} \text{ax/W} \quad \frac{\bullet h_6 : \top, \Delta_5, F_7 \wedge F_8 \vdash \Delta_9}{\vdash : \top, \Delta_5 \vdash \Delta_9} \text{hCut}
\end{array}$$

- Case rule \vee_L

$$\begin{array}{c}
\frac{h_3 : \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9, F_5}{\bullet h_3 : \top, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9, F_5} \top_L \quad \frac{h_6 : \top, \Delta_{10}, F_5, F_7 \vdash \Delta_9 \quad h_6 : \top, \Delta_{10}, F_5, F_8 \vdash \Delta_9}{\bullet h_6 : (\top, \Delta_{10}, F_7 \vee F_8), F_5 \vdash \Delta_9} \vee_L}{\vdash : \top, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9} \text{Cut} \\
\rightarrow \\
\frac{h_3 : \top, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9, F_5}{\vdash : \top, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9} \text{ax/W} \quad \frac{\bullet h_6 : \top, \Delta_{10}, F_5, F_7 \vee F_8 \vdash \Delta_9}{\vdash : \top, \Delta_{10}, F_7 \vee F_8 \vdash \Delta_9} \text{hCut} \\
\\
\frac{h_3 : \Delta_5 \vdash \Delta_9, F_7 \vee F_8}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_9, F_7 \vee F_8} \top_L \quad \frac{h_6 : \top, \Delta_5, F_7 \vdash \Delta_9 \quad h_6 : \top, \Delta_5, F_8 \vdash \Delta_9}{\bullet h_6 : (\top, \Delta_5), F_7 \vee F_8 \vdash \Delta_9} \vee_L}{\vdash : \top, \Delta_5 \vdash \Delta_9} \text{Cut} \\
\rightarrow \\
\frac{h_3 : \top, \Delta_5 \vdash \Delta_9, F_7 \vee F_8}{\vdash : \top, \Delta_5 \vdash \Delta_9} \text{ax/W} \quad \frac{\bullet h_6 : \top, \Delta_5, F_7 \vee F_8 \vdash \Delta_9}{\vdash : \top, \Delta_5 \vdash \Delta_9} \text{hCut}
\end{array}$$

- Case rule \perp_L

$$\begin{array}{c}
\frac{h_3 : \Delta_5 \vdash \Delta_7, \perp}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_7, \perp} \top_L \quad \frac{}{\bullet h_6 : (\top, \Delta_5), \perp \vdash \Delta_7} \perp_L}{\vdash : \top, \Delta_5 \vdash \Delta_7} \text{Cut} \\
\rightarrow \\
\frac{h_3 : \top, \Delta_5 \vdash \perp, \Delta_7}{\vdash : \top, \Delta_5 \vdash \Delta_7} \text{ax/W} \quad \frac{\bullet h_6 : \perp, \top, \Delta_5 \vdash \Delta_7}{\vdash : \top, \Delta_5 \vdash \Delta_7} \perp_L \text{hCut} \\
\\
\frac{h_3 : \perp, \Delta_8 \vdash \Delta_7, F_5}{\bullet h_3 : \top, \perp, \Delta_8 \vdash \Delta_7, F_5} \top_L \quad \frac{}{\bullet h_6 : (\top, \perp, \Delta_8), F_5 \vdash \Delta_7} \perp_L}{\vdash : \top, \perp, \Delta_8 \vdash \Delta_7} \text{Cut} \\
\rightarrow \\
\frac{}{\vdash : \perp, \top, \Delta_8 \vdash \Delta_7} \perp_L
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_3 : \Delta_5 \vdash (\Delta_8, p_7), p_7}{\bullet h_3 : \top, \Delta_5 \vdash (\Delta_8, p_7), p_7} \top_L \quad \frac{}{\bullet h_6 : (\top, \Delta_5), p_7 \vdash \Delta_8, p_7} I}{\vdash : \top, \Delta_5 \vdash \Delta_8, p_7} \text{Cut} \\
\rightarrow \\
\frac{h_3 : \top, \Delta_5 \vdash \Delta_8, p_7, p_7}{\vdash : \top, \Delta_5 \vdash \Delta_8, p_7} \text{ax/W} \quad \frac{\bullet h_6 : \top, \Delta_5, p_7 \vdash \Delta_8, p_7}{\vdash : \top, \Delta_5 \vdash \Delta_8, p_7} I \text{hCut} \\
\\
\frac{h_3 : \Delta_9, p_7 \vdash (\Delta_8, p_7), F_5}{\bullet h_3 : \top, \Delta_9, p_7 \vdash (\Delta_8, p_7), F_5} \top_L \quad \frac{}{\bullet h_6 : (\top, \Delta_9, p_7), F_5 \vdash \Delta_8, p_7} I}{\vdash : \top, \Delta_9, p_7 \vdash \Delta_8, p_7} \text{Cut} \\
\rightarrow \\
\frac{}{\vdash : \top, \Delta_9, p_7 \vdash \Delta_8, p_7} I
\end{array}$$

