Modal Logic S4 (K+T+4)

May 19, 2021

Contents

1 Height preserving admissibility of weakening on the left

- Case(s) rule $\to_R$

$$\dfrac{\mathtt{h}_1 : \mathtt{F}_4, \Delta_2 \vdash \mathtt{F}_5, \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \to \mathtt{F}_5}\ {\to_R} \quad \to \quad \dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_3, \mathtt{F}_5}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4, \mathtt{F}_W \vdash \Delta_3, \mathtt{F}_5}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_3, \mathtt{F}_4 \to \mathtt{F}_5}\ {\to_R}$$

- Case(s) rule $\wedge_R$

$$\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4, \Delta_3 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5, \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5}\ {\wedge_R} \quad \to \quad \dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_3, \mathtt{F}_4}\ {\scriptstyle \mathtt{IH}} \quad \dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_5}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_3, \mathtt{F}_5}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5}\ {\wedge_R}$$

- Case(s) rule $\vee_R$

$$\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4, \mathtt{F}_5, \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5}\ {\vee_R} \quad \to \quad \dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5}\ {\vee_R}$$

- Case(s) rule $\bot_R$

$$\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3}\ {\bot_R} \quad \to \quad \dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_3}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \bot, \Delta_3}\ {\bot_R}$$

- Case(s) rule $\top_R$

$$\dfrac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3}\ {\top_R} \quad \to \quad \dfrac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \top, \Delta_3}\ {\top_R}$$

- Case(s) rule $A4$

$$\dfrac{\mathtt{h}_1 : \Box\Gamma_2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Box\Gamma_2, \Delta_3 \vdash \Delta_4, []\mathtt{F}_5}\ {A4} \quad \to \quad \dfrac{\overline{\mathtt{h}_1 : \Box\Gamma_2 \vdash \mathtt{F}_5}\ {\scriptstyle \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_3, \mathtt{F}_W, \Box\Gamma_2 \vdash \Delta_4, []\mathtt{F}_5}\ {A4}$$

- Case(s) rule $\to_L$

$$\dfrac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 : \mathtt{F}_4, \Delta_2 \vdash \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \Delta_5}\ {\to_L} \quad \to \quad \dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_5, \mathtt{F}_3}\ {\scriptstyle \mathtt{IH}} \quad \dfrac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4, \mathtt{F}_W \vdash \Delta_5}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \Delta_5}\ {\to_L}$$

- Case(s) rule $\wedge_L$

$$\dfrac{\mathtt{h}_1 : \mathtt{F}_3, \mathtt{F}_4, \Delta_2 \vdash \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5}\ {\wedge_L} \quad \to \quad \dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_W \vdash \Delta_5}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5}\ {\wedge_L}$$

- Case(s) rule $\vee_L$

$$\dfrac{\mathtt{h}_1 : \mathtt{F}_3, \Delta_2 \vdash \Delta_5 \quad \mathtt{h}_1 : \mathtt{F}_4, \Delta_2 \vdash \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5}\ {\vee_L} \quad \to \quad \dfrac{\dfrac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_W \vdash \Delta_5}\ {\scriptstyle \mathtt{IH}} \quad \dfrac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}\ {\scriptstyle \mathtt{ax}}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4, \mathtt{F}_W \vdash \Delta_5}\ {\scriptstyle \mathtt{IH}}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_W, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5}\ {\vee_L}$$

- Case(s) rule AT

$$\frac{\mathtt{h}_1 : \mathtt{F}_3, \Delta_2, []\mathtt{F}_3 \vdash \Delta_4}{\bullet\mathtt{h}_1 : \Delta_2, []\mathtt{F}_3 \vdash \Delta_4}\ AT \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, []\mathtt{F}_3 \vdash \Delta_4}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_W, []\mathtt{F}_3 \vdash \Delta_4}\ \mathtt{IH}}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_W, []\mathtt{F}_3 \vdash \Delta_4}\ AT$$

- Case(s) rule $\bot_L$

$$\frac{}{\bullet\mathtt{h}_1 : \bot, \Delta_2 \vdash \Delta_3}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_1 : \bot, \Delta_2, \mathtt{F}_W \vdash \Delta_3}\ \bot_L$$

- Case(s) rule I

$$\frac{}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{p}_3 \vdash \Delta_4, \mathtt{p}_3}\ I \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_W, \mathtt{p}_3 \vdash \Delta_4, \mathtt{p}_3}\ I$$

- Case(s) rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet\mathtt{h}_1 : \top, \Delta_2 \vdash \Delta_3}\ \top_L \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_W \vdash \Delta_3}\ \mathtt{IH}}{\bullet\mathtt{h}_1 : \top, \Delta_2, \mathtt{F}_W \vdash \Delta_3}\ \top_L$$

2 Height preserving admissibility of weakening on the right

- Case(s) rule $\rightarrow_R$

$$\frac{\mathtt{h}_1 : \mathtt{F}_4, \Delta_2 \vdash \mathtt{F}_5, \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \rightarrow \mathtt{F}_5} \rightarrow_R \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_3, \mathtt{F}_5}\,\mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_3, \mathtt{F}_5, \mathtt{F}_W}\,\mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_W, \mathtt{F}_4 \rightarrow \mathtt{F}_5} \rightarrow_R$$

- Case(s) rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4, \Delta_3 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5, \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4}\,\mathtt{ax}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_W}\,\mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_5}\,\mathtt{ax}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_5, \mathtt{F}_W}\,\mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_W, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R$$

- Case(s) rule $\vee_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4, \mathtt{F}_5, \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5}\,\mathtt{ax}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_W}\,\mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_W, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R$$

- Case(s) rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3} \bot_R \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\,\mathtt{ax}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_W}\,\mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3, \mathtt{F}_W} \bot_R$$

- Case(s) rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3} \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3, \mathtt{F}_W} \top_R$$

- Case(s) rule $A4$

$$\frac{\mathtt{h}_1 : \Box\Gamma_2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Box\Gamma_2, \Delta_3 \vdash \Delta_4, []\mathtt{F}_5} A4 \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_2 \vdash \mathtt{F}_5}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_3, \Box\Gamma_2 \vdash \Delta_4, \mathtt{F}_W, []\mathtt{F}_5} A4$$

- Case(s) rule $\rightarrow_L$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 : \mathtt{F}_4, \Delta_2 \vdash \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \Delta_5} \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}\,\mathtt{ax}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_W}\,\mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_W}\,\mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_W} \rightarrow_L$$

- Case(s) rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_3, \mathtt{F}_4, \Delta_2 \vdash \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5} \wedge_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_W}\,\mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_W} \wedge_L$$

- Case(s) rule $\vee_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_3, \Delta_2 \vdash \Delta_5 \quad \mathtt{h}_1 : \mathtt{F}_4, \Delta_2 \vdash \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5} \vee_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5}\,\mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_W}\,\mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_W}\,\mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_W} \vee_L$$

- Case(s) rule AT

$$\frac{\mathtt{h}_1 : \mathtt{F}_3, \Delta_2, []\mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_2, []\mathtt{F}_3 \vdash \Delta_4} AT \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, []\mathtt{F}_3 \vdash \Delta_4}\,\mathtt{ax}}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, []\mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_W}\,\mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_2, []\mathtt{F}_3 \vdash \Delta_4, \mathtt{F}_W} AT$$

- Case(s) rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \Delta_3}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \Delta_3, \mathtt{F}_W}\ \bot_L$$

- Case(s) rule I

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_3 \vdash \Delta_4, \mathtt{p}_3}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_3 \vdash \Delta_4, \mathtt{F}_W, \mathtt{p}_3}\ I$$

- Case(s) rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \Delta_3}\ \top_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ \mathtt{ax}}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_W}\ \mathtt{IH}}{\bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \Delta_3, \mathtt{F}_W}\ \top_L$$

3 Measure of derivations

- Case(s) rule $\rightarrow_R$

$$\frac{\mathtt{h}_1 : \mathtt{F}_4, \Delta_2 \vdash \mathtt{F}_5, \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \rightarrow \mathtt{F}_5} \rightarrow_R \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_3, \mathtt{F}_5}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_3, \mathtt{F}_5}\,\mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \rightarrow \mathtt{F}_5} \rightarrow_R$$

- Case(s) rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4, \Delta_3 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5, \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4}\,\mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_5}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_5}\,\mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R$$

- Case(s) rule $\vee_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4, \mathtt{F}_5, \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5}\,\mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R$$

- Case(s) rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3} \bot_R \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\,\mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3} \bot_R$$

- Case(s) rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3} \top_R \quad \rightarrow \quad \frac{}{\bullet\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3} \top_R$$

- Case(s) rule $A4$

$$\frac{\mathtt{h}_1 : \Box\Gamma_2 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_1 : \Box\Gamma_2, \Delta_3 \vdash \Delta_4, []\mathtt{F}_5} A4 \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_2 \vdash \mathtt{F}_5}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Box\Gamma_2 \vdash \mathtt{F}_5}\,\mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_3, \Box\Gamma_2 \vdash \Delta_4, []\mathtt{F}_5} A4$$

- Case(s) rule $\rightarrow_L$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 : \mathtt{F}_4, \Delta_2 \vdash \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \Delta_5} \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}\,\mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \Delta_5} \rightarrow_L$$

- Case(s) rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_3, \mathtt{F}_4, \Delta_2 \vdash \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5} \wedge_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5} \wedge_L$$

- Case(s) rule $\vee_L$

$$\frac{\mathtt{h}_1 : \mathtt{F}_3, \Delta_2 \vdash \Delta_5 \quad \mathtt{h}_1 : \mathtt{F}_4, \Delta_2 \vdash \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5} \vee_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5}\,\mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5} \vee_L$$

- Case(s) rule AT

$$\frac{\mathtt{h}_1 : \mathtt{F}_3, \Delta_2, []\mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_1 : \Delta_2, []\mathtt{F}_3 \vdash \Delta_4} AT \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, []\mathtt{F}_3 \vdash \Delta_4}\,\mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3, []\mathtt{F}_3 \vdash \Delta_4}\,\mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \Delta_2, []\mathtt{F}_3 \vdash \Delta_4} AT$$

- Case(s) rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \Delta_3}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \Delta_3}\ \bot_L$$

- Case(s) rule I

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_3 \vdash \Delta_4, \mathtt{p}_3}\ I \quad \rightarrow \quad \frac{}{\bullet\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_3 \vdash \Delta_4, \mathtt{p}_3}\ I$$

- Case(s) rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \Delta_3}\ \top_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ \mathtt{IH}}{\bullet\bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \Delta_3}\ \top_L$$

4 Invertibility of Rules

4.1 Status of $\rightarrow_R$: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_3 : \mathtt{F}_5, \Delta_4 \vdash \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2), \mathtt{F}_5 \rightarrow \mathtt{F}_6} \rightarrow_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \rightarrow \mathtt{F}_6} \rightarrow_R$$

$$\frac{\mathtt{h}_1 : \mathtt{F}_4, \Delta_2 \vdash \mathtt{F}_5, \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \rightarrow \mathtt{F}_5} \rightarrow_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_3, \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_3, \mathtt{F}_5}\ \mathtt{H}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2), \mathtt{F}_5 \wedge \mathtt{F}_6} \wedge_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \wedge \mathtt{F}_6} \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5, \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2), \mathtt{F}_5 \vee \mathtt{F}_6} \vee_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5, \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vee \mathtt{F}_6} \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2} \bot_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \bot, \Delta_5, \mathtt{F}_2} \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_1 \rightarrow \mathtt{F}_2} \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_1 \vdash \top, \Delta_5, \mathtt{F}_2} \top_R$$

- Case rule $A4$

$$\frac{\mathtt{h}_3 : \Box\Gamma_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : \Box\Gamma_4, \Delta_5 \vdash (\Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2), []\mathtt{F}_6} A4 \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Box\Gamma_4 \vdash \mathtt{F}_6}\ \mathtt{ax}}{\bullet \mathtt{h}_3 : \Delta_5, \mathtt{F}_1, \Box\Gamma_4 \vdash \Delta_7, \mathtt{F}_2, []\mathtt{F}_6} A4$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_4 : \Delta_5 \vdash \mathtt{F}_6, \Delta_1, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \quad \mathtt{h}_4 : \mathtt{F}_7, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \rightarrow \mathtt{F}_3}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \rightarrow \mathtt{F}_3} \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_5, \mathtt{F}_2 \vdash \Delta_1, \mathtt{F}_3, \mathtt{F}_6}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_4 : \Delta_5, \mathtt{F}_2, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_2, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_3} \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_4 : \mathtt{F}_6, \mathtt{F}_7, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \rightarrow \mathtt{F}_3}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \rightarrow \mathtt{F}_3} \wedge_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_5, \mathtt{F}_2, \mathtt{F}_6, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_2, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_3} \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_4 : \mathtt{F}_6, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \quad \mathtt{h}_4 : \mathtt{F}_7, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3} \vee_L \quad \to \quad \frac{\overline{\mathtt{h}_4 : \Delta_5, \mathtt{F}_2, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_3}^{\,\mathtt{ax/ind}} \quad \overline{\mathtt{h}_4 : \Delta_5, \mathtt{F}_2, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_3}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_2, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_3} \vee_L$$

- Case rule AT

$$\frac{\mathtt{h}_4 : \mathtt{F}_6, \Delta_5, []\mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3}{\bullet \mathtt{h}_4 : \Delta_5, []\mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3} AT \quad \to \quad \frac{\overline{\mathtt{h}_4 : \Delta_5, \mathtt{F}_2, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_3}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_2, []\mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_3} AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_4 : \bot, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3} \bot_L \quad \to \quad \frac{}{\bullet \mathtt{h}_4 : \bot, \Delta_5, \mathtt{F}_2 \vdash \Delta_1, \mathtt{F}_3} \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_4 \vdash \mathtt{p}_5, \Delta_6, \mathtt{F}_1 \to \mathtt{F}_2} I \quad \to \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_1, \mathtt{p}_5 \vdash \Delta_6, \mathtt{F}_2, \mathtt{p}_5} I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_4 : \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3}{\bullet \mathtt{h}_4 : \top, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3} \top_L \quad \to \quad \frac{\overline{\mathtt{h}_4 : \Delta_5, \mathtt{F}_2 \vdash \Delta_1, \mathtt{F}_3}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \top, \Delta_5, \mathtt{F}_2 \vdash \Delta_1, \mathtt{F}_3} \top_L$$

4.2 Status of $\wedge_R$: (Left Premise): `Invertible`

- Case rule $\to_R$

$$\frac{\mathtt{h}_3 : \mathtt{F}_5, \Delta_4 \vdash \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_5 \to \mathtt{F}_6} \to_R \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_6}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \to \mathtt{F}_6} \to_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_5 \wedge \mathtt{F}_6} \wedge_R \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_5}^{\,\mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_6}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \wedge \mathtt{F}_6} \wedge_R$$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4, \Delta_3 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5, \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R \quad \to \quad \frac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4}^{\,\mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4}\, {}^{\mathtt{H}}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5, \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_5 \vee \mathtt{F}_6} \vee_R \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_5, \mathtt{F}_6}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \vee \mathtt{F}_6} \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2} \bot_R \quad \to \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_1}^{\,\mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_1} \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}\ \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_1}\ \top_R$$

- Case rule $A4$

$$\frac{\mathtt{h}_3 : \Box\Gamma_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : \Box\Gamma_4, \Delta_5 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), []\mathtt{F}_6}\ A4 \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Box\Gamma_4 \vdash \mathtt{F}_6}\ \mathtt{ax}}{\bullet \mathtt{h}_3 : \Delta_5, \Box\Gamma_4 \vdash \Delta_7, \mathtt{F}_1, []\mathtt{F}_6}\ A4$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_4 : \Delta_5 \vdash \mathtt{F}_6, \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \quad \mathtt{h}_4 : \mathtt{F}_7, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}\ \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_5 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_6}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_4 : \Delta_5, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_4 : \mathtt{F}_6, \mathtt{F}_7, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}\ \wedge_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_4 : \mathtt{F}_6, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \quad \mathtt{h}_4 : \mathtt{F}_7, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}\ \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_4 : \Delta_5, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2}\ \vee_L$$

- Case rule AT

$$\frac{\mathtt{h}_4 : \mathtt{F}_6, \Delta_5, []\mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}{\bullet \mathtt{h}_4 : \Delta_5, []\mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}\ AT \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_5, []\mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2}\ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_4 : \bot, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_4 : \bot, \Delta_5 \vdash \Delta_1, \mathtt{F}_2}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_4 \vdash \mathtt{p}_5, \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{p}_5 \vdash \Delta_6, \mathtt{F}_1, \mathtt{p}_5}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_4 : \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}{\bullet \mathtt{h}_4 : \top, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}\ \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_5 \vdash \Delta_1, \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \top, \Delta_5 \vdash \Delta_1, \mathtt{F}_2}\ \top_L$$

4.3 Status of $\wedge_R$ (Right Premise): : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_3 : \mathtt{F}_5, \Delta_4 \vdash \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_5 \rightarrow \mathtt{F}_6} \rightarrow_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_6}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \rightarrow \mathtt{F}_6} \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_5 \wedge \mathtt{F}_6} \wedge_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5}^{\ ax/ind} \quad \overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_6}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \wedge \mathtt{F}_6} \wedge_R$$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4, \Delta_3 \quad \mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_5, \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_5}^{\ ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_5} \ \mathtt{H}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5, \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_5 \vee \mathtt{F}_6} \vee_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5, \mathtt{F}_6}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vee \mathtt{F}_6} \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2} \bot_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_2}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_2} \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2} \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_2} \top_R$$

- Case rule $A4$

$$\frac{\mathtt{h}_3 : \Box\Gamma_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : \Box\Gamma_4, \Delta_5 \vdash (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), []\mathtt{F}_6} A4 \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Box\Gamma_4 \vdash \mathtt{F}_6}^{\ ax}}{\bullet \mathtt{h}_3 : \Delta_5, \Box\Gamma_4 \vdash \Delta_7, \mathtt{F}_2, []\mathtt{F}_6} A4$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_4 : \Delta_5 \vdash \mathtt{F}_6, \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \quad \mathtt{h}_4 : \mathtt{F}_7, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3} \rightarrow_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_5 \vdash \Delta_1, \mathtt{F}_3, \mathtt{F}_6}^{\ ax/ind} \quad \overline{\mathtt{h}_4 : \Delta_5, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_3}^{\ ax/ind}}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_3} \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_4 : \mathtt{F}_6, \mathtt{F}_7, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3} \wedge_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_3}^{\ ax/ind}}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_3} \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_4 : \mathtt{F}_6, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \quad \mathtt{h}_4 : \mathtt{F}_7, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3} \vee_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_3}^{\ ax/ind} \quad \overline{\mathtt{h}_4 : \Delta_5, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_3}^{\ ax/ind}}{\bullet \mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_3} \vee_L$$

- Case rule AT

$$\frac{\mathtt{h}_4 : \mathtt{F}_6, \Delta_5, []\mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}{\bullet \mathtt{h}_4 : \Delta_5, []\mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}\ AT \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Delta_5, []\mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_3}\ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_4 : \bot, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_4 : \bot, \Delta_5 \vdash \Delta_1, \mathtt{F}_3}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_5, \Delta_4 \vdash \mathtt{p}_5, \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{p}_5 \vdash \Delta_6, \mathtt{F}_2, \mathtt{p}_5}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_4 : \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}{\bullet \mathtt{h}_4 : \top, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3}\ \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_5 \vdash \Delta_1, \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \top, \Delta_5 \vdash \Delta_1, \mathtt{F}_3}\ \top_L$$

4.4 Status of $\vee_R$: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_3 : \mathtt{F}_5, \Delta_4 \vdash \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_5 \rightarrow \mathtt{F}_6}\ \rightarrow_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5 \rightarrow \mathtt{F}_6}\ \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \quad \mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5, \mathtt{F}_6, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5, \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R$$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_4, \mathtt{F}_5, \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5}\ \vee_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5}\ \mathtt{H}$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2}\ \bot_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_5, \mathtt{F}_1, \mathtt{F}_2}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \bot, \Delta_5, \mathtt{F}_1, \mathtt{F}_2}\ \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2}\ \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_4 \vdash \top, \Delta_5, \mathtt{F}_1, \mathtt{F}_2}\ \top_R$$

- Case rule $A4$

$$\frac{\mathtt{h}_3 : \Box\Gamma_4 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_3 : \Box\Gamma_4, \Delta_5 \vdash (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), []\mathtt{F}_6}\ A4 \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Box\Gamma_4 \vdash \mathtt{F}_6}\ \mathtt{ax}}{\bullet\mathtt{h}_3 : \Delta_5, \Box\Gamma_4 \vdash \Delta_7, \mathtt{F}_1, \mathtt{F}_2, []\mathtt{F}_6}\ A4$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_4 : \Delta_5 \vdash \mathtt{F}_6, \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \quad \mathtt{h}_4 : \mathtt{F}_7, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3}{\bullet\mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3}\ \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_5 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_3, \mathtt{F}_6}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_4 : \Delta_5, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \rightarrow \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_3}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_4 : \mathtt{F}_6, \mathtt{F}_7, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3}{\bullet\mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3}\ \wedge_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_3}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_4 : \mathtt{F}_6, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \quad \mathtt{h}_4 : \mathtt{F}_7, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3}{\bullet\mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3}\ \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_3}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_4 : \Delta_5, \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_4 : \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_3}\ \vee_L$$

- Case rule AT

$$\frac{\mathtt{h}_4 : \mathtt{F}_6, \Delta_5, []\mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3}{\bullet\mathtt{h}_4 : \Delta_5, []\mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3}\ AT \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_5, \mathtt{F}_6, []\mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_4 : \Delta_5, []\mathtt{F}_6 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_3}\ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet\mathtt{h}_4 : \bot, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_4 : \bot, \Delta_5 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_3}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet\mathtt{h}_3 : \mathtt{p}_5, \Delta_4 \vdash \mathtt{p}_5, \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2}\ I \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_3 : \Delta_4, \mathtt{p}_5 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_2, \mathtt{p}_5}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_4 : \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3}{\bullet\mathtt{h}_4 : \top, \Delta_5 \vdash \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3}\ \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_5 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_4 : \top, \Delta_5 \vdash \Delta_1, \mathtt{F}_2, \mathtt{F}_3}\ \top_L$$

4.5 Status of $\bot_R$: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_1 : \mathtt{F}_3, \Delta_2 \vdash \bot, \mathtt{F}_4, \Delta_5}{\bullet\mathtt{h}_1 : \Delta_2 \vdash (\bot, \Delta_5), \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ \rightarrow_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \bot, \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \bot, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\bot, \Delta_5), \mathtt{F}_3 \wedge \mathtt{F}_4}\ \wedge_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \wedge \mathtt{F}_4}\ \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \bot, \mathtt{F}_3, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\bot, \Delta_5), \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \Delta_3}\ \bot_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ \mathtt{H}$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \bot, \Delta_3}\ \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3}\ \top_R$$

- Case rule $A4$

$$\frac{\mathtt{h}_1 : \Box\Gamma_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Box\Gamma_2, \Delta_3 \vdash (\bot, \Delta_5), []\mathtt{F}_4}\ A4 \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_2 \vdash \mathtt{F}_4}\ \mathtt{ax}}{\bullet \mathtt{h}_1 : \Delta_3, \Box\Gamma_2 \vdash \Delta_5, []\mathtt{F}_4}\ A4$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \bot, \mathtt{F}_4, \Delta_1 \quad \mathtt{h}_2 : \mathtt{F}_5, \Delta_3 \vdash \bot, \Delta_1}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \bot, \Delta_1}\ \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \mathtt{F}_4}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_5 \vdash \Delta_1}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_1}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_2 : \mathtt{F}_4, \mathtt{F}_5, \Delta_3 \vdash \bot, \Delta_1}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \bot, \Delta_1}\ \wedge_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_1}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_1}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_2 : \mathtt{F}_4, \Delta_3 \vdash \bot, \Delta_1 \quad \mathtt{h}_2 : \mathtt{F}_5, \Delta_3 \vdash \bot, \Delta_1}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \bot, \Delta_1}\ \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash \Delta_1}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_5 \vdash \Delta_1}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_1}\ \vee_L$$

- Case rule AT

$$\frac{\mathtt{h}_2 : \mathtt{F}_4, \Delta_3, []\mathtt{F}_4 \vdash \bot, \Delta_1}{\bullet \mathtt{h}_2 : \Delta_3, []\mathtt{F}_4 \vdash \bot, \Delta_1}\ AT \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4, []\mathtt{F}_4 \vdash \Delta_1}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_2 : \Delta_3, []\mathtt{F}_4 \vdash \Delta_1}\ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_2 : \bot, \Delta_3 \vdash \bot, \Delta_1}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_2 : \bot, \Delta_3 \vdash \Delta_1}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_1 : \mathtt{p}_3, \Delta_2 \vdash \mathtt{p}_3, \bot, \Delta_4}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{p}_3 \vdash \Delta_4, \mathtt{p}_3}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \bot, \Delta_1}{\bullet \mathtt{h}_2 : \top, \Delta_3 \vdash \bot, \Delta_1}\ \top_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1}^{\ ax/ind}}{\bullet \mathtt{h}_2 : \top, \Delta_3 \vdash \Delta_1}\ \top_L$$

4.6 Status of $\top_R$: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_1 : \mathtt{F}_3, \Delta_2 \vdash \top, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\top, \Delta_5), \mathtt{F}_3 \rightarrow \mathtt{F}_4}\ \rightarrow_R \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \top, \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 : \Delta_2 \vdash \top, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\top, \Delta_5), \mathtt{F}_3 \wedge \mathtt{F}_4}\ \wedge_R \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \top, \mathtt{F}_3, \mathtt{F}_4, \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2 \vdash (\top, \Delta_5), \mathtt{F}_3 \vee \mathtt{F}_4}\ \vee_R \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \bot, \top, \Delta_3}\ \bot_R \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \top, \Delta_3}\ \top_R \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $A4$

$$\frac{\mathtt{h}_1 : \Box\Gamma_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Box\Gamma_2, \Delta_3 \vdash (\top, \Delta_5), []\mathtt{F}_4}\ A4 \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \top, \mathtt{F}_4, \Delta_1 \quad \mathtt{h}_2 : \mathtt{F}_5, \Delta_3 \vdash \top, \Delta_1}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \top, \Delta_1}\ \rightarrow_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_2 : \mathtt{F}_4, \mathtt{F}_5, \Delta_3 \vdash \top, \Delta_1}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \top, \Delta_1}\ \wedge_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_2 : \mathtt{F}_4, \Delta_3 \vdash \top, \Delta_1 \quad \mathtt{h}_2 : \mathtt{F}_5, \Delta_3 \vdash \top, \Delta_1}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \top, \Delta_1} \vee_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule AT

$$\frac{\mathtt{h}_2 : \mathtt{F}_4, \Delta_3, []\mathtt{F}_4 \vdash \top, \Delta_1}{\bullet \mathtt{h}_2 : \Delta_3, []\mathtt{F}_4 \vdash \top, \Delta_1} AT \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_2 : \bot, \Delta_3 \vdash \top, \Delta_1} \bot_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_1 : \mathtt{p}_3, \Delta_2 \vdash \mathtt{p}_3, \top, \Delta_4} I \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \top, \Delta_1}{\bullet \mathtt{h}_2 : \top, \Delta_3 \vdash \top, \Delta_1} \top_L \quad \rightarrow \quad \texttt{trivial}$$

4.7 Status of $A4$: : `Non invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_4 : \Box\Gamma_1, \mathtt{F}_5, \Delta_2 \vdash \mathtt{F}_6, \Delta_7, []\mathtt{F}_3}{\bullet \mathtt{h}_4 : \Box\Gamma_1, \Delta_2 \vdash (\Delta_7, []\mathtt{F}_3), \mathtt{F}_5 \rightarrow \mathtt{F}_6} \rightarrow_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Box\Gamma_1 \vdash \mathtt{F}_3}\ \texttt{ax/ind}}{\bullet \mathtt{h}_4 : \Box\Gamma_1 \vdash \mathtt{F}_3}\ \texttt{H}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_4 : \Box\Gamma_1, \Delta_2 \vdash \mathtt{F}_5, \Delta_7, []\mathtt{F}_3 \quad \mathtt{h}_4 : \Box\Gamma_1, \Delta_2 \vdash \mathtt{F}_6, \Delta_7, []\mathtt{F}_3}{\bullet \mathtt{h}_4 : \Box\Gamma_1, \Delta_2 \vdash (\Delta_7, []\mathtt{F}_3), \mathtt{F}_5 \wedge \mathtt{F}_6} \wedge_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Box\Gamma_1 \vdash \mathtt{F}_3}\ \texttt{ax/ind}}{\bullet \mathtt{h}_4 : \Box\Gamma_1 \vdash \mathtt{F}_3}\ \texttt{H}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_4 : \Box\Gamma_1, \Delta_2 \vdash \mathtt{F}_5, \mathtt{F}_6, \Delta_7, []\mathtt{F}_3}{\bullet \mathtt{h}_4 : \Box\Gamma_1, \Delta_2 \vdash (\Delta_7, []\mathtt{F}_3), \mathtt{F}_5 \vee \mathtt{F}_6} \vee_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Box\Gamma_1 \vdash \mathtt{F}_3}\ \texttt{ax/ind}}{\bullet \mathtt{h}_4 : \Box\Gamma_1 \vdash \mathtt{F}_3}\ \texttt{H}$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_4 : \Box\Gamma_1, \Delta_2 \vdash \Delta_5, []\mathtt{F}_3}{\bullet \mathtt{h}_4 : \Box\Gamma_1, \Delta_2 \vdash \bot, \Delta_5, []\mathtt{F}_3} \bot_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Box\Gamma_1 \vdash \mathtt{F}_3}\ \texttt{ax/ind}}{\bullet \mathtt{h}_4 : \Box\Gamma_1 \vdash \mathtt{F}_3}\ \texttt{H}$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_4 : \Box\Gamma_1, \Delta_2 \vdash \top, \Delta_5, []\mathtt{F}_3}\ \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_4 : \Box\Gamma_1 \vdash \mathtt{F}_3}\ \mathtt{fail}$$

- Case rule $A4$

$$\frac{\mathtt{h}_2 : \Box\Gamma_5, \Box\Gamma_6 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_2 : (\Box\Gamma_5, \Box\Gamma_6), \Box\Gamma_7, \Delta_8 \vdash (\Delta_4, []\mathtt{F}_1), []\mathtt{F}_3}\ A4 \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_2 : \Box\Gamma_5, \Box\Gamma_7 \vdash \mathtt{F}_1}\ \mathtt{fail}$$

$$\frac{\mathtt{h}_1 : \Box\Gamma_4, \Box\Gamma_5 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_1 : (\Box\Gamma_4, \Box\Gamma_5), \Box\Gamma_6, \Delta_7 \vdash \Delta_2, []\mathtt{F}_3}\ A4 \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Box\Gamma_4, \Box\Gamma_6 \vdash \mathtt{F}_3}\ \mathtt{fail}$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_4 : \Box\Gamma_1, \Delta_7 \vdash \mathtt{F}_5, \Delta_2, []\mathtt{F}_3 \quad \mathtt{h}_4 : \Box\Gamma_1, \mathtt{F}_6, \Delta_7 \vdash \Delta_2, []\mathtt{F}_3}{\bullet \mathtt{h}_4 : (\Box\Gamma_1, \Delta_7), \mathtt{F}_5 \rightarrow \mathtt{F}_6 \vdash \Delta_2, []\mathtt{F}_3}\ \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Box\Gamma_1 \vdash \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Box\Gamma_1 \vdash \mathtt{F}_3}\ \mathtt{H}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_4 : \Box\Gamma_1, \mathtt{F}_5, \mathtt{F}_6, \Delta_7 \vdash \Delta_2, []\mathtt{F}_3}{\bullet \mathtt{h}_4 : (\Box\Gamma_1, \Delta_7), \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_2, []\mathtt{F}_3}\ \wedge_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Box\Gamma_1 \vdash \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Box\Gamma_1 \vdash \mathtt{F}_3}\ \mathtt{H}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_4 : \Box\Gamma_1, \mathtt{F}_5, \Delta_7 \vdash \Delta_2, []\mathtt{F}_3 \quad \mathtt{h}_4 : \Box\Gamma_1, \mathtt{F}_6, \Delta_7 \vdash \Delta_2, []\mathtt{F}_3}{\bullet \mathtt{h}_4 : (\Box\Gamma_1, \Delta_7), \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_2, []\mathtt{F}_3}\ \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Box\Gamma_1 \vdash \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Box\Gamma_1 \vdash \mathtt{F}_3}\ \mathtt{H}$$

- Case rule AT

$$\frac{\mathtt{h}_4 : \Box\Gamma_6, \mathtt{F}_5, \Delta_1, []\mathtt{F}_5 \vdash \Delta_2, []\mathtt{F}_3}{\bullet \mathtt{h}_4 : (\Box\Gamma_6, \Delta_1), []\mathtt{F}_5 \vdash \Delta_2, []\mathtt{F}_3}\ AT \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Box\Gamma_6, []\mathtt{F}_5 \vdash \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Box\Gamma_6, []\mathtt{F}_5 \vdash \mathtt{F}_3}\ \mathtt{H}$$

$$\frac{\mathtt{h}_4 : \Box\Gamma_1, \mathtt{F}_5, \Delta_6, []\mathtt{F}_5 \vdash \Delta_2, []\mathtt{F}_3}{\bullet \mathtt{h}_4 : (\Box\Gamma_1, \Delta_6), []\mathtt{F}_5 \vdash \Delta_2, []\mathtt{F}_3}\ AT \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Box\Gamma_1 \vdash \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Box\Gamma_1 \vdash \mathtt{F}_3}\ \mathtt{H}$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_4 : \bot, \Box\Gamma_1, \Delta_5 \vdash \Delta_2, []\mathtt{F}_3}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_4 : \Box\Gamma_1 \vdash \mathtt{F}_3}\ \mathtt{fail}$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_4, \Box\Gamma_1, \Delta_6 \vdash \mathtt{p}_4, \Delta_5, []\mathtt{F}_2}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Box\Gamma_1 \vdash \mathtt{F}_2}\ \mathtt{fail}$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_4 : \Box\Gamma_1, \Delta_5 \vdash \Delta_2, []\mathtt{F}_3}{\bullet \mathtt{h}_4 : \top, \Box\Gamma_1, \Delta_5 \vdash \Delta_2, []\mathtt{F}_3}\ \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_4 : \Box\Gamma_1 \vdash \mathtt{F}_3}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_4 : \Box\Gamma_1 \vdash \mathtt{F}_3}\ \mathtt{H}$$

4.8 Status of $\rightarrow_L$: (Left Premise): `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_4 : \mathtt{F}_6, \Delta_1, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \mathtt{F}_7, \Delta_5}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \rightarrow \mathtt{F}_7} \rightarrow_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_1, \mathtt{F}_6 \vdash \Delta_5, \mathtt{F}_2, \mathtt{F}_7}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1 \vdash \Delta_5, \mathtt{F}_2, \mathtt{F}_6 \rightarrow \mathtt{F}_7} \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \mathtt{F}_6, \Delta_5 \quad \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \mathtt{F}_7, \Delta_5}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7} \wedge_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_1 \vdash \Delta_5, \mathtt{F}_2, \mathtt{F}_6}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_4 : \Delta_1 \vdash \Delta_5, \mathtt{F}_2, \mathtt{F}_7}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1 \vdash \Delta_5, \mathtt{F}_2, \mathtt{F}_6 \wedge \mathtt{F}_7} \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_5}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_1 \vdash \Delta_5, \mathtt{F}_2, \mathtt{F}_6, \mathtt{F}_7}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1 \vdash \Delta_5, \mathtt{F}_2, \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_5}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \bot, \Delta_5} \bot_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_1 \vdash \Delta_5, \mathtt{F}_2}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1 \vdash \bot, \Delta_5, \mathtt{F}_2} \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \top, \Delta_5} \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_4 : \Delta_1 \vdash \top, \Delta_5, \mathtt{F}_2} \top_R$$

- Case rule $A4$

$$\frac{\mathtt{h}_3 : \Box \Gamma_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : \Box \Gamma_4, \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_5, []\mathtt{F}_6} A4 \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Box \Gamma_4 \vdash \mathtt{F}_6}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_3 : \Delta_7, \Box \Gamma_4 \vdash \Delta_5, \mathtt{F}_1, []\mathtt{F}_6} A4$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \mathtt{F}_4, \Delta_6 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2), \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6} \rightarrow_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7 \vdash \Delta_6, \mathtt{F}_1, \mathtt{F}_4}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1} \rightarrow_L$$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 : \mathtt{F}_4, \Delta_2 \vdash \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \Delta_5} \rightarrow_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_5, \mathtt{F}_3}\ {}^{\mathtt{H}}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6} \wedge_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1} \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_6 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \rightarrow \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6} \vee_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vdash \Delta_6, \mathtt{F}_1}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6, \mathtt{F}_1} \vee_L$$

- Case rule AT

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_6, []\mathtt{F}_4, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_5}{\bullet \mathtt{h}_3 : (\Delta_6, \mathtt{F}_1 \to \mathtt{F}_2), []\mathtt{F}_4 \vdash \Delta_5}\ AT \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_4, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_1}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_6, []\mathtt{F}_4 \vdash \Delta_5, \mathtt{F}_1}\ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_4}\ \bot_L \quad \to \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_4, \mathtt{F}_1}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_4, \Delta_6, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{p}_4, \Delta_5}\ I \quad \to \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{p}_4 \vdash \Delta_5, \mathtt{F}_1, \mathtt{p}_4}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_4}\ \top_L \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5 \vdash \Delta_4, \mathtt{F}_1}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_4, \mathtt{F}_1}\ \top_L$$

4.9 Status of $\to_L$ (Right Premise): : `Invertible`

- Case rule $\to_R$

$$\frac{\mathtt{h}_4 : \mathtt{F}_6, \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_7, \Delta_5}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7}\ \to_R \quad \to \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_3, \mathtt{F}_6 \vdash \Delta_5, \mathtt{F}_7}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \to \mathtt{F}_7}\ \to_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_6, \Delta_5 \quad \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_7, \Delta_5}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \wedge_R \quad \to \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6}\ {\scriptstyle \mathtt{ax/ind}} \quad \dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_7}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_5}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7}\ \vee_R \quad \to \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_7}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7}\ \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \Delta_5}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \bot, \Delta_5}\ \bot_R \quad \to \quad \frac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \Delta_5}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \bot, \Delta_5}\ \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \to \mathtt{F}_3 \vdash \top, \Delta_5}\ \top_R \quad \to \quad \frac{}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \top, \Delta_5}\ \top_R$$

- Case rule $A4$

$$\frac{\mathtt{h}_3 : \Box\Gamma_4 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_3 : \Box\Gamma_4, \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_5, []\mathtt{F}_6}\ A4 \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Box\Gamma_4 \vdash \mathtt{F}_6}\ \mathtt{ax}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \Box\Gamma_4 \vdash \Delta_5, []\mathtt{F}_6}\ A4$$

- Case rule $\to_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{F}_4, \Delta_6 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6}{\bullet\mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \to \mathtt{F}_2), \mathtt{F}_4 \to \mathtt{F}_5 \vdash \Delta_6}\ \to_L \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_6}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \to \mathtt{F}_5 \vdash \Delta_6}\ \to_L$$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_1 : \mathtt{F}_4, \Delta_2 \vdash \Delta_5}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \Delta_5}\ \to_L \quad \to \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}\ \mathtt{ax}}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}\ \mathtt{H}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6}{\bullet\mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \to \mathtt{F}_2), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6}\ \wedge_L \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_6}{\bullet\mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \to \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6}\ \vee_L \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vdash \Delta_6}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_6}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6}\ \vee_L$$

- Case rule AT

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_6, []\mathtt{F}_4, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_5}{\bullet\mathtt{h}_3 : (\Delta_6, \mathtt{F}_1 \to \mathtt{F}_2), []\mathtt{F}_4 \vdash \Delta_5}\ AT \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_2, \mathtt{F}_4, []\mathtt{F}_4 \vdash \Delta_5}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_6, \mathtt{F}_2, []\mathtt{F}_4 \vdash \Delta_5}\ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet\mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_4}\ \bot_L \quad \to \quad \frac{}{\bullet\mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet\mathtt{h}_3 : \mathtt{p}_4, \Delta_6, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \mathtt{p}_4, \Delta_5}\ I \quad \to \quad \frac{}{\bullet\mathtt{h}_3 : \Delta_6, \mathtt{F}_2, \mathtt{p}_4 \vdash \Delta_5, \mathtt{p}_4}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_4}{\bullet\mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \to \mathtt{F}_2 \vdash \Delta_4}\ \top_L \quad \to \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \top_L$$

4.10 Status of $\wedge_L$: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_4 : \mathtt{F}_6, \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \mathtt{F}_7, \Delta_5}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \rightarrow \mathtt{F}_7} \rightarrow_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3, \mathtt{F}_6 \vdash \Delta_5, \mathtt{F}_7}^{\text{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \rightarrow \mathtt{F}_7} \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \mathtt{F}_6, \Delta_5 \quad \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \mathtt{F}_7, \Delta_5}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7} \wedge_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6}^{\text{ax/ind}} \quad \overline{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_7}^{\text{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7} \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_5}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_7}^{\text{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_5}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \bot, \Delta_5} \bot_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_5}^{\text{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \bot, \Delta_5} \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \top, \Delta_5} \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_3 \vdash \top, \Delta_5} \top_R$$

- Case rule $A4$

$$\frac{\mathtt{h}_3 : \Box\Gamma_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : \Box\Gamma_4, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_5, []\mathtt{F}_6} A4 \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Box\Gamma_4 \vdash \mathtt{F}_6}^{\text{ax}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \Box\Gamma_4 \vdash \Delta_5, []\mathtt{F}_6} A4$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \mathtt{F}_4, \Delta_6 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6} \rightarrow_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4}^{\text{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_6}^{\text{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6} \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6} \wedge_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6}^{\text{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6} \wedge_L$$

$$\frac{\mathtt{h}_1 : \mathtt{F}_3, \mathtt{F}_4, \Delta_2 \vdash \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5} \wedge_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5}^{\text{ax}}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5}\ \mathtt{H}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \wedge \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6} \vee_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \vdash \Delta_6}^{\text{ax/ind}} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_6}^{\text{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6} \vee_L$$

- Case rule AT

$$\dfrac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_6, []\mathtt{F}_4, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_5}{\bullet \mathtt{h}_3 : (\Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2), []\mathtt{F}_4 \vdash \Delta_5}\ AT \quad \rightarrow \quad \dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_1, \mathtt{F}_2, \mathtt{F}_4, []\mathtt{F}_4 \vdash \Delta_5}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_1, \mathtt{F}_2, []\mathtt{F}_4 \vdash \Delta_5}\ AT$$

- Case rule $\bot_L$

$$\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_4}\ \bot_L \quad \rightarrow \quad \dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_4}\ \bot_L$$

- Case rule I

$$\dfrac{}{\bullet \mathtt{h}_3 : \mathtt{p}_4, \Delta_6, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \mathtt{p}_4, \Delta_5}\ I \quad \rightarrow \quad \dfrac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_1, \mathtt{F}_2, \mathtt{p}_4 \vdash \Delta_5, \mathtt{p}_4}\ I$$

- Case rule $\top_L$

$$\dfrac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \wedge \mathtt{F}_2 \vdash \Delta_4}\ \top_L \quad \rightarrow \quad \dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_4}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1, \mathtt{F}_2 \vdash \Delta_4}\ \top_L$$

4.11 Status of $\vee_L$: (Left Premise): `Invertible`

- Case rule $\rightarrow_R$

$$\dfrac{\mathtt{h}_4 : \mathtt{F}_6, \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \mathtt{F}_7, \Delta_5}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\ \rightarrow_R \quad \rightarrow \quad \dfrac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2, \mathtt{F}_6 \vdash \Delta_5, \mathtt{F}_7}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vdash \Delta_5, \mathtt{F}_6 \rightarrow \mathtt{F}_7}\ \rightarrow_R$$

- Case rule $\wedge_R$

$$\dfrac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \mathtt{F}_6, \Delta_5 \quad \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \mathtt{F}_7, \Delta_5}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \wedge_R \quad \rightarrow \quad \dfrac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vdash \Delta_5, \mathtt{F}_6}\ {\scriptstyle \mathtt{ax/ind}} \quad \dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vdash \Delta_5, \mathtt{F}_7}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vdash \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7}\ \wedge_R$$

- Case rule $\vee_R$

$$\dfrac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_5}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7}\ \vee_R \quad \rightarrow \quad \dfrac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_7}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vdash \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7}\ \vee_R$$

- Case rule $\bot_R$

$$\dfrac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_5}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \bot, \Delta_5}\ \bot_R \quad \rightarrow \quad \dfrac{\dfrac{}{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vdash \Delta_5}\ {\scriptstyle \mathtt{ax/ind}}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vdash \bot, \Delta_5}\ \bot_R$$

- Case rule $\top_R$

$$\dfrac{}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \top, \Delta_5}\ \top_R \quad \rightarrow \quad \dfrac{}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vdash \top, \Delta_5}\ \top_R$$

- Case rule $A4$

$$\frac{\mathtt{h}_3 : \Box\Gamma_4 \vdash \mathtt{F}_6}{\bullet\mathtt{h}_3 : \Box\Gamma_4, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_5, []\mathtt{F}_6}\ A4 \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Box\Gamma_4 \vdash \mathtt{F}_6}\ \mathtt{ax}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \Box\Gamma_4 \vdash \Delta_5, []\mathtt{F}_6}\ A4$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_4, \Delta_6 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6}{\bullet\mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6}\ \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vdash \Delta_6, \mathtt{F}_4}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \vdash \Delta_6}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6}{\bullet\mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6}\ \wedge_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6}{\bullet\mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6}\ \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \vdash \Delta_6}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_5 \vdash \Delta_6}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_1, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6}\ \vee_L$$

$$\frac{\mathtt{h}_1 : \mathtt{F}_3, \Delta_2 \vdash \Delta_5 \quad \mathtt{h}_1 : \mathtt{F}_4, \Delta_2 \vdash \Delta_5}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5}\ \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5}\ \mathtt{ax}}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vdash \Delta_5}\ \mathtt{H}$$

- Case rule AT

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_6, []\mathtt{F}_4, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_5}{\bullet\mathtt{h}_3 : (\Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2), []\mathtt{F}_4 \vdash \Delta_5}\ AT \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_1, \mathtt{F}_4, []\mathtt{F}_4 \vdash \Delta_5}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \Delta_6, \mathtt{F}_1, []\mathtt{F}_4 \vdash \Delta_5}\ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet\mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \vdash \Delta_4}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet\mathtt{h}_3 : \mathtt{p}_4, \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{p}_4, \Delta_5}\ I \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_3 : \Delta_6, \mathtt{F}_1, \mathtt{p}_4 \vdash \Delta_5, \mathtt{p}_4}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4}{\bullet\mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4}\ \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vdash \Delta_4}\ \mathtt{ax/ind}}{\bullet\mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \vdash \Delta_4}\ \top_L$$

4.12 Status of $\vee_L$ (Right Premise): : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_4 : \mathtt{F}_6, \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \mathtt{F}_7, \Delta_5}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \rightarrow \mathtt{F}_7} \rightarrow_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_1, \mathtt{F}_3, \mathtt{F}_6 \vdash \Delta_5, \mathtt{F}_7}^{\ ax/ind}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \rightarrow \mathtt{F}_7} \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \mathtt{F}_6, \Delta_5 \quad \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \mathtt{F}_7, \Delta_5}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7} \wedge_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6}^{\ ax/ind} \quad \overline{\mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_7}^{\ ax/ind}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \wedge \mathtt{F}_7} \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_5}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6, \mathtt{F}_7}^{\ ax/ind}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \Delta_5, \mathtt{F}_6 \vee \mathtt{F}_7} \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_5}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \bot, \Delta_5} \bot_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \Delta_5}^{\ ax/ind}}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \bot, \Delta_5} \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \top, \Delta_5} \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_4 : \Delta_1, \mathtt{F}_3 \vdash \top, \Delta_5} \top_R$$

- Case rule $A4$

$$\frac{\mathtt{h}_3 : \Box \Gamma_4 \vdash \mathtt{F}_6}{\bullet \mathtt{h}_3 : \Box \Gamma_4, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_5, []\mathtt{F}_6} A4 \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Box \Gamma_4 \vdash \mathtt{F}_6}^{\ ax}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \Box \Gamma_4 \vdash \Delta_5, []\mathtt{F}_6} A4$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{F}_4, \Delta_6 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6} \rightarrow_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2 \vdash \Delta_6, \mathtt{F}_4}^{\ ax/ind} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_6}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \rightarrow \mathtt{F}_5 \vdash \Delta_6} \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6} \wedge_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4, \mathtt{F}_5 \vdash \Delta_6}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_6} \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_6}{\bullet \mathtt{h}_3 : (\Delta_7, \mathtt{F}_1 \vee \mathtt{F}_2), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6} \vee_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vdash \Delta_6}^{\ ax/ind} \quad \overline{\mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_5 \vdash \Delta_6}^{\ ax/ind}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_2, \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_6} \vee_L$$

$$\frac{\mathtt{h}_1 : \mathtt{F}_3, \Delta_2 \vdash \Delta_5 \quad \mathtt{h}_1 : \mathtt{F}_4, \Delta_2 \vdash \Delta_5}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5} \vee_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}^{\ ax}}{\bullet \mathtt{h}_1 : \Delta_2, \mathtt{F}_4 \vdash \Delta_5}\ {}^{\mathtt{H}}$$

- Case rule AT

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_6, []\mathtt{F}_4, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_5}{\bullet \mathtt{h}_3 : (\Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2), []\mathtt{F}_4 \vdash \Delta_5}\ AT \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_2, \mathtt{F}_4, []\mathtt{F}_4 \vdash \Delta_5}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_2, []\mathtt{F}_4 \vdash \Delta_5}\ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_3 : \mathtt{p}_4, \Delta_6, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \mathtt{p}_4, \Delta_5}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_6, \mathtt{F}_2, \mathtt{p}_4 \vdash \Delta_5, \mathtt{p}_4}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_1 \vee \mathtt{F}_2 \vdash \Delta_4}\ \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \top, \Delta_5, \mathtt{F}_2 \vdash \Delta_4}\ \top_L$$

4.13 Status of AT: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_3 : \mathtt{F}_5, \Delta_1, []\mathtt{F}_2 \vdash \mathtt{F}_6, \Delta_4}{\bullet \mathtt{h}_3 : \Delta_1, []\mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_5 \rightarrow \mathtt{F}_6}\ \rightarrow_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, \mathtt{F}_5, []\mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_5 \rightarrow \mathtt{F}_6}\ \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_1, []\mathtt{F}_2 \vdash \mathtt{F}_5, \Delta_4 \quad \mathtt{h}_3 : \Delta_1, []\mathtt{F}_2 \vdash \mathtt{F}_6, \Delta_4}{\bullet \mathtt{h}_3 : \Delta_1, []\mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_5}\ \mathtt{ax/ind} \quad \dfrac{}{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_5 \wedge \mathtt{F}_6}\ \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_1, []\mathtt{F}_2 \vdash \mathtt{F}_5, \mathtt{F}_6, \Delta_4}{\bullet \mathtt{h}_3 : \Delta_1, []\mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_5, \mathtt{F}_6}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_4, \mathtt{F}_5 \vee \mathtt{F}_6}\ \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_1, []\mathtt{F}_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \Delta_1, []\mathtt{F}_2 \vdash \bot, \Delta_4}\ \bot_R \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_3 : \Delta_1, \mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_4}\ \mathtt{ax/ind}}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{F}_2, []\mathtt{F}_2 \vdash \bot, \Delta_4}\ \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_1, []\mathtt{F}_2 \vdash \top, \Delta_4}\ \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{F}_2, []\mathtt{F}_2 \vdash \top, \Delta_4}\ \top_R$$

