

System for Linear Logic

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Contents

1	Measure of derivations	3
2	Weakening on bang: $\vdash \Gamma, !F$ implies $\vdash \Gamma, F$.	5
3	Invertibility of Rules	6
3.1	Status of $\mathbf{1}$: : Invertible	6
3.2	Status of $!$: : Invertible	7
3.3	Status of $?W$: : Non invertible	8
3.4	Status of $?C$: : Non invertible	9
3.5	Status of $?$: : Non invertible	11
3.6	Status of $\$$: : Invertible	12
3.7	Status of $\&$: (Left Premise): Invertible	14
3.8	Status of $\&$ (Right Premise): : Invertible	15
3.9	Status of \oplus_B : : Non invertible	16
3.10	Status of \oplus_A : : Non invertible	18
3.11	Status of \perp : : Invertible	19
3.12	Status of \top : : Invertible	20
3.13	Status of I : : Invertible	22
3.14	Status of \otimes : (Left Premise): Non invertible	22
3.15	Status of \otimes (Right Premise): : Non invertible	24
4	Identity-Expansion	27
5	Cut-Elimination	28
5.1	Status of $\mathbf{1}$: OK	28
5.2	Status of $!$: OK	30
5.3	Status of $?W$: OK	33
5.4	Status of $?C$: OK	37
5.5	Status of $?$: OK	40
5.6	Status of $\$$: OK	44
5.7	Status of $\&$: OK	47
5.8	Status of \oplus_B : OK	51
5.9	Status of \oplus_A : OK	55
5.10	Status of \perp : OK	58
5.11	Status of \top : OK	62
5.12	Status of I : OK	65
5.13	Status of \otimes : OK	68

6	Cut-Elimination	74
6.1	Status of $\mathbf{1}$: OK	74
6.2	Status of $!$: OK	74
6.3	Status of $?W$: OK	77
6.4	Status of $?C$: OK	80
6.5	Status of $?$: OK	83
6.6	Status of $\$$: OK	86
6.7	Status of $\&$: OK	86
6.8	Status of \oplus_B : OK	87
6.9	Status of \oplus_A : OK	88
6.10	Status of \perp : OK	89
6.11	Status of \top : OK	89
6.12	Status of I : OK	90
6.13	Status of \otimes : OK	91

1 Measure of derivations

- Case(s) rule 1

$$\frac{}{\bullet h_1 : \vdash 1} 1 \rightarrow \frac{}{\bullet \bullet h_1 : \vdash 1} 1$$

- Case(s) rule !

$$\frac{h_1 : \vdash F_3, ?\Upsilon 2}{\bullet h_1 : \vdash ?\Upsilon 2, !F_3} ! \rightarrow \frac{\frac{h_1 : \vdash ?\Upsilon 2, F_3}{\bullet h_1 : \vdash ?\Upsilon 2, F_3} \text{IH}}{\bullet \bullet h_1 : \vdash ?\Upsilon 2, !F_3} \text{ax} !$$

- Case(s) rule ?W

$$\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \Delta_2, ?F_3} ?W \rightarrow \frac{\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \Delta_2} \text{IH}}{\bullet \bullet h_1 : \vdash \Delta_2, ?F_3} \text{ax} ?W$$

- Case(s) rule ?C

$$\frac{h_1 : \vdash \Delta_2, ?F_3, ?F_3}{\bullet h_1 : \vdash \Delta_2, ?F_3} ?C \rightarrow \frac{\frac{h_1 : \vdash \Delta_2, ?F_3, ?F_3}{\bullet h_1 : \vdash \Delta_2, ?F_3, ?F_3} \text{IH}}{\bullet \bullet h_1 : \vdash \Delta_2, ?F_3} \text{ax} ?C$$

- Case(s) rule ?

$$\frac{h_1 : \vdash \Delta_2, F_3}{\bullet h_1 : \vdash \Delta_2, ?F_3} ? \rightarrow \frac{\frac{h_1 : \vdash \Delta_2, F_3}{\bullet h_1 : \vdash \Delta_2, F_3} \text{IH}}{\bullet \bullet h_1 : \vdash \Delta_2, ?F_3} \text{ax} ?$$