- Case rule \top_L

$$\frac{\frac{\frac{h_3 : \Delta_5 \vdash \Delta_8, F_6}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_8, F_6} \top_L \quad \frac{h_7 : \Delta_5, F_6 \vdash \Delta_8}{\bullet h_7 : (\top, \Delta_5), F_6 \vdash \Delta_8} \top_L}{- : \top, \Delta_5 \vdash \Delta_8} \text{Cut}}{\rightarrow} \quad \frac{\frac{h_3 : \top, \Delta_5 \vdash \Delta_8, F_6}{\bullet h_3 : \top, \Delta_5 \vdash \Delta_8, F_6} \text{ax/W} \quad \frac{\bullet h_7 : \top, \Delta_5, F_6 \vdash \Delta_8}{- : \top, \Delta_5 \vdash \Delta_8} \text{hCut}}{- : \top, \Delta_5 \vdash \Delta_8} \text{ax/W}$$

9 Admissibility of $A \rightarrow A \wedge A$

- Case(s) rule \rightarrow_R

$$\frac{h_2 : \Delta_3, F_4 \vdash \Delta_6, F_5, F_1 \wedge F_1}{\bullet h_2 : \Delta_3 \vdash (\Delta_6, F_4 \rightarrow F_5), F_1 \wedge F_1} \rightarrow_R \rightarrow \frac{\frac{h_2 : \Delta_3, F_4 \vdash \Delta_6, F_5, F_1 \wedge F_1}{\bullet h_2 : \Delta_3 \vdash \Delta_6, F_1, F_4 \rightarrow F_5} \text{ax}}{h_2 : \Delta_3, F_4 \vdash \Delta_6, F_1, F_5} \text{IH}}{\bullet h_2 : \Delta_3 \vdash \Delta_6, F_1, F_4 \rightarrow F_5} \rightarrow_R$$

- Case(s) rule \wedge_R

$$\frac{h_2 : \Delta_3 \vdash \Delta_6, F_4, F_1 \wedge F_1 \quad h_2 : \Delta_3 \vdash \Delta_6, F_5, F_1 \wedge F_1}{\bullet h_2 : \Delta_3 \vdash (\Delta_6, F_4 \wedge F_5), F_1 \wedge F_1} \wedge_R \rightarrow \frac{\frac{h_2 : \Delta_3 \vdash \Delta_6, F_4, F_1 \wedge F_1}{h_2 : \Delta_3 \vdash \Delta_6, F_1, F_4} \text{ax}}{\bullet h_2 : \Delta_3 \vdash \Delta_6, F_1, F_4 \wedge F_5} \text{IH}}{\frac{h_2 : \Delta_3 \vdash \Delta_6, F_5, F_1 \wedge F_1}{h_2 : \Delta_3 \vdash \Delta_6, F_1, F_5} \text{ax}}{\bullet h_2 : \Delta_3 \vdash \Delta_6, F_1, F_4 \wedge F_5} \text{IH}} \wedge_R$$

$$\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4 \quad h_1 : \Delta_2 \vdash \Delta_3, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \wedge F_4} \wedge_R \rightarrow \frac{h_1 : \Delta_2 \vdash \Delta_3, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4} \text{ax} \text{ height}$$