- Case rule $A4$

$$\frac{\mathtt{h}_2 : \Box\Gamma_6, []\mathtt{F}_1 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_2 : (\Box\Gamma_6, []\mathtt{F}_1), \Delta_3 \vdash \Delta_4, []\mathtt{F}_5}\ A4 \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Box\Gamma_6, []\mathtt{F}_1 \vdash \mathtt{F}_5}\ \mathrm{ax}}{\bullet\mathtt{h}_2 : \Delta_3, \mathtt{F}_1, \Box\Gamma_6, []\mathtt{F}_1 \vdash \Delta_4, []\mathtt{F}_5}\ A4$$

$$\frac{\mathtt{h}_2 : \Box\Gamma_3 \vdash \mathtt{F}_5}{\bullet\mathtt{h}_2 : \Box\Gamma_3, \Delta_6, []\mathtt{F}_1 \vdash \Delta_4, []\mathtt{F}_5}\ A4 \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Box\Gamma_3 \vdash \mathtt{F}_5}\ \mathrm{ax}}{\bullet\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \Box\Gamma_3, []\mathtt{F}_1 \vdash \Delta_4, []\mathtt{F}_5}\ A4$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_2 : \Delta_6, []\mathtt{F}_1 \vdash \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_2 : \mathtt{F}_4, \Delta_6, []\mathtt{F}_1 \vdash \Delta_5}{\bullet\mathtt{h}_2 : (\Delta_6, []\mathtt{F}_1), \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \Delta_5}\ \rightarrow_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, []\mathtt{F}_1 \vdash \Delta_5, \mathtt{F}_3}\ \mathrm{ax/ind} \quad \dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_4, []\mathtt{F}_1 \vdash \Delta_5}\ \mathrm{ax/ind}}{\bullet\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, []\mathtt{F}_1, \mathtt{F}_3 \rightarrow \mathtt{F}_4 \vdash \Delta_5}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_2 : \mathtt{F}_3, \mathtt{F}_4, \Delta_6, []\mathtt{F}_1 \vdash \Delta_5}{\bullet\mathtt{h}_2 : (\Delta_6, []\mathtt{F}_1), \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5}\ \wedge_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_3, \mathtt{F}_4, []\mathtt{F}_1 \vdash \Delta_5}\ \mathrm{ax/ind}}{\bullet\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, []\mathtt{F}_1, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_2 : \mathtt{F}_3, \Delta_6, []\mathtt{F}_1 \vdash \Delta_5 \quad \mathtt{h}_2 : \mathtt{F}_4, \Delta_6, []\mathtt{F}_1 \vdash \Delta_5}{\bullet\mathtt{h}_2 : (\Delta_6, []\mathtt{F}_1), \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5}\ \vee_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_3, []\mathtt{F}_1 \vdash \Delta_5}\ \mathrm{ax/ind} \quad \dfrac{}{\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, \mathtt{F}_4, []\mathtt{F}_1 \vdash \Delta_5}\ \mathrm{ax/ind}}{\bullet\mathtt{h}_2 : \Delta_6, \mathtt{F}_1, []\mathtt{F}_1, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5}\ \vee_L$$

- Case rule AT

$$\frac{\mathtt{h}_2 : \mathtt{F}_3, \Delta_5, []\mathtt{F}_1, []\mathtt{F}_3 \vdash \Delta_4}{\bullet\mathtt{h}_2 : (\Delta_5, []\mathtt{F}_1), []\mathtt{F}_3 \vdash \Delta_4}\ AT \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_5, \mathtt{F}_1, \mathtt{F}_3, []\mathtt{F}_1, []\mathtt{F}_3 \vdash \Delta_4}\ \mathrm{ax/ind}}{\bullet\mathtt{h}_2 : \Delta_5, \mathtt{F}_1, []\mathtt{F}_1, []\mathtt{F}_3 \vdash \Delta_4}\ AT$$

$$\frac{\mathtt{h}_1 : \mathtt{F}_3, \Delta_2, []\mathtt{F}_3 \vdash \Delta_4}{\bullet\mathtt{h}_1 : \Delta_2, []\mathtt{F}_3 \vdash \Delta_4}\ AT \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, \mathtt{F}_3, []\mathtt{F}_3 \vdash \Delta_4}\ \mathrm{ax/ind}}{\bullet\mathtt{h}_1 : \Delta_2, \mathtt{F}_3, []\mathtt{F}_3 \vdash \Delta_4}\ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet\mathtt{h}_2 : \bot, \Delta_4, []\mathtt{F}_1 \vdash \Delta_3}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_2 : \bot, \Delta_4, \mathtt{F}_1, []\mathtt{F}_1 \vdash \Delta_3}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet\mathtt{h}_2 : \mathtt{p}_3, \Delta_5, []\mathtt{F}_1 \vdash \mathtt{p}_3, \Delta_4}\ I \quad \rightarrow \quad \frac{}{\bullet\mathtt{h}_2 : \Delta_5, \mathtt{F}_1, \mathtt{p}_3, []\mathtt{F}_1 \vdash \Delta_4, \mathtt{p}_3}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_2 : \Delta_4, []\mathtt{F}_1 \vdash \Delta_3}{\bullet\mathtt{h}_2 : \top, \Delta_4, []\mathtt{F}_1 \vdash \Delta_3}\ \top_L \quad \rightarrow \quad \frac{\dfrac{}{\mathtt{h}_2 : \Delta_4, \mathtt{F}_1, []\mathtt{F}_1 \vdash \Delta_3}\ \mathrm{ax/ind}}{\bullet\mathtt{h}_2 : \top, \Delta_4, \mathtt{F}_1, []\mathtt{F}_1 \vdash \Delta_3}\ \top_L$$

4.14 Status of $\bot_L$: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_2 : \bot, \mathtt{F}_4, \Delta_1 \vdash \mathtt{F}_5, \Delta_3}{\bullet \mathtt{h}_2 : \bot, \Delta_1 \vdash \Delta_3, \mathtt{F}_4 \rightarrow \mathtt{F}_5} \rightarrow_R \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_2 : \bot, \Delta_1 \vdash \mathtt{F}_4, \Delta_3 \quad \mathtt{h}_2 : \bot, \Delta_1 \vdash \mathtt{F}_5, \Delta_3}{\bullet \mathtt{h}_2 : \bot, \Delta_1 \vdash \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5} \wedge_R \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_2 : \bot, \Delta_1 \vdash \mathtt{F}_4, \mathtt{F}_5, \Delta_3}{\bullet \mathtt{h}_2 : \bot, \Delta_1 \vdash \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5} \vee_R \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_2 : \bot, \Delta_1 \vdash \Delta_3}{\bullet \mathtt{h}_2 : \bot, \Delta_1 \vdash \bot, \Delta_3} \bot_R \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_2 : \bot, \Delta_1 \vdash \top, \Delta_3} \top_R \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $A4$

$$\frac{\mathtt{h}_1 : \Box\Gamma_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Box\Gamma_2, \bot, \Delta_5 \vdash \Delta_3, []\mathtt{F}_4} A4 \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_1 : \bot, \Delta_5 \vdash \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 : \bot, \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\bot, \Delta_5), \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4} \rightarrow_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \bot, \mathtt{F}_2, \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\bot, \Delta_5), \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4} \wedge_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_1 : \bot, \mathtt{F}_2, \Delta_5 \vdash \Delta_4 \quad \mathtt{h}_1 : \bot, \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\bot, \Delta_5), \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4} \vee_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule AT

$$\frac{\mathtt{h}_1 : \bot, \mathtt{F}_2, \Delta_4, []\mathtt{F}_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : (\bot, \Delta_4), []\mathtt{F}_2 \vdash \Delta_3} AT \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_2 \vdash \Delta_3}\ \bot_L \quad \to \quad \texttt{trivial}$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_1 : \mathtt{p}_2, \bot, \Delta_4 \vdash \mathtt{p}_2, \Delta_3}\ I \quad \to \quad \texttt{trivial}$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2}{\bullet \mathtt{h}_1 : \top, \bot, \Delta_3 \vdash \Delta_2}\ \top_L \quad \to \quad \texttt{trivial}$$

4.15 Status of I: : `Invertible`

- Case rule $\to_R$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_1, \mathtt{p}_2 \vdash \mathtt{F}_5, \Delta_6, \mathtt{p}_2}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{p}_2 \vdash (\Delta_6, \mathtt{p}_2), \mathtt{F}_4 \to \mathtt{F}_5}\ \to_R \quad \to \quad \texttt{trivial}$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_1, \mathtt{p}_2 \vdash \mathtt{F}_4, \Delta_6, \mathtt{p}_2 \quad \mathtt{h}_3 : \Delta_1, \mathtt{p}_2 \vdash \mathtt{F}_5, \Delta_6, \mathtt{p}_2}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{p}_2 \vdash (\Delta_6, \mathtt{p}_2), \mathtt{F}_4 \wedge \mathtt{F}_5}\ \wedge_R \quad \to \quad \texttt{trivial}$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_1, \mathtt{p}_2 \vdash \mathtt{F}_4, \mathtt{F}_5, \Delta_6, \mathtt{p}_2}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{p}_2 \vdash (\Delta_6, \mathtt{p}_2), \mathtt{F}_4 \vee \mathtt{F}_5}\ \vee_R \quad \to \quad \texttt{trivial}$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_1, \mathtt{p}_2 \vdash \Delta_4, \mathtt{p}_2}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{p}_2 \vdash \bot, \Delta_4, \mathtt{p}_2}\ \bot_R \quad \to \quad \texttt{trivial}$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_1, \mathtt{p}_2 \vdash \top, \Delta_4, \mathtt{p}_2}\ \top_R \quad \to \quad \texttt{trivial}$$

- Case rule $A4$

$$\frac{\mathtt{h}_2 : \Box\Gamma_3 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_2 : \Box\Gamma_3, \Delta_6, \mathtt{p}_1 \vdash (\Delta_5, \mathtt{p}_1), []\mathtt{F}_4}\ A4 \quad \to \quad \texttt{trivial}$$

- Case rule $\to_L$

$$\frac{\mathtt{h}_3 : \Delta_6, \mathtt{p}_1 \vdash \mathtt{F}_4, \Delta_2, \mathtt{p}_1 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_6, \mathtt{p}_1 \vdash \Delta_2, \mathtt{p}_1}{\bullet \mathtt{h}_3 : (\Delta_6, \mathtt{p}_1), \mathtt{F}_4 \to \mathtt{F}_5 \vdash \Delta_2, \mathtt{p}_1}\ \to_L \quad \to \quad \texttt{trivial}$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \mathtt{F}_5, \Delta_6, \mathtt{p}_1 \vdash \Delta_2, \mathtt{p}_1}{\bullet \mathtt{h}_3 : (\Delta_6, \mathtt{p}_1), \mathtt{F}_4 \wedge \mathtt{F}_5 \vdash \Delta_2, \mathtt{p}_1}\ \wedge_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_6, \mathtt{p}_1 \vdash \Delta_2, \mathtt{p}_1 \quad \mathtt{h}_3 : \mathtt{F}_5, \Delta_6, \mathtt{p}_1 \vdash \Delta_2, \mathtt{p}_1}{\bullet \mathtt{h}_3 : (\Delta_6, \mathtt{p}_1), \mathtt{F}_4 \vee \mathtt{F}_5 \vdash \Delta_2, \mathtt{p}_1}\ \vee_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule AT

$$\frac{\mathtt{h}_3 : \mathtt{F}_4, \Delta_5, \mathtt{p}_1, []\mathtt{F}_4 \vdash \Delta_2, \mathtt{p}_1}{\bullet \mathtt{h}_3 : (\Delta_5, \mathtt{p}_1), []\mathtt{F}_4 \vdash \Delta_2, \mathtt{p}_1}\ AT \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_3 : \bot, \Delta_4, \mathtt{p}_1 \vdash \Delta_2, \mathtt{p}_1}\ \bot_L \quad \rightarrow \quad \texttt{trivial}$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_2 : \mathtt{p}_3, \Delta_5, \mathtt{p}_1 \vdash \mathtt{p}_3, \Delta_4, \mathtt{p}_1}\ I \quad \rightarrow \quad \texttt{trivial}$$

$$\frac{}{\bullet \mathtt{h}_1 : \mathtt{p}_3, \Delta_2 \vdash \mathtt{p}_3, \Delta_4}\ I \quad \rightarrow \quad \texttt{trivial}$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_3 : \Delta_4, \mathtt{p}_1 \vdash \Delta_2, \mathtt{p}_1}{\bullet \mathtt{h}_3 : \top, \Delta_4, \mathtt{p}_1 \vdash \Delta_2, \mathtt{p}_1}\ \top_L \quad \rightarrow \quad \texttt{trivial}$$

4.16 Status of $\top_L$: : `Invertible`

- Case rule $\rightarrow_R$

$$\frac{\mathtt{h}_2 : \top, \mathtt{F}_4, \Delta_1 \vdash \mathtt{F}_5, \Delta_3}{\bullet \mathtt{h}_2 : \top, \Delta_1 \vdash \Delta_3, \mathtt{F}_4 \rightarrow \mathtt{F}_5}\ \rightarrow_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : \Delta_1, \mathtt{F}_4 \vdash \Delta_3, \mathtt{F}_5}^{\ \texttt{ax/ind}}}{\bullet \mathtt{h}_2 : \Delta_1 \vdash \Delta_3, \mathtt{F}_4 \rightarrow \mathtt{F}_5}\ \rightarrow_R$$

- Case rule $\wedge_R$

$$\frac{\mathtt{h}_2 : \top, \Delta_1 \vdash \mathtt{F}_4, \Delta_3 \quad \mathtt{h}_2 : \top, \Delta_1 \vdash \mathtt{F}_5, \Delta_3}{\bullet \mathtt{h}_2 : \top, \Delta_1 \vdash \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5}\ \wedge_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : \Delta_1 \vdash \Delta_3, \mathtt{F}_4}^{\ \texttt{ax/ind}} \quad \overline{\mathtt{h}_2 : \Delta_1 \vdash \Delta_3, \mathtt{F}_5}^{\ \texttt{ax/ind}}}{\bullet \mathtt{h}_2 : \Delta_1 \vdash \Delta_3, \mathtt{F}_4 \wedge \mathtt{F}_5}\ \wedge_R$$

- Case rule $\vee_R$

$$\frac{\mathtt{h}_2 : \top, \Delta_1 \vdash \mathtt{F}_4, \mathtt{F}_5, \Delta_3}{\bullet \mathtt{h}_2 : \top, \Delta_1 \vdash \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5}\ \vee_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : \Delta_1 \vdash \Delta_3, \mathtt{F}_4, \mathtt{F}_5}^{\ \texttt{ax/ind}}}{\bullet \mathtt{h}_2 : \Delta_1 \vdash \Delta_3, \mathtt{F}_4 \vee \mathtt{F}_5}\ \vee_R$$

- Case rule $\bot_R$

$$\frac{\mathtt{h}_2 : \top, \Delta_1 \vdash \Delta_3}{\bullet \mathtt{h}_2 : \top, \Delta_1 \vdash \bot, \Delta_3}\ \bot_R \quad \rightarrow \quad \frac{\overline{\mathtt{h}_2 : \Delta_1 \vdash \Delta_3}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_2 : \Delta_1 \vdash \bot, \Delta_3}\ \bot_R$$

- Case rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_2 : \top, \Delta_1 \vdash \top, \Delta_3}\ \top_R \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_2 : \Delta_1 \vdash \top, \Delta_3}\ \top_R$$

- Case rule $A4$

$$\frac{\mathtt{h}_1 : \Box\Gamma_2 \vdash \mathtt{F}_4}{\bullet \mathtt{h}_1 : \Box\Gamma_2, \top, \Delta_5 \vdash \Delta_3, []\mathtt{F}_4}\ A4 \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Box\Gamma_2 \vdash \mathtt{F}_4}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_5, \Box\Gamma_2 \vdash \Delta_3, []\mathtt{F}_4}\ A4$$

- Case rule $\rightarrow_L$

$$\frac{\mathtt{h}_1 : \top, \Delta_5 \vdash \mathtt{F}_2, \Delta_4 \quad \mathtt{h}_1 : \top, \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\top, \Delta_5), \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4}\ \rightarrow_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_5 \vdash \Delta_4, \mathtt{F}_2}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \rightarrow \mathtt{F}_3 \vdash \Delta_4}\ \rightarrow_L$$

- Case rule $\wedge_L$

$$\frac{\mathtt{h}_1 : \top, \mathtt{F}_2, \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\top, \Delta_5), \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4}\ \wedge_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2, \mathtt{F}_3 \vdash \Delta_4}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \wedge \mathtt{F}_3 \vdash \Delta_4}\ \wedge_L$$

- Case rule $\vee_L$

$$\frac{\mathtt{h}_1 : \top, \mathtt{F}_2, \Delta_5 \vdash \Delta_4 \quad \mathtt{h}_1 : \top, \mathtt{F}_3, \Delta_5 \vdash \Delta_4}{\bullet \mathtt{h}_1 : (\top, \Delta_5), \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4}\ \vee_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vdash \Delta_4}^{\ \mathtt{ax/ind}} \quad \overline{\mathtt{h}_1 : \Delta_5, \mathtt{F}_3 \vdash \Delta_4}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_2 \vee \mathtt{F}_3 \vdash \Delta_4}\ \vee_L$$

- Case rule AT

$$\frac{\mathtt{h}_1 : \top, \mathtt{F}_2, \Delta_4, []\mathtt{F}_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : (\top, \Delta_4), []\mathtt{F}_2 \vdash \Delta_3}\ AT \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_4, \mathtt{F}_2, []\mathtt{F}_2 \vdash \Delta_3}^{\ \mathtt{ax/ind}}}{\bullet \mathtt{h}_1 : \Delta_4, []\mathtt{F}_2 \vdash \Delta_3}\ AT$$

- Case rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_1 : \bot, \top, \Delta_3 \vdash \Delta_2}\ \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \bot, \Delta_3 \vdash \Delta_2}\ \bot_L$$

- Case rule I

$$\frac{}{\bullet \mathtt{h}_1 : \mathtt{p}_2, \top, \Delta_4 \vdash \mathtt{p}_2, \Delta_3}\ I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{p}_2 \vdash \Delta_3, \mathtt{p}_2}\ I$$

- Case rule $\top_L$

$$\frac{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}{\bullet \mathtt{h}_1 : \top, \Delta_2 \vdash \Delta_3}\ \top_L \quad \rightarrow \quad \frac{\overline{\mathtt{h}_1 : \Delta_2 \vdash \Delta_3}^{\ \mathtt{ax}}}{\bullet \mathtt{h}_1 : \Delta_2 \vdash \Delta_3}\ \mathtt{H}$$

5 Height preserving admissibility of contraction on the left

- Case(s) rule $\to_R$

$$\frac{\mathtt{h}_3 : \mathtt{F}_5, \Delta_1, \Delta_2, \Delta_2 \vdash \mathtt{F}_6, \Delta_4}{\bullet \mathtt{h}_3 : \Delta_1, \Delta_2, \Delta_2 \vdash \Delta_4, \mathtt{F}_5 \to \mathtt{F}_6} \to_R \quad \to \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_1, \Delta_2, \Delta_2, \mathtt{F}_5 \vdash \Delta_4, \mathtt{F}_6}\,\mathtt{ax}}{\mathtt{h}_3 : \Delta_1, \Delta_2, \mathtt{F}_5 \vdash \Delta_4, \mathtt{F}_6}\,\mathtt{IH}}{\bullet \mathtt{h}_3 : \Delta_1, \Delta_2 \vdash \Delta_4, \mathtt{F}_5 \to \mathtt{F}_6} \to_R$$

- Case(s) rule $\wedge_R$

$$\frac{\mathtt{h}_3 : \Delta_1, \Delta_2, \Delta_2 \vdash \mathtt{F}_5, \Delta_4 \quad \mathtt{h}_3 : \Delta_1, \Delta_2, \Delta_2 \vdash \mathtt{F}_6, \Delta_4}{\bullet \mathtt{h}_3 : \Delta_1, \Delta_2, \Delta_2 \vdash \Delta_4, \mathtt{F}_5 \wedge \mathtt{F}_6} \wedge_R \quad \to \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_1, \Delta_2, \Delta_2 \vdash \Delta_4, \mathtt{F}_5}\,\mathtt{ax}}{\mathtt{h}_3 : \Delta_1, \Delta_2 \vdash \Delta_4, \mathtt{F}_5}\,\mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_1, \Delta_2, \Delta_2 \vdash \Delta_4, \mathtt{F}_6}\,\mathtt{ax}}{\mathtt{h}_3 : \Delta_1, \Delta_2 \vdash \Delta_4, \mathtt{F}_6}\,\mathtt{IH}}{\bullet \mathtt{h}_3 : \Delta_1, \Delta_2 \vdash \Delta_4, \mathtt{F}_5 \wedge \mathtt{F}_6} \wedge_R$$

- Case(s) rule $\vee_R$

$$\frac{\mathtt{h}_3 : \Delta_1, \Delta_2, \Delta_2 \vdash \mathtt{F}_5, \mathtt{F}_6, \Delta_4}{\bullet \mathtt{h}_3 : \Delta_1, \Delta_2, \Delta_2 \vdash \Delta_4, \mathtt{F}_5 \vee \mathtt{F}_6} \vee_R \quad \to \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_1, \Delta_2, \Delta_2 \vdash \Delta_4, \mathtt{F}_5, \mathtt{F}_6}\,\mathtt{ax}}{\mathtt{h}_3 : \Delta_1, \Delta_2 \vdash \Delta_4, \mathtt{F}_5, \mathtt{F}_6}\,\mathtt{IH}}{\bullet \mathtt{h}_3 : \Delta_1, \Delta_2 \vdash \Delta_4, \mathtt{F}_5 \vee \mathtt{F}_6} \vee_R$$

- Case(s) rule $\bot_R$

$$\frac{\mathtt{h}_3 : \Delta_1, \Delta_2, \Delta_2 \vdash \Delta_4}{\bullet \mathtt{h}_3 : \Delta_1, \Delta_2, \Delta_2 \vdash \bot, \Delta_4} \bot_R \quad \to \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_1, \Delta_2, \Delta_2 \vdash \Delta_4}\,\mathtt{ax}}{\mathtt{h}_3 : \Delta_1, \Delta_2 \vdash \Delta_4}\,\mathtt{IH}}{\bullet \mathtt{h}_3 : \Delta_1, \Delta_2 \vdash \bot, \Delta_4} \bot_R$$

- Case(s) rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_3 : \Delta_1, \Delta_2, \Delta_2 \vdash \top, \Delta_4} \top_R \quad \to \quad \frac{}{\bullet \mathtt{h}_3 : \Delta_1, \Delta_2 \vdash \top, \Delta_4} \top_R$$

- Case(s) rule $A4$

$$\frac{\mathtt{h}_1 : \Box\Gamma_4, \Box\Gamma_5, \Box\Gamma_5, \Box\Gamma_6 \vdash \mathtt{F}_3}{\bullet \mathtt{h}_1 : (\Box\Gamma_4, \Delta_7), (\Box\Gamma_5, \Box\Gamma_6, \Delta_8), \Box\Gamma_5, \Box\Gamma_6, \Delta_8 \vdash \Delta_2, []\mathtt{F}_3} A4 \quad \to \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_4, \Box\Gamma_5, \Box\Gamma_5, \Box\Gamma_6 \vdash \mathtt{F}_3}\,\mathtt{ax}}{\mathtt{h}_1 : \Box\Gamma_4, \Box\Gamma_5, \Box\Gamma_6 \vdash \mathtt{F}_3}\,\mathtt{IH}}{\bullet \mathtt{h}_1 : \Delta_7, \Delta_8, \Box\Gamma_4, \Box\Gamma_5, \Box\Gamma_6 \vdash \Delta_2, []\mathtt{F}_3} A4$$

- Case(s) rule $\to_L$

$$\frac{\mathtt{h}_2 : \Delta_1, \Delta_6, \Delta_6, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_2 : \mathtt{F}_4, \Delta_1, \Delta_6, \Delta_6, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \Delta_5}{\bullet \mathtt{h}_2 : \Delta_1, (\Delta_6, \mathtt{F}_3 \to \mathtt{F}_4), \Delta_6, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \Delta_5} \to_L \quad \to \quad \dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_1, \Delta_6, \Delta_6 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_3}\,\mathtt{inv\text{-}th/ax}}{\mathtt{h}_2 : \Delta_1, \Delta_6 \vdash \Delta_5, \mathtt{F}_3, \mathtt{F}_3}\,\mathtt{IH}}{\mathtt{h}_2 : \Delta_1, \Delta_6 \vdash \Delta_5, \mathtt{F}_3}\,\mathtt{IH\text{-}Mutual} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_1, \Delta_6, \Delta_6, \mathtt{F}_4, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{inv\text{-}th/ax}}{\mathtt{h}_2 : \Delta_1, \Delta_6, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{IH}}{\bullet \mathtt{h}_2 : \Delta_1, \Delta_6, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \Delta_5} \to_L$$

$$\frac{\mathtt{h}_2 : \Delta_1, \Delta_1, \Delta_6 \vdash \mathtt{F}_3, \Delta_5 \quad \mathtt{h}_2 : \mathtt{F}_4, \Delta_1, \Delta_1, \Delta_6 \vdash \Delta_5}{\bullet \mathtt{h}_2 : (\Delta_6, \mathtt{F}_3 \to \mathtt{F}_4), \Delta_1, \Delta_1 \vdash \Delta_5} \to_L \quad \to \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_1, \Delta_1, \Delta_6 \vdash \Delta_5, \mathtt{F}_3}\,\mathtt{ax}}{\mathtt{h}_2 : \Delta_1, \Delta_6 \vdash \Delta_5, \mathtt{F}_3}\,\mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_1, \Delta_1, \Delta_6, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{ax}}{\mathtt{h}_2 : \Delta_1, \Delta_6, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{IH}}{\bullet \mathtt{h}_2 : \Delta_1, \Delta_6, \mathtt{F}_3 \to \mathtt{F}_4 \vdash \Delta_5} \to_L$$

- Case(s) rule $\wedge_L$

$$\frac{\mathtt{h}_2 : \mathtt{F}_3, \mathtt{F}_4, \Delta_1, \Delta_6, \Delta_6, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5}{\bullet \mathtt{h}_2 : \Delta_1, (\Delta_6, \mathtt{F}_3 \wedge \mathtt{F}_4), \Delta_6, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5} \wedge_L \quad \to \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_1, \Delta_6, \Delta_6, \mathtt{F}_3, \mathtt{F}_3, \mathtt{F}_4, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{inv\text{-}th/ax}}{\mathtt{h}_2 : \Delta_1, \Delta_6, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{IH}}{\bullet \mathtt{h}_2 : \Delta_1, \Delta_6, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5} \wedge_L$$

$$\frac{\mathtt{h}_2 : \mathtt{F}_3, \mathtt{F}_4, \Delta_1, \Delta_1, \Delta_6 \vdash \Delta_5}{\bullet \mathtt{h}_2 : (\Delta_6, \mathtt{F}_3 \wedge \mathtt{F}_4), \Delta_1, \Delta_1 \vdash \Delta_5} \wedge_L \quad \to \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_1, \Delta_1, \Delta_6, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{ax}}{\mathtt{h}_2 : \Delta_1, \Delta_6, \mathtt{F}_3, \mathtt{F}_4 \vdash \Delta_5}\,\mathtt{IH}}{\bullet \mathtt{h}_2 : \Delta_1, \Delta_6, \mathtt{F}_3 \wedge \mathtt{F}_4 \vdash \Delta_5} \wedge_L$$

- Case(s) rule $\vee_L$

$$\frac{\mathtt{h}_2 : \mathtt{F}_3, \Delta_1, \Delta_6, \Delta_6, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5 \quad \mathtt{h}_2 : \mathtt{F}_4, \Delta_1, \Delta_6, \Delta_6, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5}{\bullet \mathtt{h}_2 : \Delta_1, (\Delta_6, \mathtt{F}_3 \vee \mathtt{F}_4), \Delta_6, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5} \vee_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_1, \Delta_6, \Delta_6, \mathtt{F}_3, \mathtt{F}_3 \vdash \Delta_5}\,\text{inv-th/ax}}{\mathtt{h}_2 : \Delta_1, \Delta_6, \mathtt{F}_3 \vdash \Delta_5}\,\text{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_1, \Delta_6, \Delta_6, \mathtt{F}_4, \mathtt{F}_4 \vdash \Delta_5}\,\text{inv-th/ax}}{\mathtt{h}_2 : \Delta_1, \Delta_6, \mathtt{F}_4 \vdash \Delta_5}\,\text{IH}}{\bullet \mathtt{h}_2 : \Delta_1, \Delta_6, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5} \vee_L$$

$$\frac{\mathtt{h}_2 : \mathtt{F}_3, \Delta_1, \Delta_1, \Delta_6 \vdash \Delta_5 \quad \mathtt{h}_2 : \mathtt{F}_4, \Delta_1, \Delta_1, \Delta_6 \vdash \Delta_5}{\bullet \mathtt{h}_2 : (\Delta_6, \mathtt{F}_3 \vee \mathtt{F}_4), \Delta_1, \Delta_1 \vdash \Delta_5} \vee_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_1, \Delta_1, \Delta_6, \mathtt{F}_3 \vdash \Delta_5}\,\text{ax}}{\mathtt{h}_2 : \Delta_1, \Delta_6, \mathtt{F}_3 \vdash \Delta_5}\,\text{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_1, \Delta_1, \Delta_6, \mathtt{F}_4 \vdash \Delta_5}\,\text{ax}}{\mathtt{h}_2 : \Delta_1, \Delta_6, \mathtt{F}_4 \vdash \Delta_5}\,\text{IH}}{\bullet \mathtt{h}_2 : \Delta_1, \Delta_6, \mathtt{F}_3 \vee \mathtt{F}_4 \vdash \Delta_5} \vee_L$$

- Case(s) rule AT

$$\frac{\mathtt{h}_2 : \mathtt{F}_3, \Delta_1, \Delta_5, \Delta_5, []\mathtt{F}_3, []\mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_2 : \Delta_1, (\Delta_5, []\mathtt{F}_3), \Delta_5, []\mathtt{F}_3 \vdash \Delta_4} AT \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_1, \Delta_5, \Delta_5, \mathtt{F}_3, []\mathtt{F}_3, []\mathtt{F}_3 \vdash \Delta_4}\,\text{ax}}{\mathtt{h}_2 : \Delta_1, \Delta_5, \mathtt{F}_3, []\mathtt{F}_3 \vdash \Delta_4}\,\text{IH}}{\bullet \mathtt{h}_2 : \Delta_1, \Delta_5, []\mathtt{F}_3 \vdash \Delta_4} AT$$

$$\frac{\mathtt{h}_2 : \mathtt{F}_3, \Delta_1, \Delta_1, \Delta_5, []\mathtt{F}_3 \vdash \Delta_4}{\bullet \mathtt{h}_2 : (\Delta_5, []\mathtt{F}_3), \Delta_1, \Delta_1 \vdash \Delta_4} AT \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_1, \Delta_1, \Delta_5, \mathtt{F}_3, []\mathtt{F}_3 \vdash \Delta_4}\,\text{ax}}{\mathtt{h}_2 : \Delta_1, \Delta_5, \mathtt{F}_3, []\mathtt{F}_3 \vdash \Delta_4}\,\text{IH}}{\bullet \mathtt{h}_2 : \Delta_1, \Delta_5, []\mathtt{F}_3 \vdash \Delta_4} AT$$

- Case(s) rule $\bot_L$

$$\frac{}{\bullet \mathtt{h}_2 : (\bot, \Delta_4), \Delta_1, \Delta_1 \vdash \Delta_3} \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_2 : \bot, \Delta_1, \Delta_4 \vdash \Delta_3} \bot_L$$

$$\frac{}{\bullet \mathtt{h}_2 : \Delta_1, (\bot, \Delta_4), \bot, \Delta_4 \vdash \Delta_3} \bot_L \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_2 : \bot, \Delta_1, \Delta_4 \vdash \Delta_3} \bot_L$$

- Case(s) rule I

$$\frac{}{\bullet \mathtt{h}_2 : (\Delta_5, \mathtt{p}_3), \Delta_1, \Delta_1 \vdash \Delta_4, \mathtt{p}_3} I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_2 : \Delta_1, \Delta_5, \mathtt{p}_3 \vdash \Delta_4, \mathtt{p}_3} I$$

$$\frac{}{\bullet \mathtt{h}_2 : \Delta_1, (\Delta_5, \mathtt{p}_3), \Delta_5, \mathtt{p}_3 \vdash \Delta_4, \mathtt{p}_3} I \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_2 : \Delta_1, \Delta_5, \mathtt{p}_3 \vdash \Delta_4, \mathtt{p}_3} I$$

- Case(s) rule $\top_L$

$$\frac{\mathtt{h}_2 : \Delta_1, \Delta_1, \Delta_4 \vdash \Delta_3}{\bullet \mathtt{h}_2 : (\top, \Delta_4), \Delta_1, \Delta_1 \vdash \Delta_3} \top_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_1, \Delta_1, \Delta_4 \vdash \Delta_3}\,\text{ax}}{\mathtt{h}_2 : \Delta_1, \Delta_4 \vdash \Delta_3}\,\text{IH}}{\bullet \mathtt{h}_2 : \top, \Delta_1, \Delta_4 \vdash \Delta_3} \top_L$$

$$\frac{\mathtt{h}_2 : \top, \Delta_1, \Delta_4, \Delta_4 \vdash \Delta_3}{\bullet \mathtt{h}_2 : \Delta_1, (\top, \Delta_4), \top, \Delta_4 \vdash \Delta_3} \top_L \quad \rightarrow \quad \frac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_1, \Delta_4, \Delta_4 \vdash \Delta_3}\,\text{inv-th/ax}}{\mathtt{h}_2 : \Delta_1, \Delta_4 \vdash \Delta_3}\,\text{IH}}{\bullet \mathtt{h}_2 : \top, \Delta_1, \Delta_4 \vdash \Delta_3} \top_L$$

6 Height preserving admissibility of contraction on the Right

- Case(s) rule $\rightarrow_R$

$$\frac{\mathtt{h}_2 : \mathtt{F}_4, \Delta_3 \vdash \mathtt{F}_5, \Delta_1, \Delta_6, \Delta_6, \mathtt{F}_4 \rightarrow \mathtt{F}_5}{\bullet \mathtt{h}_2 : \Delta_3 \vdash \Delta_1, (\Delta_6, \mathtt{F}_4 \rightarrow \mathtt{F}_5), \Delta_6, \mathtt{F}_4 \rightarrow \mathtt{F}_5}\ {\scriptstyle \rightarrow_R} \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4, \mathtt{F}_4 \vdash \Delta_1, \Delta_6, \Delta_6, \mathtt{F}_5, \mathtt{F}_5}\ {\scriptstyle \text{inv-th/ax}}}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash \Delta_1, \Delta_6, \Delta_6, \mathtt{F}_5, \mathtt{F}_5}\ {\scriptstyle \text{IH-Mutual}}}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash \Delta_1, \Delta_6, \mathtt{F}_5}\ {\scriptstyle \text{IH}}}{\bullet \mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \Delta_6, \mathtt{F}_4 \rightarrow \mathtt{F}_5}\ {\scriptstyle \rightarrow_R}$$

$$\frac{\mathtt{h}_2 : \mathtt{F}_4, \Delta_3 \vdash \mathtt{F}_5, \Delta_1, \Delta_1, \Delta_6}{\bullet \mathtt{h}_2 : \Delta_3 \vdash (\Delta_6, \mathtt{F}_4 \rightarrow \mathtt{F}_5), \Delta_1, \Delta_1}\ {\scriptstyle \rightarrow_R} \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash \Delta_1, \Delta_1, \Delta_6, \mathtt{F}_5}\ {\scriptstyle \text{ax}}}{\mathtt{h}_2 : \Delta_3, \mathtt{F}_4 \vdash \Delta_1, \Delta_6, \mathtt{F}_5}\ {\scriptstyle \text{IH}}}{\bullet \mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \Delta_6, \mathtt{F}_4 \rightarrow \mathtt{F}_5}\ {\scriptstyle \rightarrow_R}$$

- Case(s) rule $\wedge_R$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_4, \Delta_1, \Delta_6, \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5 \quad \mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_5, \Delta_1, \Delta_6, \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5}{\bullet \mathtt{h}_2 : \Delta_3 \vdash \Delta_1, (\Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5), \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5}\ {\scriptstyle \wedge_R} \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \Delta_6, \Delta_6, \mathtt{F}_4, \mathtt{F}_4}\ {\scriptstyle \text{inv-th/ax}}}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \Delta_6, \mathtt{F}_4}\ {\scriptstyle \text{IH}} \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \Delta_6, \Delta_6, \mathtt{F}_5, \mathtt{F}_5}\ {\scriptstyle \text{inv-th/ax}}}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \Delta_6, \mathtt{F}_5}\ {\scriptstyle \text{IH}}}{\bullet \mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5}\ {\scriptstyle \wedge_R}$$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_4, \Delta_1, \Delta_1, \Delta_6 \quad \mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_5, \Delta_1, \Delta_1, \Delta_6}{\bullet \mathtt{h}_2 : \Delta_3 \vdash (\Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5), \Delta_1, \Delta_1}\ {\scriptstyle \wedge_R} \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \Delta_1, \Delta_6, \mathtt{F}_4}\ {\scriptstyle \text{ax}}}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \Delta_6, \mathtt{F}_4}\ {\scriptstyle \text{IH}} \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \Delta_1, \Delta_6, \mathtt{F}_5}\ {\scriptstyle \text{ax}}}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \Delta_6, \mathtt{F}_5}\ {\scriptstyle \text{IH}}}{\bullet \mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \Delta_6, \mathtt{F}_4 \wedge \mathtt{F}_5}\ {\scriptstyle \wedge_R}$$

- Case(s) rule $\vee_R$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_4, \mathtt{F}_5, \Delta_1, \Delta_6, \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5}{\bullet \mathtt{h}_2 : \Delta_3 \vdash \Delta_1, (\Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5), \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5}\ {\scriptstyle \vee_R} \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \Delta_6, \Delta_6, \mathtt{F}_4, \mathtt{F}_4, \mathtt{F}_5, \mathtt{F}_5}\ {\scriptstyle \text{inv-th/ax}}}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \Delta_6, \mathtt{F}_4, \mathtt{F}_5}\ {\scriptstyle \text{IH}}}{\bullet \mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5}\ {\scriptstyle \vee_R}$$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \mathtt{F}_4, \mathtt{F}_5, \Delta_1, \Delta_1, \Delta_6}{\bullet \mathtt{h}_2 : \Delta_3 \vdash (\Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5), \Delta_1, \Delta_1}\ {\scriptstyle \vee_R} \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \Delta_1, \Delta_6, \mathtt{F}_4, \mathtt{F}_5}\ {\scriptstyle \text{ax}}}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \Delta_6, \mathtt{F}_4, \mathtt{F}_5}\ {\scriptstyle \text{IH}}}{\bullet \mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \Delta_6, \mathtt{F}_4 \vee \mathtt{F}_5}\ {\scriptstyle \vee_R}$$

- Case(s) rule $\bot_R$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \Delta_1, \Delta_4}{\bullet \mathtt{h}_2 : \Delta_3 \vdash (\bot, \Delta_4), \Delta_1, \Delta_1}\ {\scriptstyle \bot_R} \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \Delta_1, \Delta_4}\ {\scriptstyle \text{ax}}}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \Delta_4}\ {\scriptstyle \text{IH}}}{\bullet \mathtt{h}_2 : \Delta_3 \vdash \bot, \Delta_1, \Delta_4}\ {\scriptstyle \bot_R}$$

$$\frac{\mathtt{h}_2 : \Delta_3 \vdash \bot, \Delta_1, \Delta_4, \Delta_4}{\bullet \mathtt{h}_2 : \Delta_3 \vdash \Delta_1, (\bot, \Delta_4), \bot, \Delta_4}\ {\scriptstyle \bot_R} \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \Delta_4, \Delta_4}\ {\scriptstyle \text{inv-th/ax}}}{\mathtt{h}_2 : \Delta_3 \vdash \Delta_1, \Delta_4}\ {\scriptstyle \text{IH}}}{\bullet \mathtt{h}_2 : \Delta_3 \vdash \bot, \Delta_1, \Delta_4}\ {\scriptstyle \bot_R}$$

- Case(s) rule $\top_R$

$$\frac{}{\bullet \mathtt{h}_2 : \Delta_3 \vdash (\top, \Delta_4), \Delta_1, \Delta_1}\ {\scriptstyle \top_R} \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_2 : \Delta_3 \vdash \top, \Delta_1, \Delta_4}\ {\scriptstyle \top_R}$$

$$\frac{}{\bullet \mathtt{h}_2 : \Delta_3 \vdash \Delta_1, (\top, \Delta_4), \top, \Delta_4}\ {\scriptstyle \top_R} \quad \rightarrow \quad \frac{}{\bullet \mathtt{h}_2 : \Delta_3 \vdash \top, \Delta_1, \Delta_4}\ {\scriptstyle \top_R}$$

- Case(s) rule $A4$

$$\frac{\mathtt{h}_2 : \Box\Gamma_3 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_2 : \Box\Gamma_3, \Delta_4 \vdash \Delta_1, (\Delta_6, []\mathtt{F}_5), \Delta_6, []\mathtt{F}_5}\ {\scriptstyle A4} \quad \rightarrow \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Box\Gamma_3 \vdash \mathtt{F}_5}\ {\scriptstyle \text{ax}}}{\bullet \mathtt{h}_2 : \Delta_4, \Box\Gamma_3 \vdash \Delta_1, \Delta_6, []\mathtt{F}_5}\ {\scriptstyle A4}$$

$$\frac{\mathtt{h}_2 : \Box\Gamma_3 \vdash \mathtt{F}_5}{\bullet \mathtt{h}_2 : \Box\Gamma_3, \Delta_4 \vdash (\Delta_6, []\mathtt{F}_5), \Delta_1, \Delta_1}\ {\scriptstyle A4} \quad \rightarrow \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Box\Gamma_3 \vdash \mathtt{F}_5}\ {\scriptstyle \text{ax}}}{\bullet \mathtt{h}_2 : \Delta_4, \Box\Gamma_3 \vdash \Delta_1, \Delta_6, []\mathtt{F}_5}\ {\scriptstyle A4}$$

- Case(s) rule $\rightarrow_L$

$$\dfrac{\mathtt{h}_3 : \Delta_4 \vdash \mathtt{F}_5, \Delta_1, \Delta_2, \Delta_2 \quad \mathtt{h}_3 : \mathtt{F}_6, \Delta_4 \vdash \Delta_1, \Delta_2, \Delta_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \rightarrow \mathtt{F}_6 \vdash \Delta_1, \Delta_2, \Delta_2}\ \rightarrow_L \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_1, \Delta_2, \Delta_2, \mathtt{F}_5}\ \mathtt{ax}}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_1, \Delta_2, \mathtt{F}_5}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \Delta_1, \Delta_2, \Delta_2}\ \mathtt{ax}}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \Delta_1, \Delta_2}\ \mathtt{IH}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \rightarrow \mathtt{F}_6 \vdash \Delta_1, \Delta_2}\ \rightarrow_L$$

- Case(s) rule $\wedge_L$

$$\dfrac{\mathtt{h}_3 : \mathtt{F}_5, \mathtt{F}_6, \Delta_4 \vdash \Delta_1, \Delta_2, \Delta_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_1, \Delta_2, \Delta_2}\ \wedge_L \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \Delta_1, \Delta_2, \Delta_2}\ \mathtt{ax}}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, \mathtt{F}_6 \vdash \Delta_1, \Delta_2}\ \mathtt{IH}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_1, \Delta_2}\ \wedge_L$$

- Case(s) rule $\vee_L$

$$\dfrac{\mathtt{h}_3 : \mathtt{F}_5, \Delta_4 \vdash \Delta_1, \Delta_2, \Delta_2 \quad \mathtt{h}_3 : \mathtt{F}_6, \Delta_4 \vdash \Delta_1, \Delta_2, \Delta_2}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_1, \Delta_2, \Delta_2}\ \vee_L \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_1, \Delta_2, \Delta_2}\ \mathtt{ax}}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vdash \Delta_1, \Delta_2}\ \mathtt{IH} \quad \dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \Delta_1, \Delta_2, \Delta_2}\ \mathtt{ax}}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_6 \vdash \Delta_1, \Delta_2}\ \mathtt{IH}}{\bullet \mathtt{h}_3 : \Delta_4, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_1, \Delta_2}\ \vee_L$$

- Case(s) rule AT

$$\dfrac{\mathtt{h}_3 : \mathtt{F}_5, \Delta_4, []\mathtt{F}_5 \vdash \Delta_1, \Delta_2, \Delta_2}{\bullet \mathtt{h}_3 : \Delta_4, []\mathtt{F}_5 \vdash \Delta_1, \Delta_2, \Delta_2}\ AT \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, []\mathtt{F}_5 \vdash \Delta_1, \Delta_2, \Delta_2}\ \mathtt{ax}}{\mathtt{h}_3 : \Delta_4, \mathtt{F}_5, []\mathtt{F}_5 \vdash \Delta_1, \Delta_2}\ \mathtt{IH}}{\bullet \mathtt{h}_3 : \Delta_4, []\mathtt{F}_5 \vdash \Delta_1, \Delta_2}\ AT$$

- Case(s) rule $\bot_L$

$$\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_4 \vdash \Delta_1, \Delta_2, \Delta_2}\ \bot_L \quad \rightarrow \quad \dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_4 \vdash \Delta_1, \Delta_2}\ \bot_L$$

- Case(s) rule I

$$\dfrac{}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{p}_4 \vdash \Delta_1, (\Delta_5, \mathtt{p}_4), \Delta_5, \mathtt{p}_4}\ I \quad \rightarrow \quad \dfrac{}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{p}_4 \vdash \Delta_1, \Delta_5, \mathtt{p}_4}\ I$$

$$\dfrac{}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{p}_4 \vdash (\Delta_5, \mathtt{p}_4), \Delta_1, \Delta_1}\ I \quad \rightarrow \quad \dfrac{}{\bullet \mathtt{h}_2 : \Delta_3, \mathtt{p}_4 \vdash \Delta_1, \Delta_5, \mathtt{p}_4}\ I$$

- Case(s) rule $\top_L$

$$\dfrac{\mathtt{h}_3 : \Delta_4 \vdash \Delta_1, \Delta_2, \Delta_2}{\bullet \mathtt{h}_3 : \top, \Delta_4 \vdash \Delta_1, \Delta_2, \Delta_2}\ \top_L \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_1, \Delta_2, \Delta_2}\ \mathtt{ax}}{\mathtt{h}_3 : \Delta_4 \vdash \Delta_1, \Delta_2}\ \mathtt{IH}}{\bullet \mathtt{h}_3 : \top, \Delta_4 \vdash \Delta_1, \Delta_2}\ \top_L$$

7 Identity-Expansion

$$\dfrac{\dfrac{\dfrac{\dfrac{}{- : \mathtt{F}_0 \vdash \mathtt{F}_0}\,\mathtt{IH}}{- : \mathtt{F}_0, \Box\mathtt{F}_0 \vdash \mathtt{F}_0}\,W}{- : \Box\mathtt{F}_0 \vdash \mathtt{F}_0}\,AT}{- : \Box\mathtt{F}_0 \vdash \Box\mathtt{F}_0}\,A4$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{- : \mathtt{F}_0 \vdash \mathtt{F}_0}\,\mathtt{IH}}{- : \mathtt{F}_0 \vdash \mathtt{F}_0, \mathtt{F}_1}\,W \quad \dfrac{\dfrac{}{- : \mathtt{F}_1 \vdash \mathtt{F}_1}\,\mathtt{IH}}{- : \mathtt{F}_1 \vdash \mathtt{F}_0, \mathtt{F}_1}\,W}{- : \mathtt{F}_0 \vee \mathtt{F}_1 \vdash \mathtt{F}_0, \mathtt{F}_1}\,\vee_L}{- : \mathtt{F}_0 \vee \mathtt{F}_1 \vdash \mathtt{F}_0 \vee \mathtt{F}_1}\,\vee_R$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{- : \mathtt{F}_0 \vdash \mathtt{F}_0}\,\mathtt{IH}}{- : \mathtt{F}_0, \mathtt{F}_1 \vdash \mathtt{F}_0}\,W \quad \dfrac{\dfrac{}{- : \mathtt{F}_1 \vdash \mathtt{F}_1}\,\mathtt{IH}}{- : \mathtt{F}_0, \mathtt{F}_1 \vdash \mathtt{F}_1}\,W}{- : \mathtt{F}_0, \mathtt{F}_1 \vdash \mathtt{F}_0 \wedge \mathtt{F}_1}\,\wedge_R}{- : \mathtt{F}_0 \wedge \mathtt{F}_1 \vdash \mathtt{F}_0 \wedge \mathtt{F}_1}\,\wedge_L$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{- : \mathtt{F}_0 \vdash \mathtt{F}_0}\,\mathtt{IH}}{- : \mathtt{F}_0 \vdash \mathtt{F}_0, \mathtt{F}_1}\,W \quad \dfrac{\dfrac{}{- : \mathtt{F}_1 \vdash \mathtt{F}_1}\,\mathtt{IH}}{- : \mathtt{F}_0, \mathtt{F}_1 \vdash \mathtt{F}_1}\,W}{- : \mathtt{F}_0, \mathtt{F}_0 \to \mathtt{F}_1 \vdash \mathtt{F}_1}\,\to_L}{- : \mathtt{F}_0 \to \mathtt{F}_1 \vdash \mathtt{F}_0 \to \mathtt{F}_1}\,\to_R$$

$$\dfrac{}{- : \top \vdash \top}\,\top_R$$

$$\dfrac{}{- : \bot \vdash \bot}\,\bot_L$$

8 Cut-Elimination

8.1 Status of $\to_R$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_6 \vdash \mathtt{F}_8, \Delta_{10}, \mathtt{F}_{11} \to \mathtt{F}_{12}}{\bullet \mathtt{h}_1 : \Delta_6 \vdash (\Delta_{10}, \mathtt{F}_{11} \to \mathtt{F}_{12}), \mathtt{F}_7 \to \mathtt{F}_8}\to_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{11}, \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_{12}, \Delta_{10}}{\bullet \mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \to \mathtt{F}_{12}}\to_R}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11} \to \mathtt{F}_{12}}\,\mathtt{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_{11}, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_{12}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_{12}, \mathtt{F}_7 \to \mathtt{F}_8}\to_R \quad \dfrac{}{\mathtt{h}_9 : \Delta_6, \mathtt{F}_{11}, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12}}\,\mathtt{ax/W}}{- : \Delta_6, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_{12}}\,\mathtt{hCut}}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11} \to \mathtt{F}_{12}}\to_R$$

$$\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_9, \Delta_8 \vdash \mathtt{F}_7, \mathtt{F}_{10}, \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13}}{\bullet \mathtt{h}_2 : \Delta_8 \vdash ((\Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13}), \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_7}\to_R \quad \dfrac{\mathtt{h}_{11} : \mathtt{F}_7, \mathtt{F}_{12}, \Delta_8 \vdash \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_9 \to \mathtt{F}_{10}}{\bullet \mathtt{h}_{11} : \Delta_8, \mathtt{F}_7 \vdash (\Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13}), \mathtt{F}_9 \to \mathtt{F}_{10}}\to_R}{- : \Delta_8 \vdash (\Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13}), \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_8, \mathtt{F}_{12}, \mathtt{F}_9 \vdash \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{13}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_2 : \Delta_8, \mathtt{F}_{12} \vdash \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_9 \to \mathtt{F}_{10}}\to_R \quad \dfrac{}{\mathtt{h}_{11} : \Delta_8, \mathtt{F}_{12}, \mathtt{F}_7 \vdash \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_{12} \vdash \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_8 \vdash \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13}, \mathtt{F}_9 \to \mathtt{F}_{10}}\to_R$$

$$\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_{11}, \Delta_8 \vdash \mathtt{F}_7, \mathtt{F}_{12}, \Delta_{10}}{\bullet \mathtt{h}_2 : \Delta_8 \vdash (\Delta_{10}, \mathtt{F}_{11} \to \mathtt{F}_{12}), \mathtt{F}_7}\to_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_7, \mathtt{F}_{11}, \Delta_8 \vdash \mathtt{F}_{12}, \Delta_{10}}{\bullet \mathtt{h}_9 : \Delta_8, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_{11} \to \mathtt{F}_{12}}\to_R}{- : \Delta_8 \vdash \Delta_{10}, \mathtt{F}_{11} \to \mathtt{F}_{12}}\,\mathtt{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_8, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_{12}, \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_8, \mathtt{F}_{11}, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_{12}}\,\mathtt{ax/W}}{\bullet \mathtt{h}_9 : \Delta_8, \mathtt{F}_{11}, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_{12}}\,\mathtt{H}}{- : \Delta_8, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_{12}}\,\mathtt{hCut}}{- : \Delta_8 \vdash \Delta_{10}, \mathtt{F}_{11} \to \mathtt{F}_{12}}\to_R$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_6 \vdash \mathtt{F}_8, \Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}{\bullet \mathtt{h}_1 : \Delta_6 \vdash (\Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}), \mathtt{F}_7 \to \mathtt{F}_8}\to_R \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_{11}, \Delta_{10} \quad \mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_{12}, \Delta_{10}}{\bullet \mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\wedge_R}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\,\mathtt{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \to \mathtt{F}_8}\to_R \quad \dfrac{}{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11}}\,\mathtt{ax/W}}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11}}\,\mathtt{hCut} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_{12}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{12}, \mathtt{F}_7 \to \mathtt{F}_8}\to_R \quad \dfrac{}{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12}}\,\mathtt{ax/}}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{12}}\,\mathtt{hCu}}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\wedge_R$$

$$\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_9, \Delta_8 \vdash \mathtt{F}_7, \mathtt{F}_{10}, \Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13}}{\bullet \mathtt{h}_2 : \Delta_8 \vdash ((\Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13}), \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_7}\to_R \quad \dfrac{\mathtt{h}_{11} : \mathtt{F}_7, \Delta_8 \vdash \mathtt{F}_{12}, \Delta_{14}, \mathtt{F}_9 \to \mathtt{F}_{10} \quad \mathtt{h}_{11} : \mathtt{F}_7, \Delta_8 \vdash \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_9 \to \mathtt{F}_{10}}{\bullet \mathtt{h}_{11} : \Delta_8, \mathtt{F}_7 \vdash (\Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13}), \mathtt{F}_9 \to \mathtt{F}_{10}}\wedge_R}{- : \Delta_8 \vdash (\Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13}), \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_8, \mathtt{F}_9 \vdash \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_{12} \wedge \mathtt{F}_{13}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_{11} : \Delta_8, \mathtt{F}_7, \mathtt{F}_9 \vdash \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{12}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_{11} : \Delta_8, \mathtt{F}_7, \mathtt{F}_9 \vdash \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{13}}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_{11} : \Delta_8, \mathtt{F}_7, \mathtt{F}_9 \vdash \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{12} \wedge \mathtt{F}_{13}}\wedge_R}{- : \Delta_8, \mathtt{F}_9 \vdash \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{12} \wedge \mathtt{F}_{13}}\,\mathtt{hCut}}{- : \Delta_8 \vdash \Delta_{14}, \mathtt{F}_9 \to \mathtt{F}_{10}, \mathtt{F}_{12} \wedge \mathtt{F}_{13}}\to_R$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_6 \vdash \mathtt{F}_8, \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}{\bullet\mathtt{h}_1 : \Delta_6 \vdash (\Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}), \mathtt{F}_7 \to \mathtt{F}_8}\,{\to_R} \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \mathtt{F}_{11}, \mathtt{F}_{12}, \Delta_{10}}{\bullet\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}\,{\vee_R}}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}\,\mathtt{Cut}}{\begin{array}{c}\to\\ - : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}\end{array}}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_7 \to \mathtt{F}_8}\,{\to_R} \quad \dfrac{}{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_{12}}\,\mathtt{ax/W}}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_{12}}\,\mathtt{hCut}}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}\,{\vee_R}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_9, \Delta_8 \vdash \mathtt{F}_7, \mathtt{F}_{10}, \Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13}}{\bullet\mathtt{h}_2 : \Delta_8 \vdash ((\Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13}), \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_{11} : \mathtt{F}_7, \Delta_8 \vdash \mathtt{F}_{12}, \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_9 \to \mathtt{F}_{10}}{\bullet\mathtt{h}_{11} : \Delta_8, \mathtt{F}_7 \vdash (\Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13}), \mathtt{F}_9 \to \mathtt{F}_{10}}\,{\vee_R}}{- : \Delta_8 \vdash (\Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13}), \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{Cut}}{\begin{array}{c}\to\\ - : \Delta_8 \vdash \Delta_{14}, \mathtt{F}_9 \to \mathtt{F}_{10}, \mathtt{F}_{12} \vee \mathtt{F}_{13}\end{array}}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_8, \mathtt{F}_9 \vdash \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_{12} \vee \mathtt{F}_{13}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_{11} : \Delta_8, \mathtt{F}_7, \mathtt{F}_9 \vdash \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{12}, \mathtt{F}_{13}}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_{11} : \Delta_8, \mathtt{F}_7, \mathtt{F}_9 \vdash \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{12} \vee \mathtt{F}_{13}}\,{\vee_R}}{- : \Delta_8, \mathtt{F}_9 \vdash \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{12} \vee \mathtt{F}_{13}}\,\mathtt{hCut}}{- : \Delta_8 \vdash \Delta_{14}, \mathtt{F}_9 \to \mathtt{F}_{10}, \mathtt{F}_{12} \vee \mathtt{F}_{13}}\,{\to_R}$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_6 \vdash \mathtt{F}_8, \bot, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_6 \vdash (\bot, \Delta_{10}), \mathtt{F}_7 \to \mathtt{F}_8}\,{\to_R} \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_{10}}{\bullet\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \bot, \Delta_{10}}\,{\bot_R}}{- : \Delta_6 \vdash \bot, \Delta_{10}}\,\mathtt{Cut}}{\begin{array}{c}\to\\ - : \Delta_6 \vdash \bot, \Delta_{10}\end{array}}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_6 \vdash \bot, \Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \bot, \Delta_{10}}\,\mathtt{ax/W}}{- : \Delta_6 \vdash \bot, \Delta_{10}}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_9, \Delta_8 \vdash \mathtt{F}_7, \mathtt{F}_{10}, \bot, \Delta_{12}}{\bullet\mathtt{h}_2 : \Delta_8 \vdash ((\bot, \Delta_{12}), \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_{11} : \mathtt{F}_7, \Delta_8 \vdash \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10}}{\bullet\mathtt{h}_{11} : \Delta_8, \mathtt{F}_7 \vdash (\bot, \Delta_{12}), \mathtt{F}_9 \to \mathtt{F}_{10}}\,{\bot_R}}{- : \Delta_8 \vdash (\bot, \Delta_{12}), \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{Cut}}{\begin{array}{c}\to\\ - : \Delta_8 \vdash \bot, \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10}\end{array}}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_2 : \Delta_8 \vdash \bot, \Delta_{12}, \mathtt{F}_7, \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_{11} : \Delta_8, \mathtt{F}_7 \vdash \bot, \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_8 \vdash \bot, \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_7, \Delta_6 \vdash \mathtt{F}_8, \top, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_6 \vdash (\top, \Delta_{10}), \mathtt{F}_7 \to \mathtt{F}_8}\,{\to_R} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \top, \Delta_{10}}\,{\top_R}}{- : \Delta_6 \vdash \top, \Delta_{10}}\,\mathtt{Cut}}{\begin{array}{c}\to\\ - : \Delta_6 \vdash \top, \Delta_{10}\end{array}}\,{\top_R}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_9, \Delta_8 \vdash \mathtt{F}_7, \mathtt{F}_{10}, \top, \Delta_{12}}{\bullet\mathtt{h}_2 : \Delta_8 \vdash ((\top, \Delta_{12}), \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_7}\,{\to_R} \quad \dfrac{}{\bullet\mathtt{h}_{11} : \Delta_8, \mathtt{F}_7 \vdash (\top, \Delta_{12}), \mathtt{F}_9 \to \mathtt{F}_{10}}\,{\top_R}}{- : \Delta_8 \vdash (\top, \Delta_{12}), \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{Cut}}{\begin{array}{c}\to\\ - : \Delta_8 \vdash \top, \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10}\end{array}}\,{\top_R}$$