- Case(s) rule \$

$$\frac{h_1 : \vdash \Delta_2, F_3, F_4}{\bullet h_1 : \vdash \Delta_2, F_3 \$F_4} \$ \rightarrow \frac{\frac{h_1 : \vdash \Delta_2, F_3, F_4}{\bullet h_1 : \vdash \Delta_2, F_3, F_4} \text{IH}}{\bullet \bullet h_1 : \vdash \Delta_2, F_3 \$F_4} \text{ax} \$$$

- Case(s) rule &

$$\frac{h_1 : \vdash \Delta_2, F_3 \quad h_1 : \vdash \Delta_2, F_4}{\bullet h_1 : \vdash \Delta_2, F_3 \&F_4} \& \rightarrow \frac{\frac{h_1 : \vdash \Delta_2, F_3}{\bullet h_1 : \vdash \Delta_2, F_3} \text{IH} \quad \frac{h_1 : \vdash \Delta_2, F_4}{\bullet h_1 : \vdash \Delta_2, F_4} \text{IH}}{\bullet \bullet h_1 : \vdash \Delta_2, F_3 \&F_4} \text{ax} \&$$

- Case(s) rule \oplus_B

$$\frac{h_1 : \vdash \Delta_2, F_4}{\bullet h_1 : \vdash \Delta_2, F_3 \oplus F_4} \oplus_B \rightarrow \frac{\frac{h_1 : \vdash \Delta_2, F_4}{\bullet h_1 : \vdash \Delta_2, F_4} \text{IH}}{\bullet \bullet h_1 : \vdash \Delta_2, F_3 \oplus F_4} \text{ax} \oplus_B$$

- Case(s) rule \oplus_A

$$\frac{h_1 : \vdash \Delta_2, F_3}{\bullet h_1 : \vdash \Delta_2, F_3 \oplus F_4} \oplus_A \rightarrow \frac{\frac{h_1 : \vdash \Delta_2, F_3}{\bullet h_1 : \vdash \Delta_2, F_3} \text{IH}}{\bullet \bullet h_1 : \vdash \Delta_2, F_3 \oplus F_4} \text{ax} \oplus_A$$

- Case(s) rule \perp

$$\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \rightarrow \frac{\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \Delta_2} \text{IH}}{\bullet \bullet h_1 : \vdash \Delta_2, \perp} \text{ax} \perp$$

- Case(s) rule \top

$$\frac{}{\bullet h_1 : \vdash \top, \Delta_2} \top \quad \rightarrow \quad \frac{}{\bullet \bullet h_1 : \vdash \Delta_2, \top} \top$$

- Case(s) rule I

$$\frac{}{\bullet h_1 : \vdash p(n_2), \neg(n_2)} I \quad \rightarrow \quad \frac{}{\bullet \bullet h_1 : \vdash p(n_2), \neg(n_2)} I$$

- Case(s) rule \otimes

$$\frac{h_1 : \vdash \Delta_2, F_4 \quad h_1 : \vdash \Delta_3, F_5}{\bullet h_1 : \vdash \Delta_2, \Delta_3, F_4 \otimes F_5} \otimes \quad \rightarrow \quad \frac{\frac{h_1 : \vdash \Delta_2, F_4}{\bullet h_1 : \vdash \Delta_2, F_4} \text{IH} \quad \frac{h_1 : \vdash \Delta_3, F_5}{\bullet h_1 : \vdash \Delta_3, F_5} \text{IH}}{\bullet \bullet h_1 : \vdash \Delta_2, \Delta_3, F_4 \otimes F_5} \otimes$$

2 Weakening on bang: $\vdash \Gamma, !F$ implies $\vdash \Gamma, F$.

- Case(s) rule 1
- Case(s) rule !