- Case(s) rule \vee_R

$$\frac{h_2 : \Delta_3 \vdash \Delta_6, F_4, F_5, F_1 \wedge F_1}{\bullet h_2 : \Delta_3 \vdash (\Delta_6, F_4 \vee F_5), F_1 \wedge F_1} \vee_R \rightarrow \frac{\frac{h_2 : \Delta_3 \vdash \Delta_6, F_4, F_5, F_1 \wedge F_1}{h_2 : \Delta_3 \vdash \Delta_6, F_1, F_4, F_5} \text{ax}}{\bullet h_2 : \Delta_3 \vdash \Delta_6, F_1, F_4 \vee F_5} \text{IH}}{\bullet h_2 : \Delta_3 \vdash \Delta_6, F_1, F_4 \vee F_5} \vee_R$$

- Case(s) rule \perp_R

$$\frac{h_2 : \Delta_3 \vdash \Delta_4, F_1 \wedge F_1}{\bullet h_2 : \Delta_3 \vdash (\perp, \Delta_4), F_1 \wedge F_1} \perp_R \rightarrow \frac{\frac{h_2 : \Delta_3 \vdash \Delta_4, F_1 \wedge F_1}{h_2 : \Delta_3 \vdash \Delta_4, F_1} \text{ax}}{\bullet h_2 : \Delta_3 \vdash \perp, \Delta_4, F_1} \text{IH}}{\bullet h_2 : \Delta_3 \vdash \perp, \Delta_4, F_1} \perp_R$$

- Case(s) rule \top_R

$$\frac{}{\bullet h_2 : \Delta_3 \vdash (\top, \Delta_4), F_1 \wedge F_1} \top_R \rightarrow \frac{}{\bullet h_2 : \Delta_3 \vdash \top, \Delta_4, F_1} \top_R$$

- Case(s) rule \rightarrow_L

$$\frac{h_3 : \Delta_4 \vdash \Delta_1, F_5, F_2 \wedge F_2 \quad h_3 : \Delta_4, F_6 \vdash \Delta_1, F_2 \wedge F_2}{\bullet h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash \Delta_1, F_2 \wedge F_2} \rightarrow_L \rightarrow \frac{\frac{h_3 : \Delta_4 \vdash \Delta_1, F_5, F_2 \wedge F_2}{h_3 : \Delta_4 \vdash \Delta_1, F_2, F_5} \text{ax}}{\bullet h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash \Delta_1, F_2} \text{IH}}{\frac{h_3 : \Delta_4, F_6 \vdash \Delta_1, F_2 \wedge F_2}{h_3 : \Delta_4, F_6 \vdash \Delta_1, F_2} \text{ax}}{\bullet h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash \Delta_1, F_2} \text{IH}} \rightarrow_L$$

- Case(s) rule \wedge_L

$$\frac{h_3 : \Delta_4, F_5, F_6 \vdash \Delta_1, F_2 \wedge F_2}{\bullet h_3 : \Delta_4, F_5 \wedge F_6 \vdash \Delta_1, F_2 \wedge F_2} \wedge_L \rightarrow \frac{\frac{h_3 : \Delta_4, F_5, F_6 \vdash \Delta_1, F_2 \wedge F_2}{h_3 : \Delta_4, F_5, F_6 \vdash \Delta_1, F_2} \text{ax}}{\bullet h_3 : \Delta_4, F_5 \wedge F_6 \vdash \Delta_1, F_2} \text{IH}}{\bullet h_3 : \Delta_4, F_5 \wedge F_6 \vdash \Delta_1, F_2} \wedge_L$$

- Case(s) rule \vee_L

$$\frac{h_3 : \Delta_4, F_5 \vdash \Delta_1, F_2 \wedge F_2 \quad h_3 : \Delta_4, F_6 \vdash \Delta_1, F_2 \wedge F_2}{\bullet h_3 : \Delta_4, F_5 \vee F_6 \vdash \Delta_1, F_2 \wedge F_2} \vee_L \rightarrow \frac{\frac{h_3 : \Delta_4, F_5 \vdash \Delta_1, F_2 \wedge F_2}{h_3 : \Delta_4, F_5 \vdash \Delta_1, F_2} \text{ax}}{\bullet h_3 : \Delta_4, F_5 \vee F_6 \vdash \Delta_1, F_2} \text{IH}}{\frac{h_3 : \Delta_4, F_6 \vdash \Delta_1, F_2 \wedge F_2}{h_3 : \Delta_4, F_6 \vdash \Delta_1, F_2} \text{ax}}{\bullet h_3 : \Delta_4, F_5 \vee F_6 \vdash \Delta_1, F_2} \text{IH}} \vee_L$$