- Case rule $A4$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Box\Gamma_9, \Delta_{12} \vdash \mathtt{F}_7, \Delta_{10}, []\mathtt{F}_{11}}{\bullet\mathtt{h}_1 : \Box\Gamma_9, \Delta_{12} \vdash (\Delta_{10}, []\mathtt{F}_{11}), \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_9 \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_8 : (\Box\Gamma_9, \Delta_{12}), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{10}, []\mathtt{F}_{11}}\,A4}{- : \Box\Gamma_9, \Delta_{12} \vdash \Delta_{10}, []\mathtt{F}_{11}}\,\mathtt{Cut}}{\begin{array}{c}\to\\ \dfrac{}{- : \Box\Gamma_9 \vdash \mathtt{F}_{11}}\,\mathtt{ax/W}\\ - : \Delta_{12}, \Box\Gamma_9 \vdash \Delta_{10}, []\mathtt{F}_{11}\end{array}}\,A4$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_8, \Box\Gamma_{14}, \Delta_{11} \vdash \Box\mathtt{F}_7, \mathtt{F}_9, \Delta_{13}, []\mathtt{F}_{12}}{\bullet\mathtt{h}_2 : \Box\Gamma_{14}, \Delta_{11} \vdash ((\Delta_{13}, []\mathtt{F}_{12}), \mathtt{F}_8 \to \mathtt{F}_9), \Box\mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_{10} : \Box\Gamma_{14}, \Box\mathtt{F}_7 \vdash \mathtt{F}_{12}}{\bullet\mathtt{h}_{10} : (\Box\Gamma_{14}, \Delta_{11}), \Box\mathtt{F}_7 \vdash (\Delta_{13}, []\mathtt{F}_{12}), \mathtt{F}_8 \to \mathtt{F}_9}\,A4}{- : \Box\Gamma_{14}, \Delta_{11} \vdash (\Delta_{13}, []\mathtt{F}_{12}), \mathtt{F}_8 \to \mathtt{F}_9}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{11}, \mathtt{F}_8, \Box\Gamma_{14} \vdash \Box\mathtt{F}_7, \Delta_{13}, \mathtt{F}_9, []\mathtt{F}_{12}}{}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_{10} : \Box\mathtt{F}_7, \Box\Gamma_{14} \vdash \mathtt{F}_{12}}\,\mathtt{ax/W}}{\bullet\mathtt{h}_{10} : \Box\mathtt{F}_7, \Delta_{11}, \mathtt{F}_8, \Box\Gamma_{14} \vdash \Delta_{13}, \mathtt{F}_9, []\mathtt{F}_{12}}\,A4}{- : \Delta_{11}, \mathtt{F}_8, \Box\Gamma_{14} \vdash \Delta_{13}, \mathtt{F}_9, []\mathtt{F}_{12}}\,\mathtt{hCut}}{- : \Delta_{11}, \Box\Gamma_{14} \vdash \Delta_{13}, []\mathtt{F}_{12}, \mathtt{F}_8 \to \mathtt{F}_9}\,{\to_R}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_8, \Box\Gamma_{11}, \Delta_{14} \vdash \mathtt{F}_7, \mathtt{F}_9, \Delta_{13}, []\mathtt{F}_{12}}{\bullet\mathtt{h}_2 : \Box\Gamma_{11}, \Delta_{14} \vdash ((\Delta_{13}, []\mathtt{F}_{12}), \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_{10} : \Box\Gamma_{11} \vdash \mathtt{F}_{12}}{\bullet\mathtt{h}_{10} : (\Box\Gamma_{11}, \Delta_{14}), \mathtt{F}_7 \vdash (\Delta_{13}, []\mathtt{F}_{12}), \mathtt{F}_8 \to \mathtt{F}_9}\,A4}{- : \Box\Gamma_{11}, \Delta_{14} \vdash (\Delta_{13}, []\mathtt{F}_{12}), \mathtt{F}_8 \to \mathtt{F}_9}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_{11} \vdash \mathtt{F}_{12}}\,\mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{11} \vdash \Delta_{13}, []\mathtt{F}_{12}, \mathtt{F}_8 \to \mathtt{F}_9}\,A4$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \mathtt{F}_7, \Delta_{11}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \mathtt{F}_9, \Delta_{11} \quad \mathtt{h}_8 : \mathtt{F}_{10}, \Delta_{12}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}{\bullet\mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}\,{\to_L}}{- : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6 \vdash \Delta_{11}, \mathtt{F}_7, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{12} \vdash \Delta_{11}, \mathtt{F}_9}\,\mathtt{hCut} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \vdash \Delta_{11}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}\,\mathtt{a}}{- : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{h} \quad {\to_L}}{- : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_8, \Delta_7 \vdash \mathtt{F}_9, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_7 \vdash \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\,{\to_R} \quad \dfrac{\mathtt{h}_6 : \Delta_7 \vdash \mathtt{F}_8, \Delta_{10} \quad \mathtt{h}_6 : \mathtt{F}_9, \Delta_7 \vdash \Delta_{10}}{\bullet\mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{10}}\,{\to_L}}{- : \Delta_7 \vdash \Delta_{10}}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{- : \Delta_7 \vdash \Delta_{10}, \mathtt{F}_8}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{- : \Delta_7, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{}{- : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{10}}\,\mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_8 \vdash \Delta_{10}}\,\mathtt{sCut}}{- : \Delta_7 \vdash \Delta_{10}}\,\mathtt{sCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_8, \Delta_{11} \vdash \mathtt{F}_{12} \to \mathtt{F}_{13}, \mathtt{F}_9, \Delta_7}{\bullet\mathtt{h}_2 : \Delta_{11} \vdash (\Delta_7, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_{12} \to \mathtt{F}_{13}}\,{\to_R} \quad \dfrac{\mathtt{h}_{10} : \Delta_{11} \vdash \mathtt{F}_{12}, \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \quad \mathtt{h}_{10} : \mathtt{F}_{13}, \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9}{\bullet\mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9}\,{\to_L}}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_{11}, \mathtt{F}_8 \vdash \Delta_7, \mathtt{F}_9, \mathtt{F}_{12} \to \mathtt{F}_{13}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_8 \vdash \Delta_7, \mathtt{F}_{12}, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_{13}, \mathtt{F}_8 \vdash \Delta_7, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_7, \mathtt{F}_9}\,{\to_L}\,\mathtt{hCut}}{- : \Delta_{11}, \mathtt{F}_8 \vdash \Delta_7, \mathtt{F}_9}$$

$$\dfrac{- : \Delta_{11}, \mathtt{F}_8 \vdash \Delta_7, \mathtt{F}_9}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9}\,{\to_R}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_9, \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \mathtt{F}_7, \mathtt{F}_{10}, \Delta_8}{\bullet\mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash (\Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_{11} : \mathtt{F}_7, \Delta_{14} \vdash \mathtt{F}_{12}, \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10} \quad \mathtt{h}_{11} : \mathtt{F}_7, \mathtt{F}_{13}, \Delta_{14} \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}{\bullet\mathtt{h}_{11} : (\Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13}), \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\,{\to_L}}{- : \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_{14}, \mathtt{F}_9, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_9 \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_{12}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_9 \vdash \Delta_8, \mathtt{F}_{10}}\,\mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_9, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_{10}}\,{\to_L}\,\mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_9, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_{10}}$$

$$\dfrac{- : \Delta_{14}, \mathtt{F}_9, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_{10}}{- : \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\,{\to_R}$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \mathtt{F}_7, \Delta_{11}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_9, \mathtt{F}_{10}, \Delta_{12}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}{\bullet \mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}\,{\wedge_L}}{- : \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{hCut}}{- : \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}}\,{\wedge_L}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_8, \Delta_{11} \vdash \mathtt{F}_{12} \wedge \mathtt{F}_{13}, \mathtt{F}_9, \Delta_7}{\bullet \mathtt{h}_2 : \Delta_{11} \vdash (\Delta_7, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_{12} \wedge \mathtt{F}_{13}}\,{\to_R} \quad \dfrac{\mathtt{h}_{10} : \mathtt{F}_{12}, \mathtt{F}_{13}, \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9}{\bullet \mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_{12} \wedge \mathtt{F}_{13} \vdash \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9}\,{\wedge_L}}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_{11}, \mathtt{F}_8 \vdash \Delta_7, \mathtt{F}_9, \mathtt{F}_{12} \wedge \mathtt{F}_{13}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_{13}, \mathtt{F}_8 \vdash \Delta_7, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_{12} \wedge \mathtt{F}_{13} \vdash \Delta_7, \mathtt{F}_9}\,{\wedge_L}}{- : \Delta_{11}, \mathtt{F}_8 \vdash \Delta_7, \mathtt{F}_9}\,\mathtt{hCut}}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9}\,{\to_R}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_9, \Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13} \vdash \mathtt{F}_7, \mathtt{F}_{10}, \Delta_8}{\bullet \mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13} \vdash (\Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_{11} : \mathtt{F}_7, \mathtt{F}_{12}, \mathtt{F}_{13}, \Delta_{14} \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}{\bullet \mathtt{h}_{11} : (\Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13}), \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\,{\wedge_L}}{- : \Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_{14}, \mathtt{F}_9, \mathtt{F}_{12} \wedge \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_{12}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_9 \vdash \Delta_8, \mathtt{F}_{10}}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_9, \mathtt{F}_{12} \wedge \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_{10}}\,{\wedge_L}}{- : \Delta_{14}, \mathtt{F}_9, \mathtt{F}_{12} \wedge \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\,{\to_R}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \mathtt{F}_7, \Delta_{11}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_9, \Delta_{12}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11} \quad \mathtt{h}_8 : \mathtt{F}_{10}, \Delta_{12}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}{\bullet \mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}\,{\vee_L}}{- : \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_6, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}\,\mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{hCut} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \vdash \Delta_{11}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{11}}\,\mathtt{a}}{- : \Delta_{12}, \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{h}}{- : \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}}\,{\vee_L}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_8, \Delta_{11} \vdash \mathtt{F}_{12} \vee \mathtt{F}_{13}, \mathtt{F}_9, \Delta_7}{\bullet \mathtt{h}_2 : \Delta_{11} \vdash (\Delta_7, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_{12} \vee \mathtt{F}_{13}}\,{\to_R} \quad \dfrac{\mathtt{h}_{10} : \mathtt{F}_{12}, \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \quad \mathtt{h}_{10} : \mathtt{F}_{13}, \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9}{\bullet \mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_{12} \vee \mathtt{F}_{13} \vdash \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9}\,{\vee_L}}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_{11}, \mathtt{F}_8 \vdash \Delta_7, \mathtt{F}_9, \mathtt{F}_{12} \vee \mathtt{F}_{13}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_8 \vdash \Delta_7, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_{13}, \mathtt{F}_8 \vdash \Delta_7, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_8, \mathtt{F}_{12} \vee \mathtt{F}_{13} \vdash \Delta_7, \mathtt{F}_9}\,{\vee_L}\,\mathtt{hCut}}{- : \Delta_{11}, \mathtt{F}_8 \vdash \Delta_7, \mathtt{F}_9}\,{\to_R}}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_9, \Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13} \vdash \mathtt{F}_7, \mathtt{F}_{10}, \Delta_8}{\bullet \mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13} \vdash (\Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_{11} : \mathtt{F}_7, \mathtt{F}_{12}, \Delta_{14} \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10} \quad \mathtt{h}_{11} : \mathtt{F}_7, \mathtt{F}_{13}, \Delta_{14} \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}{\bullet \mathtt{h}_{11} : (\Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13}), \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\,{\vee_L}}{- : \Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_{14}, \mathtt{F}_9, \mathtt{F}_{12} \vee \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_{12}, \mathtt{F}_7, \mathtt{F}_9 \vdash \Delta_8, \mathtt{F}_{10}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_9 \vdash \Delta_8, \mathtt{F}_{10}}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_9, \mathtt{F}_{12} \vee \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_{10}}\,{\vee_L}\,\mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_9, \mathtt{F}_{12} \vee \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_{10}}\,{\to_R}}{- : \Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}$$

- Case rule AT

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{11}, []\mathtt{F}_9 \vdash \mathtt{F}_7, \Delta_{10}}{\bullet\mathtt{h}_1 : \Delta_{11}, []\mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_9, \Delta_{11}, []\mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{10}}{\bullet\mathtt{h}_8 : (\Delta_{11}, []\mathtt{F}_9), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{10}}\ AT}{- : \Delta_{11}, []\mathtt{F}_9 \vdash \Delta_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_9, []\mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{11}, \mathtt{F}_9, []\mathtt{F}_9, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{10}}\ \mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_9, []\mathtt{F}_9 \vdash \Delta_{10}}\ \mathtt{hCut}}{- : \Delta_{11}, []\mathtt{F}_9 \vdash \Delta_{10}}\ AT}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_8, \Delta_{11} \vdash []\mathtt{F}_{12}, \mathtt{F}_9, \Delta_7}{\bullet\mathtt{h}_2 : \Delta_{11} \vdash (\Delta_7, \mathtt{F}_8 \to \mathtt{F}_9), []\mathtt{F}_{12}} \to_R \quad \dfrac{\mathtt{h}_{10} : \mathtt{F}_{12}, \Delta_{11}, []\mathtt{F}_{12} \vdash \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9}{\bullet\mathtt{h}_{10} : \Delta_{11}, []\mathtt{F}_{12} \vdash \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9}\ AT}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_{11}, \mathtt{F}_8 \vdash \Delta_7, \mathtt{F}_9, []\mathtt{F}_{12}}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_8, []\mathtt{F}_{12} \vdash \Delta_7, \mathtt{F}_9}\ \mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_8, []\mathtt{F}_{12} \vdash \Delta_7, \mathtt{F}_9}\ AT}{- : \Delta_{11}, \mathtt{F}_8 \vdash \Delta_7, \mathtt{F}_9}\ \mathtt{hCut}}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9}\ \to_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_9, \Delta_{13}, []\mathtt{F}_{12} \vdash \mathtt{F}_7, \mathtt{F}_{10}, \Delta_8}{\bullet\mathtt{h}_2 : \Delta_{13}, []\mathtt{F}_{12} \vdash (\Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_7} \to_R \quad \dfrac{\mathtt{h}_{11} : \mathtt{F}_7, \mathtt{F}_{12}, \Delta_{13}, []\mathtt{F}_{12} \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}{\bullet\mathtt{h}_{11} : (\Delta_{13}, []\mathtt{F}_{12}), \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\ AT}{- : \Delta_{13}, []\mathtt{F}_{12} \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_2 : \Delta_{13}, \mathtt{F}_{12}, []\mathtt{F}_{12} \vdash \Delta_8, \mathtt{F}_7, \mathtt{F}_9 \to \mathtt{F}_{10}}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_{11} : \Delta_{13}, \mathtt{F}_{12}, \mathtt{F}_7, []\mathtt{F}_{12} \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_{12}, []\mathtt{F}_{12} \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\ \mathtt{hCut}}{- : \Delta_{13}, []\mathtt{F}_{12} \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\ AT}$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \bot, \Delta_{10} \vdash \mathtt{F}_7, \Delta_9}{\bullet\mathtt{h}_1 : \bot, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{}{\bullet\mathtt{h}_8 : (\bot, \Delta_{10}), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_9}\ \bot_L}{- : \bot, \Delta_{10} \vdash \Delta_9}\ \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_{10} \vdash \Delta_9}\ \bot_L}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_8, \Delta_{11} \vdash \bot, \mathtt{F}_9, \Delta_7}{\bullet\mathtt{h}_2 : \Delta_{11} \vdash (\Delta_7, \mathtt{F}_8 \to \mathtt{F}_9), \bot} \to_R \quad \dfrac{}{\bullet\mathtt{h}_{10} : \Delta_{11}, \bot \vdash \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9}\ \bot_L}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_{11}, \mathtt{F}_8 \vdash \bot, \Delta_7, \mathtt{F}_9}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_{10} : \bot, \Delta_{11}, \mathtt{F}_8 \vdash \Delta_7, \mathtt{F}_9}\ \bot_L}{- : \Delta_{11}, \mathtt{F}_8 \vdash \Delta_7, \mathtt{F}_9}\ \mathtt{hCut}}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9}\ \to_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_9, \bot, \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_{10}, \Delta_8}{\bullet\mathtt{h}_2 : \bot, \Delta_{12} \vdash (\Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_7} \to_R \quad \dfrac{}{\bullet\mathtt{h}_{11} : (\bot, \Delta_{12}), \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\ \bot_L}{- : \bot, \Delta_{12} \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\ \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_{12} \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\ \bot_L}$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \Delta_{11}, \mathtt{p}_9 \vdash \mathtt{F}_7, \Delta_{10}, \mathtt{p}_9}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{p}_9 \vdash (\Delta_{10}, \mathtt{p}_9), \mathtt{F}_6 \to \mathtt{F}_7} \to_R \quad \dfrac{}{\bullet\mathtt{h}_8 : (\Delta_{11}, \mathtt{p}_9), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{p}_9}\ I}{- : \Delta_{11}, \mathtt{p}_9 \vdash \Delta_{10}, \mathtt{p}_9}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_{11}, \mathtt{p}_9 \vdash \Delta_{10}, \mathtt{p}_9}\ I}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_7, \Delta_{10} \vdash \mathtt{p}_{11}, \mathtt{F}_8, \Delta_{12}, \mathtt{p}_{11}}{\bullet\mathtt{h}_2 : \Delta_{10} \vdash ((\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{p}_{11}} \to_R \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{10}, \mathtt{p}_{11} \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7 \to \mathtt{F}_8}\ I}{- : \Delta_{10} \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7 \to \mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_{10}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{p}_{11}, \mathtt{p}_{11}}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{10}, \mathtt{F}_7, \mathtt{p}_{11} \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{p}_{11}}\ I}{- : \Delta_{10}, \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{p}_{11}}\ \mathtt{hCut}}{- : \Delta_{10} \vdash \Delta_{12}, \mathtt{p}_{11}, \mathtt{F}_7 \to \mathtt{F}_8}\ \to_R$$

$$\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_8, \Delta_{13}, \mathtt{p}_{11} \vdash \mathtt{F}_7, \mathtt{F}_9, \Delta_{12}, \mathtt{p}_{11}}{\bullet \mathtt{h}_2 : \Delta_{13}, \mathtt{p}_{11} \vdash ((\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_7}\,{\to_R} \quad \dfrac{}{\bullet \mathtt{h}_{10} : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_7 \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_8 \to \mathtt{F}_9}\,I}{- : \Delta_{13}, \mathtt{p}_{11} \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_8 \to \mathtt{F}_9}\,\mathtt{Cut}$$
$$\to$$
$$\dfrac{}{- : \Delta_{13}, \mathtt{p}_{11} \vdash \Delta_{12}, \mathtt{p}_{11}, \mathtt{F}_8 \to \mathtt{F}_9}\,I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \mathtt{F}_6, \top, \Delta_{10} \vdash \mathtt{F}_7, \Delta_9}{\bullet \mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_9}{\bullet \mathtt{h}_8 : (\top, \Delta_{10}), \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_9}\,\top_L}{- : \top, \Delta_{10} \vdash \Delta_9}\,\mathtt{Cut}$$
$$\to$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \to \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \top, \Delta_{10}, \mathtt{F}_6 \to \mathtt{F}_7 \vdash \Delta_9}\,\mathtt{ax/W}}{- : \top, \Delta_{10} \vdash \Delta_9}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_8, \Delta_{11} \vdash \top, \mathtt{F}_9, \Delta_7}{\bullet \mathtt{h}_2 : \Delta_{11} \vdash (\Delta_7, \mathtt{F}_8 \to \mathtt{F}_9), \top}\,{\to_R} \quad \dfrac{\mathtt{h}_{10} : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9}{\bullet \mathtt{h}_{10} : \Delta_{11}, \top \vdash \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9}\,\top_L}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9}\,\mathtt{Cut}$$
$$\to$$
$$\dfrac{}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9}\,\mathtt{ax/W}$$

$$\dfrac{\dfrac{\mathtt{h}_2 : \mathtt{F}_9, \top, \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_{10}, \Delta_8}{\bullet \mathtt{h}_2 : \top, \Delta_{12} \vdash (\Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_7}\,{\to_R} \quad \dfrac{\mathtt{h}_{11} : \mathtt{F}_7, \Delta_{12} \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}{\bullet \mathtt{h}_{11} : (\top, \Delta_{12}), \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\,\top_L}{- : \top, \Delta_{12} \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{Cut}$$
$$\to$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \top, \Delta_{12} \vdash \Delta_8, \mathtt{F}_7, \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_{11} : \top, \Delta_{12}, \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \top, \Delta_{12} \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\,\mathtt{hCut}$$

8.2 Status of $\wedge_R$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6 \vdash \mathtt{F}_7, \Delta_{10}, \mathtt{F}_{11} \to \mathtt{F}_{12} \quad \mathtt{h}_1 : \Delta_6 \vdash \mathtt{F}_8, \Delta_{10}, \mathtt{F}_{11} \to \mathtt{F}_{12}}{\bullet \mathtt{h}_1 : \Delta_6 \vdash (\Delta_{10}, \mathtt{F}_{11} \to \mathtt{F}_{12}), \mathtt{F}_7 \wedge \mathtt{F}_8}\,{\wedge_R} \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{11}, \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{12}, \Delta_{10}}{\bullet \mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \to \mathtt{F}_{12}}\,{\to_R}}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11} \to \mathtt{F}_{12}}\,\mathtt{Cut}$$
$$\to$$
$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_{12}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_1 : \Delta_6, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_{12}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_1 : \Delta_6, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_{12}, \mathtt{F}_7 \wedge \mathtt{F}_8}\,{\wedge_R} \quad \dfrac{}{\mathtt{h}_9 : \Delta_6, \mathtt{F}_{11}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12}}\,\mathtt{ax/W}}{- : \Delta_6, \mathtt{F}_{11} \vdash \Delta_{10}, \mathtt{F}_{12}}\,\mathtt{hCut}}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11} \to \mathtt{F}_{12}}\,{\to_R}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_8 \vdash \mathtt{F}_7, \mathtt{F}_9, \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \quad \mathtt{h}_2 : \Delta_8 \vdash \mathtt{F}_7, \mathtt{F}_{10}, \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13}}{\bullet \mathtt{h}_2 : \Delta_8 \vdash ((\Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13}), \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_7}\,{\wedge_R} \quad \dfrac{\mathtt{h}_{11} : \mathtt{F}_7, \mathtt{F}_{12}, \Delta_8 \vdash \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_9 \wedge \mathtt{F}_{10}}{\bullet \mathtt{h}_{11} : \Delta_8, \mathtt{F}_7 \vdash (\Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13}), \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,{\to_R}}{- : \Delta_8 \vdash (\Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13}), \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\mathtt{Cut}$$
$$\to$$
$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_8, \mathtt{F}_{12} \vdash \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_2 : \Delta_8, \mathtt{F}_{12} \vdash \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{13}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_2 : \Delta_8, \mathtt{F}_{12} \vdash \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,{\wedge_R} \quad \dfrac{}{\mathtt{h}_{11} : \Delta_8, \mathtt{F}_{12}, \mathtt{F}_7 \vdash \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_{12} \vdash \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\mathtt{hCut}}{- : \Delta_8 \vdash \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13}, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,{\to_R}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6 \vdash \mathtt{F}_7, \Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12} \quad \mathtt{h}_1 : \Delta_6 \vdash \mathtt{F}_8, \Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}{\bullet \mathtt{h}_1 : \Delta_6 \vdash (\Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}), \mathtt{F}_7 \wedge \mathtt{F}_8}\,{\wedge_R} \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{11}, \Delta_{10} \quad \mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_{12}, \Delta_{10}}{\bullet \mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\,{\wedge_R}}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\,\mathtt{Cut}$$
$$\to$$
$$\dfrac{\dfrac{\dfrac{}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_7, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{- : \Delta_6, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_8, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{- : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{- : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12}}\,\mathtt{inv\text{-}th/ax}}{- : \Delta_6, \mathtt{F}_7, \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\,{\wedge_R}}{- : \Delta_6, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\,\mathtt{sCut}}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\,\mathtt{sCut}$$

$$\dfrac{\dfrac{h_2 : \Delta_8 \vdash F_7, F_9, \Delta_{14}, F_{12} \wedge F_{13} \quad h_2 : \Delta_8 \vdash F_7, F_{10}, \Delta_{14}, F_{12} \wedge F_{13}}{\bullet h_2 : \Delta_8 \vdash ((\Delta_{14}, F_{12} \wedge F_{13}), F_9 \wedge F_{10}), F_7}\wedge_R \quad \dfrac{h_{11} : F_7, \Delta_8 \vdash F_{12}, \Delta_{14}, F_9 \wedge F_{1}\cdots}{\bullet h_{11} : \Delta_8, F_7 \vdash (\Delta\cdots}}{- : \Delta_8 \vdash (\Delta_{14}, F_{12} \wedge F_{13}), F_9 \wedge F_{10}}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{h_2 : \Delta_8 \vdash \Delta_{14}, F_{12}, F_7, F_9}\texttt{inv-th/ax} \quad \dfrac{}{h_2 : \Delta_8 \vdash \Delta_{14}, F_{10}, F_{12}, F_7}\texttt{inv-th/ax}}{\bullet h_2 : \Delta_8 \vdash \Delta_{14}, F_{12}, F_7, F_9 \wedge F_{10}}\wedge_R \quad \dfrac{}{h_{11} : \Delta_8, F_7 \vdash \Delta_{14}, F_{12}, F_9 \wedge F_{10}}\texttt{ax/W}}{- : \Delta_8 \vdash \Delta_{14}, F_{12}, F_9 \wedge F_{10}}\texttt{hCut} \quad \dfrac{\dfrac{}{h_2 : \Delta_8 \vdash \Delta_{14}, F_{13}, F_7, F_9}\texttt{inv-}\cdots}{\bullet h_2 : \Delta_8 \vdash \Delta_{1}\cdots}}{- : \Delta_8 \vdash \Delta_{14}, F_{12} \wedge F_{13}, F_9 \wedge F_{10}}$$

$$\dfrac{\dfrac{h_2 : \Delta_8 \vdash F_7, F_{11}, \Delta_{10} \quad h_2 : \Delta_8 \vdash F_7, F_{12}, \Delta_{10}}{\bullet h_2 : \Delta_8 \vdash (\Delta_{10}, F_{11} \wedge F_{12}), F_7}\wedge_R \quad \dfrac{h_9 : F_7, \Delta_8 \vdash F_{11}, \Delta_{10} \quad h_9 : F_7, \Delta_8 \vdash F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{11} \wedge F_{12}}\wedge_R}{- : \Delta_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}}\texttt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{h_2 : \Delta_8 \vdash \Delta_{10}, F_{11}, F_7}\texttt{ax/W} \quad \dfrac{\dfrac{}{h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{11}}\texttt{ax/W}}{\bullet h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{11}}\texttt{H}}{- : \Delta_8 \vdash \Delta_{10}, F_{11}}\texttt{hCut} \quad \dfrac{\dfrac{}{h_2 : \Delta_8 \vdash \Delta_{10}, F_{12}, F_7}\texttt{ax/W} \quad \dfrac{\dfrac{}{h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{12}}\texttt{ax/W}}{\bullet h_9 : \Delta_8, F_7 \vdash \Delta_{10}, F_{12}}\texttt{H}}{- : \Delta_8 \vdash \Delta_{10}, F_{12}}\texttt{hCut}}{- : \Delta_8 \vdash \Delta_{10}, F_{11} \wedge F_{12}}\wedge_R$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{h_1 : \Delta_6 \vdash F_7, \Delta_{10}, F_{11} \vee F_{12} \quad h_1 : \Delta_6 \vdash F_8, \Delta_{10}, F_{11} \vee F_{12}}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \vee F_{12}), F_7 \wedge F_8}\wedge_R \quad \dfrac{h_9 : \Delta_6, F_7 \wedge F_8 \vdash F_{11}, F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \wedge F_8 \vdash \Delta_{10}, F_{11} \vee F_{12}}\vee_R}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \vee F_{12}}\texttt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_7}\texttt{inv-th/ax} \quad \dfrac{}{h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_8}\texttt{inv-th/ax}}{\bullet h_1 : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}, F_7 \wedge F_8}\wedge_R \quad \dfrac{}{h_9 : \Delta_6, F_7 \wedge F_8 \vdash \Delta_{10}, F_{11}, F_{12}}\texttt{ax/W}}{- : \Delta_6 \vdash \Delta_{10}, F_{11}, F_{12}}\texttt{hCut}}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \vee F_{12}}\vee_R$$

$$\dfrac{\dfrac{h_2 : \Delta_8 \vdash F_7, F_9, \Delta_{14}, F_{12} \vee F_{13} \quad h_2 : \Delta_8 \vdash F_7, F_{10}, \Delta_{14}, F_{12} \vee F_{13}}{\bullet h_2 : \Delta_8 \vdash ((\Delta_{14}, F_{12} \vee F_{13}), F_9 \wedge F_{10}), F_7}\wedge_R \quad \dfrac{h_{11} : F_7, \Delta_8 \vdash F_{12}, F_{13}, \Delta_{14}, F_9 \wedge F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_9 \wedge F_{10}}\vee_R}{- : \Delta_8 \vdash (\Delta_{14}, F_{12} \vee F_{13}), F_9 \wedge F_{10}}\texttt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{h_2 : \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_7, F_9}\texttt{inv-th/ax} \quad \dfrac{}{h_2 : \Delta_8 \vdash \Delta_{14}, F_{10}, F_{12}, F_{13}, F_7}\texttt{inv-th/ax}}{\bullet h_2 : \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_7, F_9 \wedge F_{10}}\wedge_R \quad \dfrac{}{h_{11} : \Delta_8, F_7 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \wedge F_{10}}\texttt{ax/W}}{- : \Delta_8 \vdash \Delta_{14}, F_{12}, F_{13}, F_9 \wedge F_{10}}\texttt{hCut}}{- : \Delta_8 \vdash \Delta_{14}, F_9 \wedge F_{10}, F_{12} \vee F_{13}}\vee_R$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{h_1 : \Delta_6 \vdash F_7, \bot, \Delta_{10} \quad h_1 : \Delta_6 \vdash F_8, \bot, \Delta_{10}}{\bullet h_1 : \Delta_6 \vdash (\bot, \Delta_{10}), F_7 \wedge F_8}\wedge_R \quad \dfrac{h_9 : \Delta_6, F_7 \wedge F_8 \vdash \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \wedge F_8 \vdash \bot, \Delta_{10}}\bot_R}{- : \Delta_6 \vdash \bot, \Delta_{10}}\texttt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet h_1 : \Delta_6 \vdash \bot, \Delta_{10}, F_7 \wedge F_8}\texttt{ax/W} \quad \dfrac{}{h_9 : \Delta_6, F_7 \wedge F_8 \vdash \bot, \Delta_{10}}\texttt{ax/W}}{- : \Delta_6 \vdash \bot, \Delta_{10}}\texttt{hCut}$$

$$\dfrac{\dfrac{h_2 : \Delta_8 \vdash F_7, F_9, \bot, \Delta_{12} \quad h_2 : \Delta_8 \vdash F_7, F_{10}, \bot, \Delta_{12}}{\bullet h_2 : \Delta_8 \vdash ((\bot, \Delta_{12}), F_9 \wedge F_{10}), F_7}\wedge_R \quad \dfrac{h_{11} : F_7, \Delta_8 \vdash \Delta_{12}, F_9 \wedge F_{10}}{\bullet h_{11} : \Delta_8, F_7 \vdash (\bot, \Delta_{12}), F_9 \wedge F_{10}}\bot_R}{- : \Delta_8 \vdash (\bot, \Delta_{12}), F_9 \wedge F_{10}}\texttt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet h_2 : \Delta_8 \vdash \bot, \Delta_{12}, F_7, F_9 \wedge F_{10}}\texttt{ax/W} \quad \dfrac{}{h_{11} : \Delta_8, F_7 \vdash \bot, \Delta_{12}, F_9 \wedge F_{10}}\texttt{ax/W}}{- : \Delta_8 \vdash \bot, \Delta_{12}, F_9 \wedge F_{10}}\texttt{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{h_1 : \Delta_6 \vdash F_7, \top, \Delta_{10} \quad h_1 : \Delta_6 \vdash F_8, \top, \Delta_{10}}{\bullet h_1 : \Delta_6 \vdash (\top, \Delta_{10}), F_7 \wedge F_8}\wedge_R \quad \dfrac{}{\bullet h_9 : \Delta_6, F_7 \wedge F_8 \vdash \top, \Delta_{10}}\top_R}{- : \Delta_6 \vdash \top, \Delta_{10}}\texttt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_6 \vdash \top, \Delta_{10}}\top_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_8 \vdash \mathtt{F}_7, \mathtt{F}_9, \top, \Delta_{12} \quad \mathtt{h}_2 : \Delta_8 \vdash \mathtt{F}_7, \mathtt{F}_{10}, \top, \Delta_{12}}{\bullet \mathtt{h}_2 : \Delta_8 \vdash ((\top, \Delta_{12}), \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_7}\wedge_R \quad \dfrac{}{\bullet \mathtt{h}_{11} : \Delta_8, \mathtt{F}_7 \vdash (\top, \Delta_{12}), \mathtt{F}_9 \wedge \mathtt{F}_{10}}\top_R}{- : \Delta_8 \vdash (\top, \Delta_{12}), \mathtt{F}_9 \wedge \mathtt{F}_{10}}\,\mathtt{Cut}}{\to}}{- : \Delta_8 \vdash \top, \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\top_R$$

- Case rule $A4$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_9, \Delta_{12} \vdash \mathtt{F}_6, \Delta_{10}, []\mathtt{F}_{11} \quad \mathtt{h}_1 : \Box\Gamma_9, \Delta_{12} \vdash \mathtt{F}_7, \Delta_{10}, []\mathtt{F}_{11}}{\bullet \mathtt{h}_1 : \Box\Gamma_9, \Delta_{12} \vdash (\Delta_{10}, []\mathtt{F}_{11}), \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_9 \vdash \mathtt{F}_{11}}{\bullet \mathtt{h}_8 : (\Box\Gamma_9, \Delta_{12}), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_{10}, []\mathtt{F}_{11}}A4}{- : \Box\Gamma_9, \Delta_{12} \vdash \Delta_{10}, []\mathtt{F}_{11}}\,\mathtt{Cut}}{\to}}{- : \Delta_{12}, \Box\Gamma_9 \vdash \Delta_{10}, []\mathtt{F}_{11}}A4$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Box\Gamma_{14}, \Delta_{11} \vdash \Box\mathtt{F}_7, \mathtt{F}_8, \Delta_{13}, []\mathtt{F}_{12} \quad \mathtt{h}_2 : \Box\Gamma_{14}, \Delta_{11} \vdash \Box\mathtt{F}_7, \mathtt{F}_9, \Delta_{13}, []\mathtt{F}_{12}}{\bullet \mathtt{h}_2 : \Box\Gamma_{14}, \Delta_{11} \vdash ((\Delta_{13}, []\mathtt{F}_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9), \Box\mathtt{F}_7}\wedge_R \quad \dfrac{\mathtt{h}_{10} : \Box\Gamma_{14}, \Box\mathtt{F}_7 \vdash \mathtt{F}_{12}}{\bullet \mathtt{h}_{10} : (\Box\Gamma_{14}, \Delta_{11}), \Box\mathtt{F}_7 \vdash (\Delta_{13}, []\mathtt{F}_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9}}{- : \Box\Gamma_{14}, \Delta_{11} \vdash (\Delta_{13}, []\mathtt{F}_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9}}{\to}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_{11}, \Box\Gamma_{14} \vdash \Box\mathtt{F}_7, \Delta_{13}, \mathtt{F}_8, []\mathtt{F}_{12}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_{10} : \Box\mathtt{F}_7, \Box\Gamma_{14} \vdash \mathtt{F}_{12}}\,\mathtt{ax/W}}{\bullet \mathtt{h}_{10} : \Box\mathtt{F}_7, \Delta_{11}, \Box\Gamma_{14} \vdash \Delta_{13}, \mathtt{F}_8, []\mathtt{F}_{12}}A4}{- : \Delta_{11}, \Box\Gamma_{14} \vdash \Delta_{13}, \mathtt{F}_8, []\mathtt{F}_{12}}\,\mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_{11}, \Box\Gamma_{14} \vdash \Box\mathtt{F}_7, \Delta_{13}, \mathtt{F}_9, []\mathtt{F}_{12}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_{10} : \Box}}{\bullet \mathtt{h}_{10} : \Box\mathtt{F}_7, \Delta_{11}}}{- : \Delta_{11}, \Box\Gamma_{14} \vdash \Delta_{13}, \mathtt{F}_9, []\mathtt{F}_{12}}}{- : \Delta_{11}, \Box\Gamma_{14} \vdash \Delta_{13}, []\mathtt{F}_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9}\wedge$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Box\Gamma_{11}, \Delta_{14} \vdash \mathtt{F}_7, \mathtt{F}_8, \Delta_{13}, []\mathtt{F}_{12} \quad \mathtt{h}_2 : \Box\Gamma_{11}, \Delta_{14} \vdash \mathtt{F}_7, \mathtt{F}_9, \Delta_{13}, []\mathtt{F}_{12}}{\bullet \mathtt{h}_2 : \Box\Gamma_{11}, \Delta_{14} \vdash ((\Delta_{13}, []\mathtt{F}_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_7}\wedge_R \quad \dfrac{\mathtt{h}_{10} : \Box\Gamma_{11} \vdash \mathtt{F}_{12}}{\bullet \mathtt{h}_{10} : (\Box\Gamma_{11}, \Delta_{14}), \mathtt{F}_7 \vdash (\Delta_{13}, []\mathtt{F}_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9}A4}{- : \Box\Gamma_{11}, \Delta_{14} \vdash (\Delta_{13}, []\mathtt{F}_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{Cut}}{\to}}{- : \Delta_{14}, \Box\Gamma_{11} \vdash \Delta_{13}, []\mathtt{F}_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9}A4$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \mathtt{F}_6, \Delta_{11} \quad \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \mathtt{F}_7, \Delta_{11}}{\bullet \mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6 \wedge \mathtt{F}_7}\wedge_R \quad \dfrac{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \mathtt{F}_9, \Delta_{11} \quad \mathtt{h}_8 : \mathtt{F}_{10}, \Delta_{12}, \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_{11}}{\bullet \mathtt{h}_8 : (\Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_6 \wedge \mathtt{F}_7 \vdash \Delta_{11}}\to_L}{- : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{- : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{- : \Delta_{12}, \mathtt{F}_6, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{- : \Delta_{12}, \mathtt{F}_6, \mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{- : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_6, \mathtt{F}_7 \vdash \Delta_{11}}\,\mathtt{inv\text{-}th/}\to_L}{- : \Delta_{12}, \mathtt{F}_6, \mathtt{F}_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{sCut}}{- : \Delta_{12}, \mathtt{F}_6, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}}\,\mathtt{sCut}}{- : \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}}$$

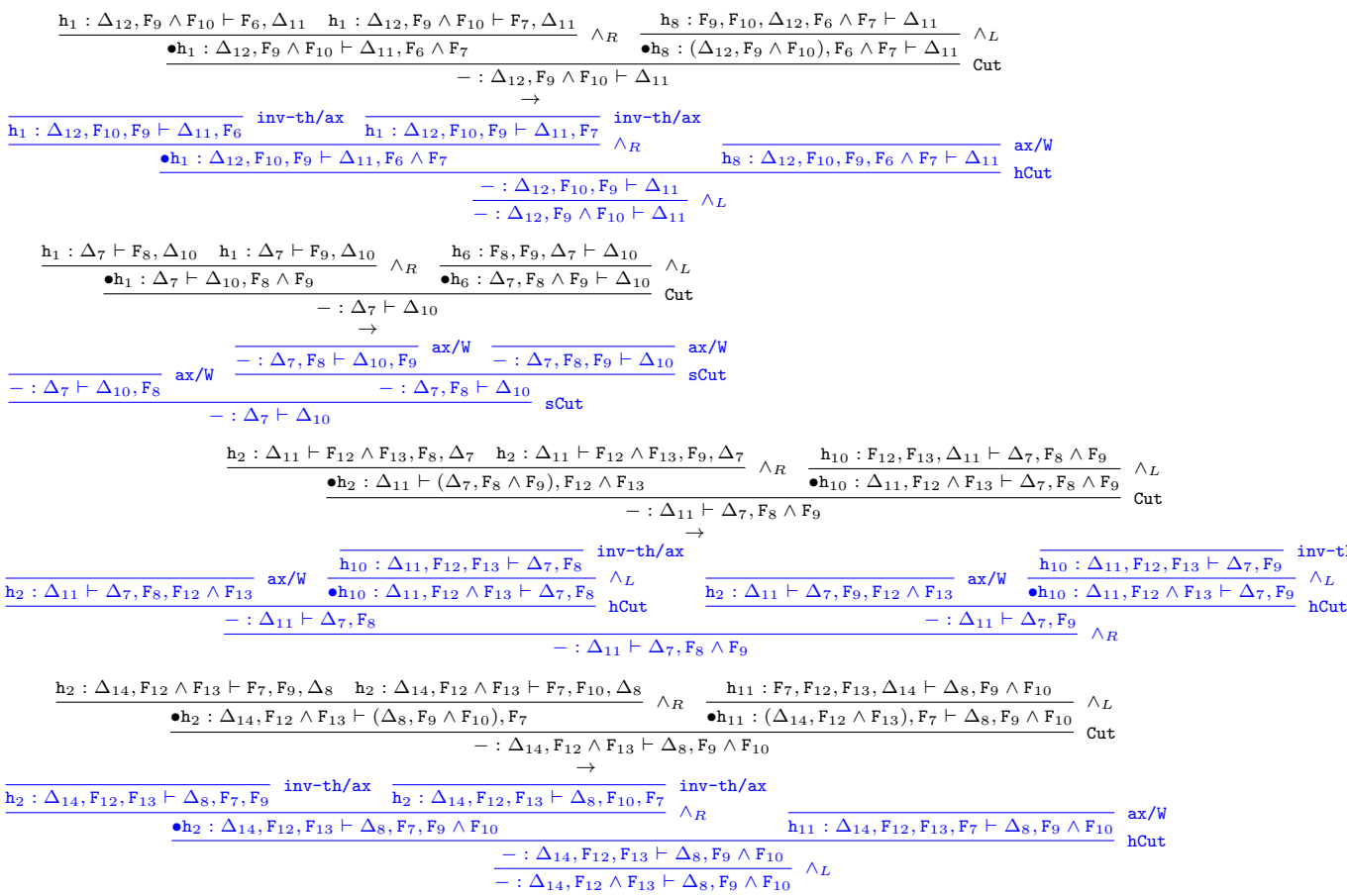
$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{11} \vdash \mathtt{F}_{12} \to \mathtt{F}_{13}, \mathtt{F}_8, \Delta_7 \quad \mathtt{h}_2 : \Delta_{11} \vdash \mathtt{F}_{12} \to \mathtt{F}_{13}, \mathtt{F}_9, \Delta_7}{\bullet \mathtt{h}_2 : \Delta_{11} \vdash (\Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_{12} \to \mathtt{F}_{13}}\wedge_R \quad \dfrac{\mathtt{h}_{10} : \Delta_{11} \vdash \mathtt{F}_{12}, \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \quad \mathtt{h}_{10} : \mathtt{F}_{13}, \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9}{\bullet \mathtt{h}_{10} : \Delta_{11}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9}\to_L}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\dfrac{}{- : \Delta_{11}, \mathtt{F}_{12} \vdash \Delta_7, \mathtt{F}_{13}, \mathtt{F}_8}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{- : \Delta_{11}, \mathtt{F}_{12} \vdash \Delta_7, \mathtt{F}_{13}, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax}}{- : \Delta_{11}, \mathtt{F}_{12} \vdash \Delta_7, \mathtt{F}_{13}, \mathtt{F}_8 \wedge \mathtt{F}_9}\wedge_R \quad \dfrac{}{- : \Delta_{11}, \mathtt{F}_{12}, \mathtt{F}_{13} \vdash \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_{12} \vdash \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{sCut}}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9}\,\mathtt{sCut}$$

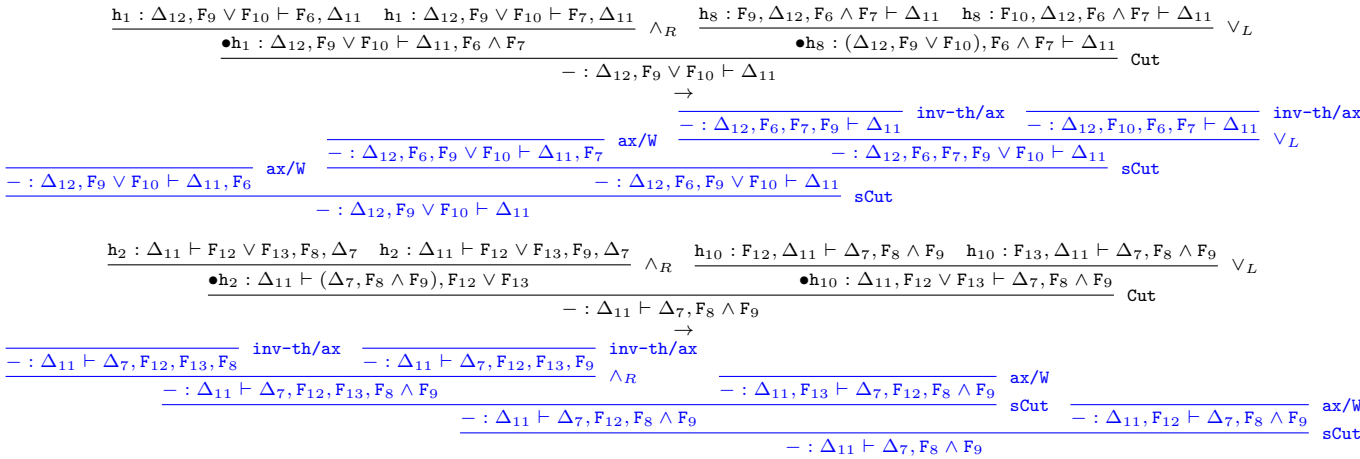
$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \mathtt{F}_7, \mathtt{F}_9, \Delta_8 \quad \mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \mathtt{F}_7, \mathtt{F}_{10}, \Delta_8}{\bullet \mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash (\Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_7}\wedge_R \quad \dfrac{\mathtt{h}_{11} : \mathtt{F}_7, \Delta_{14} \vdash \mathtt{F}_{12}, \Delta_8, \mathtt{F}_9 \wedge}{\bullet \mathtt{h}_{11} : (\Delta_{14}, \mathtt{F}_{12}}}{- : \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}}{\to}$$

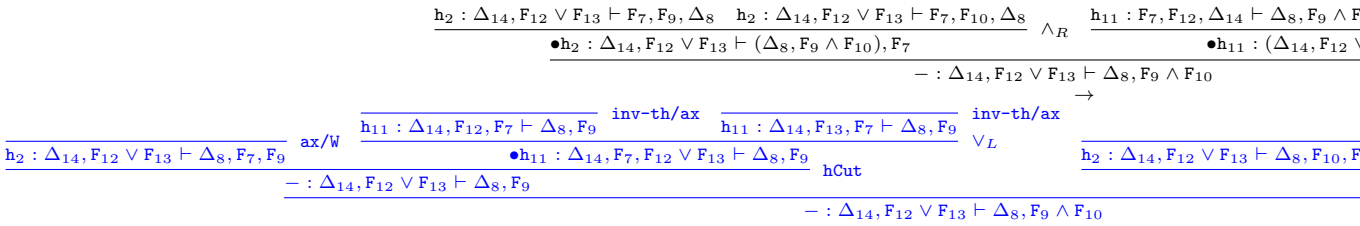
$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_7, \mathtt{F}_9}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_{12}, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_{11} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_9}\to_L}{- : \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_9}\,\mathtt{hCut} \quad \dfrac{}{\mathtt{h}_2 : \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_{10}}}{- : \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}$$