$$\frac{h_1 \vdash F_3, ?\Upsilon 2}{\bullet h_1 \vdash !F_3, ?\Upsilon 2} ! \rightarrow \frac{\overline{h_1 \vdash ?\Upsilon 2, F_3} \text{ ax}}{\bullet h_1 \vdash ?\Upsilon 2, F_3} \text{ H}$$

- Case(s) rule ?W

$$\frac{h_2 \vdash \Delta_4, !F_1}{\bullet h_2 \vdash !F_1, \Delta_4, ?F_3} ?W \rightarrow \frac{\overline{h_2 \vdash \Delta_4, !F_1} \text{ ax}}{\overline{h_2 \vdash \Delta_4, F_1} \text{ IH}} \text{ ?W}$$

- Case(s) rule ?C

$$\frac{h_2 \vdash \Delta_4, !F_1, ?F_3, ?F_3}{\bullet h_2 \vdash !F_1, \Delta_4, ?F_3} ?C \rightarrow \frac{\overline{h_2 \vdash \Delta_4, !F_1, ?F_3, ?F_3} \text{ ax}}{\overline{h_2 \vdash \Delta_4, F_1, ?F_3, ?F_3} \text{ IH}} \text{ ?C}$$

- Case(s) rule ?

$$\frac{h_2 \vdash \Delta_4, F_3, !F_1}{\bullet h_2 \vdash !F_1, \Delta_4, ?F_3} ? \rightarrow \frac{\overline{h_2 \vdash \Delta_4, F_3, !F_1} \text{ ax}}{\overline{h_2 \vdash \Delta_4, F_1, F_3} \text{ IH}} \text{ ?}$$

- Case(s) rule \$

$$\frac{h_2 \vdash \Delta_5, F_3, F_4, !F_1}{\bullet h_2 \vdash !F_1, \Delta_5, F_3 \$F_4} \$ \rightarrow \frac{\overline{h_2 \vdash \Delta_5, F_3, F_4, !F_1} \text{ ax}}{\overline{h_2 \vdash \Delta_5, F_1, F_3, F_4} \text{ IH}} \text{ \$}$$

- Case(s) rule &

$$\frac{h_2 \vdash \Delta_5, F_3, !F_1 \quad h_2 \vdash \Delta_5, F_4, !F_1}{\bullet h_2 \vdash !F_1, \Delta_5, F_3 \&F_4} \& \rightarrow \frac{\overline{h_2 \vdash \Delta_5, F_3, !F_1} \text{ ax} \quad \overline{h_2 \vdash \Delta_5, F_4, !F_1} \text{ ax}}{\overline{h_2 \vdash \Delta_5, F_1, F_3} \text{ IH} \quad \overline{h_2 \vdash \Delta_5, F_1, F_4} \text{ IH}} \text{ \&}$$

- Case(s) rule \oplus_B

$$\frac{h_2 \vdash \Delta_5, F_4, !F_1}{\bullet h_2 \vdash !F_1, \Delta_5, F_3 \oplus F_4} \oplus_B \rightarrow \frac{\overline{h_2 \vdash \Delta_5, F_4, !F_1} \text{ ax}}{\overline{h_2 \vdash \Delta_5, F_1, F_4} \text{ IH}} \text{ \oplus}_B$$

- Case(s) rule \oplus_A

$$\frac{h_2 \vdash \Delta_5, F_3, !F_1}{\bullet h_2 \vdash !F_1, \Delta_5, F_3 \oplus F_4} \oplus_A \rightarrow \frac{\overline{h_2 \vdash \Delta_5, F_3, !F_1} \text{ ax}}{\overline{h_2 \vdash \Delta_5, F_1, F_3} \text{ IH}} \text{ \oplus}_A$$

- Case(s) rule \perp

$$\frac{h_2 \vdash \Delta_3, !F_1}{\bullet h_2 \vdash !F_1, \perp, \Delta_3} \perp \rightarrow \frac{\overline{h_2 \vdash \Delta_3, !F_1} \text{ ax}}{\overline{h_2 \vdash \Delta_3, F_1} \text{ IH}} \text{ \perp}$$