- Case(s) rule \perp_L

$$\frac{}{\bullet h_3 : \perp, \Delta_4 \vdash \Delta_1, F_2 \wedge F_2} \perp_L \rightarrow \frac{}{\bullet h_3 : \perp, \Delta_4 \vdash \Delta_1, F_2} \perp_L$$

- Case(s) rule I

$$\frac{}{\bullet h_2 : \Delta_3, p_4 \vdash (\Delta_5, p_4), F_1 \wedge F_1} I \rightarrow \frac{}{\bullet h_2 : \Delta_3, p_4 \vdash \Delta_5, F_1, p_4} I$$

- Case(s) rule \top_L

$$\frac{\frac{h_3 : \Delta_4 \vdash \Delta_1, F_2 \wedge F_2}{\bullet h_3 : \top, \Delta_4 \vdash \Delta_1, F_2 \wedge F_2} \top_L}{\frac{\frac{h_3 : \Delta_4 \vdash \Delta_1, F_2 \wedge F_2}{h_3 : \Delta_4 \vdash \Delta_1, F_2} \text{IH}}{\bullet h_3 : \top, \Delta_4 \vdash \Delta_1, F_2} \top_L} \text{ax} \rightarrow$$

10 Admissibility of $A \wedge A \rightarrow A$

- Case(s) rule \rightarrow_R

$$\frac{h_2 : \Delta_3, F_4 \vdash \Delta_6, F_1, F_5}{\bullet h_2 : \Delta_3 \vdash (\Delta_6, F_4 \rightarrow F_5), F_1} \rightarrow_R \rightarrow \frac{\frac{h_2 : \Delta_3, F_4 \vdash \Delta_6, F_1, F_5}{\bullet h_2 : \Delta_3, F_4 \vdash \Delta_6, F_5, F_1 \wedge F_1} \text{IH}}{\bullet \bullet h_2 : \Delta_3 \vdash \Delta_6, F_4 \rightarrow F_5, F_1 \wedge F_1} \text{ax}}{\bullet \bullet h_2 : \Delta_3 \vdash (\Delta_6, F_4 \rightarrow F_5), F_1} \rightarrow_R$$

$$\frac{h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \rightarrow F_5} \rightarrow_R \rightarrow \frac{\frac{h_1 : \Delta_2, F_4 \vdash \Delta_3, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \rightarrow F_5} \text{ax}}{\bullet \bullet h_1 : \Delta_2 \vdash \Delta_3, (F_4 \rightarrow F_5) \wedge (F_4 \rightarrow F_5)} \rightarrow_R \wedge_R$$

- Case(s) rule \wedge_R

$$\frac{h_2 : \Delta_3 \vdash \Delta_6, F_1, F_4 \quad h_2 : \Delta_3 \vdash \Delta_6, F_1, F_5}{\bullet h_2 : \Delta_3 \vdash (\Delta_6, F_4 \wedge F_5), F_1} \wedge_R \rightarrow \frac{\frac{h_2 : \Delta_3 \vdash \Delta_6, F_1, F_4}{\bullet h_2 : \Delta_3 \vdash \Delta_6, F_4, F_1 \wedge F_1} \text{ax}}{\bullet \bullet h_2 : \Delta_3 \vdash \Delta_6, F_1 \wedge F_1, F_4 \wedge F_5} \text{IH} \quad \frac{h_2 : \Delta_3 \vdash \Delta_6, F_1, F_5}{\bullet h_2 : \Delta_3 \vdash \Delta_6, F_5, F_1 \wedge F_1} \text{ax}}{\bullet \bullet h_2 : \Delta_3 \vdash \Delta_6, F_1 \wedge F_1, F_4 \wedge F_5} \text{IH} \wedge_R$$

$$\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4 \quad h_1 : \Delta_2 \vdash \Delta_3, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \wedge F_5} \wedge_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \wedge F_5} \text{ax}}{\bullet \bullet h_1 : \Delta_2 \vdash \Delta_3, (F_4 \wedge F_5) \wedge (F_4 \wedge F_5)} \wedge_R \quad \frac{h_1 : \Delta_2 \vdash \Delta_3, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \wedge F_5} \text{ax}}{\bullet \bullet h_1 : \Delta_2 \vdash \Delta_3, (F_4 \wedge F_5) \wedge (F_4 \wedge F_5)} \wedge_R \wedge_R$$