- Case rule $\wedge_L$

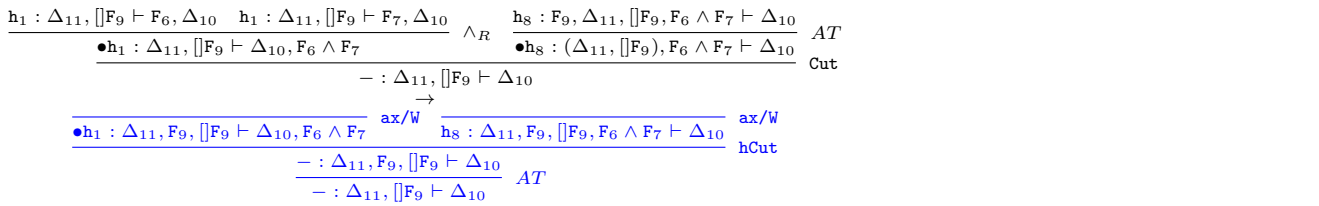


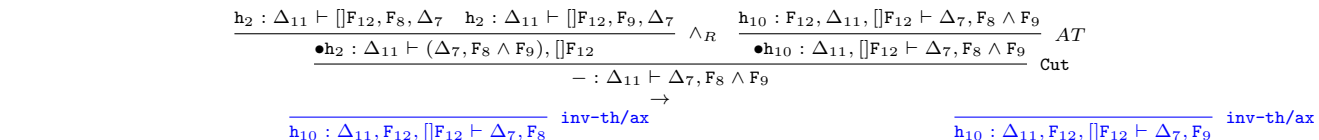
- Case rule $\vee_L$

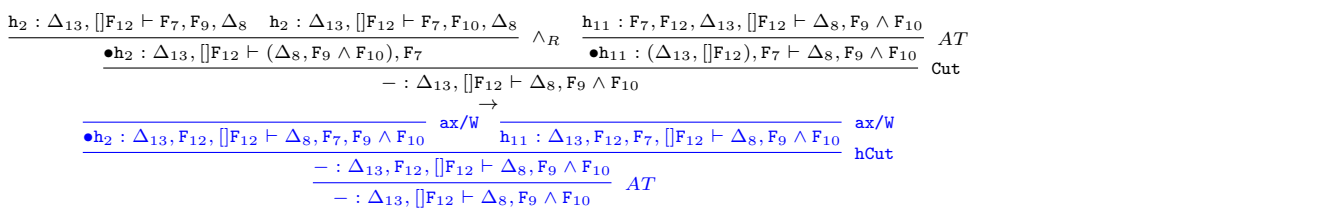




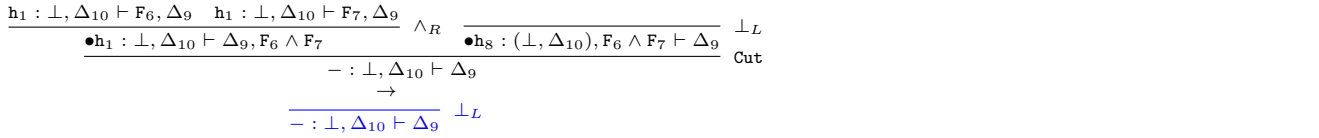
- Case rule AT

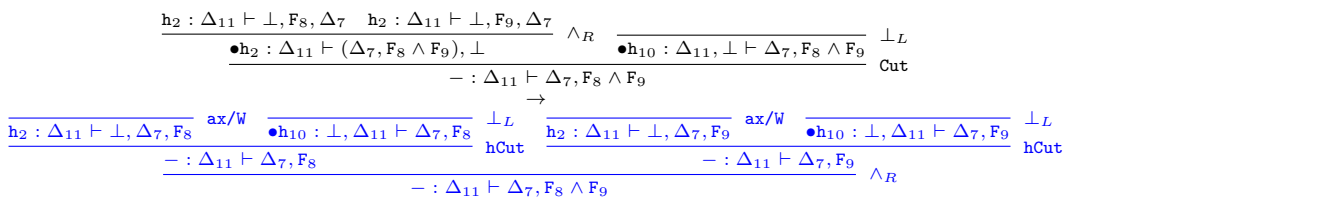


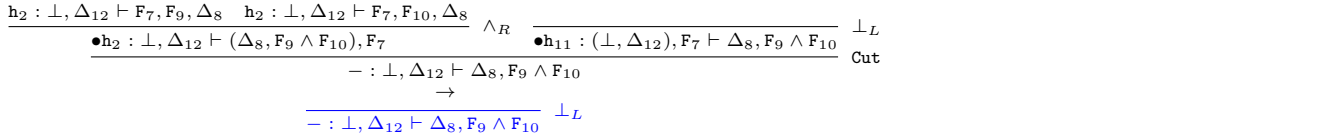




- Case rule $\bot_L$







- Case rule I

$$\dfrac{\dfrac{\dfrac{\dfrac{h_1 : \Delta_{11}, p_9 \vdash F_6, \Delta_{10}, p_9 \quad h_1 : \Delta_{11}, p_9 \vdash F_7, \Delta_{10}, p_9}{\bullet h_1 : \Delta_{11}, p_9 \vdash (\Delta_{10}, p_9), F_6 \wedge F_7}\wedge_R \quad \dfrac{}{\bullet h_8 : (\Delta_{11}, p_9), F_6 \wedge F_7 \vdash \Delta_{10}, p_9} I}{- : \Delta_{11}, p_9 \vdash \Delta_{10}, p_9}\text{Cut}}{\to}}{- : \Delta_{11}, p_9 \vdash \Delta_{10}, p_9} I$$

$$\dfrac{\dfrac{\dfrac{\dfrac{h_2 : \Delta_{10} \vdash p_{11}, F_7, \Delta_{12}, p_{11} \quad h_2 : \Delta_{10} \vdash p_{11}, F_8, \Delta_{12}, p_{11}}{\bullet h_2 : \Delta_{10} \vdash ((\Delta_{12}, p_{11}), F_7 \wedge F_8), p_{11}}\wedge_R \quad \dfrac{}{\bullet h_9 : \Delta_{10}, p_{11} \vdash (\Delta_{12}, p_{11}), F_7 \wedge F_8} I}{- : \Delta_{10} \vdash (\Delta_{12}, p_{11}), F_7 \wedge F_8}\text{Cut}}{\to}}{\dfrac{\dfrac{h_2 : \Delta_{10} \vdash \Delta_{12}, F_7, p_{11}, p_{11}}{h_2 : \Delta_{10} \vdash \Delta_{12}, F_7, p_{11}, p_{11}}\text{ax/W} \quad \dfrac{}{\bullet h_9 : \Delta_{10}, p_{11} \vdash \Delta_{12}, F_7, p_{11}} I}{- : \Delta_{10} \vdash \Delta_{12}, F_7, p_{11}}\text{hCut} \quad \dfrac{\dfrac{h_2 : \Delta_{10} \vdash \Delta_{12}, F_8, p_{11}, p_{11}}{h_2 : \Delta_{10} \vdash \Delta_{12}, F_8, p_{11}, p_{11}}\text{ax/W} \quad \dfrac{}{\bullet h_9 : \Delta_{10}, p_{11} \vdash \Delta_{12}, F_8, p_{11}} I}{- : \Delta_{10} \vdash \Delta_{12}, F_8, p_{11}}\text{hCut}}{- : \Delta_{10} \vdash \Delta_{12}, p_{11}, F_7 \wedge F_8}\wedge_R}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{h_2 : \Delta_{13}, p_{11} \vdash F_7, F_8, \Delta_{12}, p_{11} \quad h_2 : \Delta_{13}, p_{11} \vdash F_7, F_9, \Delta_{12}, p_{11}}{\bullet h_2 : \Delta_{13}, p_{11} \vdash ((\Delta_{12}, p_{11}), F_8 \wedge F_9), F_7}\wedge_R \quad \dfrac{}{\bullet h_{10} : (\Delta_{13}, p_{11}), F_7 \vdash (\Delta_{12}, p_{11}), F_8 \wedge F_9} I}{- : \Delta_{13}, p_{11} \vdash (\Delta_{12}, p_{11}), F_8 \wedge F_9}\text{Cut}}{\to}}{- : \Delta_{13}, p_{11} \vdash \Delta_{12}, p_{11}, F_8 \wedge F_9} I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\dfrac{h_1 : \top, \Delta_{10} \vdash F_6, \Delta_9 \quad h_1 : \top, \Delta_{10} \vdash F_7, \Delta_9}{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \wedge F_7}\wedge_R \quad \dfrac{h_8 : \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_9}{\bullet h_8 : (\top, \Delta_{10}), F_6 \wedge F_7 \vdash \Delta_9}\top_L}{- : \top, \Delta_{10} \vdash \Delta_9}\text{Cut}}{\to}}{\dfrac{\bullet h_1 : \top, \Delta_{10} \vdash \Delta_9, F_6 \wedge F_7}{- : \top, \Delta_{10} \vdash \Delta_9}\text{ax/W} \quad \dfrac{h_8 : \top, \Delta_{10}, F_6 \wedge F_7 \vdash \Delta_9}{}\text{ax/W}}\text{hCut}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{h_2 : \Delta_{11} \vdash \top, F_8, \Delta_7 \quad h_2 : \Delta_{11} \vdash \top, F_9, \Delta_7}{\bullet h_2 : \Delta_{11} \vdash (\Delta_7, F_8 \wedge F_9), \top}\wedge_R \quad \dfrac{h_{10} : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9}{\bullet h_{10} : \Delta_{11}, \top \vdash \Delta_7, F_8 \wedge F_9}\top_L}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9}\text{Cut}}{\to}}{- : \Delta_{11} \vdash \Delta_7, F_8 \wedge F_9}\text{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{h_2 : \top, \Delta_{12} \vdash F_7, F_9, \Delta_8 \quad h_2 : \top, \Delta_{12} \vdash F_7, F_{10}, \Delta_8}{\bullet h_2 : \top, \Delta_{12} \vdash (\Delta_8, F_9 \wedge F_{10}), F_7}\wedge_R \quad \dfrac{h_{11} : F_7, \Delta_{12} \vdash \Delta_8, F_9 \wedge F_{10}}{\bullet h_{11} : (\top, \Delta_{12}), F_7 \vdash \Delta_8, F_9 \wedge F_{10}}\top_L}{- : \top, \Delta_{12} \vdash \Delta_8, F_9 \wedge F_{10}}\text{Cut}}{\to}}{\dfrac{\bullet h_2 : \top, \Delta_{12} \vdash \Delta_8, F_7, F_9 \wedge F_{10}}{}\text{ax/W} \quad \dfrac{h_{11} : \top, \Delta_{12}, F_7 \vdash \Delta_8, F_9 \wedge F_{10}}{}\text{ax/W}}{- : \top, \Delta_{12} \vdash \Delta_8, F_9 \wedge F_{10}}\text{hCut}$$

8.3 Status of $\vee_R$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{\dfrac{h_1 : \Delta_6 \vdash F_7, F_8, \Delta_{10}, F_{11} \to F_{12}}{\bullet h_1 : \Delta_6 \vdash (\Delta_{10}, F_{11} \to F_{12}), F_7 \vee F_8}\vee_R \quad \dfrac{h_9 : F_{11}, \Delta_6, F_7 \vee F_8 \vdash F_{12}, \Delta_{10}}{\bullet h_9 : \Delta_6, F_7 \vee F_8 \vdash \Delta_{10}, F_{11} \to F_{12}}\to_R}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \to F_{12}}\text{Cut}}{\to}}{\dfrac{\dfrac{h_1 : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_7, F_8}{\bullet h_1 : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}, F_7 \vee F_8}\vee_R \quad \dfrac{}{h_9 : \Delta_6, F_{11}, F_7 \vee F_8 \vdash \Delta_{10}, F_{12}}\text{ax/W}}{- : \Delta_6, F_{11} \vdash \Delta_{10}, F_{12}}\text{hCut}}{- : \Delta_6 \vdash \Delta_{10}, F_{11} \to F_{12}}\to_R}\text{inv-th/ax}$$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Delta_8 \vdash \mathtt{F}_7, \mathtt{F}_9, \mathtt{F}_{10}, \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13}}{\bullet \mathtt{h}_2 : \Delta_8 \vdash ((\Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13}), \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_7}\ \vee_R \quad \dfrac{\mathtt{h}_{11} : \mathtt{F}_7, \mathtt{F}_{12}, \Delta_8 \vdash \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_9 \vee \mathtt{F}_{10}}{\bullet \mathtt{h}_{11} : \Delta_8, \mathtt{F}_7 \vdash (\Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13}), \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \to_R}{- : \Delta_8 \vdash (\Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13}), \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \mathtt{Cut}}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_8, \mathtt{F}_{12} \vdash \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_9}{\bullet \mathtt{h}_2 : \Delta_8, \mathtt{F}_{12} \vdash \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \vee_R \quad \dfrac{}{\mathtt{h}_{11} : \Delta_8, \mathtt{F}_{12}, \mathtt{F}_7 \vdash \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_{12} \vdash \Delta_{14}, \mathtt{F}_{13}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \mathtt{hCut}}{- : \Delta_8 \vdash \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \to_R$$

$\mathtt{inv\text{-}th/ax}$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6 \vdash \mathtt{F}_7, \mathtt{F}_8, \Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}{\bullet \mathtt{h}_1 : \Delta_6 \vdash (\Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}), \mathtt{F}_7 \vee \mathtt{F}_8}\ \vee_R \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \mathtt{F}_{11}, \Delta_{10} \quad \mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \mathtt{F}_{12}, \Delta_{10}}{\bullet \mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\ \wedge_R}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\ \mathtt{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_7, \mathtt{F}_8}{\bullet \mathtt{h}_1 : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\ \vee_R \quad \dfrac{}{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11}}\ \mathtt{ax/W}}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11}}\ \mathtt{hCut} \quad \dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{12}, \mathtt{F}_7, \mathtt{F}_8}{\bullet \mathtt{h}_1 : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}\ \vee_R \quad \dfrac{}{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12}}\ \mathtt{ax/W}}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{12}}\ \mathtt{hCut}}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\ \wedge_R$$

$\mathtt{inv\text{-}th/ax}$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Delta_8 \vdash \mathtt{F}_7, \mathtt{F}_9, \mathtt{F}_{10}, \Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13}}{\bullet \mathtt{h}_2 : \Delta_8 \vdash ((\Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13}), \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_7}\ \vee_R \quad \dfrac{\mathtt{h}_{11} : \mathtt{F}_7, \Delta_8 \vdash \mathtt{F}_{12}, \Delta_{14}, \mathtt{F}_9 \vee \mathtt{F}_{10} \quad \mathtt{h}_{11} : \mathtt{F}_7, \Delta_8 \vdash \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_9 \vee \mathtt{F}_{10}}{\bullet \mathtt{h}_{11} : \Delta_8, \mathtt{F}_7 \vdash (\Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13}), \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \wedge_R}{- : \Delta_8 \vdash (\Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13}), \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \mathtt{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_8 \vdash \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_9, \mathtt{F}_{12} \wedge \mathtt{F}_{13}}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_{11} : \Delta_8, \mathtt{F}_7 \vdash \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{12}, \mathtt{F}_9}\ \mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_{11} : \Delta_8, \mathtt{F}_7 \vdash \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{13}, \mathtt{F}_9}\ \mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_{11} : \Delta_8, \mathtt{F}_7 \vdash \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_9, \mathtt{F}_{12} \wedge \mathtt{F}_{13}}\ \wedge_R\ \mathtt{hCut}}{- : \Delta_8 \vdash \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_9, \mathtt{F}_{12} \wedge \mathtt{F}_{13}}\ \vee_R}{- : \Delta_8 \vdash \Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \vee_R$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6 \vdash \mathtt{F}_7, \mathtt{F}_8, \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}{\bullet \mathtt{h}_1 : \Delta_6 \vdash (\Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}), \mathtt{F}_7 \vee \mathtt{F}_8}\ \vee_R \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \mathtt{F}_{11}, \mathtt{F}_{12}, \Delta_{10}}{\bullet \mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}\ \vee_R}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}\ \mathtt{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_7, \mathtt{F}_8}{\bullet \mathtt{h}_1 : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_7 \vee \mathtt{F}_8}\ \vee_R \quad \dfrac{}{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_{12}}\ \mathtt{ax/W}}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_{12}}\ \mathtt{hCut}}{- : \Delta_6 \vdash \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}\ \vee_R$$

$\mathtt{inv\text{-}th/ax}$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Delta_8 \vdash \mathtt{F}_7, \mathtt{F}_9, \mathtt{F}_{10}, \Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13}}{\bullet \mathtt{h}_2 : \Delta_8 \vdash ((\Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13}), \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_7}\ \vee_R \quad \dfrac{\mathtt{h}_{11} : \mathtt{F}_7, \Delta_8 \vdash \mathtt{F}_{12}, \mathtt{F}_{13}, \Delta_{14}, \mathtt{F}_9 \vee \mathtt{F}_{10}}{\bullet \mathtt{h}_{11} : \Delta_8, \mathtt{F}_7 \vdash (\Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13}), \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \vee_R}{- : \Delta_8 \vdash (\Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13}), \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \mathtt{Cut}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_8 \vdash \Delta_{14}, \mathtt{F}_{10}, \mathtt{F}_{12}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_9}{\bullet \mathtt{h}_2 : \Delta_8 \vdash \Delta_{14}, \mathtt{F}_{12}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \vee_R \quad \dfrac{}{\mathtt{h}_{11} : \Delta_8, \mathtt{F}_7 \vdash \Delta_{14}, \mathtt{F}_{12}, \mathtt{F}_{13}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_8 \vdash \Delta_{14}, \mathtt{F}_{12}, \mathtt{F}_{13}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \mathtt{hCut}}{- : \Delta_8 \vdash \Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \vee_R$$

$\mathtt{inv\text{-}th/ax}$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Delta_8 \vdash \mathtt{F}_7, \mathtt{F}_{11}, \mathtt{F}_{12}, \Delta_{10}}{\bullet \mathtt{h}_2 : \Delta_8 \vdash (\Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}), \mathtt{F}_7}\ \vee_R \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_7, \Delta_8 \vdash \mathtt{F}_{11}, \mathtt{F}_{12}, \Delta_{10}}{\bullet \mathtt{h}_9 : \Delta_8, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}\ \vee_R}{- : \Delta_8 \vdash \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}\ \mathtt{Cut}$$

$$\to$$

$$\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_8 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_8, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_{12}}\ \mathtt{ax/W}}{\bullet \mathtt{h}_9 : \Delta_8, \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_{12}}\ \mathtt{H}\ \mathtt{hCut}}{- : \Delta_8 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_{12}}\ \vee_R$$

$$- : \Delta_8 \vdash \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6 \vdash \mathtt{F}_7, \mathtt{F}_8, \bot, \Delta_{10}}{\bullet \mathtt{h}_1 : \Delta_6 \vdash (\bot, \Delta_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}\vee_R \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_{10}}{\bullet \mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \bot, \Delta_{10}}\bot_R}{- : \Delta_6 \vdash \bot, \Delta_{10}}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_6 \vdash \bot, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \bot, \Delta_{10}}\mathtt{ax/W}}{- : \Delta_6 \vdash \bot, \Delta_{10}}\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_8 \vdash \mathtt{F}_7, \mathtt{F}_9, \mathtt{F}_{10}, \bot, \Delta_{12}}{\bullet \mathtt{h}_2 : \Delta_8 \vdash ((\bot, \Delta_{12}), \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_7}\vee_R \quad \dfrac{\mathtt{h}_{11} : \mathtt{F}_7, \Delta_8 \vdash \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10}}{\bullet \mathtt{h}_{11} : \Delta_8, \mathtt{F}_7 \vdash (\bot, \Delta_{12}), \mathtt{F}_9 \vee \mathtt{F}_{10}}\bot_R}{- : \Delta_8 \vdash (\bot, \Delta_{12}), \mathtt{F}_9 \vee \mathtt{F}_{10}}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_8 \vdash \bot, \Delta_{12}, \mathtt{F}_7, \mathtt{F}_9 \vee \mathtt{F}_{10}}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_{11} : \Delta_8, \mathtt{F}_7 \vdash \bot, \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\mathtt{ax/W}}{- : \Delta_8 \vdash \bot, \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\mathtt{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_6 \vdash \mathtt{F}_7, \mathtt{F}_8, \top, \Delta_{10}}{\bullet \mathtt{h}_1 : \Delta_6 \vdash (\top, \Delta_{10}), \mathtt{F}_7 \vee \mathtt{F}_8}\vee_R \quad \dfrac{}{\bullet \mathtt{h}_9 : \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \top, \Delta_{10}}\top_R}{- : \Delta_6 \vdash \top, \Delta_{10}}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \Delta_6 \vdash \top, \Delta_{10}}\top_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_8 \vdash \mathtt{F}_7, \mathtt{F}_9, \mathtt{F}_{10}, \top, \Delta_{12}}{\bullet \mathtt{h}_2 : \Delta_8 \vdash ((\top, \Delta_{12}), \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_7}\vee_R \quad \dfrac{}{\bullet \mathtt{h}_{11} : \Delta_8, \mathtt{F}_7 \vdash (\top, \Delta_{12}), \mathtt{F}_9 \vee \mathtt{F}_{10}}\top_R}{- : \Delta_8 \vdash (\top, \Delta_{12}), \mathtt{F}_9 \vee \mathtt{F}_{10}}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \Delta_8 \vdash \top, \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\top_R$$

- Case rule $A4$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_9, \Delta_{12} \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_{10}, []\mathtt{F}_{11}}{\bullet \mathtt{h}_1 : \Box\Gamma_9, \Delta_{12} \vdash (\Delta_{10}, []\mathtt{F}_{11}), \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_9 \vdash \mathtt{F}_{11}}{\bullet \mathtt{h}_8 : (\Box\Gamma_9, \Delta_{12}), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{10}, []\mathtt{F}_{11}}A4}{- : \Box\Gamma_9, \Delta_{12} \vdash \Delta_{10}, []\mathtt{F}_{11}}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_9 \vdash \mathtt{F}_{11}}\mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_9 \vdash \Delta_{10}, []\mathtt{F}_{11}}A4$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Box\Gamma_{14}, \Delta_{11} \vdash \Box\mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_9, \Delta_{13}, []\mathtt{F}_{12}}{\bullet \mathtt{h}_2 : \Box\Gamma_{14}, \Delta_{11} \vdash ((\Delta_{13}, []\mathtt{F}_{12}), \mathtt{F}_8 \vee \mathtt{F}_9), \Box\mathtt{F}_7}\vee_R \quad \dfrac{\mathtt{h}_{10} : \Box\Gamma_{14}, \Box\mathtt{F}_7 \vdash \mathtt{F}_{12}}{\bullet \mathtt{h}_{10} : (\Box\Gamma_{14}, \Delta_{11}), \Box\mathtt{F}_7 \vdash (\Delta_{13}, []\mathtt{F}_{12}), \mathtt{F}_8 \vee \mathtt{F}_9}A4}{- : \Box\Gamma_{14}, \Delta_{11} \vdash (\Delta_{13}, []\mathtt{F}_{12}), \mathtt{F}_8 \vee \mathtt{F}_9}\mathtt{Cut}}{\rightarrow}$$

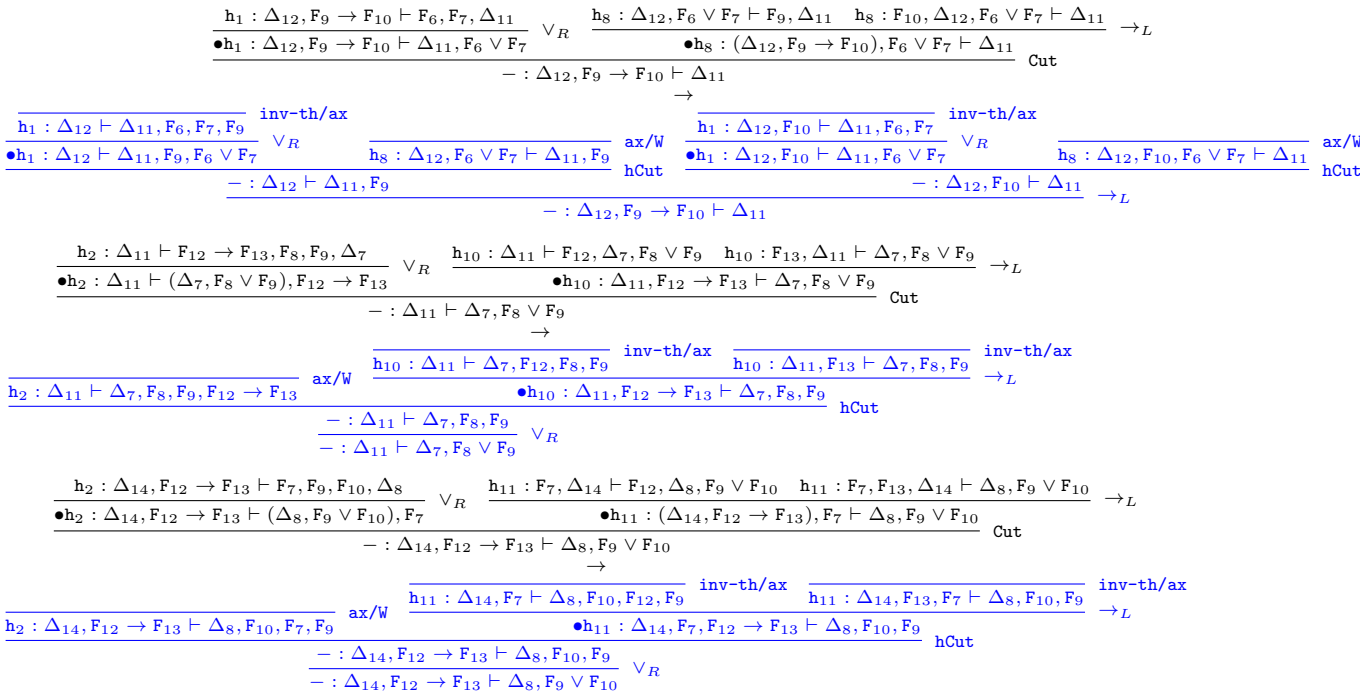
$$\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_{11}, \Box\Gamma_{14} \vdash \Box\mathtt{F}_7, \Delta_{13}, \mathtt{F}_8, \mathtt{F}_9, []\mathtt{F}_{12}}\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_{10} : \Box\mathtt{F}_7, \Box\Gamma_{14} \vdash \mathtt{F}_{12}}\mathtt{ax/W}}{\bullet \mathtt{h}_{10} : \Box\mathtt{F}_7, \Delta_{11}, \Box\Gamma_{14} \vdash \Delta_{13}, \mathtt{F}_8, \mathtt{F}_9, []\mathtt{F}_{12}}A4}{- : \Delta_{11}, \Box\Gamma_{14} \vdash \Delta_{13}, \mathtt{F}_8, \mathtt{F}_9, []\mathtt{F}_{12}}\mathtt{hCut}$$

$$\dfrac{}{- : \Delta_{11}, \Box\Gamma_{14} \vdash \Delta_{13}, []\mathtt{F}_{12}, \mathtt{F}_8 \vee \mathtt{F}_9}\vee_R$$

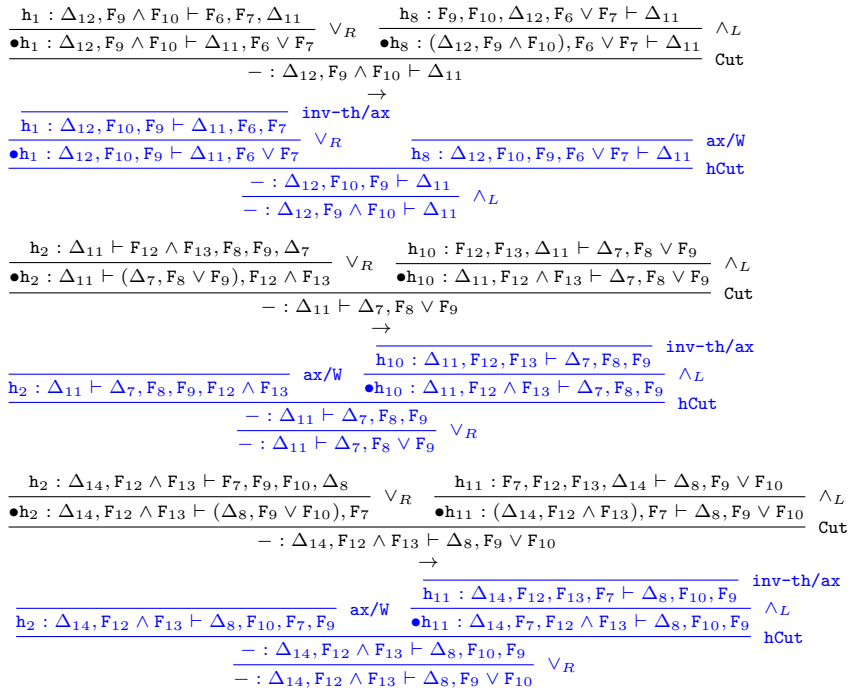
$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Box\Gamma_{11}, \Delta_{14} \vdash \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_9, \Delta_{13}, []\mathtt{F}_{12}}{\bullet \mathtt{h}_2 : \Box\Gamma_{11}, \Delta_{14} \vdash ((\Delta_{13}, []\mathtt{F}_{12}), \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_7}\vee_R \quad \dfrac{\mathtt{h}_{10} : \Box\Gamma_{11} \vdash \mathtt{F}_{12}}{\bullet \mathtt{h}_{10} : (\Box\Gamma_{11}, \Delta_{14}), \mathtt{F}_7 \vdash (\Delta_{13}, []\mathtt{F}_{12}), \mathtt{F}_8 \vee \mathtt{F}_9}A4}{- : \Box\Gamma_{11}, \Delta_{14} \vdash (\Delta_{13}, []\mathtt{F}_{12}), \mathtt{F}_8 \vee \mathtt{F}_9}\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_{11} \vdash \mathtt{F}_{12}}\mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{11} \vdash \Delta_{13}, []\mathtt{F}_{12}, \mathtt{F}_8 \vee \mathtt{F}_9}A4$$

- Case rule $\rightarrow_L$

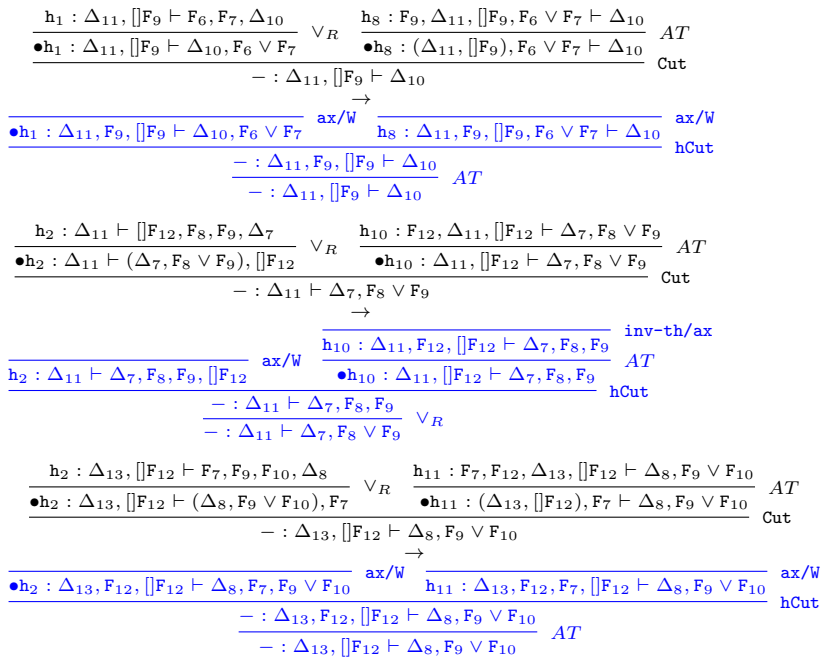


- Case rule $\wedge_L$



- Case rule $\vee_L$

- Case rule AT



- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \bot, \Delta_{10} \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_9}{\bullet\mathtt{h}_1 : \bot, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{}{\bullet\mathtt{h}_8 : (\bot, \Delta_{10}), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_9}\bot_L}{- : \bot, \Delta_{10} \vdash \Delta_9}\mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_{10} \vdash \Delta_9}\bot_L}\to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{11} \vdash \bot, \mathtt{F}_8, \mathtt{F}_9, \Delta_7}{\bullet\mathtt{h}_2 : \Delta_{11} \vdash (\Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9), \bot}\vee_R \quad \dfrac{}{\bullet\mathtt{h}_{10} : \Delta_{11}, \bot \vdash \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9}\bot_L}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9}\mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{11} \vdash \bot, \Delta_7, \mathtt{F}_8, \mathtt{F}_9}{\mathtt{h}_2 : \Delta_{11} \vdash \bot, \Delta_7, \mathtt{F}_8, \mathtt{F}_9}\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_{10} : \bot, \Delta_{11} \vdash \Delta_7, \mathtt{F}_8, \mathtt{F}_9}\bot_L}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8, \mathtt{F}_9}\mathtt{hCut}}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9}\vee_R}\to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \bot, \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_9, \mathtt{F}_{10}, \Delta_8}{\bullet\mathtt{h}_2 : \bot, \Delta_{12} \vdash (\Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_7}\vee_R \quad \dfrac{}{\bullet\mathtt{h}_{11} : (\bot, \Delta_{12}), \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\bot_L}{- : \bot, \Delta_{12} \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_{12} \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\bot_L}\to$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_{11}, \mathtt{p}_9 \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_{10}, \mathtt{p}_9}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{p}_9 \vdash (\Delta_{10}, \mathtt{p}_9), \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{}{\bullet\mathtt{h}_8 : (\Delta_{11}, \mathtt{p}_9), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_{10}, \mathtt{p}_9}I}{- : \Delta_{11}, \mathtt{p}_9 \vdash \Delta_{10}, \mathtt{p}_9}\mathtt{Cut}}{\dfrac{}{- : \Delta_{11}, \mathtt{p}_9 \vdash \Delta_{10}, \mathtt{p}_9}I}\to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{10} \vdash \mathtt{p}_{11}, \mathtt{F}_7, \mathtt{F}_8, \Delta_{12}, \mathtt{p}_{11}}{\bullet\mathtt{h}_2 : \Delta_{10} \vdash ((\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_{11}}\vee_R \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{10}, \mathtt{p}_{11} \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8}I}{- : \Delta_{10} \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7 \vee \mathtt{F}_8}\mathtt{Cut}}{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{10} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_{11}, \mathtt{p}_{11}}{\mathtt{h}_2 : \Delta_{10} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_{11}, \mathtt{p}_{11}}\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_{10}, \mathtt{p}_{11} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_{11}}I}{- : \Delta_{10} \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_{11}}\mathtt{hCut}}{- : \Delta_{10} \vdash \Delta_{12}, \mathtt{p}_{11}, \mathtt{F}_7 \vee \mathtt{F}_8}\vee_R}\to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{13}, \mathtt{p}_{11} \vdash \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_9, \Delta_{12}, \mathtt{p}_{11}}{\bullet\mathtt{h}_2 : \Delta_{13}, \mathtt{p}_{11} \vdash ((\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_7}\vee_R \quad \dfrac{}{\bullet\mathtt{h}_{10} : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_7 \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_8 \vee \mathtt{F}_9}I}{- : \Delta_{13}, \mathtt{p}_{11} \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_8 \vee \mathtt{F}_9}\mathtt{Cut}}{\dfrac{}{- : \Delta_{13}, \mathtt{p}_{11} \vdash \Delta_{12}, \mathtt{p}_{11}, \mathtt{F}_8 \vee \mathtt{F}_9}I}\to$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_{10} \vdash \mathtt{F}_6, \mathtt{F}_7, \Delta_9}{\bullet\mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7}\vee_R \quad \dfrac{\mathtt{h}_8 : \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_9}{\bullet\mathtt{h}_8 : (\top, \Delta_{10}), \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_9}\top_L}{- : \top, \Delta_{10} \vdash \Delta_9}\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \top, \Delta_{10} \vdash \Delta_9, \mathtt{F}_6 \vee \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \top, \Delta_{10}, \mathtt{F}_6 \vee \mathtt{F}_7 \vdash \Delta_9}\mathtt{ax/W}}{- : \top, \Delta_{10} \vdash \Delta_9}\mathtt{hCut}}\to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{11} \vdash \top, \mathtt{F}_8, \mathtt{F}_9, \Delta_7}{\bullet\mathtt{h}_2 : \Delta_{11} \vdash (\Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9), \top}\vee_R \quad \dfrac{\mathtt{h}_{10} : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9}{\bullet\mathtt{h}_{10} : \Delta_{11}, \top \vdash \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9}\top_L}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9}\mathtt{Cut}}{\dfrac{}{- : \Delta_{11} \vdash \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9}\mathtt{ax/W}}\to$$

$$\dfrac{\dfrac{\mathtt{h}_2 : \top, \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_9, \mathtt{F}_{10}, \Delta_8}{\bullet \mathtt{h}_2 : \top, \Delta_{12} \vdash (\Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_7}\ \vee_R \quad \dfrac{\mathtt{h}_{11} : \mathtt{F}_7, \Delta_{12} \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}{\bullet \mathtt{h}_{11} : (\top, \Delta_{12}), \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \top_L}{- : \top, \Delta_{12} \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \top, \Delta_{12} \vdash \Delta_8, \mathtt{F}_7, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_{11} : \top, \Delta_{12}, \mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \top, \Delta_{12} \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \mathtt{hCut}$$

8.4 Status of $\bot_R$: OK

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \rightarrow \mathtt{F}_8}{\bullet \mathtt{h}_1 : \Delta_4 \vdash (\Delta_6, \mathtt{F}_7 \rightarrow \mathtt{F}_8), \bot}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \bot, \mathtt{F}_7, \Delta_4 \vdash \mathtt{F}_8, \Delta_6}{\bullet \mathtt{h}_5 : \Delta_4, \bot \vdash \Delta_6, \mathtt{F}_7 \rightarrow \mathtt{F}_8}\ \rightarrow_R}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \rightarrow \mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \rightarrow \mathtt{F}_8}\ \mathtt{ax/W}$$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Delta_6 \vdash \mathtt{F}_5, \Delta_{10}, \mathtt{F}_8 \rightarrow \mathtt{F}_9}{\bullet \mathtt{h}_2 : \Delta_6 \vdash (\bot, \Delta_{10}, \mathtt{F}_8 \rightarrow \mathtt{F}_9), \mathtt{F}_5}\ \bot_R \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_5, \mathtt{F}_8, \Delta_6 \vdash \bot, \mathtt{F}_9, \Delta_{10}}{\bullet \mathtt{h}_7 : \Delta_6, \mathtt{F}_5 \vdash \bot, \Delta_{10}, \mathtt{F}_8 \rightarrow \mathtt{F}_9}\ \rightarrow_R}{- : \Delta_6 \vdash \bot, \Delta_{10}, \mathtt{F}_8 \rightarrow \mathtt{F}_9}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_6 \vdash \bot, \Delta_{10}, \mathtt{F}_5, \mathtt{F}_8 \rightarrow \mathtt{F}_9}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_7 : \Delta_6, \mathtt{F}_5 \vdash \bot, \Delta_{10}, \mathtt{F}_8 \rightarrow \mathtt{F}_9}\ \mathtt{ax/W}}{- : \Delta_6 \vdash \bot, \Delta_{10}, \mathtt{F}_8 \rightarrow \mathtt{F}_9}\ \mathtt{hCut}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8}{\bullet \mathtt{h}_1 : \Delta_4 \vdash (\Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8), \bot}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_4 \vdash \mathtt{F}_7, \Delta_6 \quad \mathtt{h}_5 : \bot, \Delta_4 \vdash \mathtt{F}_8, \Delta_6}{\bullet \mathtt{h}_5 : \Delta_4, \bot \vdash \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \wedge_R}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \mathtt{ax/W}$$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Delta_6 \vdash \mathtt{F}_5, \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}{\bullet \mathtt{h}_2 : \Delta_6 \vdash (\bot, \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_5}\ \bot_R \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_5, \Delta_6 \vdash \bot, \mathtt{F}_8, \Delta_{10} \quad \mathtt{h}_7 : \mathtt{F}_5, \Delta_6 \vdash \bot, \mathtt{F}_9, \Delta_{10}}{\bullet \mathtt{h}_7 : \Delta_6, \mathtt{F}_5 \vdash \bot, \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\ \wedge_R}{- : \Delta_6 \vdash \bot, \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_6 \vdash \bot, \Delta_{10}, \mathtt{F}_5, \mathtt{F}_8 \wedge \mathtt{F}_9}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_7 : \Delta_6, \mathtt{F}_5 \vdash \bot, \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\ \mathtt{ax/W}}{- : \Delta_6 \vdash \bot, \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\ \mathtt{hCut}$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8}{\bullet \mathtt{h}_1 : \Delta_4 \vdash (\Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8), \bot}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_4 \vdash \mathtt{F}_7, \mathtt{F}_8, \Delta_6}{\bullet \mathtt{h}_5 : \Delta_4, \bot \vdash \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8}\ \vee_R}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8}\ \mathtt{ax/W}$$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Delta_6 \vdash \mathtt{F}_5, \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}{\bullet \mathtt{h}_2 : \Delta_6 \vdash (\bot, \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_5}\ \bot_R \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_5, \Delta_6 \vdash \bot, \mathtt{F}_8, \mathtt{F}_9, \Delta_{10}}{\bullet \mathtt{h}_7 : \Delta_6, \mathtt{F}_5 \vdash \bot, \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\ \vee_R}{- : \Delta_6 \vdash \bot, \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_6 \vdash \bot, \Delta_{10}, \mathtt{F}_5, \mathtt{F}_8 \vee \mathtt{F}_9}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_7 : \Delta_6, \mathtt{F}_5 \vdash \bot, \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\ \mathtt{ax/W}}{- : \Delta_6 \vdash \bot, \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\ \mathtt{hCut}$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Delta_4 \vdash \bot, \Delta_6}{\bullet \mathtt{h}_1 : \Delta_4 \vdash (\bot, \Delta_6), \bot}\ \bot_R \quad \dfrac{\mathtt{h}_5 : \bot, \Delta_4 \vdash \Delta_6}{\bullet \mathtt{h}_5 : \Delta_4, \bot \vdash \bot, \Delta_6}\ \bot_R}{- : \Delta_4 \vdash \bot, \Delta_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_4 \vdash \bot, \Delta_6}\ \mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_6 \vdash \mathtt{F}_5, \Delta_8}{\bullet \mathtt{h}_2 : \Delta_6 \vdash (\bot, \Delta_8), \mathtt{F}_5}\ \bot_R \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_5, \Delta_6 \vdash \Delta_8}{\bullet \mathtt{h}_7 : \Delta_6, \mathtt{F}_5 \vdash \bot, \Delta_8}\ \bot_R}{- : \Delta_6 \vdash \bot, \Delta_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_6 \vdash \bot, \Delta_8, \mathtt{F}_5}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_7 : \Delta_6, \mathtt{F}_5 \vdash \bot, \Delta_8}\ \mathtt{ax/W}}{- : \Delta_6 \vdash \bot, \Delta_8}\ \mathtt{hCut}}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_4 \vdash \top, \Delta_6}{\bullet \mathtt{h}_1 : \Delta_4 \vdash (\top, \Delta_6), \bot}\ \bot_R \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_4, \bot \vdash \top, \Delta_6}\ \top_R}{- : \Delta_4 \vdash \top, \Delta_6}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_4 \vdash \top, \Delta_6}\ \top_R}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_6 \vdash \mathtt{F}_5, \top, \Delta_8}{\bullet \mathtt{h}_2 : \Delta_6 \vdash (\bot, \top, \Delta_8), \mathtt{F}_5}\ \bot_R \quad \dfrac{}{\bullet \mathtt{h}_7 : \Delta_6, \mathtt{F}_5 \vdash \bot, \top, \Delta_8}\ \top_R}{- : \Delta_6 \vdash \bot, \top, \Delta_8}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_6 \vdash \bot, \top, \Delta_8}\ \top_R}$$

- Case rule $A4$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_5, \Delta_8 \vdash \Delta_6, []\mathtt{F}_7}{\bullet \mathtt{h}_1 : \Box\Gamma_5, \Delta_8 \vdash (\Delta_6, []\mathtt{F}_7), \bot}\ \bot_R \quad \dfrac{\mathtt{h}_4 : \Box\Gamma_5 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_4 : (\Box\Gamma_5, \Delta_8), \bot \vdash \Delta_6, []\mathtt{F}_7}\ A4}{- : \Box\Gamma_5, \Delta_8 \vdash \Delta_6, []\mathtt{F}_7}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_8, \Box\Gamma_5 \vdash \Delta_6, []\mathtt{F}_7}\ \mathtt{ax/W}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Box\Gamma_{10}, \Delta_7 \vdash \Box\mathtt{F}_5, \Delta_9, []\mathtt{F}_8}{\bullet \mathtt{h}_2 : \Box\Gamma_{10}, \Delta_7 \vdash (\bot, \Delta_9, []\mathtt{F}_8), \Box\mathtt{F}_5}\ \bot_R \quad \dfrac{\mathtt{h}_6 : \Box\Gamma_{10}, \Box\mathtt{F}_5 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : (\Box\Gamma_{10}, \Delta_7), \Box\mathtt{F}_5 \vdash \bot, \Delta_9, []\mathtt{F}_8}\ A4}{- : \Box\Gamma_{10}, \Delta_7 \vdash \bot, \Delta_9, []\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_7, \Box\Gamma_{10} \vdash \bot, \Box\mathtt{F}_5, \Delta_9, []\mathtt{F}_8}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \Box\mathtt{F}_5, \Delta_7, \Box\Gamma_{10} \vdash \bot, \Delta_9, []\mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_7, \Box\Gamma_{10} \vdash \bot, \Delta_9, []\mathtt{F}_8}\ \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Box\Gamma_7, \Delta_{10} \vdash \mathtt{F}_5, \Delta_9, []\mathtt{F}_8}{\bullet \mathtt{h}_2 : \Box\Gamma_7, \Delta_{10} \vdash (\bot, \Delta_9, []\mathtt{F}_8), \mathtt{F}_5}\ \bot_R \quad \dfrac{\mathtt{h}_6 : \Box\Gamma_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : (\Box\Gamma_7, \Delta_{10}), \mathtt{F}_5 \vdash \bot, \Delta_9, []\mathtt{F}_8}\ A4}{- : \Box\Gamma_7, \Delta_{10} \vdash \bot, \Delta_9, []\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_7 \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_{10}, \Box\Gamma_7 \vdash \bot, \Delta_9, []\mathtt{F}_8}\ A4}$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_7}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_7, \bot}\ \bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \Delta_8 \vdash \mathtt{F}_5, \Delta_7 \quad \mathtt{h}_4 : \bot, \mathtt{F}_6, \Delta_8 \vdash \Delta_7}{\bullet \mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \to \mathtt{F}_6), \bot \vdash \Delta_7}\ \to_L}{- : \Delta_8, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_8, \mathtt{F}_5 \to \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{ax/W}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_7 \vdash \mathtt{F}_8 \to \mathtt{F}_9, \Delta_5}{\bullet \mathtt{h}_2 : \Delta_7 \vdash (\bot, \Delta_5), \mathtt{F}_8 \to \mathtt{F}_9}\ \bot_R \quad \dfrac{\mathtt{h}_6 : \Delta_7 \vdash \bot, \mathtt{F}_8, \Delta_5 \quad \mathtt{h}_6 : \mathtt{F}_9, \Delta_7 \vdash \bot, \Delta_5}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \bot, \Delta_5}\ \to_L}{- : \Delta_7 \vdash \bot, \Delta_5}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_7 \vdash \bot, \Delta_5, \mathtt{F}_8 \to \mathtt{F}_9}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \bot, \Delta_5}\ \mathtt{ax/W}}{- : \Delta_7 \vdash \bot, \Delta_5}\ \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_5, \Delta_6}{\bullet \mathtt{h}_2 : \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash (\bot, \Delta_6), \mathtt{F}_5}\ \bot_R \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_5, \Delta_{10} \vdash \bot, \mathtt{F}_8, \Delta_6 \quad \mathtt{h}_7 : \mathtt{F}_5, \mathtt{F}_9, \Delta_{10} \vdash \bot, \Delta_6}{\bullet \mathtt{h}_7 : (\Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_5 \vdash \bot, \Delta_6}\ \to_L}{- : \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \bot, \Delta_6}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \bot, \Delta_6, \mathtt{F}_5}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_7 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \bot, \Delta_6}\ \mathtt{ax/W}}{- : \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \bot, \Delta_6}\ \mathtt{hCut}}$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7, \bot}\ \bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \mathtt{F}_5, \mathtt{F}_6, \Delta_8 \vdash \Delta_7}{\bullet \mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6), \bot \vdash \Delta_7}\ \wedge_L}{- : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{ax/W}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_7 \vdash \mathtt{F}_8 \wedge \mathtt{F}_9, \Delta_5}{\bullet \mathtt{h}_2 : \Delta_7 \vdash (\bot, \Delta_5), \mathtt{F}_8 \wedge \mathtt{F}_9}\ \bot_R \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_8, \mathtt{F}_9, \Delta_7 \vdash \bot, \Delta_5}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \bot, \Delta_5}\ \wedge_L}{- : \Delta_7 \vdash \bot, \Delta_5}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_7 \vdash \bot, \Delta_5, \mathtt{F}_8 \wedge \mathtt{F}_9}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \bot, \Delta_5}\ \mathtt{ax/W}}{- : \Delta_7 \vdash \bot, \Delta_5}\ \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_5, \Delta_6}{\bullet \mathtt{h}_2 : \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash (\bot, \Delta_6), \mathtt{F}_5}\ \bot_R \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_5, \mathtt{F}_8, \mathtt{F}_9, \Delta_{10} \vdash \bot, \Delta_6}{\bullet \mathtt{h}_7 : (\Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_5 \vdash \bot, \Delta_6}\ \wedge_L}{- : \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \bot, \Delta_6}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \bot, \Delta_6, \mathtt{F}_5}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_7 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \bot, \Delta_6}\ \mathtt{ax/W}}{- : \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \bot, \Delta_6}\ \mathtt{hCut}}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7, \bot}\ \bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \mathtt{F}_5, \Delta_8 \vdash \Delta_7 \quad \mathtt{h}_4 : \bot, \mathtt{F}_6, \Delta_8 \vdash \Delta_7}{\bullet \mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6), \bot \vdash \Delta_7}\ \vee_L}{- : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{ax/W}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_7 \vdash \mathtt{F}_8 \vee \mathtt{F}_9, \Delta_5}{\bullet \mathtt{h}_2 : \Delta_7 \vdash (\bot, \Delta_5), \mathtt{F}_8 \vee \mathtt{F}_9}\ \bot_R \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_8, \Delta_7 \vdash \bot, \Delta_5 \quad \mathtt{h}_6 : \mathtt{F}_9, \Delta_7 \vdash \bot, \Delta_5}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \bot, \Delta_5}\ \vee_L}{- : \Delta_7 \vdash \bot, \Delta_5}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_7 \vdash \bot, \Delta_5, \mathtt{F}_8 \vee \mathtt{F}_9}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \bot, \Delta_5}\ \mathtt{ax/W}}{- : \Delta_7 \vdash \bot, \Delta_5}\ \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_5, \Delta_6}{\bullet \mathtt{h}_2 : \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash (\bot, \Delta_6), \mathtt{F}_5}\ \bot_R \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_5, \mathtt{F}_8, \Delta_{10} \vdash \bot, \Delta_6 \quad \mathtt{h}_7 : \mathtt{F}_5, \mathtt{F}_9, \Delta_{10} \vdash \bot, \Delta_6}{\bullet \mathtt{h}_7 : (\Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_5 \vdash \bot, \Delta_6}\ \vee_L}{- : \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \bot, \Delta_6}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \bot, \Delta_6, \mathtt{F}_5}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_7 : \Delta_{10}, \mathtt{F}_5, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \bot, \Delta_6}\ \mathtt{ax/W}}{- : \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \bot, \Delta_6}\ \mathtt{hCut}}$$