- Case(s) rule \top

$$\frac{}{\bullet h_2 : \vdash !F_1, \top, \Delta_3} \top \rightarrow \frac{}{\bullet h_2 : \vdash \Delta_3, F_1, \top} \top$$

- Case(s) rule I
- Case(s) rule \otimes

$$\frac{\frac{h_2 : \vdash \Delta_6, F_4, !F_1}{\bullet h_2 : \vdash !F_1, \Delta_3, \Delta_6, F_4} \otimes \quad \frac{h_2 : \vdash \Delta_3, F_5}{\bullet h_2 : \vdash \Delta_3, F_5} \otimes}{\bullet h_2 : \vdash !F_1, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \rightarrow \frac{\frac{\frac{h_2 : \vdash \Delta_6, F_4, !F_1}{\bullet h_2 : \vdash \Delta_6, F_1, F_4} \text{IH} \quad \frac{h_2 : \vdash \Delta_3, F_5}{\bullet h_2 : \vdash \Delta_3, F_5} \text{ax}}{\bullet h_2 : \vdash \Delta_3, \Delta_6, F_1, F_4 \otimes F_5} \otimes}{\bullet h_2 : \vdash \Delta_3, \Delta_6, F_1, F_4 \otimes F_5} \otimes$$

$$\frac{\frac{h_2 : \vdash \Delta_3, F_4}{\bullet h_2 : \vdash !F_1, \Delta_3, \Delta_6, F_4} \otimes \quad \frac{h_2 : \vdash \Delta_6, F_5, !F_1}{\bullet h_2 : \vdash \Delta_6, F_5, !F_1} \otimes}{\bullet h_2 : \vdash !F_1, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \rightarrow \frac{\frac{\frac{h_2 : \vdash \Delta_3, F_4}{\bullet h_2 : \vdash \Delta_3, F_4} \text{ax} \quad \frac{h_2 : \vdash \Delta_6, F_5, !F_1}{\bullet h_2 : \vdash \Delta_6, F_1, F_5} \text{IH}}{\bullet h_2 : \vdash \Delta_3, \Delta_6, F_1, F_4 \otimes F_5} \otimes}{\bullet h_2 : \vdash \Delta_3, \Delta_6, F_1, F_4 \otimes F_5} \otimes$$

3 Invertibility of Rules

3.1 Status of 1: : Invertible

- Case rule 1

$$\frac{}{\bullet h_1 : \vdash 1} 1 \rightarrow \text{trivial}$$

- Case rule !
- Case rule ? W
- Case rule ? C
- Case rule ?
- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I
- Case rule \otimes

3.2 Status of ! : Invertible

- Case rule 1

- Case rule !

$$\frac{h_1 \text{ :- } F_3, ?Y_2}{\bullet h_1 \text{ :- } ?Y_2, !F_3} ! \rightarrow \frac{\overline{h_1 \text{ :- } ?Y_2, F_3}^{\text{ax}}}{\bullet h_1 \text{ :- } ?Y_2, F_3} H$$

- Case rule ?W

$$\frac{h_2 \text{ :- } ?Y_4, !F_1}{\bullet h_2 \text{ :- } (?Y_4, !F_1), ?F_3} ?W \rightarrow \frac{\overline{h_2 \text{ :- } ?Y_4, F_1}^{\text{ax/ind}}}{\bullet h_2 \text{ :- } ?Y_4, F_1, ?F_3} ?W$$

- Case rule ?C

$$\frac{h_2 \text{ :- } ?Y_4, !F_1, ?F_3, ?F_3}{\bullet h_2 \text{ :- } (?Y_4, !F_1), ?F_3} ?C \rightarrow \frac{\overline{h_2 \text{ :- } ?Y_4, F_1, ?F_3, ?F_3}^{\text{ax/ind}}}{\bullet h_2 \text{ :- } ?Y_4, F_1, ?F_3} ?C$$

- Case rule ?