- Case(s) rule \vee_R

$$\frac{h_2 : \Delta_3 \vdash \Delta_6, F_1, F_4, F_5}{\bullet h_2 : \Delta_3 \vdash (\Delta_6, F_4 \vee F_5), F_1} \vee_R \rightarrow \frac{\frac{h_2 : \Delta_3 \vdash \Delta_6, F_1, F_4, F_5}{\bullet h_2 : \Delta_3 \vdash \Delta_6, F_4, F_5, F_1 \wedge F_1} \text{ax}}{\bullet \bullet h_2 : \Delta_3 \vdash \Delta_6, F_1 \wedge F_1, F_4 \vee F_5} \text{IH}}{\bullet \bullet h_2 : \Delta_3 \vdash \Delta_6, F_1 \wedge F_1, F_4 \vee F_5} \vee_R$$

$$\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \vee F_5} \vee_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \vee F_5} \text{ax}}{\bullet \bullet h_1 : \Delta_2 \vdash \Delta_3, (F_4 \vee F_5) \wedge (F_4 \vee F_5)} \vee_R \quad \frac{h_1 : \Delta_2 \vdash \Delta_3, F_4, F_5}{\bullet h_1 : \Delta_2 \vdash \Delta_3, F_4 \vee F_5} \text{ax}}{\bullet \bullet h_1 : \Delta_2 \vdash \Delta_3, (F_4 \vee F_5) \wedge (F_4 \vee F_5)} \vee_R \wedge_R$$

- Case(s) rule \perp_R

$$\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \Delta_3, \perp} \perp_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \text{ax}}{\bullet \bullet h_1 : \Delta_2 \vdash \Delta_3, \perp \wedge \perp} \perp_R \quad \frac{h_1 : \Delta_2 \vdash \Delta_3}{\bullet h_1 : \Delta_2 \vdash \perp, \Delta_3} \text{ax}}{\bullet \bullet h_1 : \Delta_2 \vdash \Delta_3, \perp \wedge \perp} \perp_R \wedge_R$$

$$\frac{h_2 : \Delta_3 \vdash \Delta_4, F_1}{\bullet h_2 : \Delta_3 \vdash (\perp, \Delta_4), F_1} \perp_R \rightarrow \frac{\frac{h_2 : \Delta_3 \vdash \Delta_4, F_1}{\bullet h_2 : \Delta_3 \vdash \Delta_4, F_1 \wedge F_1} \text{ax}}{\bullet \bullet h_2 : \Delta_3 \vdash \perp, \Delta_4, F_1 \wedge F_1} \text{IH}}{\bullet \bullet h_2 : \Delta_3 \vdash \perp, \Delta_4, F_1 \wedge F_1} \perp_R$$

- Case(s) rule \top_R

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \Delta_3, \top} \top_R \rightarrow \frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R \quad \frac{}{\bullet h_1 : \Delta_2 \vdash \top, \Delta_3} \top_R}{\bullet \bullet h_1 : \Delta_2 \vdash \Delta_3, \top \wedge \top} \top_R \wedge_R$$

$$\frac{}{\bullet h_2 : \Delta_3 \vdash (\top, \Delta_4), F_1} \top_R \rightarrow \frac{}{\bullet \bullet h_2 : \Delta_3 \vdash \top, \Delta_4, F_1 \wedge F_1} \top_R$$