- Case rule AT

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_7, []\mathtt{F}_5 \vdash \Delta_6}{\bullet \mathtt{h}_1 : \Delta_7, []\mathtt{F}_5 \vdash \Delta_6, \bot}\ \bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \mathtt{F}_5, \Delta_7, []\mathtt{F}_5 \vdash \Delta_6}{\bullet \mathtt{h}_4 : (\Delta_7, []\mathtt{F}_5), \bot \vdash \Delta_6}\ AT}{- : \Delta_7, []\mathtt{F}_5 \vdash \Delta_6}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_7, []\mathtt{F}_5 \vdash \Delta_6}\ \mathtt{ax/W}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_7 \vdash []\mathtt{F}_8, \Delta_5}{\bullet\mathtt{h}_2 : \Delta_7 \vdash (\bot, \Delta_5), []\mathtt{F}_8}\ \bot_R \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_8, \Delta_7, []\mathtt{F}_8 \vdash \bot, \Delta_5}{\bullet\mathtt{h}_6 : \Delta_7, []\mathtt{F}_8 \vdash \bot, \Delta_5}\ AT}{- : \Delta_7 \vdash \bot, \Delta_5}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_7 \vdash \bot, \Delta_5, []\mathtt{F}_8}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_7, []\mathtt{F}_8 \vdash \bot, \Delta_5}\ \mathtt{ax/W}}{- : \Delta_7 \vdash \bot, \Delta_5}\ \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_9, []\mathtt{F}_8 \vdash \mathtt{F}_5, \Delta_6}{\bullet\mathtt{h}_2 : \Delta_9, []\mathtt{F}_8 \vdash (\bot, \Delta_6), \mathtt{F}_5}\ \bot_R \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_5, \mathtt{F}_8, \Delta_9, []\mathtt{F}_8 \vdash \bot, \Delta_6}{\bullet\mathtt{h}_7 : (\Delta_9, []\mathtt{F}_8), \mathtt{F}_5 \vdash \bot, \Delta_6}\ AT}{- : \Delta_9, []\mathtt{F}_8 \vdash \bot, \Delta_6}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_9, []\mathtt{F}_8 \vdash \bot, \Delta_6, \mathtt{F}_5}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \Delta_9, \mathtt{F}_5, []\mathtt{F}_8 \vdash \bot, \Delta_6}\ \mathtt{ax/W}}{- : \Delta_9, []\mathtt{F}_8 \vdash \bot, \Delta_6}\ \mathtt{hCut}}$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_5 \vdash \Delta_6}{\bullet\mathtt{h}_1 : \Delta_5 \vdash \Delta_6, \bot}\ \bot_R \quad \dfrac{}{\bullet\mathtt{h}_4 : \Delta_5, \bot \vdash \Delta_6}\ \bot_L}{- : \Delta_5 \vdash \Delta_6}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_5 \vdash \Delta_6}\ \mathtt{ax/W}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_7 \vdash \bot, \Delta_5}{\bullet\mathtt{h}_2 : \Delta_7 \vdash (\bot, \Delta_5), \bot}\ \bot_R \quad \dfrac{}{\bullet\mathtt{h}_6 : \Delta_7, \bot \vdash \bot, \Delta_5}\ \bot_L}{- : \Delta_7 \vdash \bot, \Delta_5}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_7 \vdash \bot, \Delta_5}\ \mathtt{ax/W}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \bot, \Delta_8 \vdash \mathtt{F}_5, \Delta_6}{\bullet\mathtt{h}_2 : \bot, \Delta_8 \vdash (\bot, \Delta_6), \mathtt{F}_5}\ \bot_R \quad \dfrac{}{\bullet\mathtt{h}_7 : (\bot, \Delta_8), \mathtt{F}_5 \vdash \bot, \Delta_6}\ \bot_L}{- : \bot, \Delta_8 \vdash \bot, \Delta_6}\ \mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_8 \vdash \bot, \Delta_6}\ \bot_L}$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Delta_7, \mathtt{p}_5 \vdash \Delta_6, \mathtt{p}_5}{\bullet\mathtt{h}_1 : \Delta_7, \mathtt{p}_5 \vdash (\Delta_6, \mathtt{p}_5), \bot}\ \bot_R \quad \dfrac{}{\bullet\mathtt{h}_4 : (\Delta_7, \mathtt{p}_5), \bot \vdash \Delta_6, \mathtt{p}_5}\ I}{- : \Delta_7, \mathtt{p}_5 \vdash \Delta_6, \mathtt{p}_5}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_7, \mathtt{p}_5 \vdash \Delta_6, \mathtt{p}_5}\ I}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_6 \vdash \mathtt{p}_7, \Delta_8, \mathtt{p}_7}{\bullet\mathtt{h}_2 : \Delta_6 \vdash (\bot, \Delta_8, \mathtt{p}_7), \mathtt{p}_7}\ \bot_R \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{p}_7 \vdash \bot, \Delta_8, \mathtt{p}_7}\ I}{- : \Delta_6 \vdash \bot, \Delta_8, \mathtt{p}_7}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_2 : \Delta_6 \vdash \bot, \Delta_8, \mathtt{p}_7, \mathtt{p}_7}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_5 : \Delta_6, \mathtt{p}_7 \vdash \bot, \Delta_8, \mathtt{p}_7}\ I}{- : \Delta_6 \vdash \bot, \Delta_8, \mathtt{p}_7}\ \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_9, \mathtt{p}_7 \vdash \mathtt{F}_5, \Delta_8, \mathtt{p}_7}{\bullet\mathtt{h}_2 : \Delta_9, \mathtt{p}_7 \vdash (\bot, \Delta_8, \mathtt{p}_7), \mathtt{F}_5}\ \bot_R \quad \dfrac{}{\bullet\mathtt{h}_6 : (\Delta_9, \mathtt{p}_7), \mathtt{F}_5 \vdash \bot, \Delta_8, \mathtt{p}_7}\ I}{- : \Delta_9, \mathtt{p}_7 \vdash \bot, \Delta_8, \mathtt{p}_7}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_9, \mathtt{p}_7 \vdash \bot, \Delta_8, \mathtt{p}_7}\ I}$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \top, \Delta_6 \vdash \Delta_5}{\bullet\mathtt{h}_1 : \top, \Delta_6 \vdash \Delta_5, \bot}\ \bot_R \quad \dfrac{\mathtt{h}_4 : \bot, \Delta_6 \vdash \Delta_5}{\bullet\mathtt{h}_4 : (\top, \Delta_6), \bot \vdash \Delta_5}\ \top_L}{- : \top, \Delta_6 \vdash \Delta_5}\ \mathtt{Cut}}{\dfrac{}{- : \top, \Delta_6 \vdash \Delta_5}\ \mathtt{ax/W}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Delta_7 \vdash \top, \Delta_5}{\bullet \mathtt{h}_2 : \Delta_7 \vdash (\bot, \Delta_5), \top}\ \bot_R \quad \dfrac{\mathtt{h}_6 : \Delta_7 \vdash \bot, \Delta_5}{\bullet \mathtt{h}_6 : \Delta_7, \top \vdash \bot, \Delta_5}\ \top_L}{- : \Delta_7 \vdash \bot, \Delta_5}\ \mathtt{Cut}}{- : \Delta_7 \vdash \bot, \Delta_5}\ \mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \top, \Delta_8 \vdash \mathtt{F}_5, \Delta_6}{\bullet \mathtt{h}_2 : \top, \Delta_8 \vdash (\bot, \Delta_6), \mathtt{F}_5}\ \bot_R \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_5, \Delta_8 \vdash \bot, \Delta_6}{\bullet \mathtt{h}_7 : (\top, \Delta_8), \mathtt{F}_5 \vdash \bot, \Delta_6}\ \top_L}{- : \top, \Delta_8 \vdash \bot, \Delta_6}\ \mathtt{Cut}}{\dfrac{\mathtt{h}_2 : \top, \Delta_8 \vdash \bot, \Delta_6, \mathtt{F}_5}{- : \top, \Delta_8 \vdash \bot, \Delta_6}\ \mathtt{hCut}}\ \mathtt{ax/W} \quad \dfrac{\bullet \mathtt{h}_7 : \top, \Delta_8, \mathtt{F}_5 \vdash \bot, \Delta_6}{}\ \mathtt{ax/W}$$

8.5 Status of $\top_R$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_1 : \Delta_4 \vdash (\Delta_6, \mathtt{F}_7 \to \mathtt{F}_8), \top}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8}\ \top_R \quad \dfrac{\mathtt{h}_5 : \top, \mathtt{F}_7, \Delta_4 \vdash \mathtt{F}_8, \Delta_6}{\bullet \mathtt{h}_5 : \Delta_4, \top \vdash \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8}\ \to_R}{\dfrac{\bullet \mathtt{h}_1 : \Delta_4, \mathtt{F}_7 \vdash \top, \Delta_6, \mathtt{F}_8}{- : \Delta_4, \mathtt{F}_7 \vdash \Delta_6, \mathtt{F}_8}\ \top_R \quad \dfrac{\mathtt{h}_5 : \top, \Delta_4, \mathtt{F}_7 \vdash \Delta_6, \mathtt{F}_8}{}\ \mathtt{ax/W}}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \to \mathtt{F}_8}\ \to_R}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_2 : \Delta_6 \vdash (\top, \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_5}{- : \Delta_6 \vdash \top, \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\ \top_R \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_5, \mathtt{F}_8, \Delta_6 \vdash \top, \mathtt{F}_9, \Delta_{10}}{\bullet \mathtt{h}_7 : \Delta_6, \mathtt{F}_5 \vdash \top, \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\ \to_R}{- : \Delta_6 \vdash \top, \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\ \mathtt{Cut}}{- : \Delta_6 \vdash \top, \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9}\ \top_R$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_1 : \Delta_4 \vdash (\Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8), \top}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \top_R \quad \dfrac{\mathtt{h}_5 : \top, \Delta_4 \vdash \mathtt{F}_7, \Delta_6 \quad \mathtt{h}_5 : \top, \Delta_4 \vdash \mathtt{F}_8, \Delta_6}{\bullet \mathtt{h}_5 : \Delta_4, \top \vdash \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \wedge_R}{\dfrac{\bullet \mathtt{h}_1 : \Delta_4 \vdash \top, \Delta_6, \mathtt{F}_7}{}\ \top_R \quad \dfrac{\mathtt{h}_5 : \top, \Delta_4 \vdash \Delta_6, \mathtt{F}_7}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_7}\ \mathtt{ax/W}\ \mathtt{hCut} \quad \dfrac{\bullet \mathtt{h}_1 : \Delta_4 \vdash \top, \Delta_6, \mathtt{F}_8}{}\ \top_R \quad \dfrac{\mathtt{h}_5 : \top, \Delta_4 \vdash \Delta_6, \mathtt{F}_8}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_8}\ \mathtt{ax/W}\ \mathtt{hCut}}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \wedge_R}\ \mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_2 : \Delta_6 \vdash (\top, \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_5}{- : \Delta_6 \vdash \top, \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\ \top_R \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_5, \Delta_6 \vdash \top, \mathtt{F}_8, \Delta_{10} \quad \mathtt{h}_7 : \mathtt{F}_5, \Delta_6 \vdash \top, \mathtt{F}_9, \Delta_{10}}{\bullet \mathtt{h}_7 : \Delta_6, \mathtt{F}_5 \vdash \top, \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\ \wedge_R}{- : \Delta_6 \vdash \top, \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\ \mathtt{Cut}}{- : \Delta_6 \vdash \top, \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9}\ \top_R$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_1 : \Delta_4 \vdash (\Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8), \top}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8}\ \top_R \quad \dfrac{\mathtt{h}_5 : \top, \Delta_4 \vdash \mathtt{F}_7, \mathtt{F}_8, \Delta_6}{\bullet \mathtt{h}_5 : \Delta_4, \top \vdash \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8}\ \vee_R}{\dfrac{\bullet \mathtt{h}_1 : \Delta_4 \vdash \top, \Delta_6, \mathtt{F}_7, \mathtt{F}_8}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_7, \mathtt{F}_8}\ \top_R \quad \dfrac{\mathtt{h}_5 : \top, \Delta_4 \vdash \Delta_6, \mathtt{F}_7, \mathtt{F}_8}{}\ \mathtt{ax/W}}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8}\ \vee_R}\ \mathtt{hCut}}{- : \Delta_4 \vdash \Delta_6, \mathtt{F}_7 \vee \mathtt{F}_8}\ \mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_2 : \Delta_6 \vdash (\top, \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_5}{- : \Delta_6 \vdash \top, \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\ \top_R \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_5, \Delta_6 \vdash \top, \mathtt{F}_8, \mathtt{F}_9, \Delta_{10}}{\bullet \mathtt{h}_7 : \Delta_6, \mathtt{F}_5 \vdash \top, \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\ \vee_R}{- : \Delta_6 \vdash \top, \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\ \mathtt{Cut}}{- : \Delta_6 \vdash \top, \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9}\ \top_R$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_4 \vdash (\bot, \Delta_6), \top}\ \top_R \quad \dfrac{\mathtt{h}_5 : \top, \Delta_4 \vdash \Delta_6}{\bullet \mathtt{h}_5 : \Delta_4, \top \vdash \bot, \Delta_6}\ \bot_R}{- : \Delta_4 \vdash \bot, \Delta_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_4 \vdash \bot, \top, \Delta_6}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_5 : \top, \Delta_4 \vdash \bot, \Delta_6}\ \mathtt{ax/W}}{- : \Delta_4 \vdash \bot, \Delta_6}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_6 \vdash (\top, \bot, \Delta_8), \mathtt{F}_5}\ \top_R \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_5, \Delta_6 \vdash \top, \Delta_8}{\bullet \mathtt{h}_7 : \Delta_6, \mathtt{F}_5 \vdash \top, \bot, \Delta_8}\ \bot_R}{- : \Delta_6 \vdash \top, \bot, \Delta_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_6 \vdash \bot, \top, \Delta_8}\ \top_R$$

- Case rule $\top_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_4 \vdash (\top, \Delta_6), \top}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_4, \top \vdash \top, \Delta_6}\ \top_R}{- : \Delta_4 \vdash \top, \Delta_6}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_4 \vdash \top, \Delta_6}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_6 \vdash (\top, \Delta_8), \mathtt{F}_5}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_7 : \Delta_6, \mathtt{F}_5 \vdash \top, \Delta_8}\ \top_R}{- : \Delta_6 \vdash \top, \Delta_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_6 \vdash \top, \Delta_8}\ \top_R$$

- Case rule $A4$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Box\Gamma_5, \Delta_8 \vdash (\Delta_6, []\mathtt{F}_7), \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \Box\Gamma_5 \vdash \mathtt{F}_7}{\bullet \mathtt{h}_4 : (\Box\Gamma_5, \Delta_8), \top \vdash \Delta_6, []\mathtt{F}_7}\ A4}{- : \Box\Gamma_5, \Delta_8 \vdash \Delta_6, []\mathtt{F}_7}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_5 \vdash \mathtt{F}_7}\ \mathtt{ax/W}}{- : \Delta_8, \Box\Gamma_5 \vdash \Delta_6, []\mathtt{F}_7}\ A4$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Box\Gamma_{10}, \Delta_7 \vdash (\top, \Delta_9, []\mathtt{F}_8), \Box\mathtt{F}_5}\ \top_R \quad \dfrac{\mathtt{h}_6 : \Box\Gamma_{10}, \Box\mathtt{F}_5 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : (\Box\Gamma_{10}, \Delta_7), \Box\mathtt{F}_5 \vdash \top, \Delta_9, []\mathtt{F}_8}\ A4}{- : \Box\Gamma_{10}, \Delta_7 \vdash \top, \Delta_9, []\mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_7, \Box\Gamma_{10} \vdash \top, \Delta_9, []\mathtt{F}_8}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Box\Gamma_7, \Delta_{10} \vdash (\top, \Delta_9, []\mathtt{F}_8), \mathtt{F}_5}\ \top_R \quad \dfrac{\mathtt{h}_6 : \Box\Gamma_7 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : (\Box\Gamma_7, \Delta_{10}), \mathtt{F}_5 \vdash \top, \Delta_9, []\mathtt{F}_8}\ A4}{- : \Box\Gamma_7, \Delta_{10} \vdash \top, \Delta_9, []\mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \Delta_{10}, \Box\Gamma_7 \vdash \top, \Delta_9, []\mathtt{F}_8}\ \top_R$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \rightarrow \mathtt{F}_6 \vdash \Delta_7, \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \top, \Delta_8 \vdash \mathtt{F}_5, \Delta_7 \quad \mathtt{h}_4 : \top, \mathtt{F}_6, \Delta_8 \vdash \Delta_7}{\bullet \mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \rightarrow \mathtt{F}_6), \top \vdash \Delta_7}\ \rightarrow_L}{- : \Delta_8, \mathtt{F}_5 \rightarrow \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8 \vdash \top, \Delta_7, \mathtt{F}_5}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8 \vdash \Delta_7, \mathtt{F}_5}\ \mathtt{ax/W}}{- : \Delta_8 \vdash \Delta_7, \mathtt{F}_5}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_6 \vdash \top, \Delta_7}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{hCut}}{- : \Delta_8, \mathtt{F}_5 \rightarrow \mathtt{F}_6 \vdash \Delta_7}\ \rightarrow_L$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_7 \vdash (\top, \Delta_5), \mathtt{F}_8 \to \mathtt{F}_9}\ \top_R \quad \dfrac{\mathtt{h}_6 : \Delta_7 \vdash \top, \mathtt{F}_8, \Delta_5 \quad \mathtt{h}_6 : \mathtt{F}_9, \Delta_7 \vdash \top, \Delta_5}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \top, \Delta_5}\ \to_L}{- : \Delta_7 \vdash \top, \Delta_5}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_7 \vdash \top, \Delta_5}\ \top_R}\ \to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash (\top, \Delta_6), \mathtt{F}_5}\ \top_R \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_5, \Delta_{10} \vdash \top, \mathtt{F}_8, \Delta_6 \quad \mathtt{h}_7 : \mathtt{F}_5, \mathtt{F}_9, \Delta_{10} \vdash \top, \Delta_6}{\bullet \mathtt{h}_7 : (\Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_5 \vdash \top, \Delta_6}\ \to_L}{- : \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \top, \Delta_6}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_{10}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \top, \Delta_6}\ \top_R}\ \to$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7, \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \top, \mathtt{F}_5, \mathtt{F}_6, \Delta_8 \vdash \Delta_7}{\bullet \mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6), \top \vdash \Delta_7}\ \wedge_L}{- : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5, \mathtt{F}_6 \vdash \top, \Delta_7}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_5, \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_5, \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{hCut}}{- : \Delta_8, \mathtt{F}_5 \wedge \mathtt{F}_6 \vdash \Delta_7}\ \wedge_L}\ \to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_7 \vdash (\top, \Delta_5), \mathtt{F}_8 \wedge \mathtt{F}_9}\ \top_R \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_8, \mathtt{F}_9, \Delta_7 \vdash \top, \Delta_5}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \top, \Delta_5}\ \wedge_L}{- : \Delta_7 \vdash \top, \Delta_5}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_7 \vdash \top, \Delta_5}\ \top_R}\ \to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash (\top, \Delta_6), \mathtt{F}_5}\ \top_R \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_5, \mathtt{F}_8, \mathtt{F}_9, \Delta_{10} \vdash \top, \Delta_6}{\bullet \mathtt{h}_7 : (\Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_5 \vdash \top, \Delta_6}\ \wedge_L}{- : \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \top, \Delta_6}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_{10}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \top, \Delta_6}\ \top_R}\ \to$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7, \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \top, \mathtt{F}_5, \Delta_8 \vdash \Delta_7 \quad \mathtt{h}_4 : \top, \mathtt{F}_6, \Delta_8 \vdash \Delta_7}{\bullet \mathtt{h}_4 : (\Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6), \top \vdash \Delta_7}\ \vee_L}{- : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_5 \vdash \top, \Delta_7}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_5 \vdash \Delta_7}\ \mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_5 \vdash \Delta_7}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_8, \mathtt{F}_6 \vdash \top, \Delta_7}\ \top_R \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_8, \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_6 \vdash \Delta_7}\ \mathtt{hCut}}{- : \Delta_8, \mathtt{F}_5 \vee \mathtt{F}_6 \vdash \Delta_7}\ \vee_L}\ \to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_7 \vdash (\top, \Delta_5), \mathtt{F}_8 \vee \mathtt{F}_9}\ \top_R \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_8, \Delta_7 \vdash \top, \Delta_5 \quad \mathtt{h}_6 : \mathtt{F}_9, \Delta_7 \vdash \top, \Delta_5}{\bullet \mathtt{h}_6 : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \top, \Delta_5}\ \vee_L}{- : \Delta_7 \vdash \top, \Delta_5}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_7 \vdash \top, \Delta_5}\ \top_R}\ \to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash (\top, \Delta_6), \mathtt{F}_5}\ \top_R \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_5, \mathtt{F}_8, \Delta_{10} \vdash \top, \Delta_6 \quad \mathtt{h}_7 : \mathtt{F}_5, \mathtt{F}_9, \Delta_{10} \vdash \top, \Delta_6}{\bullet \mathtt{h}_7 : (\Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_5 \vdash \top, \Delta_6}\ \vee_L}{- : \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \top, \Delta_6}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_{10}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \top, \Delta_6}\ \top_R}\ \to$$

- Case rule AT

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7, []\mathtt{F}_5 \vdash \Delta_6, \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \top, \mathtt{F}_5, \Delta_7, []\mathtt{F}_5 \vdash \Delta_6}{\bullet \mathtt{h}_4 : (\Delta_7, []\mathtt{F}_5), \top \vdash \Delta_6}\ AT}{- : \Delta_7, []\mathtt{F}_5 \vdash \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7, \mathtt{F}_5, []\mathtt{F}_5 \vdash \top, \Delta_6}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_4 : \top, \Delta_7, \mathtt{F}_5, []\mathtt{F}_5 \vdash \Delta_6}\ \mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_5, []\mathtt{F}_5 \vdash \Delta_6}\ \mathtt{hCut}}{- : \Delta_7, []\mathtt{F}_5 \vdash \Delta_6}\ AT$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_7 \vdash (\top, \Delta_5), []\mathtt{F}_8}\ \top_R \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_8, \Delta_7, []\mathtt{F}_8 \vdash \top, \Delta_5}{\bullet \mathtt{h}_6 : \Delta_7, []\mathtt{F}_8 \vdash \top, \Delta_5}\ AT}{- : \Delta_7 \vdash \top, \Delta_5}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_7 \vdash \top, \Delta_5}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_9, []\mathtt{F}_8 \vdash (\top, \Delta_6), \mathtt{F}_5}\ \top_R \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_5, \mathtt{F}_8, \Delta_9, []\mathtt{F}_8 \vdash \top, \Delta_6}{\bullet \mathtt{h}_7 : (\Delta_9, []\mathtt{F}_8), \mathtt{F}_5 \vdash \top, \Delta_6}\ AT}{- : \Delta_9, []\mathtt{F}_8 \vdash \top, \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_9, []\mathtt{F}_8 \vdash \top, \Delta_6}\ \top_R$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \bot, \Delta_6 \vdash \Delta_5, \top}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_4 : (\bot, \Delta_6), \top \vdash \Delta_5}\ \bot_L}{- : \bot, \Delta_6 \vdash \Delta_5}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_6 \vdash \Delta_5}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_7 \vdash (\top, \Delta_5), \bot}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_6 : \Delta_7, \bot \vdash \top, \Delta_5}\ \bot_L}{- : \Delta_7 \vdash \top, \Delta_5}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_7 \vdash \top, \Delta_5}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \bot, \Delta_8 \vdash (\top, \Delta_6), \mathtt{F}_5}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_7 : (\bot, \Delta_8), \mathtt{F}_5 \vdash \top, \Delta_6}\ \bot_L}{- : \bot, \Delta_8 \vdash \top, \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_8 \vdash \top, \Delta_6}\ \top_R$$

- Case rule I

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_7, \mathtt{p}_5 \vdash (\Delta_6, \mathtt{p}_5), \top}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_4 : (\Delta_7, \mathtt{p}_5), \top \vdash \Delta_6, \mathtt{p}_5}\ I}{- : \Delta_7, \mathtt{p}_5 \vdash \Delta_6, \mathtt{p}_5}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_7, \mathtt{p}_5 \vdash \Delta_6, \mathtt{p}_5}\ I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_6 \vdash (\top, \Delta_8, \mathtt{p}_7), \mathtt{p}_7}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_5 : \Delta_6, \mathtt{p}_7 \vdash \top, \Delta_8, \mathtt{p}_7}\ I}{- : \Delta_6 \vdash \top, \Delta_8, \mathtt{p}_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_6 \vdash \top, \Delta_8, \mathtt{p}_7}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_9, \mathtt{p}_7 \vdash (\top, \Delta_8, \mathtt{p}_7), \mathtt{F}_5}\ \top_R \quad \dfrac{}{\bullet \mathtt{h}_6 : (\Delta_9, \mathtt{p}_7), \mathtt{F}_5 \vdash \top, \Delta_8, \mathtt{p}_7}\ I}{- : \Delta_9, \mathtt{p}_7 \vdash \top, \Delta_8, \mathtt{p}_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_9, \mathtt{p}_7 \vdash \top, \Delta_8, \mathtt{p}_7}\ \top_R$$

- Case rule $\top_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5 \vdash \Delta_6, \top}\ \top_R \quad \dfrac{\mathtt{h}_4 : \Delta_5 \vdash \Delta_6}{\bullet \mathtt{h}_4 : \Delta_5, \top \vdash \Delta_6}\ \top_L}{- : \Delta_5 \vdash \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_5 \vdash \Delta_6}\ \mathtt{ax/W}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_7 \vdash (\top, \Delta_5), \top}\ \top_R \quad \dfrac{\mathtt{h}_6 : \Delta_7 \vdash \top, \Delta_5}{\bullet \mathtt{h}_6 : \Delta_7, \top \vdash \top, \Delta_5}\ \top_L}{- : \Delta_7 \vdash \top, \Delta_5}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_7 \vdash \top, \Delta_5}\ \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \top, \Delta_8 \vdash (\top, \Delta_6), \mathtt{F}_5}\ \top_R \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_5, \Delta_8 \vdash \top, \Delta_6}{\bullet \mathtt{h}_7 : (\top, \Delta_8), \mathtt{F}_5 \vdash \top, \Delta_6}\ \top_L}{- : \top, \Delta_8 \vdash \top, \Delta_6}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \top, \Delta_8 \vdash \top, \Delta_6}\ \top_R$$

8.6 Status of $A4$: OK

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_6 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_1 : \Box\Gamma_6, \Delta_7 \vdash (\Delta_{10}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}), []\mathtt{F}_8}\ A4 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_6, \mathtt{F}_{11}, \Delta_7, []\mathtt{F}_8 \vdash \mathtt{F}_{12}, \Delta_{10}}{\bullet \mathtt{h}_9 : (\Box\Gamma_6, \Delta_7), []\mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}}\ \rightarrow_R}{- : \Box\Gamma_6, \Delta_7 \vdash \Delta_{10}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_6 \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{\bullet \mathtt{h}_1 : \Delta_7, \mathtt{F}_{11}, \Box\Gamma_6 \vdash \Delta_{10}, \mathtt{F}_{12}, []\mathtt{F}_8}\ A4 \quad \dfrac{}{\mathtt{h}_9 : \Delta_7, \mathtt{F}_{11}, \Box\Gamma_6, []\mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12}}\ \mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_{11}, \Box\Gamma_6 \vdash \Delta_{10}, \mathtt{F}_{12}}\ \mathtt{hCut}$$
$$\dfrac{}{- : \Delta_7, \Box\Gamma_6 \vdash \Delta_{10}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}}\ \rightarrow_R$$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_2 : \Box\Gamma_7, \Delta_9 \vdash ((\Delta_{14}, \mathtt{F}_{12} \rightarrow \mathtt{F}_{13}), []\mathtt{F}_{10}), \mathtt{F}_8}\ A4 \quad \dfrac{\mathtt{h}_{11} : \Box\Gamma_7, \mathtt{F}_8, \mathtt{F}_{12}, \Delta_9 \vdash \mathtt{F}_{13}, \Delta_{14}, []\mathtt{F}_{10}}{\bullet \mathtt{h}_{11} : (\Box\Gamma_7, \Delta_9), \mathtt{F}_8 \vdash (\Delta_{14}, \mathtt{F}_{12} \rightarrow \mathtt{F}_{13}), []\mathtt{F}_{10}}\ \rightarrow_R}{- : \Box\Gamma_7, \Delta_9 \vdash (\Delta_{14}, \mathtt{F}_{12} \rightarrow \mathtt{F}_{13}), []\mathtt{F}_{10}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_7 \vdash \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_9, \Box\Gamma_7 \vdash \Delta_{14}, []\mathtt{F}_{10}, \mathtt{F}_{12} \rightarrow \mathtt{F}_{13}}\ A4$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_6 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_1 : \Box\Gamma_6, \Delta_7 \vdash (\Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}), []\mathtt{F}_8}\ A4 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_6, \Delta_7, []\mathtt{F}_8 \vdash \mathtt{F}_{11}, \Delta_{10} \quad \mathtt{h}_9 : \Box\Gamma_6, \Delta_7, []\mathtt{F}_8 \vdash \mathtt{F}_{12}, \Delta_{10}}{\bullet \mathtt{h}_9 : (\Box\Gamma_6, \Delta_7), []\mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\ \wedge_R}{- : \Box\Gamma_6, \Delta_7 \vdash \Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_6 \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{\bullet \mathtt{h}_1 : \Delta_7, \Box\Gamma_6 \vdash \Delta_{10}, \mathtt{F}_{11}, []\mathtt{F}_8}\ A4 \quad \dfrac{}{\mathtt{h}_9 : \Delta_7, \Box\Gamma_6, []\mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11}}\ \mathtt{ax/W}}{- : \Delta_7, \Box\Gamma_6 \vdash \Delta_{10}, \mathtt{F}_{11}}\ \mathtt{hCut} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_6 \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{\bullet \mathtt{h}_1 : \Delta_7, \Box\Gamma_6 \vdash \Delta_{10}, \mathtt{F}_{12}, []\mathtt{F}_8}\ A4 \quad \dfrac{}{\mathtt{h}_9 : \Delta_7, \Box\Gamma_6, []\mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{12}}\ \mathtt{ax/W}}{- : \Delta_7, \Box\Gamma_6 \vdash \Delta_{10}, \mathtt{F}_{12}}\ \mathtt{hCut}}{- : \Delta_7, \Box\Gamma_6 \vdash \Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\ \wedge_R$$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_2 : \Box\Gamma_7, \Delta_9 \vdash ((\Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13}), []\mathtt{F}_{10}), \mathtt{F}_8}\ A4 \quad \dfrac{\mathtt{h}_{11} : \Box\Gamma_7, \mathtt{F}_8, \Delta_9 \vdash \mathtt{F}_{12}, \Delta_{14}, []\mathtt{F}_{10} \quad \mathtt{h}_{11} : \Box\Gamma_7, \mathtt{F}_8, \Delta_9 \vdash \mathtt{F}_{13}, \Delta_{14}, []\mathtt{F}_{10}}{\bullet \mathtt{h}_{11} : (\Box\Gamma_7, \Delta_9), \mathtt{F}_8 \vdash (\Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13}), []\mathtt{F}_{10}}\ \wedge_R}{- : \Box\Gamma_7, \Delta_9 \vdash (\Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13}), []\mathtt{F}_{10}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_7 \vdash \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_9, \Box\Gamma_7 \vdash \Delta_{14}, []\mathtt{F}_{10}, \mathtt{F}_{12} \wedge \mathtt{F}_{13}}\ A4$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_6 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_1 : \Box\Gamma_6, \Delta_7 \vdash (\Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}), []\mathtt{F}_8}\ A4 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_6, \Delta_7, []\mathtt{F}_8 \vdash \mathtt{F}_{11}, \mathtt{F}_{12}, \Delta_{10}}{\bullet \mathtt{h}_9 : (\Box\Gamma_6, \Delta_7), []\mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}\ \vee_R}{- : \Box\Gamma_6, \Delta_7 \vdash \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_6 \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{\bullet \mathtt{h}_1 : \Delta_7, \Box\Gamma_6 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_{12}, []\mathtt{F}_8}\ A4 \quad \dfrac{}{\mathtt{h}_9 : \Delta_7, \Box\Gamma_6, []\mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_{12}}\ \mathtt{ax/W}}{- : \Delta_7, \Box\Gamma_6 \vdash \Delta_{10}, \mathtt{F}_{11}, \mathtt{F}_{12}}\ \mathtt{hCut}$$
$$\dfrac{}{- : \Delta_7, \Box\Gamma_6 \vdash \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}\ \vee_R$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : \Box\Gamma_7, \Delta_9 \vdash ((\Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13}), []\mathtt{F}_{10}), \mathtt{F}_8}\ A4 \quad \dfrac{\mathtt{h}_{11} : \Box\Gamma_7, \mathtt{F}_8, \Delta_9 \vdash \mathtt{F}_{12}, \mathtt{F}_{13}, \Delta_{14}, []\mathtt{F}_{10}}{\bullet\mathtt{h}_{11} : (\Box\Gamma_7, \Delta_9), \mathtt{F}_8 \vdash (\Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13}), []\mathtt{F}_{10}}\ \vee_R}{- : \Box\Gamma_7, \Delta_9 \vdash (\Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13}), []\mathtt{F}_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_7 \vdash \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_9, \Box\Gamma_7 \vdash \Delta_{14}, []\mathtt{F}_{10}, \mathtt{F}_{12} \vee \mathtt{F}_{13}}\ A4}$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_6 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Box\Gamma_6, \Delta_7 \vdash (\bot, \Delta_{10}), []\mathtt{F}_8}\ A4 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_6, \Delta_7, []\mathtt{F}_8 \vdash \Delta_{10}}{\bullet\mathtt{h}_9 : (\Box\Gamma_6, \Delta_7), []\mathtt{F}_8 \vdash \bot, \Delta_{10}}\ \bot_R}{- : \Box\Gamma_6, \Delta_7 \vdash \bot, \Delta_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_7, \Box\Gamma_6 \vdash \bot, \Delta_{10}, []\mathtt{F}_8}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_7, \Box\Gamma_6, []\mathtt{F}_8 \vdash \bot, \Delta_{10}}\ \mathtt{ax/W}}{- : \Delta_7, \Box\Gamma_6 \vdash \bot, \Delta_{10}}\ \mathtt{hCut}}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : \Box\Gamma_7, \Delta_9 \vdash ((\bot, \Delta_{12}), []\mathtt{F}_{10}), \mathtt{F}_8}\ A4 \quad \dfrac{\mathtt{h}_{11} : \Box\Gamma_7, \mathtt{F}_8, \Delta_9 \vdash \Delta_{12}, []\mathtt{F}_{10}}{\bullet\mathtt{h}_{11} : (\Box\Gamma_7, \Delta_9), \mathtt{F}_8 \vdash (\bot, \Delta_{12}), []\mathtt{F}_{10}}\ \bot_R}{- : \Box\Gamma_7, \Delta_9 \vdash (\bot, \Delta_{12}), []\mathtt{F}_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_7 \vdash \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_9, \Box\Gamma_7 \vdash \bot, \Delta_{12}, []\mathtt{F}_{10}}\ A4}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_6 \vdash \mathtt{F}_8}{\bullet\mathtt{h}_1 : \Box\Gamma_6, \Delta_7 \vdash (\top, \Delta_{10}), []\mathtt{F}_8}\ A4 \quad \dfrac{}{\bullet\mathtt{h}_9 : (\Box\Gamma_6, \Delta_7), []\mathtt{F}_8 \vdash \top, \Delta_{10}}\ \top_R}{- : \Box\Gamma_6, \Delta_7 \vdash \top, \Delta_{10}}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_7, \Box\Gamma_6 \vdash \top, \Delta_{10}}\ \top_R}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : \Box\Gamma_7, \Delta_9 \vdash ((\top, \Delta_{12}), []\mathtt{F}_{10}), \mathtt{F}_8}\ A4 \quad \dfrac{}{\bullet\mathtt{h}_{11} : (\Box\Gamma_7, \Delta_9), \mathtt{F}_8 \vdash (\top, \Delta_{12}), []\mathtt{F}_{10}}\ \top_R}{- : \Box\Gamma_7, \Delta_9 \vdash (\top, \Delta_{12}), []\mathtt{F}_{10}}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_9, \Box\Gamma_7 \vdash \top, \Delta_{12}, []\mathtt{F}_{10}}\ \top_R}$$

- Case rule $A4$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{10}, \Box\Gamma_{12} \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash (\Delta_8, []\mathtt{F}_9), []\mathtt{F}_6}\ A4 \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{10}, \Box\Gamma_{11}, []\mathtt{F}_6 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_6 \vdash \Delta_8, []\mathtt{F}_9}\ A4}{- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash \Delta_8, []\mathtt{F}_9}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_{10}, \Box\Gamma_{12} \vdash \mathtt{F}_6}\ \mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \mathtt{F}_9, []\mathtt{F}_6}\ A4 \quad \dfrac{}{\mathtt{h}_7 : \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12}, []\mathtt{F}_6 \vdash \mathtt{F}_9}\ \mathtt{ax/W}}{- : \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \mathtt{F}_9}\ \mathtt{hCut}}{- : \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_8, []\mathtt{F}_9}\ A4$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{10}, \Box\Gamma_{12} \vdash \mathtt{F}_6}{\bullet\mathtt{h}_1 : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash (\Delta_8, []\mathtt{F}_9), []\mathtt{F}_6}\ A4 \quad \dfrac{\mathtt{h}_7 : \Box\Gamma_{10}, \Box\Gamma_{11} \vdash \mathtt{F}_9}{\bullet\mathtt{h}_7 : ((\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13}), []\mathtt{F}_6 \vdash \Delta_8, []\mathtt{F}_9}\ A4}{- : (\Box\Gamma_{10}, \Box\Gamma_{12}), \Box\Gamma_{11}, \Delta_{13} \vdash \Delta_8, []\mathtt{F}_9}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_{10}, \Box\Gamma_{11} \vdash \mathtt{F}_9}\ \mathtt{ax/W}}{- : \Delta_{13}, \Box\Gamma_{10}, \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \Delta_8, []\mathtt{F}_9}\ A4}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Box\Gamma_{12}, \Box\Gamma_{14} \vdash \mathtt{F}_8}{\bullet\mathtt{h}_2 : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash ((\Delta_{11}, []\mathtt{F}_{10}), []\mathtt{F}_8), \Box\mathtt{F}_7}\ A4 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : ((\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15}), \Box\mathtt{F}_7 \vdash (\Delta_{11}, []\mathtt{F}_{10}), []\mathtt{F}_8}\ A4}{- : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash (\Delta_{11}, []\mathtt{F}_{10}), []\mathtt{F}_8}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_{12}, \Box\Gamma_{14} \vdash \mathtt{F}_8}\ \mathtt{ax/W}}{- : \Delta_{15}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Delta_{11}, []\mathtt{F}_{10}, []\mathtt{F}_8}\ A4}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Box\Gamma_{11}, \Box\Gamma_{13} \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_9, []\mathtt{F}_{10}), \Box\mathtt{F}_7}\, A4 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14}), \Box\mathtt{F}_7 \vdash \Delta_9, []\mathtt{F}_{10}}\, A4}{- : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash \Delta_9, []\mathtt{F}_{10}}\, \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_{11}, \Box\Gamma_{13} \vdash \mathtt{F}_{10}}\, \mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Delta_9, []\mathtt{F}_{10}}\, A4$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Box\Gamma_{12}, \Box\Gamma_{14} \vdash \mathtt{F}_8}{\bullet\mathtt{h}_2 : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash ((\Delta_{11}, []\mathtt{F}_{10}), []\mathtt{F}_8), \mathtt{F}_7}\, A4 \quad \dfrac{\mathtt{h}_9 : \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_9 : ((\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15}), \mathtt{F}_7 \vdash (\Delta_{11}, []\mathtt{F}_{10}), []\mathtt{F}_8}\, A4}{- : (\Box\Gamma_{12}, \Box\Gamma_{14}), \Box\Gamma_{13}, \Delta_{15} \vdash (\Delta_{11}, []\mathtt{F}_{10}), []\mathtt{F}_8}\, \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \mathtt{F}_{10}}\, \mathtt{ax/W}}{- : \Delta_{15}, \Box\Gamma_{12}, \Box\Gamma_{13}, \Box\Gamma_{14} \vdash \Delta_{11}, []\mathtt{F}_{10}, []\mathtt{F}_8}\, A4$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Box\Gamma_{11}, \Box\Gamma_{13} \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash (\Delta_9, []\mathtt{F}_{10}), \mathtt{F}_7}\, A4 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14}), \mathtt{F}_7 \vdash \Delta_9, []\mathtt{F}_{10}}\, A4}{- : (\Box\Gamma_{11}, \Box\Gamma_{13}), \Box\Gamma_{12}, \Delta_{14} \vdash \Delta_9, []\mathtt{F}_{10}}\, \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_{11}, \Box\Gamma_{12} \vdash \mathtt{F}_{10}}\, \mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{11}, \Box\Gamma_{12}, \Box\Gamma_{13} \vdash \Delta_9, []\mathtt{F}_{10}}\, A4$$

- Case rule $\to_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_6 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_6, \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}, []\mathtt{F}_7}\, A4 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_6, \Delta_{12}, []\mathtt{F}_7 \vdash \mathtt{F}_9, \Delta_{11} \quad \mathtt{h}_8 : \Box\Gamma_6, \mathtt{F}_{10}, \Delta_{12}, []\mathtt{F}_7 \vdash \Delta_{11}}{\bullet\mathtt{h}_8 : (\Box\Gamma_6, \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10}), []\mathtt{F}_7 \vdash \Delta_{11}}\, \to_L}{- : \Box\Gamma_6, \Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}}\, \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_6 \vdash \mathtt{F}_7}\, \mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{12}, \Box\Gamma_6 \vdash \Delta_{11}, \mathtt{F}_9, []\mathtt{F}_7}\, A4 \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \Box\Gamma_6, []\mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9}\, \mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_6 \vdash \Delta_{11}, \mathtt{F}_9}\, \mathtt{hCut} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_6 \vdash \mathtt{F}_7}\, \mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10}, \Box\Gamma_6 \vdash \Delta_{11}, []\mathtt{F}_7}\, A4 \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \Box\Gamma_6, []\mathtt{F}_7 \vdash \Delta_{11}}\, \mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_{10}, \Box\Gamma_6 \vdash \Delta_{11}}\, \mathtt{hCut}}{- : \Delta_{12}, \Box\Gamma_6, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}}\, \to_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : \Box\Gamma_7, \Delta_8 \vdash (\Delta_9, []\mathtt{F}_{10}), \mathtt{F}_{12} \to \mathtt{F}_{13}}\, A4 \quad \dfrac{\mathtt{h}_{11} : \Box\Gamma_7, \Delta_8 \vdash \mathtt{F}_{12}, \Delta_9, []\mathtt{F}_{10} \quad \mathtt{h}_{11} : \Box\Gamma_7, \mathtt{F}_{13}, \Delta_8 \vdash \Delta_9, []\mathtt{F}_{10}}{\bullet\mathtt{h}_{11} : (\Box\Gamma_7, \Delta_8), \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_9, []\mathtt{F}_{10}}\, \to_L}{- : \Box\Gamma_7, \Delta_8 \vdash \Delta_9, []\mathtt{F}_{10}}\, \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_7 \vdash \mathtt{F}_{10}}\, \mathtt{ax/W}}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_9, []\mathtt{F}_{10}}\, A4$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : \Box\Gamma_7, \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash (\Delta_9, []\mathtt{F}_{10}), \mathtt{F}_8}\, A4 \quad \dfrac{\mathtt{h}_{11} : \Box\Gamma_7, \mathtt{F}_8, \Delta_{14} \vdash \mathtt{F}_{12}, \Delta_9, []\mathtt{F}_{10} \quad \mathtt{h}_{11} : \Box\Gamma_7, \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{14} \vdash \Delta_9, []\mathtt{F}_{10}}{\bullet\mathtt{h}_{11} : (\Box\Gamma_7, \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13}), \mathtt{F}_8 \vdash \Delta_9, []\mathtt{F}_{10}}\, \to_L}{- : \Box\Gamma_7, \Delta_{14}, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_9, []\mathtt{F}_{10}}\, \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_7 \vdash \mathtt{F}_{10}}\, \mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_7, \mathtt{F}_{12} \to \mathtt{F}_{13} \vdash \Delta_9, []\mathtt{F}_{10}}\, A4$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_6 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_6, \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}, []\mathtt{F}_7}\, A4 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_6, \mathtt{F}_9, \mathtt{F}_{10}, \Delta_{12}, []\mathtt{F}_7 \vdash \Delta_{11}}{\bullet\mathtt{h}_8 : (\Box\Gamma_6, \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10}), []\mathtt{F}_7 \vdash \Delta_{11}}\, \wedge_L}{- : \Box\Gamma_6, \Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}}\, \mathtt{Cut}}{\to}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_6 \vdash \mathtt{F}_7}\, \mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_9, \Box\Gamma_6 \vdash \Delta_{11}, []\mathtt{F}_7}\, A4 \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_9, \Box\Gamma_6, []\mathtt{F}_7 \vdash \Delta_{11}}\, \mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_{10}, \mathtt{F}_9, \Box\Gamma_6 \vdash \Delta_{11}}\, \mathtt{hCut}}{- : \Delta_{12}, \Box\Gamma_6, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}}\, \wedge_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : \Box\Gamma_7, \Delta_8 \vdash (\Delta_9, []\mathtt{F}_{10}), \mathtt{F}_{12} \wedge \mathtt{F}_{13}}\ A4 \quad \dfrac{\mathtt{h}_{11} : \Box\Gamma_7, \mathtt{F}_{12}, \mathtt{F}_{13}, \Delta_8 \vdash \Delta_9, []\mathtt{F}_{10}}{\bullet\mathtt{h}_{11} : (\Box\Gamma_7, \Delta_8), \mathtt{F}_{12} \wedge \mathtt{F}_{13} \vdash \Delta_9, []\mathtt{F}_{10}}\ \wedge_L}{- : \Box\Gamma_7, \Delta_8 \vdash \Delta_9, []\mathtt{F}_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_7 \vdash \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_9, []\mathtt{F}_{10}}\ A4}\ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : \Box\Gamma_7, \Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13} \vdash (\Delta_9, []\mathtt{F}_{10}), \mathtt{F}_8}\ A4 \quad \dfrac{\mathtt{h}_{11} : \Box\Gamma_7, \mathtt{F}_8, \mathtt{F}_{12}, \mathtt{F}_{13}, \Delta_{14} \vdash \Delta_9, []\mathtt{F}_{10}}{\bullet\mathtt{h}_{11} : (\Box\Gamma_7, \Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13}), \mathtt{F}_8 \vdash \Delta_9, []\mathtt{F}_{10}}\ \wedge_L}{- : \Box\Gamma_7, \Delta_{14}, \mathtt{F}_{12} \wedge \mathtt{F}_{13} \vdash \Delta_9, []\mathtt{F}_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_7 \vdash \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_7, \mathtt{F}_{12} \wedge \mathtt{F}_{13} \vdash \Delta_9, []\mathtt{F}_{10}}\ A4}\ \to$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_6 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_6, \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}, []\mathtt{F}_7}\ A4 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_6, \mathtt{F}_9, \Delta_{12}, []\mathtt{F}_7 \vdash \Delta_{11} \quad \mathtt{h}_8 : \Box\Gamma_6, \mathtt{F}_{10}, \Delta_{12}, []\mathtt{F}_7 \vdash \Delta_{11}}{\bullet\mathtt{h}_8 : (\Box\Gamma_6, \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10}), []\mathtt{F}_7 \vdash \Delta_{11}}\ \mathtt{Cut}}{- : \Box\Gamma_6, \Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}}\ \vee_L}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_6 \vdash \mathtt{F}_7}\ \mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_9, \Box\Gamma_6 \vdash \Delta_{11}, []\mathtt{F}_7}\ A4 \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_9, \Box\Gamma_6, []\mathtt{F}_7 \vdash \Delta_{11}}\ \mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_9, \Box\Gamma_6 \vdash \Delta_{11}}\ \mathtt{hCut} \quad \dfrac{\dfrac{\dfrac{}{\mathtt{h}_1 : \Box\Gamma_6 \vdash \mathtt{F}_7}\ \mathtt{ax/W}}{\bullet\mathtt{h}_1 : \Delta_{12}, \mathtt{F}_{10}, \Box\Gamma_6 \vdash \Delta_{11}, []\mathtt{F}_7}\ A4 \quad \dfrac{}{\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_{10}, \Box\Gamma_6, []\mathtt{F}_7 \vdash \Delta_{11}}\ \mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_{10}, \Box\Gamma_6 \vdash \Delta_{11}}\ \mathtt{hCut}}{- : \Delta_{12}, \Box\Gamma_6, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}}\ \vee_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : \Box\Gamma_7, \Delta_8 \vdash (\Delta_9, []\mathtt{F}_{10}), \mathtt{F}_{12} \vee \mathtt{F}_{13}}\ A4 \quad \dfrac{\mathtt{h}_{11} : \Box\Gamma_7, \mathtt{F}_{12}, \Delta_8 \vdash \Delta_9, []\mathtt{F}_{10} \quad \mathtt{h}_{11} : \Box\Gamma_7, \mathtt{F}_{13}, \Delta_8 \vdash \Delta_9, []\mathtt{F}_{10}}{\bullet\mathtt{h}_{11} : (\Box\Gamma_7, \Delta_8), \mathtt{F}_{12} \vee \mathtt{F}_{13} \vdash \Delta_9, []\mathtt{F}_{10}}\ \vee_L}{- : \Box\Gamma_7, \Delta_8 \vdash \Delta_9, []\mathtt{F}_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_7 \vdash \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_9, []\mathtt{F}_{10}}\ A4}\ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : \Box\Gamma_7, \Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13} \vdash (\Delta_9, []\mathtt{F}_{10}), \mathtt{F}_8}\ A4 \quad \dfrac{\mathtt{h}_{11} : \Box\Gamma_7, \mathtt{F}_8, \mathtt{F}_{12}, \Delta_{14} \vdash \Delta_9, []\mathtt{F}_{10} \quad \mathtt{h}_{11} : \Box\Gamma_7, \mathtt{F}_8, \mathtt{F}_{13}, \Delta_{14} \vdash \Delta_9, []\mathtt{F}_{10}}{\bullet\mathtt{h}_{11} : (\Box\Gamma_7, \Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13}), \mathtt{F}_8 \vdash \Delta_9, []\mathtt{F}_{10}}\ \vee_L}{- : \Box\Gamma_7, \Delta_{14}, \mathtt{F}_{12} \vee \mathtt{F}_{13} \vdash \Delta_9, []\mathtt{F}_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_7 \vdash \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_7, \mathtt{F}_{12} \vee \mathtt{F}_{13} \vdash \Delta_9, []\mathtt{F}_{10}}\ A4}\ \to$$