$$\frac{h_2 \text{ :- } F_3, ?Y_4, !F_1}{\bullet h_2 \text{ :- } (?Y_4, !F_1), ?F_3} ? \rightarrow \frac{\overline{h_2 \text{ :- } ?Y_4, F_3, !F_1}^{\text{ax}}}{\bullet h_2 \text{ :- } ?Y_4, !F_1, ?F_3} ? \quad \frac{\overline{h_2 \text{ :- } ?Y_4, F_1, ?F_3}^{\text{ax/ind}}}{\bullet h_2 \text{ :- } ?Y_4, F_1, ?F_3} ?W!$$

- Case rule \$

- Case rule &

- Case rule \oplus_B

- Case rule \oplus_A

- Case rule \perp

- Case rule \top

- Case rule I

- Case rule \otimes

3.3 Status of $?W$: : Non invertible

- Case rule 1

- Case rule !

$$\frac{h_2 \text{ :- } F_3, ?\Upsilon_4, ?F_1}{\bullet h_2 \text{ :- } (?\Upsilon_4, ?F_1), !F_3} ! \rightarrow \frac{\overline{h_2 \text{ :- } ?\Upsilon_4, F_3}}{\bullet h_2 \text{ :- } ?\Upsilon_4, !F_3} \text{ ax/ind} !$$

- Case rule $?W$

$$\frac{h_2 \text{ :- } \Delta_4, ?F_1}{\bullet h_2 \text{ :- } (\Delta_4, ?F_1), ?F_3} ?W \rightarrow \frac{\overline{h_2 \text{ :- } \Delta_4}}{\bullet h_2 \text{ :- } \Delta_4, ?F_3} \text{ ax/ind} ?W$$

$$\frac{h_1 \text{ :- } \Delta_2}{\bullet h_1 \text{ :- } \Delta_2, ?F_3} ?W \rightarrow \frac{\overline{h_1 \text{ :- } \Delta_2}}{\bullet h_1 \text{ :- } \Delta_2} \text{ ax} \text{ H}$$

- Case rule $?C$

$$\frac{h_2 \text{ :- } \Delta_4, ?F_1, ?F_3, ?F_3}{\bullet h_2 \text{ :- } (\Delta_4, ?F_1), ?F_3} ?C \rightarrow \frac{\overline{h_2 \text{ :- } \Delta_4, ?F_3, ?F_3}}{\bullet h_2 \text{ :- } \Delta_4, ?F_3} \text{ ax/ind} ?C$$

$$\frac{h_1 \text{ :- } \Delta_2, ?F_3, ?F_3}{\bullet h_1 \text{ :- } \Delta_2, ?F_3} ?C \rightarrow \overline{\bullet h_1 \text{ :- } \Delta_2} \text{ fail}$$

- Case rule ?

$$\frac{h_2 \text{ :- } \Delta_4, F_3, ?F_1}{\bullet h_2 \text{ :- } (\Delta_4, ?F_1), ?F_3} ? \rightarrow \frac{\overline{h_2 \text{ :- } \Delta_4, F_3}}{\bullet h_2 \text{ :- } \Delta_4, ?F_3} \text{ ax/ind} ?$$

$$\frac{h_1 \text{ :- } \Delta_2, F_3}{\bullet h_1 \text{ :- } \Delta_2, ?F_3} ? \rightarrow \overline{\bullet h_1 \text{ :- } \Delta_2} \text{ fail}$$

- Case rule \$

$$\frac{h_2 \text{ :- } \Delta_5, F_3, F_4, ?F_1}{\bullet h_2 \text{ :- } (\Delta_5, ?F_1), F_3 \$F_4} \$ \rightarrow \frac{\overline{h_2 \text{ :- } \Delta_5, F_3, F_4}}{\bullet h_2 \text{ :- } \Delta_5, F_3 \$F_4} \text{ ax/ind} \$$$

- Case rule &

$$\frac{h_2 \text{ :- } \Delta_5, F_3, ?F_1 \quad h_2 \text{ :- } \Delta_5, F_4, ?F_1}{\bullet h_2 \text{ :- } (\Delta_5, ?F_1), F_3 \&F_4} \& \rightarrow \frac{\overline{h_2 \text{ :- } \Delta_5, F_3} \quad \text{ax/ind} \quad \overline{h_2 \text{ :- } \Delta_5, F_4} \quad \text{ax/ind}}{\bullet h_2 \text{ :- } \Delta_5, F_3 \&F_4} \&$$