- Case(s) rule \rightarrow_L

$$\frac{h_3 : \Delta_4 \vdash \Delta_1, F_2, F_5 \quad h_3 : \Delta_4, F_6 \vdash \Delta_1, F_2}{\bullet h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash \Delta_1, F_2} \rightarrow_L \rightarrow \frac{\frac{h_3 : \Delta_4 \vdash \Delta_1, F_2, F_5}{\bullet h_3 : \Delta_4 \vdash \Delta_1, F_5, F_2 \wedge F_2} \text{ax}}{\bullet \bullet h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash \Delta_1, F_2 \wedge F_2} \text{IH} \quad \frac{h_3 : \Delta_4, F_6 \vdash \Delta_1, F_2}{\bullet h_3 : \Delta_4, F_6 \vdash \Delta_1, F_2 \wedge F_2} \text{ax}}{\bullet \bullet h_3 : \Delta_4, F_5 \rightarrow F_6 \vdash \Delta_1, F_2 \wedge F_2} \text{IH} \rightarrow_L$$

- Case(s) rule \wedge_L

$$\frac{\frac{h_3 : \Delta_4, F_5, F_6 \vdash \Delta_1, F_2}{\bullet h_3 : \Delta_4, F_5 \wedge F_6 \vdash \Delta_1, F_2} \wedge_L}{\frac{\frac{h_3 : \Delta_4, F_5, F_6 \vdash \Delta_1, F_2}{\bullet h_3 : \Delta_4, F_5, F_6 \vdash \Delta_1, F_2} \text{ ax}}{\bullet h_3 : \Delta_4, F_5, F_6 \vdash \Delta_1, F_2 \wedge F_2} \text{ IH}}{\bullet \bullet h_3 : \Delta_4, F_5 \wedge F_6 \vdash \Delta_1, F_2 \wedge F_2} \wedge_L} \rightarrow$$

- Case(s) rule \vee_L

$$\frac{\frac{h_3 : \Delta_4, F_5 \vdash \Delta_1, F_2}{\bullet h_3 : \Delta_4, F_5 \vee F_6 \vdash \Delta_1, F_2} \vee_L}{\frac{\frac{h_3 : \Delta_4, F_5 \vdash \Delta_1, F_2}{\bullet h_3 : \Delta_4, F_5 \vdash \Delta_1, F_2} \text{ ax}}{\bullet h_3 : \Delta_4, F_5 \vdash \Delta_1, F_2 \wedge F_2} \text{ IH} \quad \frac{h_3 : \Delta_4, F_6 \vdash \Delta_1, F_2}{\bullet h_3 : \Delta_4, F_6 \vdash \Delta_1, F_2 \wedge F_2} \text{ IH}}{\bullet \bullet h_3 : \Delta_4, F_5 \vee F_6 \vdash \Delta_1, F_2 \wedge F_2} \vee_L} \rightarrow$$

- Case(s) rule \perp_L

$$\frac{\frac{}{\bullet h_3 : \perp, \Delta_4 \vdash \Delta_1, F_2} \perp_L}{\bullet \bullet h_3 : \perp, \Delta_4 \vdash \Delta_1, F_2 \wedge F_2} \perp_L} \rightarrow$$

- Case(s) rule I

$$\frac{\frac{}{\bullet h_2 : \Delta_3, P_4 \vdash (\Delta_5, P_4), F_1} I}{\bullet h_1 : \Delta_2, P_3 \vdash \Delta_4, P_3} I} \rightarrow \frac{\frac{\frac{}{\bullet h_2 : \Delta_3, P_4 \vdash \Delta_5, P_4, F_1 \wedge F_1} I}{\bullet \bullet h_1 : \Delta_2, P_3 \vdash \Delta_4, P_3} I} \wedge_R}{\bullet \bullet h_1 : \Delta_2, P_3 \vdash \Delta_4, P_3} I} \wedge_R$$

- Case(s) rule \top_L

$$\frac{\frac{h_3 : \Delta_4 \vdash \Delta_1, F_2}{\bullet h_3 : \top, \Delta_4 \vdash \Delta_1, F_2} \top_L}{\frac{\frac{h_3 : \Delta_4 \vdash \Delta_1, F_2}{\bullet h_3 : \Delta_4 \vdash \Delta_1, F_2} \text{ ax}}{\bullet h_3 : \Delta_4 \vdash \Delta_1, F_2 \wedge F_2} \text{ IH}}{\bullet \bullet h_3 : \top, \Delta_4 \vdash \Delta_1, F_2 \wedge F_2} \top_L} \rightarrow$$

Note: This rule does not preserve the height of the derivation but it produces proofs of at most $s(n)$ steps. See the specification in `prop-and-A.maude`.