- Case rule AT

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_{11}, []\mathtt{F}_9 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : (\Box\Gamma_{11}, []\mathtt{F}_9), \Delta_6 \vdash \Delta_{10}, []\mathtt{F}_7}\ A4 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{11}, \mathtt{F}_9, \Delta_6, []\mathtt{F}_7, []\mathtt{F}_9 \vdash \Delta_{10}}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{11}, []\mathtt{F}_9), \Delta_6), []\mathtt{F}_7 \vdash \Delta_{10}}\ AT}{- : (\Box\Gamma_{11}, []\mathtt{F}_9), \Delta_6 \vdash \Delta_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_6, \mathtt{F}_9, \Box\Gamma_{11}, []\mathtt{F}_9 \vdash \Delta_{10}, []\mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_6, \mathtt{F}_9, \Box\Gamma_{11}, []\mathtt{F}_7, []\mathtt{F}_9 \vdash \Delta_{10}}\ \mathtt{ax/W}}{- : \Delta_6, \mathtt{F}_9, \Box\Gamma_{11}, []\mathtt{F}_9 \vdash \Delta_{10}}\ \mathtt{hCut}}{- : \Delta_6, \Box\Gamma_{11}, []\mathtt{F}_9 \vdash \Delta_{10}}\ AT}\ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_6 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_6, \Delta_{11}, []\mathtt{F}_9 \vdash \Delta_{10}, []\mathtt{F}_7}\ A4 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_6, \mathtt{F}_9, \Delta_{11}, []\mathtt{F}_7, []\mathtt{F}_9 \vdash \Delta_{10}}{\bullet\mathtt{h}_8 : (\Box\Gamma_6, \Delta_{11}, []\mathtt{F}_9), []\mathtt{F}_7 \vdash \Delta_{10}}\ AT}{- : \Box\Gamma_6, \Delta_{11}, []\mathtt{F}_9 \vdash \Delta_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_{11}, \mathtt{F}_9, \Box\Gamma_6, []\mathtt{F}_9 \vdash \Delta_{10}, []\mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_{11}, \mathtt{F}_9, \Box\Gamma_6, []\mathtt{F}_7, []\mathtt{F}_9 \vdash \Delta_{10}}\ \mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_9, \Box\Gamma_6, []\mathtt{F}_9 \vdash \Delta_{10}}\ \mathtt{hCut}}{- : \Delta_{11}, \Box\Gamma_6, []\mathtt{F}_9 \vdash \Delta_{10}}\ AT}\ \to$$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_6 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_1 : \Box\Gamma_6, \Delta_7 \vdash \Delta_{10}, []\mathtt{F}_9}\ A4 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_6, \mathtt{F}_9, \Delta_7, []\mathtt{F}_9 \vdash \Delta_{10}}{\bullet\mathtt{h}_8 : (\Box\Gamma_6, \Delta_7), []\mathtt{F}_9 \vdash \Delta_{10}}\ AT}{- : \Box\Gamma_6, \Delta_7 \vdash \Delta_{10}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : \Delta_7, \Box\Gamma_6 \vdash \Delta_{10}, \mathtt{F}_9}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \Delta_7, \mathtt{F}_9, \Box\Gamma_6 \vdash \Delta_{10}, []\mathtt{F}_9}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \Delta_7, \mathtt{F}_9, \Box\Gamma_6, []\mathtt{F}_9 \vdash \Delta_{10}}\ \mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_9, \Box\Gamma_6 \vdash \Delta_{10}}\ \mathtt{hCut}}{- : \Delta_7, \Box\Gamma_6 \vdash \Delta_{10}}\ \mathtt{sCut}$$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Box\Gamma_{13}, []\mathtt{F}_{12} \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : (\Box\Gamma_{13}, []\mathtt{F}_{12}), \Delta_8 \vdash (\Delta_9, []\mathtt{F}_{10}), \mathtt{F}_7}\ A4 \quad \dfrac{\mathtt{h}_{11} : \Box\Gamma_{13}, \mathtt{F}_7, \mathtt{F}_{12}, \Delta_8, []\mathtt{F}_{12} \vdash \Delta_9, []\mathtt{F}_{10}}{\bullet\mathtt{h}_{11} : ((\Box\Gamma_{13}, []\mathtt{F}_{12}), \Delta_8), \mathtt{F}_7 \vdash \Delta_9, []\mathtt{F}_{10}}\ AT}{- : (\Box\Gamma_{13}, []\mathtt{F}_{12}), \Delta_8 \vdash \Delta_9, []\mathtt{F}_{10}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_{13}, []\mathtt{F}_{12} \vdash \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_8, \Box\Gamma_{13}, []\mathtt{F}_{12} \vdash \Delta_9, []\mathtt{F}_{10}}\ A4$$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : \Box\Gamma_7, \Delta_8 \vdash (\Delta_9, []\mathtt{F}_{10}), []\mathtt{F}_{12}}\ A4 \quad \dfrac{\mathtt{h}_{11} : \Box\Gamma_7, \mathtt{F}_{12}, \Delta_8, []\mathtt{F}_{12} \vdash \Delta_9, []\mathtt{F}_{10}}{\bullet\mathtt{h}_{11} : (\Box\Gamma_7, \Delta_8), []\mathtt{F}_{12} \vdash \Delta_9, []\mathtt{F}_{10}}\ AT}{- : \Box\Gamma_7, \Delta_8 \vdash \Delta_9, []\mathtt{F}_{10}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_7 \vdash \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_9, []\mathtt{F}_{10}}\ A4$$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : \Box\Gamma_7, \Delta_{13}, []\mathtt{F}_{12} \vdash (\Delta_9, []\mathtt{F}_{10}), \mathtt{F}_8}\ A4 \quad \dfrac{\mathtt{h}_{11} : \Box\Gamma_7, \mathtt{F}_8, \mathtt{F}_{12}, \Delta_{13}, []\mathtt{F}_{12} \vdash \Delta_9, []\mathtt{F}_{10}}{\bullet\mathtt{h}_{11} : (\Box\Gamma_7, \Delta_{13}, []\mathtt{F}_{12}), \mathtt{F}_8 \vdash \Delta_9, []\mathtt{F}_{10}}\ AT}{- : \Box\Gamma_7, \Delta_{13}, []\mathtt{F}_{12} \vdash \Delta_9, []\mathtt{F}_{10}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_7 \vdash \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_{13}, \Box\Gamma_7, []\mathtt{F}_{12} \vdash \Delta_9, []\mathtt{F}_{10}}\ A4$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_6 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_6, \bot, \Delta_{10} \vdash \Delta_9, []\mathtt{F}_7}\ A4 \quad \dfrac{}{\bullet\mathtt{h}_8 : (\Box\Gamma_6, \bot, \Delta_{10}), []\mathtt{F}_7 \vdash \Delta_9}\ \bot_L}{- : \Box\Gamma_6, \bot, \Delta_{10} \vdash \Delta_9}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_{10}, \Box\Gamma_6 \vdash \Delta_9}\ \bot_L$$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : \Box\Gamma_7, \Delta_8 \vdash (\Delta_9, []\mathtt{F}_{10}), \bot}\ A4 \quad \dfrac{}{\bullet\mathtt{h}_{11} : (\Box\Gamma_7, \Delta_8), \bot \vdash \Delta_9, []\mathtt{F}_{10}}\ \bot_L}{- : \Box\Gamma_7, \Delta_8 \vdash \Delta_9, []\mathtt{F}_{10}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : \Box\Gamma_7 \vdash \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_9, []\mathtt{F}_{10}}\ A4$$

$$\dfrac{\dfrac{\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : \Box\Gamma_7, \bot, \Delta_{12} \vdash (\Delta_9, []\mathtt{F}_{10}), \mathtt{F}_8}\ A4 \quad \dfrac{}{\bullet\mathtt{h}_{11} : (\Box\Gamma_7, \bot, \Delta_{12}), \mathtt{F}_8 \vdash \Delta_9, []\mathtt{F}_{10}}\ \bot_L}{- : \Box\Gamma_7, \bot, \Delta_{12} \vdash \Delta_9, []\mathtt{F}_{10}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_{12}, \Box\Gamma_7 \vdash \Delta_9, []\mathtt{F}_{10}}\ \bot_L$$

- Case rule I

$$\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_6 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_6, \Delta_{11}, \mathtt{p}_9 \vdash (\Delta_{10}, \mathtt{p}_9), []\mathtt{F}_7}\ A4 \quad \dfrac{}{\bullet\mathtt{h}_8 : (\Box\Gamma_6, \Delta_{11}, \mathtt{p}_9), []\mathtt{F}_7 \vdash \Delta_{10}, \mathtt{p}_9}\ I}{- : \Box\Gamma_6, \Delta_{11}, \mathtt{p}_9 \vdash \Delta_{10}, \mathtt{p}_9}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_{11}, \Box\Gamma_6, \mathtt{p}_9 \vdash \Delta_{10}, \mathtt{p}_9}\ I$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_2 : \Box\Gamma_7, \Delta_8 \vdash ((\Delta_{12}, \mathtt{p}_{11}), []\mathtt{F}_9), \mathtt{p}_{11}}\ A4 \quad \dfrac{}{\bullet\mathtt{h}_{10} : (\Box\Gamma_7, \Delta_8), \mathtt{p}_{11} \vdash (\Delta_{12}, \mathtt{p}_{11}), []\mathtt{F}_9}\ I}{- : \Box\Gamma_7, \Delta_8 \vdash (\Delta_{12}, \mathtt{p}_{11}), []\mathtt{F}_9}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_7 \vdash \mathtt{F}_9}\ \mathtt{ax/W}}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_{12}, \mathtt{p}_{11}, []\mathtt{F}_9}\ A4}\ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_9}{\bullet\mathtt{h}_2 : \Box\Gamma_7, \Delta_{13}, \mathtt{p}_{11} \vdash ((\Delta_{12}, \mathtt{p}_{11}), []\mathtt{F}_9), \mathtt{F}_8}\ A4 \quad \dfrac{}{\bullet\mathtt{h}_{10} : (\Box\Gamma_7, \Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_8 \vdash (\Delta_{12}, \mathtt{p}_{11}), []\mathtt{F}_9}\ I}{- : \Box\Gamma_7, \Delta_{13}, \mathtt{p}_{11} \vdash (\Delta_{12}, \mathtt{p}_{11}), []\mathtt{F}_9}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_{13}, \Box\Gamma_7, \mathtt{p}_{11} \vdash \Delta_{12}, \mathtt{p}_{11}, []\mathtt{F}_9}\ I}\ \to$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_1 : \Box\Gamma_6 \vdash \mathtt{F}_7}{\bullet\mathtt{h}_1 : \Box\Gamma_6, \top, \Delta_{10} \vdash \Delta_9, []\mathtt{F}_7}\ A4 \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_6, \Delta_{10}, []\mathtt{F}_7 \vdash \Delta_9}{\bullet\mathtt{h}_8 : (\Box\Gamma_6, \top, \Delta_{10}), []\mathtt{F}_7 \vdash \Delta_9}\ \top_L}{- : \Box\Gamma_6, \top, \Delta_{10} \vdash \Delta_9}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\bullet\mathtt{h}_1 : \top, \Delta_{10}, \Box\Gamma_6 \vdash \Delta_9, []\mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \top, \Delta_{10}, \Box\Gamma_6, []\mathtt{F}_7 \vdash \Delta_9}\ \mathtt{ax/W}}{- : \top, \Delta_{10}, \Box\Gamma_6 \vdash \Delta_9}\ \mathtt{hCut}}\ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : \Box\Gamma_7, \Delta_8 \vdash (\Delta_9, []\mathtt{F}_{10}), \top}\ A4 \quad \dfrac{\mathtt{h}_{11} : \Box\Gamma_7, \Delta_8 \vdash \Delta_9, []\mathtt{F}_{10}}{\bullet\mathtt{h}_{11} : (\Box\Gamma_7, \Delta_8), \top \vdash \Delta_9, []\mathtt{F}_{10}}\ \top_L}{- : \Box\Gamma_7, \Delta_8 \vdash \Delta_9, []\mathtt{F}_{10}}\ \mathtt{Cut}}{\dfrac{}{- : \Delta_8, \Box\Gamma_7 \vdash \Delta_9, []\mathtt{F}_{10}}\ \mathtt{ax/W}}\ \to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_2 : \Box\Gamma_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_2 : \Box\Gamma_7, \top, \Delta_{12} \vdash (\Delta_9, []\mathtt{F}_{10}), \mathtt{F}_8}\ A4 \quad \dfrac{\mathtt{h}_{11} : \Box\Gamma_7, \mathtt{F}_8, \Delta_{12} \vdash \Delta_9, []\mathtt{F}_{10}}{\bullet\mathtt{h}_{11} : (\Box\Gamma_7, \top, \Delta_{12}), \mathtt{F}_8 \vdash \Delta_9, []\mathtt{F}_{10}}\ \top_L}{- : \Box\Gamma_7, \top, \Delta_{12} \vdash \Delta_9, []\mathtt{F}_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_7 \vdash \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \top, \Delta_{12}, \Box\Gamma_7 \vdash \Delta_9, []\mathtt{F}_{10}}\ A4}\ \to$$

8.7 Status of $\to_L$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_8 \vdash \mathtt{F}_7, \mathtt{F}_9, \Delta_{12}, \mathtt{F}_{13} \to \mathtt{F}_{14} \quad \mathtt{h}_3 : \mathtt{F}_{10}, \Delta_8 \vdash \mathtt{F}_7, \Delta_{12}, \mathtt{F}_{13} \to \mathtt{F}_{14}}{\bullet\mathtt{h}_3 : \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash (\Delta_{12}, \mathtt{F}_{13} \to \mathtt{F}_{14}), \mathtt{F}_7}\ \to_L \quad \dfrac{\mathtt{h}_{11} : \mathtt{F}_7, \mathtt{F}_{13}, \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \mathtt{F}_{14}, \Delta_{12}}{\bullet\mathtt{h}_{11} : (\Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{13} \to \mathtt{F}_{14}}\ \to_R}{- : \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \to \mathtt{F}_{14}}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_8, \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_7, \mathtt{F}_9}\ \mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_3 : \Delta_8, \mathtt{F}_{10}, \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_7}\ \mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_3 : \Delta_8, \mathtt{F}_{13}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_7}\ \to_L \quad \dfrac{}{\mathtt{h}_{11} : \Delta_8, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{14}}\ \mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_{13}, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{14}}\ \mathtt{hCut}}{- : \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \to \mathtt{F}_{14}}\ \to_R}\ \to$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_8 \vdash \mathtt{F}_7, \mathtt{F}_9, \Delta_{12}, \mathtt{F}_{13} \wedge \mathtt{F}_{14} \quad \mathtt{h}_3 : \mathtt{F}_{10}, \Delta_8 \vdash \mathtt{F}_7, \Delta_{12}, \mathtt{F}_{13} \wedge \mathtt{F}_{14}}{\bullet\mathtt{h}_3 : \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash (\Delta_{12}, \mathtt{F}_{13} \wedge \mathtt{F}_{14}), \mathtt{F}_7}\ \to_L \quad \dfrac{\mathtt{h}_{11} : \mathtt{F}_7, \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \mathtt{F}_{13}, \Delta}{\bullet\mathtt{h}_{11} : (\Delta_8, \mathtt{F}_9 \to}}{- : \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \wedge \mathtt{F}_{14}}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_8 \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_9}\ \mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_3 : \Delta_8, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_7}\ \mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_3 : \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_7}\ \to_L \quad \dfrac{}{\mathtt{h}_{11} : \Delta_8, \mathtt{F}_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}}\ \mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_8 \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_7, \mathtt{F}_9}\ \mathtt{inv}}{\bullet\mathtt{h}_3 : \Delta_8, \mathtt{F}_9 -}}{- : \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \wedge \mathtt{F}_{14}}\ \to$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_8 \vdash \mathtt{F}_7, \mathtt{F}_9, \Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14} \quad \mathtt{h}_3 : \mathtt{F}_{10}, \Delta_8 \vdash \mathtt{F}_7, \Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}}{\bullet\mathtt{h}_3 : \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash (\Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}), \mathtt{F}_7}\ \to_L \quad \dfrac{\mathtt{h}_{11} : \mathtt{F}_7, \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \mathtt{F}_{13}, \mathtt{F}_{14}, \Delta_{12}}{\bullet\mathtt{h}_{11} : (\Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}}\ \vee_R}{- : \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}}\ \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_8 \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}, \mathtt{F}_7, \mathtt{F}_9}{\bullet\mathtt{h}_3 : \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}, \mathtt{F}_7}\ \mathtt{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_3 : \Delta_8, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}, \mathtt{F}_7}{}\ \mathtt{inv\text{-}th/ax}}{- : \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}}\ \to_L \quad \dfrac{}{\mathtt{h}_{11} : \Delta_8, \mathtt{F}_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}}\ \mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}}\ \vee_R}\ \mathtt{hCut}$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_8 \vdash \mathtt{F}_7, \mathtt{F}_9, \bot, \Delta_{12} \quad \mathtt{h}_3 : \mathtt{F}_{10}, \Delta_8 \vdash \mathtt{F}_7, \bot, \Delta_{12}}{\bullet\mathtt{h}_3 : \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash (\bot, \Delta_{12}), \mathtt{F}_7}\ \to_L \quad \dfrac{\mathtt{h}_{11} : \mathtt{F}_7, \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{12}}{\bullet\mathtt{h}_{11} : (\Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_7 \vdash \bot, \Delta_{12}}\ \bot_R}{- : \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \bot, \Delta_{12}}\ \mathtt{Cut}}{\dfrac{\dfrac{\bullet\mathtt{h}_3 : \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \bot, \Delta_{12}, \mathtt{F}_7}{}\ \mathtt{ax/W} \quad \dfrac{\mathtt{h}_{11} : \Delta_8, \mathtt{F}_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \bot, \Delta_{12}}{}\ \mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \bot, \Delta_{12}}\ \mathtt{hCut}}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_8 \vdash \mathtt{F}_7, \mathtt{F}_9, \top, \Delta_{12} \quad \mathtt{h}_3 : \mathtt{F}_{10}, \Delta_8 \vdash \mathtt{F}_7, \top, \Delta_{12}}{\bullet\mathtt{h}_3 : \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash (\top, \Delta_{12}), \mathtt{F}_7}\ \to_L \quad \dfrac{}{\bullet\mathtt{h}_{11} : (\Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_7 \vdash \top, \Delta_{12}}\ \top_R}{- : \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \top, \Delta_{12}}\ \mathtt{Cut}}{- : \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \top, \Delta_{12}}\ \top_R$$

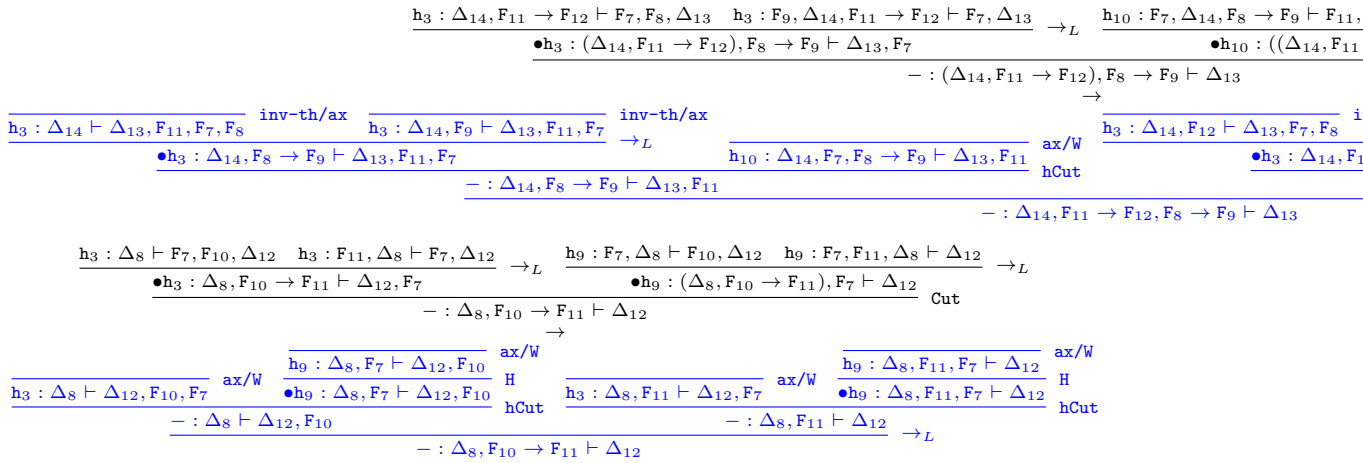
- Case rule $A4$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_{14}, \Box\Gamma_{13} \vdash \Box\mathtt{F}_7, \mathtt{F}_8, \Delta_{11}, []\mathtt{F}_{12}}{}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_{10} : \Box\mathtt{F}_7, \Box\Gamma_{13} \vdash \mathtt{F}_{12}}\ \mathtt{ax/W}}{\bullet\mathtt{h}_{10} : \Box\mathtt{F}_7, \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, \mathtt{F}_8, []\mathtt{F}_{12}}\ A4}{- : \Delta_{14}, \Box\Gamma_{13} \vdash \Delta_{11}, \mathtt{F}_8, []\mathtt{F}_{12}}\ \mathtt{hCut} \quad \dfrac{\mathtt{h}_3 : \Delta_{14}, \mathtt{F}_9, \Box\Gamma_{13} \vdash \Box\mathtt{F}_7, \Delta_{11}, []\mathtt{F}_{12}}{}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_{10} : \Box}}{\bullet\mathtt{h}_{10} : \Box\mathtt{F}_7, \Delta_{14}}}{- : \Delta_{14}, \mathtt{F}_9, \Box\Gamma_{13} \vdash \Delta_{11}, []\mathtt{F}_{12}}\ \to$$

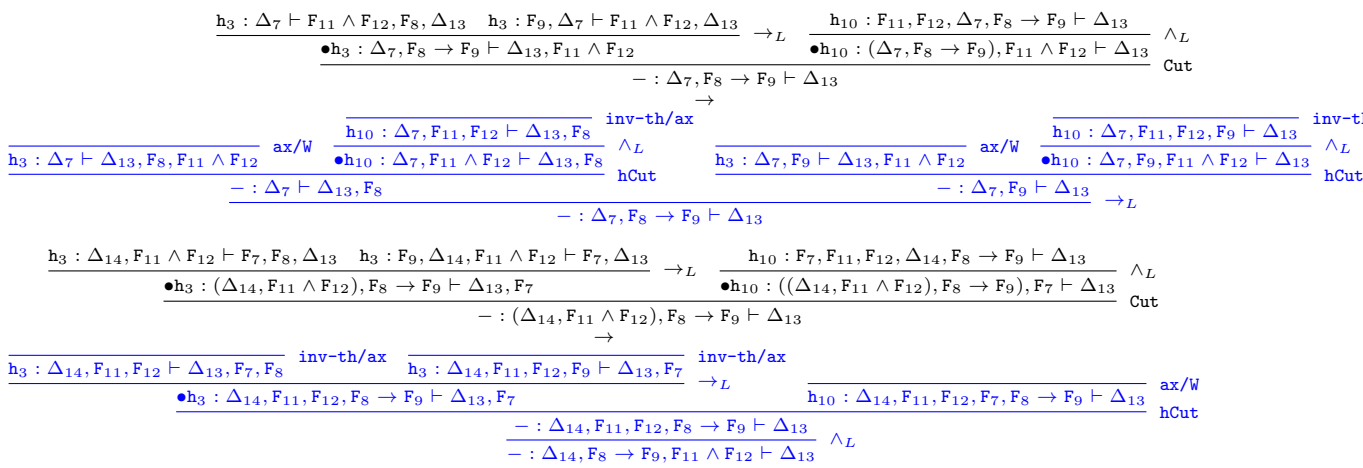
$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Box\Gamma_{11}, \Delta_{14} \vdash \mathtt{F}_7, \mathtt{F}_8, \Delta_{12}, []\mathtt{F}_{13} \quad \mathtt{h}_3 : \mathtt{F}_9, \Box\Gamma_{11}, \Delta_{14} \vdash \mathtt{F}_7, \Delta_{12}, []\mathtt{F}_{13}}{\bullet\mathtt{h}_3 : (\Box\Gamma_{11}, \Delta_{14}), \mathtt{F}_8 \to \mathtt{F}_9 \vdash (\Delta_{12}, []\mathtt{F}_{13}), \mathtt{F}_7}\ \to_L \quad \dfrac{\mathtt{h}_{10} : \Box\Gamma_{11} \vdash \mathtt{F}_{13}}{\bullet\mathtt{h}_{10} : ((\Box\Gamma_{11}, \Delta_{14}), \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_7 \vdash \Delta_{12}, []\mathtt{F}_{13}}\ A4}{- : (\Box\Gamma_{11}, \Delta_{14}), \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{12}, []\mathtt{F}_{13}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_{11} \vdash \mathtt{F}_{13}}\ \mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{12}, []\mathtt{F}_{13}}\ A4}$$

- Case rule $\to_L$

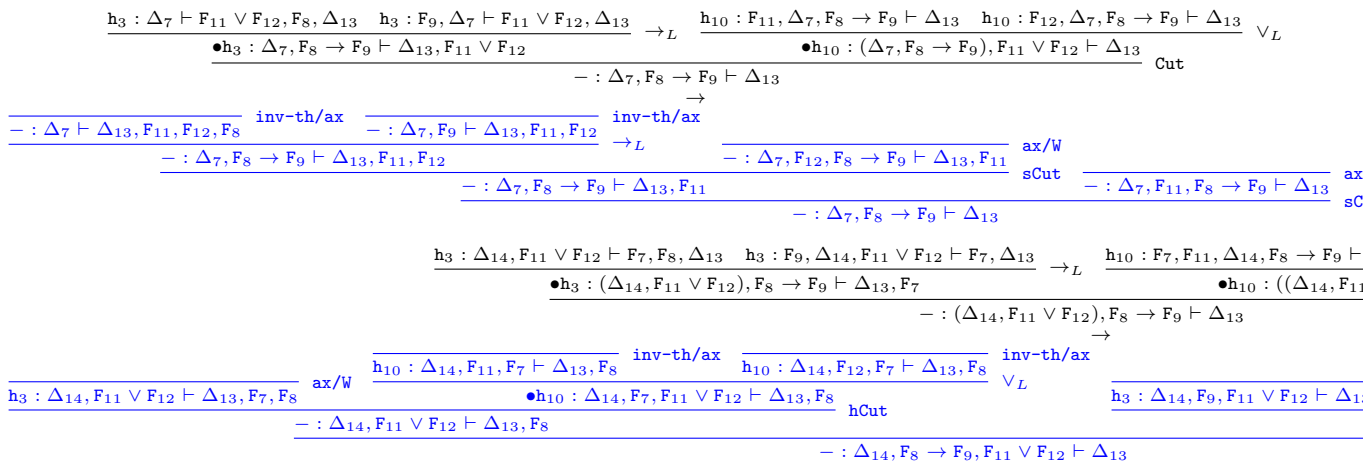
$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7 \vdash \mathtt{F}_{11} \to \mathtt{F}_{12}, \mathtt{F}_8, \Delta_{13} \quad \mathtt{h}_3 : \mathtt{F}_9, \Delta_7 \vdash \mathtt{F}_{11} \to \mathtt{F}_{12}, \Delta_{13}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11} \to \mathtt{F}_{12}}\ \to_L \quad \dfrac{\mathtt{h}_{10} : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \mathtt{F}_{11}, \Delta_{13} \quad \mathtt{h}_{10} : \mathtt{F}_{12}, \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{13}}{\bullet\mathtt{h}_{10} : (\Delta_7, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_{11} \to \mathtt{F}_{12} \vdash \Delta_{13}}\ \to_L}{- : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{13}}\ \mathtt{Cut}}{\dfrac{\dfrac{- : \Delta_7, \mathtt{F}_{11} \vdash \Delta_{13}, \mathtt{F}_{12}, \mathtt{F}_8}{}\ \mathtt{inv\text{-}th/ax} \quad \dfrac{- : \Delta_7, \mathtt{F}_{11}, \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{12}}{}\ \mathtt{inv\text{-}th/ax}}{- : \Delta_7, \mathtt{F}_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{12}}\ \to_L \quad \dfrac{}{- : \Delta_7, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{13}}\ \mathtt{ax}}{- : \Delta_7, \mathtt{F}_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{13}}\ \mathtt{sC}}{\dfrac{}{- : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11}}\ \mathtt{ax/W}}\ \mathtt{sCut}$$



- Case rule $\wedge_L$



- Case rule $\vee_L$



- Case rule AT

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7 \vdash \Box\mathtt{F}_{11}, \mathtt{F}_8, \Delta_{12} \quad \mathtt{h}_3 : \mathtt{F}_9, \Delta_7 \vdash \Box\mathtt{F}_{11}, \Delta_{12}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{12}, \Box\mathtt{F}_{11}} \to_L \quad \dfrac{\mathtt{h}_{10} : \mathtt{F}_{11}, \Delta_7, \Box\mathtt{F}_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{12}}{\bullet\mathtt{h}_{10} : (\Delta_7, \mathtt{F}_8 \to \mathtt{F}_9), \Box\mathtt{F}_{11} \vdash \Delta_{12}}\, AT}{- : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{12}}\, \mathtt{Cut}}{}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7 \vdash \Delta_{12}, \mathtt{F}_8, \Box\mathtt{F}_{11}}\, \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_{10} : \Delta_7, \mathtt{F}_{11}, \Box\mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8}\, \mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_{10} : \Delta_7, \Box\mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_8}\, AT}{- : \Delta_7 \vdash \Delta_{12}, \mathtt{F}_8}\, \mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \vdash \Delta_{12}, \Box\mathtt{F}_{11}}\, \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_{10} : \Delta_7, \mathtt{F}_{11}, \mathtt{F}_9, \Box\mathtt{F}_{11} \vdash \Delta_{12}}\, \mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_{10} : \Delta_7, \mathtt{F}_9, \Box\mathtt{F}_{11} \vdash \Delta_{12}}\, AT}{- : \Delta_7, \mathtt{F}_9 \vdash \Delta_{12}}\, \mathtt{hCut}}{- : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{12}}\, \to_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_{13}, \Box\mathtt{F}_{11} \vdash \mathtt{F}_7, \mathtt{F}_8, \Delta_{12} \quad \mathtt{h}_3 : \mathtt{F}_9, \Delta_{13}, \Box\mathtt{F}_{11} \vdash \mathtt{F}_7, \Delta_{12}}{\bullet\mathtt{h}_3 : (\Delta_{13}, \Box\mathtt{F}_{11}), \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{F}_7} \to_L \quad \dfrac{\mathtt{h}_{10} : \mathtt{F}_7, \mathtt{F}_{11}, \Delta_{13}, \Box\mathtt{F}_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{12}}{\bullet\mathtt{h}_{10} : ((\Delta_{13}, \Box\mathtt{F}_{11}), \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_7 \vdash \Delta_{12}}\, AT}{- : (\Delta_{13}, \Box\mathtt{F}_{11}), \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{12}}\, \mathtt{Cut}}{}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\bullet\mathtt{h}_3 : \Delta_{13}, \mathtt{F}_{11}, \Box\mathtt{F}_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{F}_7}\, \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_{10} : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7, \Box\mathtt{F}_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{12}}\, \mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_{11}, \Box\mathtt{F}_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{12}}\, \mathtt{hCut}}{- : \Delta_{13}, \Box\mathtt{F}_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{12}}\, AT$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7 \vdash \bot, \mathtt{F}_8, \Delta_{11} \quad \mathtt{h}_3 : \mathtt{F}_9, \Delta_7 \vdash \bot, \Delta_{11}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}, \bot} \to_L \quad \dfrac{}{\bullet\mathtt{h}_{10} : (\Delta_7, \mathtt{F}_8 \to \mathtt{F}_9), \bot \vdash \Delta_{11}}\, \bot_L}{- : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}}\, \mathtt{Cut}}{}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7 \vdash \bot, \Delta_{11}, \mathtt{F}_8}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_{10} : \bot, \Delta_7 \vdash \Delta_{11}, \mathtt{F}_8}\, \bot_L}{- : \Delta_7 \vdash \Delta_{11}, \mathtt{F}_8}\, \mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \vdash \bot, \Delta_{11}}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_{10} : \bot, \Delta_7, \mathtt{F}_9 \vdash \Delta_{11}}\, \bot_L}{- : \Delta_7, \mathtt{F}_9 \vdash \Delta_{11}}\, \mathtt{hCut}}{- : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}}\, \to_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \bot, \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_8, \Delta_{11} \quad \mathtt{h}_3 : \mathtt{F}_9, \bot, \Delta_{12} \vdash \mathtt{F}_7, \Delta_{11}}{\bullet\mathtt{h}_3 : (\bot, \Delta_{12}), \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_7} \to_L \quad \dfrac{}{\bullet\mathtt{h}_{10} : ((\bot, \Delta_{12}), \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_7 \vdash \Delta_{11}}\, \bot_L}{- : (\bot, \Delta_{12}), \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}}\, \mathtt{Cut}}{}$$

$$\to$$

$$\dfrac{}{- : \bot, \Delta_{12}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}}\, \bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7 \vdash \mathtt{p}_{11}, \mathtt{F}_8, \Delta_{12}, \mathtt{p}_{11} \quad \mathtt{h}_3 : \mathtt{F}_9, \Delta_7 \vdash \mathtt{p}_{11}, \Delta_{12}, \mathtt{p}_{11}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{p}_{11}} \to_L \quad \dfrac{}{\bullet\mathtt{h}_{10} : (\Delta_7, \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{p}_{11} \vdash \Delta_{12}, \mathtt{p}_{11}}\, I}{- : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}}\, \mathtt{Cut}}{}$$

$$\to$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7 \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{p}_{11}, \mathtt{p}_{11}}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_{10} : \Delta_7, \mathtt{p}_{11} \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{p}_{11}}\, I}{- : \Delta_7 \vdash \Delta_{12}, \mathtt{F}_8, \mathtt{p}_{11}}\, \mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}, \mathtt{p}_{11}}\, \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_{10} : \Delta_7, \mathtt{F}_9, \mathtt{p}_{11} \vdash \Delta_{12}, \mathtt{p}_{11}}\, I}{- : \Delta_7, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}}\, \mathtt{hCut}}{- : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}}\, \to_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_{13}, \mathtt{p}_{11} \vdash \mathtt{F}_7, \mathtt{F}_8, \Delta_{12}, \mathtt{p}_{11} \quad \mathtt{h}_3 : \mathtt{F}_9, \Delta_{13}, \mathtt{p}_{11} \vdash \mathtt{F}_7, \Delta_{12}, \mathtt{p}_{11}}{\bullet\mathtt{h}_3 : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_8 \to \mathtt{F}_9 \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7} \to_L \quad \dfrac{}{\bullet\mathtt{h}_{10} : ((\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{p}_{11}}\, I}{- : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}}\, \mathtt{Cut}}{}$$

$$\to$$

$$\dfrac{}{- : \Delta_{13}, \mathtt{p}_{11}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}}\, I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7 \vdash \top, \mathtt{F}_8, \Delta_{11} \quad \mathtt{h}_3 : \mathtt{F}_9, \Delta_7 \vdash \top, \Delta_{11}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}, \top} \to_L \quad \dfrac{\mathtt{h}_{10} : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}}{\bullet\mathtt{h}_{10} : (\Delta_7, \mathtt{F}_8 \to \mathtt{F}_9), \top \vdash \Delta_{11}}\, \top_L}{- : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}}\, \mathtt{Cut}}{}$$

$$\to$$

$$\dfrac{}{- : \Delta_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}}\, \mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \top, \Delta_{12} \vdash \mathtt{F}_7, \mathtt{F}_8, \Delta_{11} \quad \mathtt{h}_3 : \mathtt{F}_9, \top, \Delta_{12} \vdash \mathtt{F}_7, \Delta_{11}}{\bullet \mathtt{h}_3 : (\top, \Delta_{12}), \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_7}\,{\to_L} \quad \dfrac{\mathtt{h}_{10} : \mathtt{F}_7, \Delta_{12}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}}{\bullet \mathtt{h}_{10} : ((\top, \Delta_{12}), \mathtt{F}_8 \to \mathtt{F}_9), \mathtt{F}_7 \vdash \Delta_{11}}\,{\top_L}}{- : (\top, \Delta_{12}), \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \top, \Delta_{12}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_{10} : \top, \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{ax/W}}{- : \top, \Delta_{12}, \mathtt{F}_8 \to \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{hCut}}$$

8.8 Status of $\wedge_L$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_9, \mathtt{F}_{10}, \Delta_8 \vdash \mathtt{F}_7, \Delta_{12}, \mathtt{F}_{13} \to \mathtt{F}_{14}}{\bullet \mathtt{h}_3 : \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash (\Delta_{12}, \mathtt{F}_{13} \to \mathtt{F}_{14}), \mathtt{F}_7}\,{\wedge_L} \quad \dfrac{\mathtt{h}_{11} : \mathtt{F}_7, \mathtt{F}_{13}, \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \mathtt{F}_{14}, \Delta_{12}}{\bullet \mathtt{h}_{11} : (\Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{13} \to \mathtt{F}_{14}}\,{\to_R}}{- : \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \to \mathtt{F}_{14}}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_8, \mathtt{F}_{10}, \mathtt{F}_{13}, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_3 : \Delta_8, \mathtt{F}_{13}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_7}\,{\wedge_L} \quad \dfrac{}{\mathtt{h}_{11} : \Delta_8, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{14}}\,\mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_{13}, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{14}}\,\mathtt{hCut}}{- : \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \to \mathtt{F}_{14}}\,{\to_R}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_9, \mathtt{F}_{10}, \Delta_8 \vdash \mathtt{F}_7, \Delta_{12}, \mathtt{F}_{13} \wedge \mathtt{F}_{14}}{\bullet \mathtt{h}_3 : \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash (\Delta_{12}, \mathtt{F}_{13} \wedge \mathtt{F}_{14}), \mathtt{F}_7}\,{\wedge_L} \quad \dfrac{\mathtt{h}_{11} : \mathtt{F}_7, \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \mathtt{F}_{13}, \Delta_{12} \quad \mathtt{h}_{11} : \mathtt{F}_7, \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \mathtt{F}_{14}, \Delta_{12}}{\bullet \mathtt{h}_{11} : (\Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{13} \wedge \mathtt{F}_{14}}\,{\wedge_R}}{- : \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \wedge \mathtt{F}_{14}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_8, \mathtt{F}_{10}, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{F}_7, \mathtt{F}_{13} \wedge \mathtt{F}_{14}}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_{11} : \Delta_8, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{F}_{13}}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_{11} : \Delta_8, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{F}_{14}}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_{11} : \Delta_8, \mathtt{F}_{10}, \mathtt{F}_7, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{F}_{13} \wedge \mathtt{F}_{14}}\,{\wedge_R}\,\mathtt{hCut}}{- : \Delta_8, \mathtt{F}_{10}, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{F}_{13} \wedge \mathtt{F}_{14}}\,{\wedge_L}}{- : \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \wedge \mathtt{F}_{14}}$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_9, \mathtt{F}_{10}, \Delta_8 \vdash \mathtt{F}_7, \Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}}{\bullet \mathtt{h}_3 : \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash (\Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}), \mathtt{F}_7}\,{\wedge_L} \quad \dfrac{\mathtt{h}_{11} : \mathtt{F}_7, \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \mathtt{F}_{13}, \mathtt{F}_{14}, \Delta_{12}}{\bullet \mathtt{h}_{11} : (\Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}}\,{\vee_R}}{- : \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}}\,\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_8, \mathtt{F}_{10}, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}, \mathtt{F}_7}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_3 : \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}, \mathtt{F}_7}\,{\wedge_L} \quad \dfrac{}{\mathtt{h}_{11} : \Delta_8, \mathtt{F}_7, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}}\,\mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}}\,\mathtt{hCut}}{- : \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}}\,{\vee_R}$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_9, \mathtt{F}_{10}, \Delta_8 \vdash \mathtt{F}_7, \bot, \Delta_{12}}{\bullet \mathtt{h}_3 : \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash (\bot, \Delta_{12}), \mathtt{F}_7}\,{\wedge_L} \quad \dfrac{\mathtt{h}_{11} : \mathtt{F}_7, \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{12}}{\bullet \mathtt{h}_{11} : (\Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_7 \vdash \bot, \Delta_{12}}\,{\bot_R}}{- : \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \bot, \Delta_{12}}\,\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \bot, \Delta_{12}, \mathtt{F}_7}\,\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_{11} : \Delta_8, \mathtt{F}_7, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \bot, \Delta_{12}}\,\mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \bot, \Delta_{12}}\,\mathtt{hCut}}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_9, \mathtt{F}_{10}, \Delta_8 \vdash \mathtt{F}_7, \top, \Delta_{12}}{\bullet \mathtt{h}_3 : \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash (\top, \Delta_{12}), \mathtt{F}_7}\,{\wedge_L} \quad \dfrac{}{\bullet \mathtt{h}_{11} : (\Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_7 \vdash \top, \Delta_{12}}\,{\top_R}}{- : \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \top, \Delta_{12}}\,\mathtt{Cut}}{- : \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \top, \Delta_{12}}\,{\top_R}$$

- Case rule $A4$

$$\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \mathtt{F}_9, \Box\Gamma_{13}, \Delta_{14} \vdash \Box\mathtt{F}_7, \Delta_{11}, []\mathtt{F}_{12}}{\bullet\mathtt{h}_3 : (\Box\Gamma_{13}, \Delta_{14}), \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash (\Delta_{11}, []\mathtt{F}_{12}), \Box\mathtt{F}_7}\ \wedge_L \quad \dfrac{\mathtt{h}_{10} : \Box\Gamma_{13}, \Box\mathtt{F}_7 \vdash \mathtt{F}_{12}}{\bullet\mathtt{h}_{10} : ((\Box\Gamma_{13}, \Delta_{14}), \mathtt{F}_8 \wedge \mathtt{F}_9), \Box\mathtt{F}_7 \vdash \Delta_{11}, []\mathtt{F}_{12}}\ A4}{- : (\Box\Gamma_{13}, \Delta_{14}), \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{11}, []\mathtt{F}_{12}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_{14}, \mathtt{F}_8, \mathtt{F}_9, \Box\Gamma_{13} \vdash \Box\mathtt{F}_7, \Delta_{11}, []\mathtt{F}_{12}}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_{10} : \Box\mathtt{F}_7, \Box\Gamma_{13} \vdash \mathtt{F}_{12}}\ \mathtt{ax/W}}{\bullet\mathtt{h}_{10} : \Box\mathtt{F}_7, \Delta_{14}, \mathtt{F}_8, \mathtt{F}_9, \Box\Gamma_{13} \vdash \Delta_{11}, []\mathtt{F}_{12}}\ A4}{- : \Delta_{14}, \mathtt{F}_8, \mathtt{F}_9, \Box\Gamma_{13} \vdash \Delta_{11}, []\mathtt{F}_{12}}\ \mathtt{hCut}}{- : \Delta_{14}, \Box\Gamma_{13}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{11}, []\mathtt{F}_{12}}\ \wedge_L$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \mathtt{F}_9, \Box\Gamma_{11}, \Delta_{14} \vdash \mathtt{F}_7, \Delta_{12}, []\mathtt{F}_{13}}{\bullet\mathtt{h}_3 : (\Box\Gamma_{11}, \Delta_{14}), \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash (\Delta_{12}, []\mathtt{F}_{13}), \mathtt{F}_7}\ \wedge_L \quad \dfrac{\mathtt{h}_{10} : \Box\Gamma_{11} \vdash \mathtt{F}_{13}}{\bullet\mathtt{h}_{10} : ((\Box\Gamma_{11}, \Delta_{14}), \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_7 \vdash \Delta_{12}, []\mathtt{F}_{13}}\ A4}{- : (\Box\Gamma_{11}, \Delta_{14}), \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{12}, []\mathtt{F}_{13}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_{11} \vdash \mathtt{F}_{13}}\ \mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{12}, []\mathtt{F}_{13}}\ A4$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \mathtt{F}_9, \Delta_7 \vdash \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}, \Delta_{13}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}}\ \wedge_L \quad \dfrac{\mathtt{h}_{10} : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{11}, \Delta_{13} \quad \mathtt{h}_{10} : \mathtt{F}_{12}, \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}{\bullet\mathtt{h}_{10} : (\Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_{11} \rightarrow \mathtt{F}_{12} \vdash \Delta_{13}}\ \rightarrow_L}{- : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_{10} : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11}}\ \mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_{10} : \Delta_7, \mathtt{F}_{12}, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}}\ \mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_{10} : \Delta_7, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12} \vdash \Delta_{13}}\ \rightarrow_L}{- : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}}\ \mathtt{hCut}}{- : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}\ \wedge_L$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \mathtt{F}_9, \Delta_{14}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12} \vdash \mathtt{F}_7, \Delta_{13}}{\bullet\mathtt{h}_3 : (\Delta_{14}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_7}\ \wedge_L \quad \dfrac{\mathtt{h}_{10} : \mathtt{F}_7, \Delta_{14}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \mathtt{F}_{11}, \Delta_{13} \quad \mathtt{h}_{10} : \mathtt{F}_7, \mathtt{F}_{12}, \Delta_{14}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}{\bullet\mathtt{h}_{10} : ((\Delta_{14}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_7 \vdash \Delta_{13}}\ \rightarrow_L}{- : (\Delta_{14}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_{14}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12} \vdash \Delta_{13}, \mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_{10} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11}}\ \mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_{10} : \Delta_{14}, \mathtt{F}_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}}\ \mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_{10} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12} \vdash \Delta_{13}}\ \rightarrow_L}{- : \Delta_{14}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12} \vdash \Delta_{13}}\ \mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}\ \wedge_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \mathtt{F}_9, \Delta_7 \vdash \mathtt{F}_{11} \wedge \mathtt{F}_{12}, \Delta_{13}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\ \wedge_L \quad \dfrac{\mathtt{h}_{10} : \mathtt{F}_{11}, \mathtt{F}_{12}, \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}{\bullet\mathtt{h}_{10} : (\Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_{11} \wedge \mathtt{F}_{12} \vdash \Delta_{13}}\ \wedge_L}{- : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_{10} : \Delta_7, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}}\ \mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_{10} : \Delta_7, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{11} \wedge \mathtt{F}_{12} \vdash \Delta_{13}}\ \wedge_L}{- : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}}\ \mathtt{hCut}}{- : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}\ \wedge_L$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \mathtt{F}_9, \Delta_{14}, \mathtt{F}_{11} \wedge \mathtt{F}_{12} \vdash \mathtt{F}_7, \Delta_{13}}{\bullet\mathtt{h}_3 : (\Delta_{14}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_7}\ \wedge_L \quad \dfrac{\mathtt{h}_{10} : \mathtt{F}_7, \mathtt{F}_{11}, \mathtt{F}_{12}, \Delta_{14}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}{\bullet\mathtt{h}_{10} : ((\Delta_{14}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_7 \vdash \Delta_{13}}\ \wedge_L}{- : (\Delta_{14}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_{14}, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_7}{\bullet\mathtt{h}_3 : \Delta_{14}, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_7}\ \wedge_L}\ \mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_{10} : \Delta_{14}, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}\ \mathtt{ax/W}}{- : \Delta_{14}, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}\ \mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}\ \wedge_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_8 \vdash \mathtt{F}_7, \Delta_{12}}{\bullet \mathtt{h}_3 : \Delta_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7}\wedge_L \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_7, \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_8 \vdash \Delta_{12}}{\bullet \mathtt{h}_9 : (\Delta_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{F}_7 \vdash \Delta_{12}}\wedge_L}{- : \Delta_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_8, \mathtt{F}_{10}, \mathtt{F}_{11} \vdash \Delta_{12}, \mathtt{F}_7}{}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_9 : \Delta_8, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vdash \Delta_{12}}\,\mathtt{ax/W}}{\bullet \mathtt{h}_9 : \Delta_8, \mathtt{F}_{10}, \mathtt{F}_{11}, \mathtt{F}_7 \vdash \Delta_{12}}\,\mathtt{H}}{- : \Delta_8, \mathtt{F}_{10}, \mathtt{F}_{11} \vdash \Delta_{12}}\,\mathtt{hCut}}{- : \Delta_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_{12}}\wedge_L$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \mathtt{F}_9, \Delta_7 \vdash \mathtt{F}_{11} \vee \mathtt{F}_{12}, \Delta_{13}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}\wedge_L \quad \dfrac{\mathtt{h}_{10} : \mathtt{F}_{11}, \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13} \quad \mathtt{h}_{10} : \mathtt{F}_{12}, \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}{\bullet \mathtt{h}_{10} : (\Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_{11} \vee \mathtt{F}_{12} \vdash \Delta_{13}}\vee_L}{- : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}{}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_{10} : \Delta_7, \mathtt{F}_{11}, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}}{}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_{10} : \Delta_7, \mathtt{F}_{12}, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}}{}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_{10} : \Delta_7, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{11} \vee \mathtt{F}_{12} \vdash \Delta_{13}}\vee_L}{- : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}}\,\mathtt{hCut}}{- : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}\wedge_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \mathtt{F}_9, \Delta_{14}, \mathtt{F}_{11} \vee \mathtt{F}_{12} \vdash \mathtt{F}_7, \Delta_{13}}{\bullet \mathtt{h}_3 : (\Delta_{14}, \mathtt{F}_{11} \vee \mathtt{F}_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_7}\wedge_L \quad \dfrac{\mathtt{h}_{10} : \mathtt{F}_7, \mathtt{F}_{11}, \Delta_{14}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13} \quad \mathtt{h}_{10} : \mathtt{F}_7, \mathtt{F}_{12}, \Delta_{14}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}{\bullet \mathtt{h}_{10} : ((\Delta_{14}, \mathtt{F}_{11} \vee \mathtt{F}_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_7 \vdash \Delta_{13}}\vee_L}{- : (\Delta_{14}, \mathtt{F}_{11} \vee \mathtt{F}_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{13}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_{14}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{11} \vee \mathtt{F}_{12} \vdash \Delta_{13}, \mathtt{F}_7}{}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_{10} : \Delta_{14}, \mathtt{F}_{11}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}}{}\,\mathtt{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_{10} : \Delta_{14}, \mathtt{F}_{12}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{13}}{}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_{10} : \Delta_{14}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{11} \vee \mathtt{F}_{12} \vdash \Delta_{13}}\vee_L}{- : \Delta_{14}, \mathtt{F}_8, \mathtt{F}_9, \mathtt{F}_{11} \vee \mathtt{F}_{12} \vdash \Delta_{13}}\,\mathtt{hCut}}{- : \Delta_{14}, \mathtt{F}_8 \wedge \mathtt{F}_9, \mathtt{F}_{11} \vee \mathtt{F}_{12} \vdash \Delta_{13}}\wedge_L$$

- Case rule AT

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \mathtt{F}_9, \Delta_7 \vdash []\mathtt{F}_{11}, \Delta_{12}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{12}, []\mathtt{F}_{11}}\wedge_L \quad \dfrac{\mathtt{h}_{10} : \mathtt{F}_{11}, \Delta_7, []\mathtt{F}_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{12}}{\bullet \mathtt{h}_{10} : (\Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9), []\mathtt{F}_{11} \vdash \Delta_{12}}\,AT}{- : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{12}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{12}, []\mathtt{F}_{11}}{}\,\mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_{10} : \Delta_7, \mathtt{F}_{11}, \mathtt{F}_8, \mathtt{F}_9, []\mathtt{F}_{11} \vdash \Delta_{12}}{}\,\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_{10} : \Delta_7, \mathtt{F}_8, \mathtt{F}_9, []\mathtt{F}_{11} \vdash \Delta_{12}}\,AT}{- : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{12}}\,\mathtt{hCut}}{- : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{12}}\wedge_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \mathtt{F}_9, \Delta_{13}, []\mathtt{F}_{11} \vdash \mathtt{F}_7, \Delta_{12}}{\bullet \mathtt{h}_3 : (\Delta_{13}, []\mathtt{F}_{11}), \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{F}_7}\wedge_L \quad \dfrac{\mathtt{h}_{10} : \mathtt{F}_7, \mathtt{F}_{11}, \Delta_{13}, []\mathtt{F}_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{12}}{\bullet \mathtt{h}_{10} : ((\Delta_{13}, []\mathtt{F}_{11}), \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_7 \vdash \Delta_{12}}\,AT}{- : (\Delta_{13}, []\mathtt{F}_{11}), \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{12}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet \mathtt{h}_3 : \Delta_{13}, \mathtt{F}_{11}, []\mathtt{F}_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{F}_7}{}\,\mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_{10} : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7, []\mathtt{F}_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{12}}\,\mathtt{ax/W}}{}\,\mathtt{hCut}}{- : \Delta_{13}, \mathtt{F}_{11}, []\mathtt{F}_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{12}}\,AT}{- : \Delta_{13}, []\mathtt{F}_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{12}}$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \mathtt{F}_9, \Delta_7 \vdash \bot, \Delta_{11}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{11}, \bot}\wedge_L \quad \dfrac{}{\bullet \mathtt{h}_{10} : (\Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9), \bot \vdash \Delta_{11}}\bot_L}{- : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \bot, \Delta_{11}}{}\,\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_{10} : \bot, \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{11}}\bot_L}{- : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{11}}\,\mathtt{hCut}}{- : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{11}}\wedge_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \mathtt{F}_9, \bot, \Delta_{12} \vdash \mathtt{F}_7, \Delta_{11}}{\bullet \mathtt{h}_3 : (\bot, \Delta_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_7}\wedge_L \quad \dfrac{}{\bullet \mathtt{h}_{10} : ((\bot, \Delta_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_7 \vdash \Delta_{11}}\bot_L}{- : (\bot, \Delta_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{11}}\mathtt{Cut}}{\dfrac{}{- : \bot, \Delta_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{11}}\bot_L}\to$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \mathtt{F}_9, \Delta_7 \vdash \mathtt{p}_{11}, \Delta_{12}, \mathtt{p}_{11}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{p}_{11}}\wedge_L \quad \dfrac{}{\bullet \mathtt{h}_{10} : (\Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{p}_{11} \vdash \Delta_{12}, \mathtt{p}_{11}}I}{- : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}}\mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}, \mathtt{p}_{11}}\mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_{10} : \Delta_7, \mathtt{F}_8, \mathtt{F}_9, \mathtt{p}_{11} \vdash \Delta_{12}, \mathtt{p}_{11}}I}{- : \Delta_7, \mathtt{F}_8, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}}\mathtt{hCut}}{- : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}}\wedge_L}\to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \mathtt{F}_9, \Delta_{13}, \mathtt{p}_{11} \vdash \mathtt{F}_7, \Delta_{12}, \mathtt{p}_{11}}{\bullet \mathtt{h}_3 : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7}\wedge_L \quad \dfrac{}{\bullet \mathtt{h}_{10} : ((\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{p}_{11}}I}{- : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}}\mathtt{Cut}}{\dfrac{}{- : \Delta_{13}, \mathtt{p}_{11}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}}I}\to$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \mathtt{F}_9, \Delta_7 \vdash \top, \Delta_{11}}{\bullet \mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{11}, \top}\wedge_L \quad \dfrac{\mathtt{h}_{10} : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{11}}{\bullet \mathtt{h}_{10} : (\Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9), \top \vdash \Delta_{11}}\top_L}{- : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{11}}\mathtt{Cut}}{\dfrac{}{- : \Delta_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{11}}\mathtt{ax/W}}\to$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \mathtt{F}_9, \top, \Delta_{12} \vdash \mathtt{F}_7, \Delta_{11}}{\bullet \mathtt{h}_3 : (\top, \Delta_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_7}\wedge_L \quad \dfrac{\mathtt{h}_{10} : \mathtt{F}_7, \Delta_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{11}}{\bullet \mathtt{h}_{10} : ((\top, \Delta_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9), \mathtt{F}_7 \vdash \Delta_{11}}\top_L}{- : (\top, \Delta_{12}), \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{11}}\mathtt{Cut}}{\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \top, \Delta_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_{10} : \top, \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{11}}\mathtt{ax/W}}{- : \top, \Delta_{12}, \mathtt{F}_8 \wedge \mathtt{F}_9 \vdash \Delta_{11}}\mathtt{hCut}}\to$$