- Case rule \oplus_B

$$\frac{h_2 \text{ :- } \Delta_5, F_4, ?F_1}{\bullet h_2 \text{ :- } (\Delta_5, ?F_1), F_3 \oplus F_4} \oplus_B \rightarrow \frac{\overline{h_2 \text{ :- } \Delta_5, F_4} \quad \text{ax/ind}}{\bullet h_2 \text{ :- } \Delta_5, F_3 \oplus F_4} \oplus_B$$

- Case rule \oplus_A

$$\frac{h_2 : \vdash \Delta_5, F_3, ?F_1}{\bullet h_2 : \vdash (\Delta_5, ?F_1), F_3 \oplus F_4} \oplus_A \rightarrow \frac{\overline{h_2 : \vdash \Delta_5, F_3} \text{ ax/ind}}{\bullet h_2 : \vdash \Delta_5, F_3 \oplus F_4} \oplus_A$$

- Case rule \perp

$$\frac{h_2 : \vdash \Delta_3, ?F_1}{\bullet h_2 : \vdash \perp, \Delta_3, ?F_1} \perp \rightarrow \frac{\overline{h_2 : \vdash \Delta_3} \text{ ax/ind}}{\bullet h_2 : \vdash \Delta_3, \perp} \perp$$

- Case rule \top

$$\overline{\bullet h_2 : \vdash \top, \Delta_3, ?F_1} \top \rightarrow \overline{\bullet h_2 : \vdash \Delta_3, \top} \top$$

- Case rule I

- Case rule \otimes

$$\frac{h_2 : \vdash \Delta_6, F_4, ?F_1 \quad h_2 : \vdash \Delta_3, F_5}{\bullet h_2 : \vdash (\Delta_6, ?F_1), \Delta_3, F_4 \otimes F_5} \otimes \rightarrow \frac{\overline{h_2 : \vdash \Delta_6, F_4} \text{ ax/ind} \quad \overline{h_2 : \vdash \Delta_3, F_5} \text{ ax}}{\bullet h_2 : \vdash \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes$$

$$\frac{h_2 : \vdash \Delta_3, F_4 \quad h_2 : \vdash \Delta_6, F_5, ?F_1}{\bullet h_2 : \vdash \Delta_3, (\Delta_6, ?F_1), F_4 \otimes F_5} \otimes \rightarrow \frac{\overline{h_2 : \vdash \Delta_3, F_4} \text{ ax} \quad \overline{h_2 : \vdash \Delta_6, F_5} \text{ ax/ind}}{\bullet h_2 : \vdash \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes$$

3.4 Status of $?C$: : Non invertible

- Case rule 1

- Case rule $!$

$$\frac{h_2 : \vdash F_3, ?\Upsilon_4, ?F_1}{\bullet h_2 : \vdash (? \Upsilon_4, ?F_1), !F_3} ! \rightarrow \frac{\overline{h_2 : \vdash ?\Upsilon_4, F_3, ?F_1, ?F_1} \text{ ax/ind}}{\bullet h_2 : \vdash ?\Upsilon_4, !F_3, ?F_1, ?F_1} !$$

- Case rule $?W$

$$\frac{h_2 : \vdash \Delta_4, ?F_1}{\bullet h_2 : \vdash (\Delta_4, ?F_1), ?F_3} ?W \rightarrow \frac{\overline{h_2 : \vdash \Delta_4, ?F_1, ?F_1} \text{ ax/ind}}{\bullet h_2 : \vdash \Delta_4, ?F_1, ?F_1, ?F_3} ?W$$

$$\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \Delta_2, ?F_3} ?W \rightarrow \overline{\bullet h_1 : \vdash \Delta_2, ?F_3, ?F_3} \text{ fail}$$