8.9 Status of $\vee_L$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_9, \Delta_8 \vdash \mathtt{F}_7, \Delta_{12}, \mathtt{F}_{13} \to \mathtt{F}_{14} \quad \mathtt{h}_3 : \mathtt{F}_{10}, \Delta_8 \vdash \mathtt{F}_7, \Delta_{12}, \mathtt{F}_{13} \to \mathtt{F}_{14}}{\bullet \mathtt{h}_3 : \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash (\Delta_{12}, \mathtt{F}_{13} \to \mathtt{F}_{14}), \mathtt{F}_7}\vee_L \quad \dfrac{\mathtt{h}_{11} : \mathtt{F}_7, \mathtt{F}_{13}, \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \mathtt{F}_{14}, \Delta_{12}}{\bullet \mathtt{h}_{11} : (\Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{13} \to \mathtt{F}_{14}}\to_R}{- : \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \to \mathtt{F}_{14}}\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_8, \mathtt{F}_{13}, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_7}\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_3 : \Delta_8, \mathtt{F}_{10}, \mathtt{F}_{13} \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_7}\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_3 : \Delta_8, \mathtt{F}_{13}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_7}\vee_L \quad \dfrac{}{\mathtt{h}_{11} : \Delta_8, \mathtt{F}_{13}, \mathtt{F}_7, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{14}}\mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_{13}, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{14}}\mathtt{hCut}}{- : \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \to \mathtt{F}_{14}}\to_R}\to$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_9, \Delta_8 \vdash \mathtt{F}_7, \Delta_{12}, \mathtt{F}_{13} \wedge \mathtt{F}_{14} \quad \mathtt{h}_3 : \mathtt{F}_{10}, \Delta_8 \vdash \mathtt{F}_7, \Delta_{12}, \mathtt{F}_{13} \wedge \mathtt{F}_{14}}{\bullet \mathtt{h}_3 : \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash (\Delta_{12}, \mathtt{F}_{13} \wedge \mathtt{F}_{14}), \mathtt{F}_7}\vee_L \quad \dfrac{\mathtt{h}_{11} : \mathtt{F}_7, \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \mathtt{F}_{13}, \Delta_{12}}{\bullet \mathtt{h}_{11} : (\Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{13} \wedge \mathtt{F}_{14}}\wedge_R}{- : \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \wedge \mathtt{F}_{14}}\mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_8, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_7}\mathtt{inv\text{-}th/ax} \quad \dfrac{}{\mathtt{h}_3 : \Delta_8, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_7}\mathtt{inv\text{-}th/ax}}{\bullet \mathtt{h}_3 : \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_7}\vee_L \quad \dfrac{}{\mathtt{h}_{11} : \Delta_8, \mathtt{F}_7, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}}\mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}}\mathtt{hCut} \quad \dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_8, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{F}_{14}, \mathtt{F}_7}\mathtt{inv\text{-}}}{\bullet \mathtt{h}_3 : \Delta_8, \mathtt{F}_9 \vee}}{- : \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \wedge \mathtt{F}_{14}}}\to$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_9, \Delta_8 \vdash \mathtt{F}_7, \Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14} \quad \mathtt{h}_3 : \mathtt{F}_{10}, \Delta_8 \vdash \mathtt{F}_7, \Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}}{\bullet\mathtt{h}_3 : \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash (\Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}), \mathtt{F}_7}\ \vee_L \quad \dfrac{\mathtt{h}_{11} : \mathtt{F}_7, \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \mathtt{F}_{13}, \mathtt{F}_{14}, \Delta_{12}}{\bullet\mathtt{h}_{11} : (\Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}}\ \vee_R}{- : \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}}\ \mathtt{Cut}}{\rightarrow}}{\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_8, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}, \mathtt{F}_7}{\mathtt{h}_3 : \Delta_8, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}, \mathtt{F}_7}\ \mathtt{inv\text{-}th/ax} \quad \dfrac{\mathtt{h}_3 : \Delta_8, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}, \mathtt{F}_7}{\mathtt{h}_3 : \Delta_8, \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}, \mathtt{F}_7}\ \mathtt{inv\text{-}th/ax}}{\bullet\mathtt{h}_3 : \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}, \mathtt{F}_7}\ \vee_L \quad \dfrac{\mathtt{h}_{11} : \Delta_8, \mathtt{F}_7, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}}{}\ \mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13}, \mathtt{F}_{14}}\ \mathtt{hCut}}{- : \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{12}, \mathtt{F}_{13} \vee \mathtt{F}_{14}}\ \vee_R$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_9, \Delta_8 \vdash \mathtt{F}_7, \bot, \Delta_{12} \quad \mathtt{h}_3 : \mathtt{F}_{10}, \Delta_8 \vdash \mathtt{F}_7, \bot, \Delta_{12}}{\bullet\mathtt{h}_3 : \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash (\bot, \Delta_{12}), \mathtt{F}_7}\ \vee_L \quad \dfrac{\mathtt{h}_{11} : \mathtt{F}_7, \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{12}}{\bullet\mathtt{h}_{11} : (\Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_7 \vdash \bot, \Delta_{12}}\ \bot_R}{- : \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \bot, \Delta_{12}}\ \mathtt{Cut}}{\rightarrow}}{\dfrac{\dfrac{\bullet\mathtt{h}_3 : \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \bot, \Delta_{12}, \mathtt{F}_7}{}\ \mathtt{ax/W} \quad \dfrac{\mathtt{h}_{11} : \Delta_8, \mathtt{F}_7, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \bot, \Delta_{12}}{}\ \mathtt{ax/W}}{- : \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \bot, \Delta_{12}}\ \mathtt{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_9, \Delta_8 \vdash \mathtt{F}_7, \top, \Delta_{12} \quad \mathtt{h}_3 : \mathtt{F}_{10}, \Delta_8 \vdash \mathtt{F}_7, \top, \Delta_{12}}{\bullet\mathtt{h}_3 : \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash (\top, \Delta_{12}), \mathtt{F}_7}\ \vee_L \quad \dfrac{}{\bullet\mathtt{h}_{11} : (\Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_7 \vdash \top, \Delta_{12}}\ \top_R}{- : \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \top, \Delta_{12}}\ \mathtt{Cut}}{\rightarrow}}{- : \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \top, \Delta_{12}}\ \top_R$$

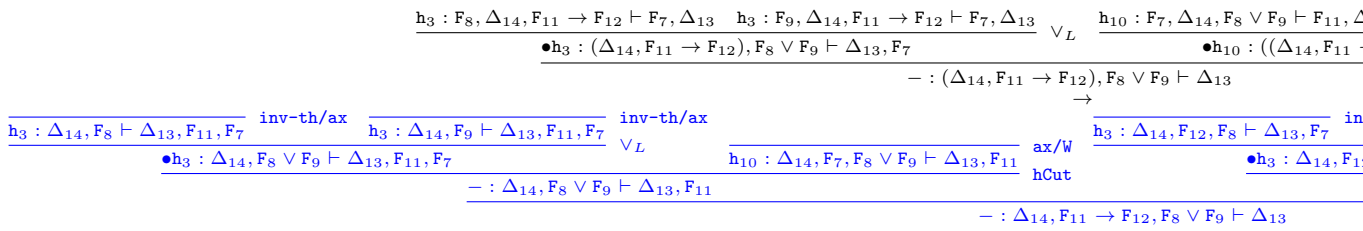
- Case rule $A4$

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \Box\Gamma_{13}, \Delta_{14} \vdash \Box\mathtt{F}_7, \Delta_{11}, []\mathtt{F}_{12} \quad \mathtt{h}_3 : \mathtt{F}_9, \Box\Gamma_{13}, \Delta_{14} \vdash \Box\mathtt{F}_7, \Delta_{11}, []\mathtt{F}_{12}}{\bullet\mathtt{h}_3 : (\Box\Gamma_{13}, \Delta_{14}), \mathtt{F}_8 \vee \mathtt{F}_9 \vdash (\Delta_{11}, []\mathtt{F}_{12}), \Box\mathtt{F}_7}\ \vee_L \quad \dfrac{\mathtt{h}_{10} : \Box\Gamma_{13}, \Box\mathtt{F}_7 \vdash \mathtt{F}_{12}}{\bullet\mathtt{h}_{10} : ((\Box\Gamma_{13}, \Delta_{14}), \mathtt{F}_8 \vee \mathtt{F}_9), \Box\mathtt{F}_7 \vdash \Delta_{11}, []\mathtt{F}_{1}}\ \mathtt{Cut}}{- : (\Box\Gamma_{13}, \Delta_{14}), \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{11}, []\mathtt{F}_{12}}}{\rightarrow}}{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_{14}, \mathtt{F}_8, \Box\Gamma_{13} \vdash \Box\mathtt{F}_7, \Delta_{11}, []\mathtt{F}_{12}}{}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_{10} : \Box\mathtt{F}_7, \Box\Gamma_{13} \vdash \mathtt{F}_{12}}\ \mathtt{ax/W}}{\bullet\mathtt{h}_{10} : \Box\mathtt{F}_7, \Delta_{14}, \mathtt{F}_8, \Box\Gamma_{13} \vdash \Delta_{11}, []\mathtt{F}_{12}}\ A4}{- : \Delta_{14}, \mathtt{F}_8, \Box\Gamma_{13} \vdash \Delta_{11}, []\mathtt{F}_{12}}\ \mathtt{hCut} \quad \dfrac{\mathtt{h}_3 : \Delta_{14}, \mathtt{F}_9, \Box\Gamma_{13} \vdash \Box\mathtt{F}_7, \Delta_{11}, []\mathtt{F}_{12}}{}\ \mathtt{ax/W} \quad \dfrac{\dfrac{}{\mathtt{h}_{10} : \Box}}{\bullet\mathtt{h}_{10} : \Box\mathtt{F}_7, \Delta_{14}}}{- : \Delta_{14}, \mathtt{F}_9, \Box\Gamma_{13} \vdash \Delta_{11}, []\mathtt{F}_{12}}\ \vee$$

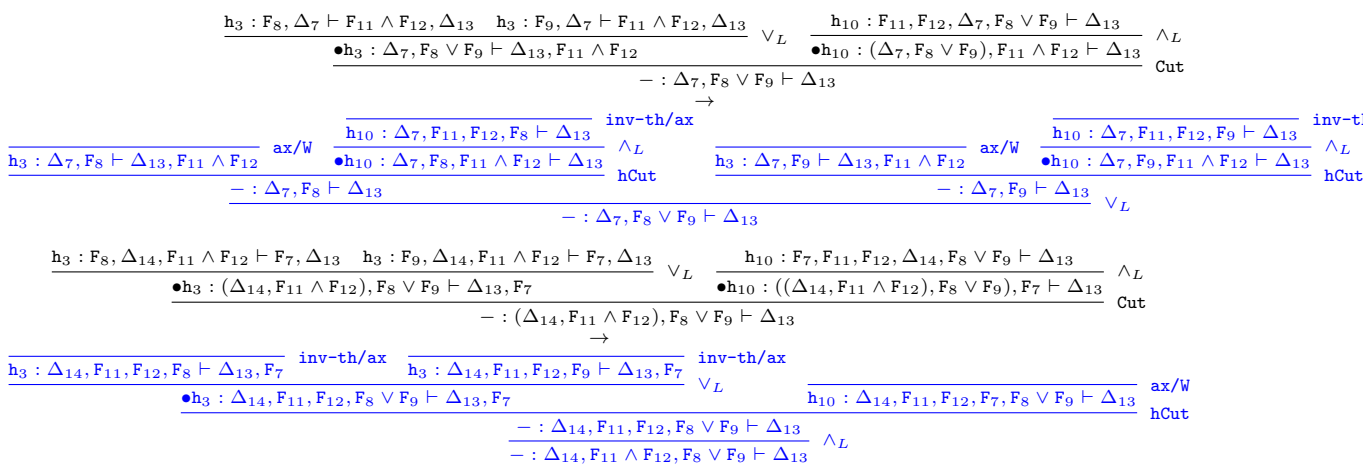
$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \Box\Gamma_{11}, \Delta_{14} \vdash \mathtt{F}_7, \Delta_{12}, []\mathtt{F}_{13} \quad \mathtt{h}_3 : \mathtt{F}_9, \Box\Gamma_{11}, \Delta_{14} \vdash \mathtt{F}_7, \Delta_{12}, []\mathtt{F}_{13}}{\bullet\mathtt{h}_3 : (\Box\Gamma_{11}, \Delta_{14}), \mathtt{F}_8 \vee \mathtt{F}_9 \vdash (\Delta_{12}, []\mathtt{F}_{13}), \mathtt{F}_7}\ \vee_L \quad \dfrac{\mathtt{h}_{10} : \Box\Gamma_{11} \vdash \mathtt{F}_{13}}{\bullet\mathtt{h}_{10} : ((\Box\Gamma_{11}, \Delta_{14}), \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_7 \vdash \Delta_{12}, []\mathtt{F}_{13}}\ A4}{- : (\Box\Gamma_{11}, \Delta_{14}), \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{12}, []\mathtt{F}_{13}}\ \mathtt{Cut}}{\rightarrow}}{\dfrac{\dfrac{}{- : \Box\Gamma_{11} \vdash \mathtt{F}_{13}}\ \mathtt{ax/W}}{- : \Delta_{14}, \Box\Gamma_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{12}, []\mathtt{F}_{13}}\ A4}$$

- Case rule $\rightarrow_L$

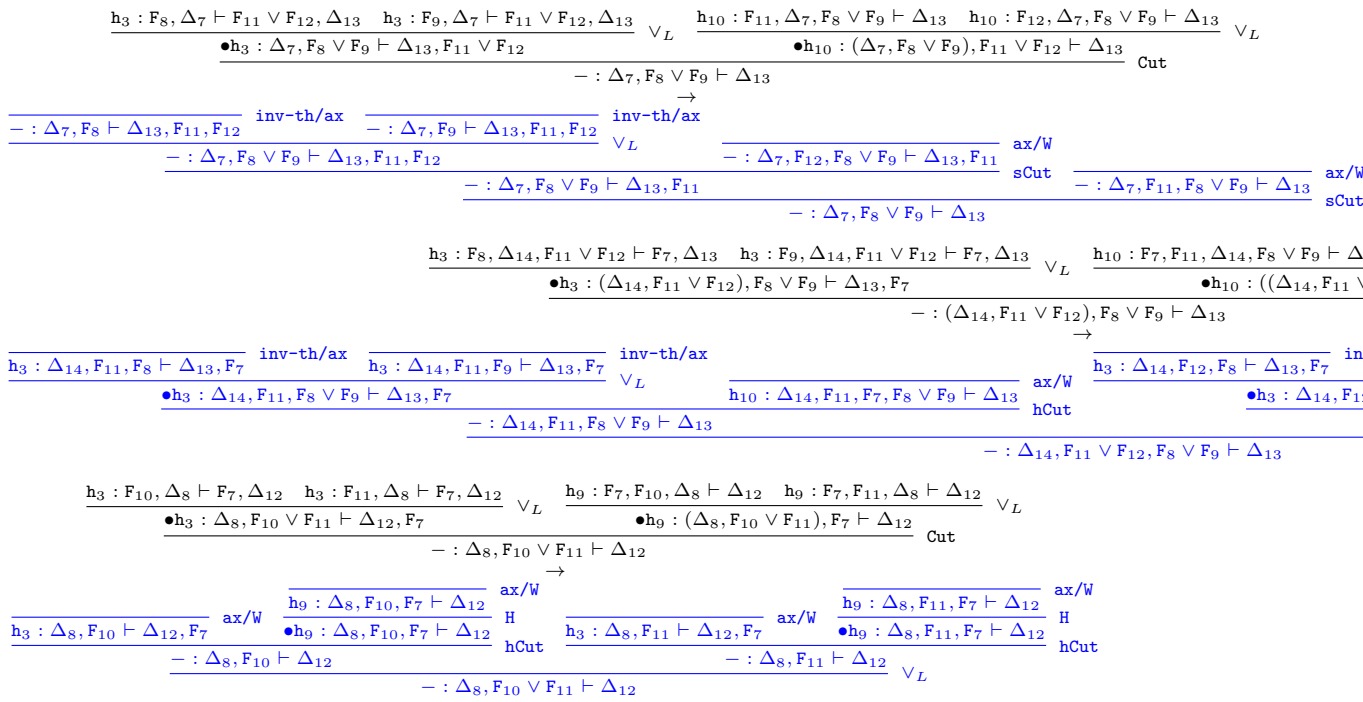
$$\dfrac{\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \Delta_7 \vdash \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}, \Delta_{13} \quad \mathtt{h}_3 : \mathtt{F}_9, \Delta_7 \vdash \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}, \Delta_{13}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}}\ \vee_L \quad \dfrac{\mathtt{h}_{10} : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \mathtt{F}_{11}, \Delta_{13} \quad \mathtt{h}_{10} : \mathtt{F}_{12}, \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}}{\bullet\mathtt{h}_{10} : (\Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_{11} \rightarrow \mathtt{F}_{12} \vdash \Delta_{13}}\ \rightarrow_L}{- : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}}\ \mathtt{Cut}}{\rightarrow}}{\dfrac{\dfrac{\dfrac{\dfrac{}{- : \Delta_7, \mathtt{F}_{11}, \mathtt{F}_8 \vdash \Delta_{13}, \mathtt{F}_{12}}\ \mathtt{inv\text{-}th/ax} \quad \dfrac{}{- : \Delta_7, \mathtt{F}_{11}, \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{12}}\ \mathtt{inv\text{-}th/ax}}{- : \Delta_7, \mathtt{F}_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{12}}\ \vee_L \quad \dfrac{}{- : \Delta_7, \mathtt{F}_{11}, \mathtt{F}_{12}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}}\ \mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}}\ \mathtt{sCut}}{- : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}, \mathtt{F}_{11}}\ \mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{13}}\ \mathtt{sCut}$$



- Case rule $\wedge_L$



- Case rule $\vee_L$



- Case rule AT

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \Delta_7 \vdash []\mathtt{F}_{11}, \Delta_{12} \quad \mathtt{h}_3 : \mathtt{F}_9, \Delta_7 \vdash []\mathtt{F}_{11}, \Delta_{12}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{12}, []\mathtt{F}_{11}} \vee_L \quad \dfrac{\mathtt{h}_{10} : \mathtt{F}_{11}, \Delta_7, []\mathtt{F}_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{12}}{\bullet\mathtt{h}_{10} : (\Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9), []\mathtt{F}_{11} \vdash \Delta_{12}} AT}{- : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{12}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \vdash \Delta_{12}, []\mathtt{F}_{11}}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \vdash \Delta_{12}, []\mathtt{F}_{11}}\ \mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_{10} : \Delta_7, \mathtt{F}_{11}, \mathtt{F}_8, []\mathtt{F}_{11} \vdash \Delta_{12}}{\bullet\mathtt{h}_{10} : \Delta_7, \mathtt{F}_8, []\mathtt{F}_{11} \vdash \Delta_{12}} AT}{\mathtt{h}_{10} : \Delta_7, \mathtt{F}_{11}, \mathtt{F}_8, []\mathtt{F}_{11} \vdash \Delta_{12}}\ \mathtt{inv\text{-}th/ax}}{- : \Delta_7, \mathtt{F}_8 \vdash \Delta_{12}} \mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \vdash \Delta_{12}, []\mathtt{F}_{11}}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \vdash \Delta_{12}, []\mathtt{F}_{11}}\ \mathtt{ax/W} \quad \dfrac{\dfrac{\mathtt{h}_{10} : \Delta_7, \mathtt{F}_{11}, \mathtt{F}_9, []\mathtt{F}_{11} \vdash \Delta_{12}}{\bullet\mathtt{h}_{10} : \Delta_7, \mathtt{F}_9, []\mathtt{F}_{11} \vdash \Delta_{12}} AT}{\mathtt{h}_{10} : \Delta_7, \mathtt{F}_{11}, \mathtt{F}_9, []\mathtt{F}_{11} \vdash \Delta_{12}}\ \mathtt{inv\text{-}th/ax}}{- : \Delta_7, \mathtt{F}_9 \vdash \Delta_{12}} \mathtt{hCut}}{- : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{12}} \vee_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \Delta_{13}, []\mathtt{F}_{11} \vdash \mathtt{F}_7, \Delta_{12} \quad \mathtt{h}_3 : \mathtt{F}_9, \Delta_{13}, []\mathtt{F}_{11} \vdash \mathtt{F}_7, \Delta_{12}}{\bullet\mathtt{h}_3 : (\Delta_{13}, []\mathtt{F}_{11}), \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{F}_7} \vee_L \quad \dfrac{\mathtt{h}_{10} : \mathtt{F}_7, \mathtt{F}_{11}, \Delta_{13}, []\mathtt{F}_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{12}}{\bullet\mathtt{h}_{10} : ((\Delta_{13}, []\mathtt{F}_{11}), \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_7 \vdash \Delta_{12}} AT}{- : (\Delta_{13}, []\mathtt{F}_{11}), \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{12}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\bullet\mathtt{h}_3 : \Delta_{13}, \mathtt{F}_{11}, []\mathtt{F}_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{F}_7}{\bullet\mathtt{h}_3 : \Delta_{13}, \mathtt{F}_{11}, []\mathtt{F}_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{F}_7}\ \mathtt{ax/W} \quad \dfrac{\mathtt{h}_{10} : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7, []\mathtt{F}_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{12}}{\mathtt{h}_{10} : \Delta_{13}, \mathtt{F}_{11}, \mathtt{F}_7, []\mathtt{F}_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{12}}\ \mathtt{ax/W}}{- : \Delta_{13}, \mathtt{F}_{11}, []\mathtt{F}_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{12}} \mathtt{hCut}}{- : \Delta_{13}, []\mathtt{F}_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{12}} AT$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \Delta_7 \vdash \bot, \Delta_{11} \quad \mathtt{h}_3 : \mathtt{F}_9, \Delta_7 \vdash \bot, \Delta_{11}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{11}, \bot} \vee_L \quad \dfrac{}{\bullet\mathtt{h}_{10} : (\Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9), \bot \vdash \Delta_{11}} \bot_L}{- : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{11}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \vdash \bot, \Delta_{11}}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \vdash \bot, \Delta_{11}}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_{10} : \bot, \Delta_7, \mathtt{F}_8 \vdash \Delta_{11}} \bot_L}{- : \Delta_7, \mathtt{F}_8 \vdash \Delta_{11}} \mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \vdash \bot, \Delta_{11}}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \vdash \bot, \Delta_{11}}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_{10} : \bot, \Delta_7, \mathtt{F}_9 \vdash \Delta_{11}} \bot_L}{- : \Delta_7, \mathtt{F}_9 \vdash \Delta_{11}} \mathtt{hCut}}{- : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{11}} \vee_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \bot, \Delta_{12} \vdash \mathtt{F}_7, \Delta_{11} \quad \mathtt{h}_3 : \mathtt{F}_9, \bot, \Delta_{12} \vdash \mathtt{F}_7, \Delta_{11}}{\bullet\mathtt{h}_3 : (\bot, \Delta_{12}), \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_7} \vee_L \quad \dfrac{}{\bullet\mathtt{h}_{10} : ((\bot, \Delta_{12}), \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_7 \vdash \Delta_{11}} \bot_L}{- : (\bot, \Delta_{12}), \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{11}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \bot, \Delta_{12}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{11}} \bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \Delta_7 \vdash \mathtt{p}_{11}, \Delta_{12}, \mathtt{p}_{11} \quad \mathtt{h}_3 : \mathtt{F}_9, \Delta_7 \vdash \mathtt{p}_{11}, \Delta_{12}, \mathtt{p}_{11}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{p}_{11}} \vee_L \quad \dfrac{}{\bullet\mathtt{h}_{10} : (\Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{p}_{11} \vdash \Delta_{12}, \mathtt{p}_{11}} I}{- : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{p}_{11}, \mathtt{p}_{11}}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{p}_{11}, \mathtt{p}_{11}}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_{10} : \Delta_7, \mathtt{F}_8, \mathtt{p}_{11} \vdash \Delta_{12}, \mathtt{p}_{11}} I}{- : \Delta_7, \mathtt{F}_8 \vdash \Delta_{12}, \mathtt{p}_{11}} \mathtt{hCut} \quad \dfrac{\dfrac{\mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}, \mathtt{p}_{11}}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}, \mathtt{p}_{11}}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_{10} : \Delta_7, \mathtt{F}_9, \mathtt{p}_{11} \vdash \Delta_{12}, \mathtt{p}_{11}} I}{- : \Delta_7, \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}} \mathtt{hCut}}{- : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}} \vee_L$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \Delta_{13}, \mathtt{p}_{11} \vdash \mathtt{F}_7, \Delta_{12}, \mathtt{p}_{11} \quad \mathtt{h}_3 : \mathtt{F}_9, \Delta_{13}, \mathtt{p}_{11} \vdash \mathtt{F}_7, \Delta_{12}, \mathtt{p}_{11}}{\bullet\mathtt{h}_3 : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_8 \vee \mathtt{F}_9 \vdash (\Delta_{12}, \mathtt{p}_{11}), \mathtt{F}_7} \vee_L \quad \dfrac{}{\bullet\mathtt{h}_{10} : ((\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_7 \vdash \Delta_{12}, \mathtt{p}_{11}} I}{- : (\Delta_{13}, \mathtt{p}_{11}), \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \Delta_{13}, \mathtt{p}_{11}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{12}, \mathtt{p}_{11}} I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \Delta_7 \vdash \top, \Delta_{11} \quad \mathtt{h}_3 : \mathtt{F}_9, \Delta_7 \vdash \top, \Delta_{11}}{\bullet\mathtt{h}_3 : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{11}, \top} \vee_L \quad \dfrac{\mathtt{h}_{10} : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{11}}{\bullet\mathtt{h}_{10} : (\Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9), \top \vdash \Delta_{11}} \top_L}{- : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{11}} \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \Delta_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{11}}\ \mathtt{ax/W}$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \top, \Delta_{12} \vdash \mathtt{F}_7, \Delta_{11} \quad \mathtt{h}_3 : \mathtt{F}_9, \top, \Delta_{12} \vdash \mathtt{F}_7, \Delta_{11}}{\bullet\mathtt{h}_3 : (\top, \Delta_{12}), \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_7}\vee_L \quad \dfrac{\mathtt{h}_{10} : \mathtt{F}_7, \Delta_{12}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{11}}{\bullet\mathtt{h}_{10} : ((\top, \Delta_{12}), \mathtt{F}_8 \vee \mathtt{F}_9), \mathtt{F}_7 \vdash \Delta_{11}}\top_L}{- : (\top, \Delta_{12}), \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{11}}\mathtt{Cut}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_3 : \top, \Delta_{12}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{11}, \mathtt{F}_7}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_{10} : \top, \Delta_{12}, \mathtt{F}_7, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{11}}\mathtt{ax/W}}{- : \top, \Delta_{12}, \mathtt{F}_8 \vee \mathtt{F}_9 \vdash \Delta_{11}}\mathtt{hCut}$$

8.10 Status of AT: OK

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \Delta_7, []\mathtt{F}_8 \vdash \mathtt{F}_6, \Delta_{10}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}}{\bullet\mathtt{h}_3 : \Delta_7, []\mathtt{F}_8 \vdash (\Delta_{10}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}), \mathtt{F}_6}AT \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_6, \mathtt{F}_{11}, \Delta_7, []\mathtt{F}_8 \vdash \mathtt{F}_{12}, \Delta_{10}}{\bullet\mathtt{h}_9 : (\Delta_7, []\mathtt{F}_8), \mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}}\rightarrow_R}{- : \Delta_7, []\mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}}\mathtt{Cut}$$

$$\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_8, []\mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_6, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}}\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_7, \mathtt{F}_6, \mathtt{F}_8, []\mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}}\mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_8, []\mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}}\mathtt{hCut}$$

$$\dfrac{- : \Delta_7, \mathtt{F}_8, []\mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}}{- : \Delta_7, []\mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \rightarrow \mathtt{F}_{12}}AT$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \Delta_7, []\mathtt{F}_8 \vdash \mathtt{F}_6, \Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}{\bullet\mathtt{h}_3 : \Delta_7, []\mathtt{F}_8 \vdash (\Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}), \mathtt{F}_6}AT \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_6, \Delta_7, []\mathtt{F}_8 \vdash \mathtt{F}_{11}, \Delta_{10} \quad \mathtt{h}_9 : \mathtt{F}_6, \Delta_7, []\mathtt{F}_8 \vdash \mathtt{F}_{12}, \Delta_{10}}{\bullet\mathtt{h}_9 : (\Delta_7, []\mathtt{F}_8), \mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\wedge_R}{- : \Delta_7, []\mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\mathtt{Cut}$$

$$\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_8, []\mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_6, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_7, \mathtt{F}_6, \mathtt{F}_8, []\mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_8, []\mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}\mathtt{hCut}$$

$$\dfrac{- : \Delta_7, \mathtt{F}_8, []\mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}{- : \Delta_7, []\mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \wedge \mathtt{F}_{12}}AT$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \Delta_7, []\mathtt{F}_8 \vdash \mathtt{F}_6, \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}{\bullet\mathtt{h}_3 : \Delta_7, []\mathtt{F}_8 \vdash (\Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}), \mathtt{F}_6}AT \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_6, \Delta_7, []\mathtt{F}_8 \vdash \mathtt{F}_{11}, \mathtt{F}_{12}, \Delta_{10}}{\bullet\mathtt{h}_9 : (\Delta_7, []\mathtt{F}_8), \mathtt{F}_6 \vdash \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}\vee_R}{- : \Delta_7, []\mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}\mathtt{Cut}$$

$$\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_8, []\mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_6, \mathtt{F}_{11} \vee \mathtt{F}_{12}}\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_9 : \Delta_7, \mathtt{F}_6, \mathtt{F}_8, []\mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}\mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_8, []\mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}\mathtt{hCut}$$

$$\dfrac{- : \Delta_7, \mathtt{F}_8, []\mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}{- : \Delta_7, []\mathtt{F}_8 \vdash \Delta_{10}, \mathtt{F}_{11} \vee \mathtt{F}_{12}}AT$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \Delta_7, []\mathtt{F}_8 \vdash \mathtt{F}_6, \bot, \Delta_{10}}{\bullet\mathtt{h}_3 : \Delta_7, []\mathtt{F}_8 \vdash (\bot, \Delta_{10}), \mathtt{F}_6}AT \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_6, \Delta_7, []\mathtt{F}_8 \vdash \Delta_{10}}{\bullet\mathtt{h}_9 : (\Delta_7, []\mathtt{F}_8), \mathtt{F}_6 \vdash \bot, \Delta_{10}}\bot_R}{- : \Delta_7, []\mathtt{F}_8 \vdash \bot, \Delta_{10}}\mathtt{Cut}$$

$$\dfrac{\dfrac{}{\bullet\mathtt{h}_3 : \Delta_7, []\mathtt{F}_8 \vdash \bot, \Delta_{10}, \mathtt{F}_6}\mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_9 : \Delta_7, \mathtt{F}_6, []\mathtt{F}_8 \vdash \bot, \Delta_{10}}\mathtt{ax/W}}{- : \Delta_7, []\mathtt{F}_8 \vdash \bot, \Delta_{10}}\mathtt{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_8, \Delta_7, []\mathtt{F}_8 \vdash \mathtt{F}_6, \top, \Delta_{10}}{\bullet\mathtt{h}_3 : \Delta_7, []\mathtt{F}_8 \vdash (\top, \Delta_{10}), \mathtt{F}_6}AT \quad \dfrac{}{\bullet\mathtt{h}_9 : (\Delta_7, []\mathtt{F}_8), \mathtt{F}_6 \vdash \top, \Delta_{10}}\top_R}{- : \Delta_7, []\mathtt{F}_8 \vdash \top, \Delta_{10}}\mathtt{Cut}$$

$$\dfrac{}{- : \Delta_7, []\mathtt{F}_8 \vdash \top, \Delta_{10}}\top_R$$

- Case rule $A4$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_7, (\Box\Gamma_{12}, \Delta_9), []\mathtt{F}_7 \vdash \Box\mathtt{F}_6, \Delta_{10}, []\mathtt{F}_{11}}{\bullet\mathtt{h}_3 : (\Box\Gamma_{12}, \Delta_9), []\mathtt{F}_7 \vdash (\Delta_{10}, []\mathtt{F}_{11}), \Box\mathtt{F}_6}\ AT \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{12}, \Box\mathtt{F}_6, []\mathtt{F}_7 \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{12}, \Delta_9), []\mathtt{F}_7), \Box\mathtt{F}_6 \vdash \Delta_{10}, []\mathtt{F}_{11}}\ A4}{- : (\Box\Gamma_{12}, \Delta_9), []\mathtt{F}_7 \vdash \Delta_{10}, []\mathtt{F}_{11}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_9, \mathtt{F}_7, \Box\Gamma_{12}, []\mathtt{F}_7 \vdash \Box\mathtt{F}_6, \Delta_{10}, []\mathtt{F}_{11}}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_8 : \Box\mathtt{F}_6, \Delta_9, \mathtt{F}_7, \Box\Gamma_{12}, []\mathtt{F}_7 \vdash \Delta_{10}, []\mathtt{F}_{11}}\ \mathtt{ax/W}}{- : \Delta_9, \mathtt{F}_7, \Box\Gamma_{12}, []\mathtt{F}_7 \vdash \Delta_{10}, []\mathtt{F}_{11}}\ \mathtt{hCut}}{- : \Delta_9, \Box\Gamma_{12}, []\mathtt{F}_7 \vdash \Delta_{10}, []\mathtt{F}_{11}}\ AT}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_7, (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_7 \vdash \mathtt{F}_6, \Delta_9, []\mathtt{F}_{10}}{\bullet\mathtt{h}_3 : (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_7 \vdash (\Delta_9, []\mathtt{F}_{10}), \mathtt{F}_6}\ AT \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{11}, []\mathtt{F}_7 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_7), \mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_{10}}\ A4}{- : (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_7 \vdash \Delta_9, []\mathtt{F}_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_{11}, []\mathtt{F}_7 \vdash \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_7 \vdash \Delta_9, []\mathtt{F}_{10}}\ A4}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_7, (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_7 \vdash \Box\mathtt{F}_6, \Delta_9, []\mathtt{F}_{10}}{\bullet\mathtt{h}_3 : (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_7 \vdash (\Delta_9, []\mathtt{F}_{10}), \Box\mathtt{F}_6}\ AT \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_{11}, \Box\mathtt{F}_6 \vdash \mathtt{F}_{10}}{\bullet\mathtt{h}_8 : ((\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_7), \Box\mathtt{F}_6 \vdash \Delta_9, []\mathtt{F}_{10}}\ A4}{- : (\Box\Gamma_{11}, \Delta_{12}), []\mathtt{F}_7 \vdash \Delta_9, []\mathtt{F}_{10}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_{12}, \mathtt{F}_7, \Box\Gamma_{11}, []\mathtt{F}_7 \vdash \Box\mathtt{F}_6, \Delta_9, []\mathtt{F}_{10}}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_8 : \Box\mathtt{F}_6, \Delta_{12}, \mathtt{F}_7, \Box\Gamma_{11}, []\mathtt{F}_7 \vdash \Delta_9, []\mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_7, \Box\Gamma_{11}, []\mathtt{F}_7 \vdash \Delta_9, []\mathtt{F}_{10}}\ \mathtt{hCut}}{- : \Delta_{12}, \Box\Gamma_{11}, []\mathtt{F}_7 \vdash \Delta_9, []\mathtt{F}_{10}}\ AT}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_7, (\Box\Gamma_9, \Delta_{12}), []\mathtt{F}_7 \vdash \mathtt{F}_6, \Delta_{10}, []\mathtt{F}_{11}}{\bullet\mathtt{h}_3 : (\Box\Gamma_9, \Delta_{12}), []\mathtt{F}_7 \vdash (\Delta_{10}, []\mathtt{F}_{11}), \mathtt{F}_6}\ AT \quad \dfrac{\mathtt{h}_8 : \Box\Gamma_9 \vdash \mathtt{F}_{11}}{\bullet\mathtt{h}_8 : ((\Box\Gamma_9, \Delta_{12}), []\mathtt{F}_7), \mathtt{F}_6 \vdash \Delta_{10}, []\mathtt{F}_{11}}\ A4}{- : (\Box\Gamma_9, \Delta_{12}), []\mathtt{F}_7 \vdash \Delta_{10}, []\mathtt{F}_{11}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{- : \Box\Gamma_9 \vdash \mathtt{F}_{11}}\ \mathtt{ax/W}}{- : \Delta_{12}, \Box\Gamma_9, []\mathtt{F}_7 \vdash \Delta_{10}, []\mathtt{F}_{11}}\ A4}$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_7, \Delta_6, []\mathtt{F}_7 \vdash \mathtt{F}_9 \to \mathtt{F}_{10}, \Delta_{11}}{\bullet\mathtt{h}_3 : \Delta_6, []\mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}}\ AT \quad \dfrac{\mathtt{h}_8 : \Delta_6, []\mathtt{F}_7 \vdash \mathtt{F}_9, \Delta_{11} \quad \mathtt{h}_8 : \mathtt{F}_{10}, \Delta_6, []\mathtt{F}_7 \vdash \Delta_{11}}{\bullet\mathtt{h}_8 : (\Delta_6, []\mathtt{F}_7), \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}}\ \to_L}{- : \Delta_6, []\mathtt{F}_7 \vdash \Delta_{11}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_7, []\mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9 \to \mathtt{F}_{10}}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_8 : \Delta_6, \mathtt{F}_7, []\mathtt{F}_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}}\ \mathtt{ax/W}}{- : \Delta_6, \mathtt{F}_7, []\mathtt{F}_7 \vdash \Delta_{11}}\ \mathtt{hCut}}{- : \Delta_6, []\mathtt{F}_7 \vdash \Delta_{11}}\ AT}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_7, (\Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10}), []\mathtt{F}_7 \vdash \mathtt{F}_6, \Delta_{11}}{\bullet\mathtt{h}_3 : (\Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10}), []\mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_6}\ AT \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_6, \Delta_{12}, []\mathtt{F}_7 \vdash \mathtt{F}_9, \Delta_{11} \quad \mathtt{h}_8 : \mathtt{F}_6, \mathtt{F}_{10}, \Delta_{12}, []\mathtt{F}_7 \vdash \Delta_{11}}{\bullet\mathtt{h}_8 : ((\Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10}), []\mathtt{F}_7), \mathtt{F}_6 \vdash \Delta_{11}}\ \to_L}{- : (\Delta_{12}, \mathtt{F}_9 \to \mathtt{F}_{10}), []\mathtt{F}_7 \vdash \Delta_{11}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_{12}, \mathtt{F}_7, []\mathtt{F}_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6, \mathtt{F}_7, []\mathtt{F}_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}}\ \mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_7, []\mathtt{F}_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}}\ \mathtt{hCut}}{- : \Delta_{12}, []\mathtt{F}_7, \mathtt{F}_9 \to \mathtt{F}_{10} \vdash \Delta_{11}}\ AT}$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_7, \Delta_6, []\mathtt{F}_7 \vdash \mathtt{F}_9 \wedge \mathtt{F}_{10}, \Delta_{11}}{\bullet\mathtt{h}_3 : \Delta_6, []\mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\ AT \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_9, \mathtt{F}_{10}, \Delta_6, []\mathtt{F}_7 \vdash \Delta_{11}}{\bullet\mathtt{h}_8 : (\Delta_6, []\mathtt{F}_7), \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}}\ \wedge_L}{- : \Delta_6, []\mathtt{F}_7 \vdash \Delta_{11}}\ \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_7, []\mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_8 : \Delta_6, \mathtt{F}_7, []\mathtt{F}_7, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}}\ \mathtt{ax/W}}{- : \Delta_6, \mathtt{F}_7, []\mathtt{F}_7 \vdash \Delta_{11}}\ \mathtt{hCut}}{- : \Delta_6, []\mathtt{F}_7 \vdash \Delta_{11}}\ AT}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_7, (\Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10}), []\mathtt{F}_7 \vdash \mathtt{F}_6, \Delta_{11}}{\bullet\mathtt{h}_3 : (\Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10}), []\mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_6}\ AT \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_6, \mathtt{F}_9, \mathtt{F}_{10}, \Delta_{12}, []\mathtt{F}_7 \vdash \Delta_{11}}{\bullet\mathtt{h}_8 : ((\Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10}), []\mathtt{F}_7), \mathtt{F}_6 \vdash \Delta_{11}}\ \wedge_L}{- : (\Delta_{12}, \mathtt{F}_9 \wedge \mathtt{F}_{10}), []\mathtt{F}_7 \vdash \Delta_{11}}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_{12}, \mathtt{F}_7, []\mathtt{F}_7, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6, \mathtt{F}_7, []\mathtt{F}_7, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}}\ \mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_7, []\mathtt{F}_7, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}}\ \mathtt{hCut}}{- : \Delta_{12}, []\mathtt{F}_7, \mathtt{F}_9 \wedge \mathtt{F}_{10} \vdash \Delta_{11}}\ AT$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_7, \Delta_6, []\mathtt{F}_7 \vdash \mathtt{F}_9 \vee \mathtt{F}_{10}, \Delta_{11}}{\bullet\mathtt{h}_3 : \Delta_6, []\mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ AT \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_9, \Delta_6, []\mathtt{F}_7 \vdash \Delta_{11} \quad \mathtt{h}_8 : \mathtt{F}_{10}, \Delta_6, []\mathtt{F}_7 \vdash \Delta_{11}}{\bullet\mathtt{h}_8 : (\Delta_6, []\mathtt{F}_7), \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}}\ \vee_L}{- : \Delta_6, []\mathtt{F}_7 \vdash \Delta_{11}}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_7, []\mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_8 : \Delta_6, \mathtt{F}_7, []\mathtt{F}_7, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}}\ \mathtt{ax/W}}{- : \Delta_6, \mathtt{F}_7, []\mathtt{F}_7 \vdash \Delta_{11}}\ \mathtt{hCut}}{- : \Delta_6, []\mathtt{F}_7 \vdash \Delta_{11}}\ AT$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_7, (\Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10}), []\mathtt{F}_7 \vdash \mathtt{F}_6, \Delta_{11}}{\bullet\mathtt{h}_3 : (\Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10}), []\mathtt{F}_7 \vdash \Delta_{11}, \mathtt{F}_6}\ AT \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_6, \mathtt{F}_9, \Delta_{12}, []\mathtt{F}_7 \vdash \Delta_{11} \quad \mathtt{h}_8 : \mathtt{F}_6, \mathtt{F}_{10}, \Delta_{12}, []\mathtt{F}_7 \vdash \Delta_{11}}{\bullet\mathtt{h}_8 : ((\Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10}), []\mathtt{F}_7), \mathtt{F}_6 \vdash \Delta_{11}}\ \vee_L}{- : (\Delta_{12}, \mathtt{F}_9 \vee \mathtt{F}_{10}), []\mathtt{F}_7 \vdash \Delta_{11}}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_{12}, \mathtt{F}_7, []\mathtt{F}_7, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}, \mathtt{F}_6}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_8 : \Delta_{12}, \mathtt{F}_6, \mathtt{F}_7, []\mathtt{F}_7, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}}\ \mathtt{ax/W}}{- : \Delta_{12}, \mathtt{F}_7, []\mathtt{F}_7, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}}\ \mathtt{hCut}}{- : \Delta_{12}, []\mathtt{F}_7, \mathtt{F}_9 \vee \mathtt{F}_{10} \vdash \Delta_{11}}\ AT$$

- Case rule AT

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_7, \Delta_6, []\mathtt{F}_7 \vdash []\mathtt{F}_9, \Delta_{10}}{\bullet\mathtt{h}_3 : \Delta_6, []\mathtt{F}_7 \vdash \Delta_{10}, []\mathtt{F}_9}\ AT \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_9, \Delta_6, []\mathtt{F}_7, []\mathtt{F}_9 \vdash \Delta_{10}}{\bullet\mathtt{h}_8 : (\Delta_6, []\mathtt{F}_7), []\mathtt{F}_9 \vdash \Delta_{10}}\ AT}{- : \Delta_6, []\mathtt{F}_7 \vdash \Delta_{10}}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_7, []\mathtt{F}_7 \vdash \Delta_{10}, []\mathtt{F}_9}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_8 : \Delta_6, \mathtt{F}_7, []\mathtt{F}_7, []\mathtt{F}_9 \vdash \Delta_{10}}\ \mathtt{ax/W}}{- : \Delta_6, \mathtt{F}_7, []\mathtt{F}_7 \vdash \Delta_{10}}\ \mathtt{hCut}}{- : \Delta_6, []\mathtt{F}_7 \vdash \Delta_{10}}\ AT$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_7, (\Delta_{11}, []\mathtt{F}_9), []\mathtt{F}_7 \vdash \mathtt{F}_6, \Delta_{10}}{\bullet\mathtt{h}_3 : (\Delta_{11}, []\mathtt{F}_9), []\mathtt{F}_7 \vdash \Delta_{10}, \mathtt{F}_6}\ AT \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_6, \mathtt{F}_9, \Delta_{11}, []\mathtt{F}_7, []\mathtt{F}_9 \vdash \Delta_{10}}{\bullet\mathtt{h}_8 : ((\Delta_{11}, []\mathtt{F}_9), []\mathtt{F}_7), \mathtt{F}_6 \vdash \Delta_{10}}\ AT}{- : (\Delta_{11}, []\mathtt{F}_9), []\mathtt{F}_7 \vdash \Delta_{10}}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_{11}, \mathtt{F}_7, []\mathtt{F}_7, []\mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_6}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_8 : \Delta_{11}, \mathtt{F}_6, \mathtt{F}_7, []\mathtt{F}_7, []\mathtt{F}_9 \vdash \Delta_{10}}\ \mathtt{ax/W}}{- : \Delta_{11}, \mathtt{F}_7, []\mathtt{F}_7, []\mathtt{F}_9 \vdash \Delta_{10}}\ \mathtt{hCut}}{- : \Delta_{11}, []\mathtt{F}_7, []\mathtt{F}_9 \vdash \Delta_{10}}\ AT$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_9, \Delta_7, []\mathtt{F}_9 \vdash \mathtt{F}_6, \Delta_{10}}{\bullet\mathtt{h}_3 : \Delta_7, []\mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_6}\ AT \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_6, \mathtt{F}_9, \Delta_7, []\mathtt{F}_9 \vdash \Delta_{10}}{\bullet\mathtt{h}_8 : (\Delta_7, []\mathtt{F}_9), \mathtt{F}_6 \vdash \Delta_{10}}\ AT}{- : \Delta_7, []\mathtt{F}_9 \vdash \Delta_{10}}\ \mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_7, \mathtt{F}_9, []\mathtt{F}_9 \vdash \Delta_{10}, \mathtt{F}_6}\ \mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_8 : \Delta_7, \mathtt{F}_6, \mathtt{F}_9, []\mathtt{F}_9 \vdash \Delta_{10}}\ \mathtt{ax/W}}{- : \Delta_7, \mathtt{F}_9, []\mathtt{F}_9 \vdash \Delta_{10}}\ \mathtt{hCut}}{- : \Delta_7, []\mathtt{F}_9 \vdash \Delta_{10}}\ AT$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_7, \Delta_6, []\mathtt{F}_7 \vdash \bot, \Delta_9}{\bullet \mathtt{h}_3 : \Delta_6, []\mathtt{F}_7 \vdash \Delta_9, \bot}\, AT \quad \dfrac{}{\bullet \mathtt{h}_8 : (\Delta_6, []\mathtt{F}_7), \bot \vdash \Delta_9}\, \bot_L}{- : \Delta_6, []\mathtt{F}_7 \vdash \Delta_9}\, \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_7, []\mathtt{F}_7 \vdash \bot, \Delta_9}\, \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_8 : \bot, \Delta_6, \mathtt{F}_7, []\mathtt{F}_7 \vdash \Delta_9}\, \bot_L}{- : \Delta_6, \mathtt{F}_7, []\mathtt{F}_7 \vdash \Delta_9}\, \mathtt{hCut} \quad AT}{- : \Delta_6, []\mathtt{F}_7 \vdash \Delta_9}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_7, (\bot, \Delta_{10}), []\mathtt{F}_7 \vdash \mathtt{F}_6, \Delta_9}{\bullet \mathtt{h}_3 : (\bot, \Delta_{10}), []\mathtt{F}_7 \vdash \Delta_9, \mathtt{F}_6}\, AT \quad \dfrac{}{\bullet \mathtt{h}_8 : ((\bot, \Delta_{10}), []\mathtt{F}_7), \mathtt{F}_6 \vdash \Delta_9}\, \bot_L}{- : (\bot, \Delta_{10}), []\mathtt{F}_7 \vdash \Delta_9}\, \mathtt{Cut}}{- : \bot, \Delta_{10}, []\mathtt{F}_7 \vdash \Delta_9}\, \bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_7, \Delta_6, []\mathtt{F}_7 \vdash \mathtt{p}_9, \Delta_{10}, \mathtt{p}_9}{\bullet \mathtt{h}_3 : \Delta_6, []\mathtt{F}_7 \vdash (\Delta_{10}, \mathtt{p}_9), \mathtt{p}_9}\, AT \quad \dfrac{}{\bullet \mathtt{h}_8 : (\Delta_6, []\mathtt{F}_7), \mathtt{p}_9 \vdash \Delta_{10}, \mathtt{p}_9}\, I}{- : \Delta_6, []\mathtt{F}_7 \vdash \Delta_{10}, \mathtt{p}_9}\, \mathtt{Cut}}{\dfrac{\dfrac{}{\mathtt{h}_3 : \Delta_6, \mathtt{F}_7, []\mathtt{F}_7 \vdash \Delta_{10}, \mathtt{p}_9, \mathtt{p}_9}\, \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_8 : \Delta_6, \mathtt{F}_7, \mathtt{p}_9, []\mathtt{F}_7 \vdash \Delta_{10}, \mathtt{p}_9}\, I}{- : \Delta_6, \mathtt{F}_7, []\mathtt{F}_7 \vdash \Delta_{10}, \mathtt{p}_9}\, \mathtt{hCut} \quad AT}{- : \Delta_6, []\mathtt{F}_7 \vdash \Delta_{10}, \mathtt{p}_9}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_7, (\Delta_{11}, \mathtt{p}_9), []\mathtt{F}_7 \vdash \mathtt{F}_6, \Delta_{10}, \mathtt{p}_9}{\bullet \mathtt{h}_3 : (\Delta_{11}, \mathtt{p}_9), []\mathtt{F}_7 \vdash (\Delta_{10}, \mathtt{p}_9), \mathtt{F}_6}\, AT \quad \dfrac{}{\bullet \mathtt{h}_8 : ((\Delta_{11}, \mathtt{p}_9), []\mathtt{F}_7), \mathtt{F}_6 \vdash \Delta_{10}, \mathtt{p}_9}\, I}{- : (\Delta_{11}, \mathtt{p}_9), []\mathtt{F}_7 \vdash \Delta_{10}, \mathtt{p}_9}\, \mathtt{Cut}}{- : \Delta_{11}, \mathtt{p}_9, []\mathtt{F}_7 \vdash \Delta_{10}, \mathtt{p}_9}\, I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_7, \Delta_6, []\mathtt{F}_7 \vdash \top, \Delta_9}{\bullet \mathtt{h}_3 : \Delta_6, []\mathtt{F}_7 \vdash \Delta_9, \top}\, AT \quad \dfrac{\mathtt{h}_8 : \Delta_6, []\mathtt{F}_7 \vdash \Delta_9}{\bullet \mathtt{h}_8 : (\Delta_6, []\mathtt{F}_7), \top \vdash \Delta_9}\, \top_L}{- : \Delta_6, []\mathtt{F}_7 \vdash \Delta_9}\, \mathtt{Cut}}{- : \Delta_6, []\mathtt{F}_7 \vdash \Delta_9}\, \mathtt{ax/W}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \mathtt{F}_7, (\top, \Delta_{10}), []\mathtt{F}_7 \vdash \mathtt{F}_6, \Delta_9}{\bullet \mathtt{h}_3 : (\top, \Delta_{10}), []\mathtt{F}_7 \vdash \Delta_9, \mathtt{F}_6}\, AT \quad \dfrac{\mathtt{h}_8 : \mathtt{F}_6, \Delta_{10}, []\mathtt{F}_7 \vdash \Delta_9}{\bullet \mathtt{h}_8 : ((\top, \Delta_{10}), []\mathtt{F}_7), \mathtt{F}_6 \vdash \Delta_9}\, \top_L}{- : (\top, \Delta_{10}), []\mathtt{F}_7 \vdash \Delta_9}\, \mathtt{Cut}}{\dfrac{}{\bullet \mathtt{h}_3 : \top, \Delta_{10}, []\mathtt{F}_7 \vdash \Delta_9, \mathtt{F}_6}\, \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_8 : \top, \Delta_{10}, \mathtt{F}_6, []\mathtt{F}_7 \vdash \Delta_9}\, \mathtt{ax/W}}{- : \top, \Delta_{10}, []\mathtt{F}_7 \vdash \Delta_9}\, \mathtt{hCut}$$

8.11 Status of $\bot_L$: OK

- Case rule $\to_R$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_6 \vdash (\Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}), \mathtt{F}_5}\, \bot_L \quad \dfrac{\mathtt{h}_7 : \bot, \mathtt{F}_5, \mathtt{F}_9, \Delta_6 \vdash \mathtt{F}_{10}, \Delta_8}{\bullet \mathtt{h}_7 : (\bot, \Delta_6), \mathtt{F}_5 \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\, \to_R}{- : \bot, \Delta_6 \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\, \mathtt{Cut}}{- : \bot, \Delta_6 \vdash \Delta_8, \mathtt{F}_9 \to \mathtt{F}_{10}}\, \bot_L$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_6 \vdash (\Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_5}\ \bot_L \quad \dfrac{\mathtt{h}_7 : \bot, \mathtt{F}_5, \Delta_6 \vdash \mathtt{F}_9, \Delta_8 \quad \mathtt{h}_7 : \bot, \mathtt{F}_5, \Delta_6 \vdash \mathtt{F}_{10}, \Delta_8}{\bullet \mathtt{h}_7 : (\bot, \Delta_6), \mathtt{F}_5 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\ \wedge_R}{- : \bot, \Delta_6 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_6 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\ \bot_L$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_6 \vdash (\Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_5}\ \bot_L \quad \dfrac{\mathtt{h}_7 : \bot, \mathtt{F}_5, \Delta_6 \vdash \mathtt{F}_9, \mathtt{F}_{10}, \Delta_8}{\bullet \mathtt{h}_7 : (\bot, \Delta_6), \mathtt{F}_5 \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \vee_R}{- : \bot, \Delta_6 \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_6 \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \bot_L$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_6 \vdash (\bot, \Delta_8), \mathtt{F}_5}\ \bot_L \quad \dfrac{\mathtt{h}_7 : \bot, \mathtt{F}_5, \Delta_6 \vdash \Delta_8}{\bullet \mathtt{h}_7 : (\bot, \Delta_6), \mathtt{F}_5 \vdash \bot, \Delta_8}\ \bot_R}{- : \bot, \Delta_6 \vdash \bot, \Delta_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_6 \vdash \bot, \Delta_8}\ \bot_L$$

- Case rule $\top_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_6 \vdash (\top, \Delta_8), \mathtt{F}_5}\ \bot_L \quad \dfrac{}{\bullet \mathtt{h}_7 : (\bot, \Delta_6), \mathtt{F}_5 \vdash \top, \Delta_8}\ \top_R}{- : \bot, \Delta_6 \vdash \top, \Delta_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_6 \vdash \top, \Delta_8}\ \top_R$$

- Case rule $A4$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Box\Gamma_9, \Delta_{10} \vdash (\Delta_7, []\mathtt{F}_8), \Box\mathtt{F}_5}\ \bot_L \quad \dfrac{\mathtt{h}_6 : \Box\Gamma_9, \Box\mathtt{F}_5 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : (\bot, \Box\Gamma_9, \Delta_{10}), \Box\mathtt{F}_5 \vdash \Delta_7, []\mathtt{F}_8}\ A4}{- : \bot, \Box\Gamma_9, \Delta_{10} \vdash \Delta_7, []\mathtt{F}_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_7, []\mathtt{F}_8}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Box\Gamma_7, \Delta_{10} \vdash (\Delta_8, []\mathtt{F}_9), \mathtt{F}_5}\ \bot_L \quad \dfrac{\mathtt{h}_6 : \Box\Gamma_7 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_6 : (\bot, \Box\Gamma_7, \Delta_{10}), \mathtt{F}_5 \vdash \Delta_8, []\mathtt{F}_9}\ A4}{- : \bot, \Box\Gamma_7, \Delta_{10} \vdash \Delta_8, []\mathtt{F}_9}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_{10}, \Box\Gamma_7 \vdash \Delta_8, []\mathtt{F}_9}\ \bot_L$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_9, \mathtt{F}_7 \rightarrow \mathtt{F}_8}\ \bot_L \quad \dfrac{\mathtt{h}_6 : \bot, \Delta_5 \vdash \mathtt{F}_7, \Delta_9 \quad \mathtt{h}_6 : \bot, \mathtt{F}_8, \Delta_5 \vdash \Delta_9}{\bullet \mathtt{h}_6 : (\bot, \Delta_5), \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_9}\ \rightarrow_L}{- : \bot, \Delta_5 \vdash \Delta_9}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_5 \vdash \Delta_9}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_{10}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_5}\ \bot_L \quad \dfrac{\mathtt{h}_6 : \bot, \mathtt{F}_5, \Delta_{10} \vdash \mathtt{F}_7, \Delta_9 \quad \mathtt{h}_6 : \bot, \mathtt{F}_5, \mathtt{F}_8, \Delta_{10} \vdash \Delta_9}{\bullet \mathtt{h}_6 : (\bot, \Delta_{10}, \mathtt{F}_7 \rightarrow \mathtt{F}_8), \mathtt{F}_5 \vdash \Delta_9}\ \rightarrow_L}{- : \bot, \Delta_{10}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_9}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_{10}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_9}\ \bot_L$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_9, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \bot_L \quad \dfrac{\mathtt{h}_6 : \bot, \mathtt{F}_7, \mathtt{F}_8, \Delta_5 \vdash \Delta_9}{\bullet \mathtt{h}_6 : (\bot, \Delta_5), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9}\ \wedge_L}{- : \bot, \Delta_5 \vdash \Delta_9}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_5 \vdash \Delta_9}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_5}\ \bot_L \quad \dfrac{\mathtt{h}_6 : \bot, \mathtt{F}_5, \mathtt{F}_7, \mathtt{F}_8, \Delta_{10} \vdash \Delta_9}{\bullet \mathtt{h}_6 : (\bot, \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_5 \vdash \Delta_9}\ \wedge_L}{- : \bot, \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9}\ \bot_L$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_9, \mathtt{F}_7 \vee \mathtt{F}_8}\ \bot_L \quad \dfrac{\mathtt{h}_6 : \bot, \mathtt{F}_7, \Delta_5 \vdash \Delta_9 \quad \mathtt{h}_6 : \bot, \mathtt{F}_8, \Delta_5 \vdash \Delta_9}{\bullet \mathtt{h}_6 : (\bot, \Delta_5), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9}\ \vee_L}{- : \bot, \Delta_5 \vdash \Delta_9}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_5 \vdash \Delta_9}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_5}\ \bot_L \quad \dfrac{\mathtt{h}_6 : \bot, \mathtt{F}_5, \mathtt{F}_7, \Delta_{10} \vdash \Delta_9 \quad \mathtt{h}_6 : \bot, \mathtt{F}_5, \mathtt{F}_8, \Delta_{10} \vdash \Delta_9}{\bullet \mathtt{h}_6 : (\bot, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_5 \vdash \Delta_9}\ \vee_L}{- : \bot, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9}\ \bot_L$$

- Case rule AT

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_8, []\mathtt{F}_7}\ \bot_L \quad \dfrac{\mathtt{h}_6 : \bot, \mathtt{F}_7, \Delta_5, []\mathtt{F}_7 \vdash \Delta_8}{\bullet \mathtt{h}_6 : (\bot, \Delta_5), []\mathtt{F}_7 \vdash \Delta_8}\ AT}{- : \bot, \Delta_5 \vdash \Delta_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_5 \vdash \Delta_8}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_9, []\mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_5}\ \bot_L \quad \dfrac{\mathtt{h}_6 : \bot, \mathtt{F}_5, \mathtt{F}_7, \Delta_9, []\mathtt{F}_7 \vdash \Delta_8}{\bullet \mathtt{h}_6 : (\bot, \Delta_9, []\mathtt{F}_7), \mathtt{F}_5 \vdash \Delta_8}\ AT}{- : \bot, \Delta_9, []\mathtt{F}_7 \vdash \Delta_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_9, []\mathtt{F}_7 \vdash \Delta_8}\ \bot_L$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_6 \vdash \Delta_8, \mathtt{F}_5}\ \bot_L \quad \dfrac{}{\bullet \mathtt{h}_7 : (\bot, \Delta_6), \mathtt{F}_5 \vdash \Delta_8}\ \bot_L}{- : \bot, \Delta_6 \vdash \Delta_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_6 \vdash \Delta_8}\ \bot_L$$

- Case rule I

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{p}_7}\ \bot_L \quad \dfrac{}{\bullet \mathtt{h}_6 : (\bot, \Delta_5), \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\ I}{- : \bot, \Delta_5 \vdash \Delta_8, \mathtt{p}_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_5 \vdash \Delta_8, \mathtt{p}_7}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_9, \mathtt{p}_7 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{F}_5}\ \bot_L \quad \dfrac{}{\bullet \mathtt{h}_6 : (\bot, \Delta_9, \mathtt{p}_7), \mathtt{F}_5 \vdash \Delta_8, \mathtt{p}_7}\ I}{- : \bot, \Delta_9, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_9, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\ \bot_L$$

- Case rule $\top_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \Delta_5 \vdash \Delta_7, \top}\ \bot_L \quad \dfrac{\mathtt{h}_6 : \bot, \Delta_5 \vdash \Delta_7}{\bullet \mathtt{h}_6 : (\bot, \Delta_5), \top \vdash \Delta_7}\ \top_L}{- : \bot, \Delta_5 \vdash \Delta_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \Delta_5 \vdash \Delta_7}\ \bot_L$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_3 : \bot, \top, \Delta_8 \vdash \Delta_7, \mathtt{F}_5}\ \bot_L \quad \dfrac{\mathtt{h}_6 : \bot, \mathtt{F}_5, \Delta_8 \vdash \Delta_7}{\bullet \mathtt{h}_6 : (\bot, \top, \Delta_8), \mathtt{F}_5 \vdash \Delta_7}\ \top_L}{- : \bot, \top, \Delta_8 \vdash \Delta_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \bot, \top, \Delta_8 \vdash \Delta_7}\ \bot_L$$

8.12 Status of I: OK

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{p}_6 \vdash (\Delta_8, \mathtt{F}_9 \rightarrow \mathtt{F}_{10}), \mathtt{p}_6}\ I \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_9, \Delta_5, \mathtt{p}_6, \mathtt{p}_6 \vdash \mathtt{F}_{10}, \Delta_8}{\bullet \mathtt{h}_7 : (\Delta_5, \mathtt{p}_6), \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9 \rightarrow \mathtt{F}_{10}}\ \rightarrow_R}{- : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9 \rightarrow \mathtt{F}_{10}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{F}_9, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{p}_6}\ I \quad \dfrac{}{\mathtt{h}_7 : \Delta_5, \mathtt{F}_9, \mathtt{p}_6, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_5, \mathtt{F}_9, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_{10}}\ \mathtt{hCut}$$
$$\dfrac{}{- : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9 \rightarrow \mathtt{F}_{10}}\ \rightarrow_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_7, \mathtt{p}_8 \vdash ((\Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}), \mathtt{p}_8), \mathtt{F}_6}\ I \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_6, \mathtt{F}_{10}, \Delta_7, \mathtt{p}_8 \vdash \mathtt{F}_{11}, \Delta_{12}, \mathtt{p}_8}{\bullet \mathtt{h}_9 : (\Delta_7, \mathtt{p}_8), \mathtt{F}_6 \vdash (\Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}), \mathtt{p}_8}\ \rightarrow_R}{- : \Delta_7, \mathtt{p}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}), \mathtt{p}_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_7, \mathtt{p}_8 \vdash \Delta_{12}, \mathtt{p}_8, \mathtt{F}_{10} \rightarrow \mathtt{F}_{11}}\ I$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{p}_6 \vdash (\Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{p}_6}\ I \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{p}_6, \mathtt{p}_6 \vdash \mathtt{F}_9, \Delta_8 \quad \mathtt{h}_7 : \Delta_5, \mathtt{p}_6, \mathtt{p}_6 \vdash \mathtt{F}_{10}, \Delta_8}{\bullet \mathtt{h}_7 : (\Delta_5, \mathtt{p}_6), \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\ \wedge_R}{- : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9, \mathtt{p}_6}\ I \quad \dfrac{}{\mathtt{h}_7 : \Delta_5, \mathtt{p}_6, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9}\ \mathtt{ax/W}}{- : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9}\ \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{p}_6}\ I \quad \dfrac{}{\mathtt{h}_7 : \Delta_5, \mathtt{p}_6, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_{10}}\ \mathtt{hCut}}{- : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\ \wedge_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_7, \mathtt{p}_8 \vdash ((\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{p}_8), \mathtt{F}_6}\ I \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_6, \Delta_7, \mathtt{p}_8 \vdash \mathtt{F}_{10}, \Delta_{12}, \mathtt{p}_8 \quad \mathtt{h}_9 : \mathtt{F}_6, \Delta_7, \mathtt{p}_8 \vdash \mathtt{F}_{11}, \Delta_{12}, \mathtt{p}_8}{\bullet \mathtt{h}_9 : (\Delta_7, \mathtt{p}_8), \mathtt{F}_6 \vdash (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{p}_8}\ \wedge_R}{- : \Delta_7, \mathtt{p}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{p}_8}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_7, \mathtt{p}_8 \vdash \Delta_{12}, \mathtt{p}_8, \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\ I$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{p}_6 \vdash (\Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{p}_6}\ I \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{p}_6, \mathtt{p}_6 \vdash \mathtt{F}_9, \mathtt{F}_{10}, \Delta_8}{\bullet \mathtt{h}_7 : (\Delta_5, \mathtt{p}_6), \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \vee_R}{- : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_9, \mathtt{p}_6}\ I \quad \dfrac{}{\mathtt{h}_7 : \Delta_5, \mathtt{p}_6, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_9}\ \mathtt{ax/W}}{- : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_{10}, \mathtt{F}_9}\ \mathtt{hCut}$$
$$\dfrac{}{- : \Delta_5, \mathtt{p}_6 \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \vee_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_7, \mathtt{p}_8 \vdash ((\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{p}_8), \mathtt{F}_6}\, I \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_6, \Delta_7, \mathtt{p}_8 \vdash \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_{12}, \mathtt{p}_8}{\bullet \mathtt{h}_9 : (\Delta_7, \mathtt{p}_8), \mathtt{F}_6 \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{p}_8}\, \vee_R}{- : \Delta_7, \mathtt{p}_8 \vdash (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{p}_8}\, \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_7, \mathtt{p}_8 \vdash \Delta_{12}, \mathtt{p}_8, \mathtt{F}_{10} \vee \mathtt{F}_{11}}\, I$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{p}_6 \vdash (\bot, \Delta_8), \mathtt{p}_6}\, I \quad \dfrac{\mathtt{h}_7 : \Delta_5, \mathtt{p}_6, \mathtt{p}_6 \vdash \Delta_8}{\bullet \mathtt{h}_7 : (\Delta_5, \mathtt{p}_6), \mathtt{p}_6 \vdash \bot, \Delta_8}\, \bot_R}{- : \Delta_5, \mathtt{p}_6 \vdash \bot, \Delta_8}\, \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{p}_6 \vdash \bot, \Delta_8, \mathtt{p}_6}\, \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_7 : \Delta_5, \mathtt{p}_6, \mathtt{p}_6 \vdash \bot, \Delta_8}\, \mathtt{ax/W}}{- : \Delta_5, \mathtt{p}_6 \vdash \bot, \Delta_8}\, \mathtt{hCut}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_7, \mathtt{p}_8 \vdash ((\bot, \Delta_{10}), \mathtt{p}_8), \mathtt{F}_6}\, I \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_6, \Delta_7, \mathtt{p}_8 \vdash \Delta_{10}, \mathtt{p}_8}{\bullet \mathtt{h}_9 : (\Delta_7, \mathtt{p}_8), \mathtt{F}_6 \vdash (\bot, \Delta_{10}), \mathtt{p}_8}\, \bot_R}{- : \Delta_7, \mathtt{p}_8 \vdash (\bot, \Delta_{10}), \mathtt{p}_8}\, \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_7, \mathtt{p}_8 \vdash \bot, \Delta_{10}, \mathtt{p}_8}\, I$$

- Case rule $\top_R$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{p}_6 \vdash (\top, \Delta_8), \mathtt{p}_6}\, I \quad \dfrac{}{\bullet \mathtt{h}_7 : (\Delta_5, \mathtt{p}_6), \mathtt{p}_6 \vdash \top, \Delta_8}\, \top_R}{- : \Delta_5, \mathtt{p}_6 \vdash \top, \Delta_8}\, \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_5, \mathtt{p}_6 \vdash \top, \Delta_8}\, \top_R$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_7, \mathtt{p}_8 \vdash ((\top, \Delta_{10}), \mathtt{p}_8), \mathtt{F}_6}\, I \quad \dfrac{}{\bullet \mathtt{h}_9 : (\Delta_7, \mathtt{p}_8), \mathtt{F}_6 \vdash (\top, \Delta_{10}), \mathtt{p}_8}\, \top_R}{- : \Delta_7, \mathtt{p}_8 \vdash (\top, \Delta_{10}), \mathtt{p}_8}\, \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_7, \mathtt{p}_8 \vdash \top, \Delta_{10}, \mathtt{p}_8}\, \top_R$$

- Case rule $A4$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Box \Gamma_7, \Delta_{10}), \mathtt{p}_5 \vdash (\Delta_8, []\mathtt{F}_9), \mathtt{p}_5}\, I \quad \dfrac{\mathtt{h}_6 : \Box \Gamma_7 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_6 : ((\Box \Gamma_7, \Delta_{10}), \mathtt{p}_5), \mathtt{p}_5 \vdash \Delta_8, []\mathtt{F}_9}\, A4}{- : (\Box \Gamma_7, \Delta_{10}), \mathtt{p}_5 \vdash \Delta_8, []\mathtt{F}_9}\, \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{- : \Box \Gamma_7 \vdash \mathtt{F}_9}\, \mathtt{ax/W}}{- : \Delta_{10}, \Box \Gamma_7, \mathtt{p}_5 \vdash \Delta_8, []\mathtt{F}_9}\, A4$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : (\Box \Gamma_{11}, \Delta_{12}), \mathtt{p}_7 \vdash ((\Delta_{10}, []\mathtt{F}_9), \mathtt{p}_7), \Box \mathtt{F}_6}\, I \quad \dfrac{\mathtt{h}_8 : \Box \Gamma_{11}, \Box \mathtt{F}_6 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_8 : ((\Box \Gamma_{11}, \Delta_{12}), \mathtt{p}_7), \Box \mathtt{F}_6 \vdash (\Delta_{10}, []\mathtt{F}_9), \mathtt{p}_7}\, A4}{- : (\Box \Gamma_{11}, \Delta_{12}), \mathtt{p}_7 \vdash (\Delta_{10}, []\mathtt{F}_9), \mathtt{p}_7}\, \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_{12}, \Box \Gamma_{11}, \mathtt{p}_7 \vdash \Delta_{10}, \mathtt{p}_7, []\mathtt{F}_9}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : (\Box \Gamma_9, \Delta_{12}), \mathtt{p}_7 \vdash ((\Delta_{11}, []\mathtt{F}_{10}), \mathtt{p}_7), \mathtt{F}_6}\, I \quad \dfrac{\mathtt{h}_8 : \Box \Gamma_9 \vdash \mathtt{F}_{10}}{\bullet \mathtt{h}_8 : ((\Box \Gamma_9, \Delta_{12}), \mathtt{p}_7), \mathtt{F}_6 \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{p}_7}\, A4}{- : (\Box \Gamma_9, \Delta_{12}), \mathtt{p}_7 \vdash (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{p}_7}\, \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_{12}, \Box \Gamma_9, \mathtt{p}_7 \vdash \Delta_{11}, \mathtt{p}_7, []\mathtt{F}_{10}}\, I$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{p}_5 \vdash \Delta_9, \mathtt{p}_5}\, I \quad \dfrac{\mathtt{h}_6 : \Delta_{10}, \mathtt{p}_5, \mathtt{p}_5 \vdash \mathtt{F}_7, \Delta_9 \quad \mathtt{h}_6 : \mathtt{F}_8, \Delta_{10}, \mathtt{p}_5, \mathtt{p}_5 \vdash \Delta_9}{\bullet \mathtt{h}_6 : ((\Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{p}_5), \mathtt{p}_5 \vdash \Delta_9}\, \to_L}{- : (\Delta_{10}, \mathtt{F}_7 \to \mathtt{F}_8), \mathtt{p}_5 \vdash \Delta_9}\, \mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10}, \mathtt{p}_5 \vdash \Delta_9, \mathtt{F}_7, \mathtt{p}_5}\, I \quad \dfrac{}{\mathtt{h}_6 : \Delta_{10}, \mathtt{p}_5, \mathtt{p}_5 \vdash \Delta_9, \mathtt{F}_7}\, \mathtt{ax/W}}{- : \Delta_{10}, \mathtt{p}_5 \vdash \Delta_9, \mathtt{F}_7}\, \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10}, \mathtt{F}_8, \mathtt{p}_5 \vdash \Delta_9, \mathtt{p}_5}\, I \quad \dfrac{}{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_8, \mathtt{p}_5, \mathtt{p}_5 \vdash \Delta_9}\, \mathtt{ax/W}}{- : \Delta_{10}, \mathtt{F}_8, \mathtt{p}_5 \vdash \Delta_9}\, \mathtt{hCut}}{- : \Delta_{10}, \mathtt{p}_5, \mathtt{F}_7 \to \mathtt{F}_8 \vdash \Delta_9}\, \to_L$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{p}_7 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{F}_{10} \to \mathtt{F}_{11}}\, I \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{p}_7 \vdash \mathtt{F}_{10}, \Delta_8, \mathtt{p}_7 \quad \mathtt{h}_9 : \mathtt{F}_{11}, \Delta_6, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}{\bullet \mathtt{h}_9 : (\Delta_6, \mathtt{p}_7), \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_8, \mathtt{p}_7}\, \to_L}{- : \Delta_6, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, \mathtt{Cut}}{- : \Delta_6, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, I$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : (\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{p}_7 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{F}_6}\, I \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_6, \Delta_{12}, \mathtt{p}_7 \vdash \mathtt{F}_{10}, \Delta_8, \mathtt{p}_7 \quad \mathtt{h}_9 : \mathtt{F}_6, \mathtt{F}_{11}, \Delta_{12}, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}{\bullet \mathtt{h}_9 : ((\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{p}_7), \mathtt{F}_6 \vdash \Delta_8, \mathtt{p}_7}\, \to_L}{- : (\Delta_{12}, \mathtt{F}_{10} \to \mathtt{F}_{11}), \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, \mathtt{Cut}}{- : \Delta_{12}, \mathtt{p}_7, \mathtt{F}_{10} \to \mathtt{F}_{11} \vdash \Delta_8, \mathtt{p}_7}\, I$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_5 \vdash \Delta_9, \mathtt{p}_5}\, I \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_7, \mathtt{F}_8, \Delta_{10}, \mathtt{p}_5, \mathtt{p}_5 \vdash \Delta_9}{\bullet \mathtt{h}_6 : ((\Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_5), \mathtt{p}_5 \vdash \Delta_9}\, \wedge_L}{- : (\Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{p}_5 \vdash \Delta_9}\, \mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_5 \vdash \Delta_9, \mathtt{p}_5}\, I \quad \dfrac{}{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_5, \mathtt{p}_5 \vdash \Delta_9}\, \mathtt{ax/W}}{- : \Delta_{10}, \mathtt{F}_7, \mathtt{F}_8, \mathtt{p}_5 \vdash \Delta_9}\, \mathtt{hCut}}{- : \Delta_{10}, \mathtt{p}_5, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9}\, \wedge_L$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{p}_7 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{F}_{10} \wedge \mathtt{F}_{11}}\, I \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_6, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}{\bullet \mathtt{h}_9 : (\Delta_6, \mathtt{p}_7), \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_8, \mathtt{p}_7}\, \wedge_L}{- : \Delta_6, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, \mathtt{Cut}}{- : \Delta_6, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, I$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{p}_7 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{F}_6}\, I \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_6, \mathtt{F}_{10}, \mathtt{F}_{11}, \Delta_{12}, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}{\bullet \mathtt{h}_9 : ((\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{p}_7), \mathtt{F}_6 \vdash \Delta_8, \mathtt{p}_7}\, \wedge_L}{- : (\Delta_{12}, \mathtt{F}_{10} \wedge \mathtt{F}_{11}), \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, \mathtt{Cut}}{- : \Delta_{12}, \mathtt{p}_7, \mathtt{F}_{10} \wedge \mathtt{F}_{11} \vdash \Delta_8, \mathtt{p}_7}\, I$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_5 \vdash \Delta_9, \mathtt{p}_5}\, I \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_7, \Delta_{10}, \mathtt{p}_5, \mathtt{p}_5 \vdash \Delta_9 \quad \mathtt{h}_6 : \mathtt{F}_8, \Delta_{10}, \mathtt{p}_5, \mathtt{p}_5 \vdash \Delta_9}{\bullet \mathtt{h}_6 : ((\Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_5), \mathtt{p}_5 \vdash \Delta_9}\, \vee_L}{- : (\Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{p}_5 \vdash \Delta_9}\, \mathtt{Cut}$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10}, \mathtt{F}_7, \mathtt{p}_5 \vdash \Delta_9, \mathtt{p}_5}\, I \quad \dfrac{}{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_7, \mathtt{p}_5, \mathtt{p}_5 \vdash \Delta_9}\, \mathtt{ax/W}}{- : \Delta_{10}, \mathtt{F}_7, \mathtt{p}_5 \vdash \Delta_9}\, \mathtt{hCut} \quad \dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_{10}, \mathtt{F}_8, \mathtt{p}_5 \vdash \Delta_9, \mathtt{p}_5}\, I \quad \dfrac{}{\mathtt{h}_6 : \Delta_{10}, \mathtt{F}_8, \mathtt{p}_5, \mathtt{p}_5 \vdash \Delta_9}\, \mathtt{ax/W}}{- : \Delta_{10}, \mathtt{F}_8, \mathtt{p}_5 \vdash \Delta_9}\, \mathtt{hCut}}{- : \Delta_{10}, \mathtt{p}_5, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9}\, \vee_L$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{p}_7 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{F}_{10} \vee \mathtt{F}_{11}}\, I \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \Delta_6, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7 \quad \mathtt{h}_9 : \mathtt{F}_{11}, \Delta_6, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}{\bullet \mathtt{h}_9 : (\Delta_6, \mathtt{p}_7), \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_8, \mathtt{p}_7}\, \vee_L}{- : \Delta_6, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, \mathtt{Cut}}{- : \Delta_6, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, I$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{p}_7 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{F}_6}\, I \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_6, \mathtt{F}_{10}, \Delta_{12}, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7 \quad \mathtt{h}_9 : \mathtt{F}_6, \mathtt{F}_{11}, \Delta_{12}, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}{\bullet \mathtt{h}_9 : ((\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{p}_7), \mathtt{F}_6 \vdash \Delta_8, \mathtt{p}_7}\, \vee_L}{- : (\Delta_{12}, \mathtt{F}_{10} \vee \mathtt{F}_{11}), \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, \mathtt{Cut}}{- : \Delta_{12}, \mathtt{p}_7, \mathtt{F}_{10} \vee \mathtt{F}_{11} \vdash \Delta_8, \mathtt{p}_7}\, I$$

- Case rule AT

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_9, []\mathtt{F}_7), \mathtt{p}_5 \vdash \Delta_8, \mathtt{p}_5}\, I \quad \dfrac{\mathtt{h}_6 : \mathtt{F}_7, \Delta_9, \mathtt{p}_5, \mathtt{p}_5, []\mathtt{F}_7 \vdash \Delta_8}{\bullet \mathtt{h}_6 : ((\Delta_9, []\mathtt{F}_7), \mathtt{p}_5), \mathtt{p}_5 \vdash \Delta_8}\, AT}{- : (\Delta_9, []\mathtt{F}_7), \mathtt{p}_5 \vdash \Delta_8}\, \mathtt{Cut}}{\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_9, \mathtt{F}_7, \mathtt{p}_5, []\mathtt{F}_7 \vdash \Delta_8, \mathtt{p}_5}\, I \quad \mathtt{h}_6 : \Delta_9, \mathtt{F}_7, \mathtt{p}_5, \mathtt{p}_5, []\mathtt{F}_7 \vdash \Delta_8}{- : \Delta_9, \mathtt{F}_7, \mathtt{p}_5, []\mathtt{F}_7 \vdash \Delta_8}\, \mathtt{hCut}}{- : \Delta_9, \mathtt{p}_5, []\mathtt{F}_7 \vdash \Delta_8}\, AT}{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{p}_7 \vdash (\Delta_8, \mathtt{p}_7), []\mathtt{F}_{10}}\, I \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_{10}, \Delta_6, \mathtt{p}_7, []\mathtt{F}_{10} \vdash \Delta_8, \mathtt{p}_7}{\bullet \mathtt{h}_9 : (\Delta_6, \mathtt{p}_7), []\mathtt{F}_{10} \vdash \Delta_8, \mathtt{p}_7}\, AT}{- : \Delta_6, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, \mathtt{Cut}}{- : \Delta_6, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, I$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{p}_7 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{F}_6}\, I \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_6, \mathtt{F}_{10}, \Delta_{11}, \mathtt{p}_7, []\mathtt{F}_{10} \vdash \Delta_8, \mathtt{p}_7}{\bullet \mathtt{h}_9 : ((\Delta_{11}, []\mathtt{F}_{10}), \mathtt{p}_7), \mathtt{F}_6 \vdash \Delta_8, \mathtt{p}_7}\, AT}{- : (\Delta_{11}, []\mathtt{F}_{10}), \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, \mathtt{Cut}}{- : \Delta_{11}, \mathtt{p}_7, []\mathtt{F}_{10} \vdash \Delta_8, \mathtt{p}_7}\, I$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\bot, \Delta_8), \mathtt{p}_5 \vdash \Delta_7, \mathtt{p}_5}\, I \quad \dfrac{}{\bullet \mathtt{h}_6 : ((\bot, \Delta_8), \mathtt{p}_5) \vdash \Delta_7}\, \bot_L}{- : (\bot, \Delta_8), \mathtt{p}_5 \vdash \Delta_7}\, \mathtt{Cut}}{- : \bot, \Delta_8, \mathtt{p}_5 \vdash \Delta_7}\, \bot_L$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{p}_7 \vdash (\Delta_8, \mathtt{p}_7), \bot}\, I \quad \dfrac{}{\bullet \mathtt{h}_9 : (\Delta_6, \mathtt{p}_7), \bot \vdash \Delta_8, \mathtt{p}_7}\, \bot_L}{- : \Delta_6, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, \mathtt{Cut}}{- : \Delta_6, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, I$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : (\bot, \Delta_{10}), \mathtt{p}_7 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{F}_6}\, I \quad \dfrac{}{\bullet \mathtt{h}_9 : ((\bot, \Delta_{10}), \mathtt{p}_7), \mathtt{F}_6 \vdash \Delta_8, \mathtt{p}_7}\, \bot_L}{- : (\bot, \Delta_{10}), \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, \mathtt{Cut}}{- : \bot, \Delta_{10}, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, \bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \Delta_5, \mathtt{p}_7 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{p}_7}\, I \quad \dfrac{}{\bullet \mathtt{h}_6 : (\Delta_5, \mathtt{p}_7), \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, I}{- : \Delta_5, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, \mathtt{Cut}}{- : \Delta_5, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, I$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\Delta_9, \mathtt{p}_7), \mathtt{p}_5 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{p}_5}\, I \quad \dfrac{}{\bullet \mathtt{h}_6 : ((\Delta_9, \mathtt{p}_7), \mathtt{p}_5), \mathtt{p}_5 \vdash \Delta_8, \mathtt{p}_7}\, I}{- : (\Delta_9, \mathtt{p}_7), \mathtt{p}_5 \vdash \Delta_8, \mathtt{p}_7}\, \mathtt{Cut}}{- : \Delta_9, \mathtt{p}_5, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\, I$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{p}_7 \vdash ((\Delta_{10}, \mathtt{p}_9), \mathtt{p}_7), \mathtt{p}_9}\, I \quad \dfrac{}{\bullet \mathtt{h}_8 : (\Delta_6, \mathtt{p}_7), \mathtt{p}_9 \vdash (\Delta_{10}, \mathtt{p}_9), \mathtt{p}_7}\, I}{- : \Delta_6, \mathtt{p}_7 \vdash (\Delta_{10}, \mathtt{p}_9), \mathtt{p}_7}\, \mathtt{Cut}}{- : \Delta_6, \mathtt{p}_7 \vdash \Delta_{10}, \mathtt{p}_7, \mathtt{p}_9}\, I$$

$$\dfrac{\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : (\Delta_{11}, \mathtt{p}_9), \mathtt{p}_7 \vdash ((\Delta_{10}, \mathtt{p}_9), \mathtt{p}_7), \mathtt{F}_6}\, I \quad \dfrac{}{\bullet \mathtt{h}_8 : ((\Delta_{11}, \mathtt{p}_9), \mathtt{p}_7), \mathtt{F}_6 \vdash (\Delta_{10}, \mathtt{p}_9), \mathtt{p}_7}\, I}{- : (\Delta_{11}, \mathtt{p}_9), \mathtt{p}_7 \vdash (\Delta_{10}, \mathtt{p}_9), \mathtt{p}_7}\, \mathtt{Cut}}{- : \Delta_{11}, \mathtt{p}_7, \mathtt{p}_9 \vdash \Delta_{10}, \mathtt{p}_7, \mathtt{p}_9}\, I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_7, \mathtt{p}_9 \vdash (\Delta_{10}, \mathtt{p}_9), \mathtt{F}_6}\ I \quad \dfrac{}{\bullet \mathtt{h}_8 : (\Delta_7, \mathtt{p}_9), \mathtt{F}_6 \vdash \Delta_{10}, \mathtt{p}_9}\ I}{- : \Delta_7, \mathtt{p}_9 \vdash \Delta_{10}, \mathtt{p}_9}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_7, \mathtt{p}_9 \vdash \Delta_{10}, \mathtt{p}_9}\ I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : (\top, \Delta_8), \mathtt{p}_5 \vdash \Delta_7, \mathtt{p}_5}\ I \quad \dfrac{\mathtt{h}_6 : \Delta_8, \mathtt{p}_5, \mathtt{p}_5 \vdash \Delta_7}{\bullet \mathtt{h}_6 : ((\top, \Delta_8), \mathtt{p}_5), \mathtt{p}_5 \vdash \Delta_7}\ \top_L}{- : (\top, \Delta_8), \mathtt{p}_5 \vdash \Delta_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{\bullet \mathtt{h}_1 : \top, \Delta_8, \mathtt{p}_5 \vdash \Delta_7, \mathtt{p}_5}\ \mathtt{ax/W} \quad \dfrac{}{\mathtt{h}_6 : \top, \Delta_8, \mathtt{p}_5, \mathtt{p}_5 \vdash \Delta_7}\ \mathtt{ax/W}}{- : \top, \Delta_8, \mathtt{p}_5 \vdash \Delta_7}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : \Delta_6, \mathtt{p}_7 \vdash (\Delta_8, \mathtt{p}_7), \top}\ I \quad \dfrac{\mathtt{h}_9 : \Delta_6, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}{\bullet \mathtt{h}_9 : (\Delta_6, \mathtt{p}_7), \top \vdash \Delta_8, \mathtt{p}_7}\ \top_L}{- : \Delta_6, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \Delta_6, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\ I$$

$$\dfrac{\dfrac{}{\bullet \mathtt{h}_2 : (\top, \Delta_{10}), \mathtt{p}_7 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{F}_6}\ I \quad \dfrac{\mathtt{h}_9 : \mathtt{F}_6, \Delta_{10}, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}{\bullet \mathtt{h}_9 : ((\top, \Delta_{10}), \mathtt{p}_7), \mathtt{F}_6 \vdash \Delta_8, \mathtt{p}_7}\ \top_L}{- : (\top, \Delta_{10}), \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{}{- : \top, \Delta_{10}, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\ I$$

8.13 Status of $\top_L$: OK

- Case rule $\rightarrow_R$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_6 \vdash \mathtt{F}_5, \Delta_8, \mathtt{F}_9 \rightarrow \mathtt{F}_{10}}{\bullet \mathtt{h}_3 : \top, \Delta_6 \vdash (\Delta_8, \mathtt{F}_9 \rightarrow \mathtt{F}_{10}), \mathtt{F}_5}\ \top_L \quad \dfrac{\mathtt{h}_7 : \top, \mathtt{F}_5, \mathtt{F}_9, \Delta_6 \vdash \mathtt{F}_{10}, \Delta_8}{\bullet \mathtt{h}_7 : (\top, \Delta_6), \mathtt{F}_5 \vdash \Delta_8, \mathtt{F}_9 \rightarrow \mathtt{F}_{10}}\ \rightarrow_R}{- : \top, \Delta_6 \vdash \Delta_8, \mathtt{F}_9 \rightarrow \mathtt{F}_{10}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{\mathtt{h}_3 : \top, \Delta_6 \vdash \Delta_8, \mathtt{F}_5, \mathtt{F}_9 \rightarrow \mathtt{F}_{10}}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_7 : \top, \Delta_6, \mathtt{F}_5 \vdash \Delta_8, \mathtt{F}_9 \rightarrow \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \top, \Delta_6 \vdash \Delta_8, \mathtt{F}_9 \rightarrow \mathtt{F}_{10}}\ \mathtt{hCut}$$

- Case rule $\wedge_R$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_6 \vdash \mathtt{F}_5, \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}{\bullet \mathtt{h}_3 : \top, \Delta_6 \vdash (\Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}), \mathtt{F}_5}\ \top_L \quad \dfrac{\mathtt{h}_7 : \top, \mathtt{F}_5, \Delta_6 \vdash \mathtt{F}_9, \Delta_8 \quad \mathtt{h}_7 : \top, \mathtt{F}_5, \Delta_6 \vdash \mathtt{F}_{10}, \Delta_8}{\bullet \mathtt{h}_7 : (\top, \Delta_6), \mathtt{F}_5 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\ \wedge_R}{- : \top, \Delta_6 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{\mathtt{h}_3 : \top, \Delta_6 \vdash \Delta_8, \mathtt{F}_5, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_7 : \top, \Delta_6, \mathtt{F}_5 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \top, \Delta_6 \vdash \Delta_8, \mathtt{F}_9 \wedge \mathtt{F}_{10}}\ \mathtt{hCut}$$

- Case rule $\vee_R$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_6 \vdash \mathtt{F}_5, \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}{\bullet \mathtt{h}_3 : \top, \Delta_6 \vdash (\Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}), \mathtt{F}_5}\ \top_L \quad \dfrac{\mathtt{h}_7 : \top, \mathtt{F}_5, \Delta_6 \vdash \mathtt{F}_9, \mathtt{F}_{10}, \Delta_8}{\bullet \mathtt{h}_7 : (\top, \Delta_6), \mathtt{F}_5 \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \vee_R}{- : \top, \Delta_6 \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \mathtt{Cut}$$
$$\rightarrow$$
$$\dfrac{\dfrac{}{\mathtt{h}_3 : \top, \Delta_6 \vdash \Delta_8, \mathtt{F}_5, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_7 : \top, \Delta_6, \mathtt{F}_5 \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \mathtt{ax/W}}{- : \top, \Delta_6 \vdash \Delta_8, \mathtt{F}_9 \vee \mathtt{F}_{10}}\ \mathtt{hCut}$$

- Case rule $\bot_R$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_6 \vdash \mathtt{F}_5, \bot, \Delta_8}{\bullet \mathtt{h}_3 : \top, \Delta_6 \vdash (\bot, \Delta_8), \mathtt{F}_5}\ \top_L \quad \dfrac{\mathtt{h}_7 : \top, \mathtt{F}_5, \Delta_6 \vdash \Delta_8}{\bullet \mathtt{h}_7 : (\top, \Delta_6), \mathtt{F}_5 \vdash \bot, \Delta_8}\ \bot_R}{- : \top, \Delta_6 \vdash \bot, \Delta_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_3 : \top, \Delta_6 \vdash \bot, \Delta_8, \mathtt{F}_5}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_7 : \top, \Delta_6, \mathtt{F}_5 \vdash \bot, \Delta_8}\ \mathtt{ax/W}}{- : \top, \Delta_6 \vdash \bot, \Delta_8}\ \mathtt{hCut}$$

- Case rule $\top_R$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_6 \vdash \mathtt{F}_5, \top, \Delta_8}{\bullet \mathtt{h}_3 : \top, \Delta_6 \vdash (\top, \Delta_8), \mathtt{F}_5}\ \top_L \quad \dfrac{}{\bullet \mathtt{h}_7 : (\top, \Delta_6), \mathtt{F}_5 \vdash \top, \Delta_8}\ \top_R}{- : \top, \Delta_6 \vdash \top, \Delta_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{}{- : \top, \Delta_6 \vdash \top, \Delta_8}\ \top_R$$

- Case rule $A4$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Box\Gamma_9, \Delta_{10} \vdash \Box\mathtt{F}_5, \Delta_7, []\mathtt{F}_8}{\bullet \mathtt{h}_3 : \top, \Box\Gamma_9, \Delta_{10} \vdash (\Delta_7, []\mathtt{F}_8), \Box\mathtt{F}_5}\ \top_L \quad \dfrac{\mathtt{h}_6 : \Box\Gamma_9, \Box\mathtt{F}_5 \vdash \mathtt{F}_8}{\bullet \mathtt{h}_6 : (\top, \Box\Gamma_9, \Delta_{10}), \Box\mathtt{F}_5 \vdash \Delta_7, []\mathtt{F}_8}\ A4}{- : \top, \Box\Gamma_9, \Delta_{10} \vdash \Delta_7, []\mathtt{F}_8}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_3 : \top, \Delta_{10}, \Box\Gamma_9 \vdash \Box\mathtt{F}_5, \Delta_7, []\mathtt{F}_8}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \top, \Box\mathtt{F}_5, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_7, []\mathtt{F}_8}\ \mathtt{ax/W}}{- : \top, \Delta_{10}, \Box\Gamma_9 \vdash \Delta_7, []\mathtt{F}_8}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Box\Gamma_7, \Delta_{10} \vdash \mathtt{F}_5, \Delta_8, []\mathtt{F}_9}{\bullet \mathtt{h}_3 : \top, \Box\Gamma_7, \Delta_{10} \vdash (\Delta_8, []\mathtt{F}_9), \mathtt{F}_5}\ \top_L \quad \dfrac{\mathtt{h}_6 : \Box\Gamma_7 \vdash \mathtt{F}_9}{\bullet \mathtt{h}_6 : (\top, \Box\Gamma_7, \Delta_{10}), \mathtt{F}_5 \vdash \Delta_8, []\mathtt{F}_9}\ A4}{- : \top, \Box\Gamma_7, \Delta_{10} \vdash \Delta_8, []\mathtt{F}_9}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{- : \Box\Gamma_7 \vdash \mathtt{F}_9}\ \mathtt{ax/W}}{- : \top, \Delta_{10}, \Box\Gamma_7 \vdash \Delta_8, []\mathtt{F}_9}\ A4$$

- Case rule $\rightarrow_L$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_5 \vdash \mathtt{F}_7 \rightarrow \mathtt{F}_8, \Delta_9}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_9, \mathtt{F}_7 \rightarrow \mathtt{F}_8}\ \top_L \quad \dfrac{\mathtt{h}_6 : \top, \Delta_5 \vdash \mathtt{F}_7, \Delta_9 \quad \mathtt{h}_6 : \top, \mathtt{F}_8, \Delta_5 \vdash \Delta_9}{\bullet \mathtt{h}_6 : (\top, \Delta_5), \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_9}\ \rightarrow_L}{- : \top, \Delta_5 \vdash \Delta_9}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_9, \mathtt{F}_7 \rightarrow \mathtt{F}_8}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \top, \Delta_5, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_9}\ \mathtt{ax/W}}{- : \top, \Delta_5 \vdash \Delta_9}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_{10}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \mathtt{F}_5, \Delta_9}{\bullet \mathtt{h}_3 : \top, \Delta_{10}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_5}\ \top_L \quad \dfrac{\mathtt{h}_6 : \top, \mathtt{F}_5, \Delta_{10} \vdash \mathtt{F}_7, \Delta_9 \quad \mathtt{h}_6 : \top, \mathtt{F}_5, \mathtt{F}_8, \Delta_{10} \vdash \Delta_9}{\bullet \mathtt{h}_6 : (\top, \Delta_{10}, \mathtt{F}_7 \rightarrow \mathtt{F}_8), \mathtt{F}_5 \vdash \Delta_9}\ \rightarrow_L}{- : \top, \Delta_{10}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_9}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_3 : \top, \Delta_{10}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_5}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \top, \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_9}\ \mathtt{ax/W}}{- : \top, \Delta_{10}, \mathtt{F}_7 \rightarrow \mathtt{F}_8 \vdash \Delta_9}\ \mathtt{hCut}$$

- Case rule $\wedge_L$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_5 \vdash \mathtt{F}_7 \wedge \mathtt{F}_8, \Delta_9}{\bullet \mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_9, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \top_L \quad \dfrac{\mathtt{h}_6 : \top, \mathtt{F}_7, \mathtt{F}_8, \Delta_5 \vdash \Delta_9}{\bullet \mathtt{h}_6 : (\top, \Delta_5), \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9}\ \wedge_L}{- : \top, \Delta_5 \vdash \Delta_9}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_9, \mathtt{F}_7 \wedge \mathtt{F}_8}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \top, \Delta_5, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9}\ \mathtt{ax/W}}{- : \top, \Delta_5 \vdash \Delta_9}\ \mathtt{hCut}$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \mathtt{F}_5, \Delta_9}{\bullet \mathtt{h}_3 : \top, \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_5}\ \top_L \quad \dfrac{\mathtt{h}_6 : \top, \mathtt{F}_5, \mathtt{F}_7, \mathtt{F}_8, \Delta_{10} \vdash \Delta_9}{\bullet \mathtt{h}_6 : (\top, \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8), \mathtt{F}_5 \vdash \Delta_9}\ \wedge_L}{- : \top, \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9}\ \mathtt{Cut}$$

$$\rightarrow$$

$$\dfrac{\dfrac{}{\mathtt{h}_3 : \top, \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_5}\ \mathtt{ax/W} \quad \dfrac{}{\bullet \mathtt{h}_6 : \top, \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9}\ \mathtt{ax/W}}{- : \top, \Delta_{10}, \mathtt{F}_7 \wedge \mathtt{F}_8 \vdash \Delta_9}\ \mathtt{hCut}$$

- Case rule $\vee_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_5 \vdash \mathtt{F}_7 \vee \mathtt{F}_8, \Delta_9}{\bullet\mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_9, \mathtt{F}_7 \vee \mathtt{F}_8}\,\top_L \quad \dfrac{\mathtt{h}_6 : \top, \mathtt{F}_7, \Delta_5 \vdash \Delta_9 \quad \mathtt{h}_6 : \top, \mathtt{F}_8, \Delta_5 \vdash \Delta_9}{\bullet\mathtt{h}_6 : (\top, \Delta_5), \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9}\,\vee_L}{- : \top, \Delta_5 \vdash \Delta_9}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_9, \mathtt{F}_7 \vee \mathtt{F}_8}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_6 : \top, \Delta_5, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9}{}\,\mathtt{ax/W}}{- : \top, \Delta_5 \vdash \Delta_9}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \mathtt{F}_5, \Delta_9}{\bullet\mathtt{h}_3 : \top, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_5}\,\top_L \quad \dfrac{\mathtt{h}_6 : \top, \mathtt{F}_5, \mathtt{F}_7, \Delta_{10} \vdash \Delta_9 \quad \mathtt{h}_6 : \top, \mathtt{F}_5, \mathtt{F}_8, \Delta_{10} \vdash \Delta_9}{\bullet\mathtt{h}_6 : (\top, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8), \mathtt{F}_5 \vdash \Delta_9}\,\vee_L}{- : \top, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \top, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9, \mathtt{F}_5}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_6 : \top, \Delta_{10}, \mathtt{F}_5, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9}{}\,\mathtt{ax/W}}{- : \top, \Delta_{10}, \mathtt{F}_7 \vee \mathtt{F}_8 \vdash \Delta_9}\,\mathtt{hCut}$$

- Case rule AT

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_5 \vdash []\mathtt{F}_7, \Delta_8}{\bullet\mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_8, []\mathtt{F}_7}\,\top_L \quad \dfrac{\mathtt{h}_6 : \top, \mathtt{F}_7, \Delta_5, []\mathtt{F}_7 \vdash \Delta_8}{\bullet\mathtt{h}_6 : (\top, \Delta_5), []\mathtt{F}_7 \vdash \Delta_8}\,AT}{- : \top, \Delta_5 \vdash \Delta_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_8, []\mathtt{F}_7}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_6 : \top, \Delta_5, []\mathtt{F}_7 \vdash \Delta_8}{}\,\mathtt{ax/W}}{- : \top, \Delta_5 \vdash \Delta_8}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_9, []\mathtt{F}_7 \vdash \mathtt{F}_5, \Delta_8}{\bullet\mathtt{h}_3 : \top, \Delta_9, []\mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_5}\,\top_L \quad \dfrac{\mathtt{h}_6 : \top, \mathtt{F}_5, \mathtt{F}_7, \Delta_9, []\mathtt{F}_7 \vdash \Delta_8}{\bullet\mathtt{h}_6 : (\top, \Delta_9, []\mathtt{F}_7), \mathtt{F}_5 \vdash \Delta_8}\,AT}{- : \top, \Delta_9, []\mathtt{F}_7 \vdash \Delta_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \top, \Delta_9, []\mathtt{F}_7 \vdash \Delta_8, \mathtt{F}_5}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_6 : \top, \Delta_9, \mathtt{F}_5, []\mathtt{F}_7 \vdash \Delta_8}{}\,\mathtt{ax/W}}{- : \top, \Delta_9, []\mathtt{F}_7 \vdash \Delta_8}\,\mathtt{hCut}$$

- Case rule $\bot_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_5 \vdash \bot, \Delta_7}{\bullet\mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_7, \bot}\,\top_L \quad \dfrac{}{\bullet\mathtt{h}_6 : (\top, \Delta_5), \bot \vdash \Delta_7}\,\bot_L}{- : \top, \Delta_5 \vdash \Delta_7}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \top, \Delta_5 \vdash \bot, \Delta_7}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_6 : \bot, \top, \Delta_5 \vdash \Delta_7}{}\,\bot_L}{- : \top, \Delta_5 \vdash \Delta_7}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \bot, \Delta_8 \vdash \mathtt{F}_5, \Delta_7}{\bullet\mathtt{h}_3 : \top, \bot, \Delta_8 \vdash \Delta_7, \mathtt{F}_5}\,\top_L \quad \dfrac{}{\bullet\mathtt{h}_6 : (\top, \bot, \Delta_8), \mathtt{F}_5 \vdash \Delta_7}\,\bot_L}{- : \top, \bot, \Delta_8 \vdash \Delta_7}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \bot, \top, \Delta_8 \vdash \Delta_7}\,\bot_L$$

- Case rule I

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_5 \vdash \mathtt{p}_7, \Delta_8, \mathtt{p}_7}{\bullet\mathtt{h}_3 : \top, \Delta_5 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{p}_7}\,\top_L \quad \dfrac{}{\bullet\mathtt{h}_6 : (\top, \Delta_5), \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\,I}{- : \top, \Delta_5 \vdash \Delta_8, \mathtt{p}_7}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{\mathtt{h}_3 : \top, \Delta_5 \vdash \Delta_8, \mathtt{p}_7, \mathtt{p}_7}{}\,\mathtt{ax/W} \quad \dfrac{\bullet\mathtt{h}_6 : \top, \Delta_5, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}{}\,I}{- : \top, \Delta_5 \vdash \Delta_8, \mathtt{p}_7}\,\mathtt{hCut}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_9, \mathtt{p}_7 \vdash \mathtt{F}_5, \Delta_8, \mathtt{p}_7}{\bullet\mathtt{h}_3 : \top, \Delta_9, \mathtt{p}_7 \vdash (\Delta_8, \mathtt{p}_7), \mathtt{F}_5}\,\top_L \quad \dfrac{}{\bullet\mathtt{h}_6 : (\top, \Delta_9, \mathtt{p}_7), \mathtt{F}_5 \vdash \Delta_8, \mathtt{p}_7}\,I}{- : \top, \Delta_9, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{}{- : \top, \Delta_9, \mathtt{p}_7 \vdash \Delta_8, \mathtt{p}_7}\,I$$

- Case rule $\top_L$

$$\dfrac{\dfrac{\dfrac{\mathtt{h}_3 : \Delta_6 \vdash \mathtt{F}_5, \Delta_8}{\bullet\mathtt{h}_3 : \top, \Delta_6 \vdash \Delta_8, \mathtt{F}_5}\,\top_L \quad \dfrac{\mathtt{h}_7 : \mathtt{F}_5, \Delta_6 \vdash \Delta_8}{\bullet\mathtt{h}_7 : (\top, \Delta_6), \mathtt{F}_5 \vdash \Delta_8}\,\top_L}{- : \top, \Delta_6 \vdash \Delta_8}\,\mathtt{Cut}}{\rightarrow}$$

$$\dfrac{\dfrac{}{\mathtt{h}_3 : \top, \Delta_6 \vdash \Delta_8, \mathtt{F}_5}\,\mathtt{ax/W} \quad \dfrac{}{\bullet\mathtt{h}_7 : \top, \Delta_6, \mathtt{F}_5 \vdash \Delta_8}\,\mathtt{ax/W}}{- : \top, \Delta_6 \vdash \Delta_8}\,\mathtt{hCut}$$