- Case rule $?C$

$$\frac{h_2 : \vdash \Delta_4, ?F_1, ?F_3, ?F_3}{\bullet h_2 : \vdash (\Delta_4, ?F_1), ?F_3} ?C \rightarrow \frac{\overline{h_2 : \vdash \Delta_4, ?F_1, ?F_1, ?F_3, ?F_3} \text{ ax/ind}}{\bullet h_2 : \vdash \Delta_4, ?F_1, ?F_1, ?F_3} ?C$$

$$\frac{h_1 : \vdash \Delta_2, ?F_3, ?F_3}{\bullet h_1 : \vdash \Delta_2, ?F_3} ?C \rightarrow \frac{\overline{h_1 : \vdash \Delta_2, ?F_3, ?F_3, ?F_3} \text{ ax/ind}}{\bullet h_1 : \vdash \Delta_2, ?F_3, ?F_3} ?C$$

- Case rule ?

$$\frac{h_2 \text{ :- } \Delta_4, F_3, ?F_1}{\bullet h_2 \text{ :- } (\Delta_4, ?F_1), ?F_3} ? \rightarrow \frac{h_2 \text{ :- } \Delta_4, F_3, ?F_1}{\bullet h_2 \text{ :- } \Delta_4, ?F_1, ?F_1, ?F_3} \text{ ax/ind} ?$$

$$\frac{h_1 \text{ :- } \Delta_2, F_3}{\bullet h_1 \text{ :- } \Delta_2, ?F_3} ? \rightarrow \frac{}{\bullet h_1 \text{ :- } \Delta_2, ?F_3, ?F_3} \text{ fail}$$

- Case rule \$

$$\frac{h_2 \text{ :- } \Delta_5, F_3, F_4, ?F_1}{\bullet h_2 \text{ :- } (\Delta_5, ?F_1), F_3 \& F_4} \$ \rightarrow \frac{h_2 \text{ :- } \Delta_5, F_3, F_4, ?F_1}{\bullet h_2 \text{ :- } \Delta_5, ?F_1, ?F_1, F_3 \& F_4} \text{ ax/ind} \$$$

- Case rule &

$$\frac{h_2 \text{ :- } \Delta_5, F_3, ?F_1 \quad h_2 \text{ :- } \Delta_5, F_4, ?F_1}{\bullet h_2 \text{ :- } (\Delta_5, ?F_1), F_3 \& F_4} \& \rightarrow \frac{h_2 \text{ :- } \Delta_5, F_3, ?F_1}{\bullet h_2 \text{ :- } \Delta_5, ?F_1, ?F_1, F_3 \& F_4} \text{ ax/ind} \quad \frac{h_2 \text{ :- } \Delta_5, F_4, ?F_1}{\bullet h_2 \text{ :- } \Delta_5, ?F_1, ?F_1, F_3 \& F_4} \text{ ax/ind} \&$$

- Case rule \oplus_B

$$\frac{h_2 \text{ :- } \Delta_5, F_4, ?F_1}{\bullet h_2 \text{ :- } (\Delta_5, ?F_1), F_3 \oplus F_4} \oplus_B \rightarrow \frac{h_2 \text{ :- } \Delta_5, F_4, ?F_1}{\bullet h_2 \text{ :- } \Delta_5, ?F_1, ?F_1, F_3 \oplus F_4} \text{ ax/ind} \oplus_B$$

- Case rule \oplus_A

$$\frac{h_2 \text{ :- } \Delta_5, F_3, ?F_1}{\bullet h_2 \text{ :- } (\Delta_5, ?F_1), F_3 \oplus F_4} \oplus_A \rightarrow \frac{h_2 \text{ :- } \Delta_5, F_3, ?F_1}{\bullet h_2 \text{ :- } \Delta_5, ?F_1, ?F_1, F_3 \oplus F_4} \text{ ax/ind} \oplus_A$$

- Case rule \perp

$$\frac{h_2 \text{ :- } \Delta_3, ?F_1}{\bullet h_2 \text{ :- } \perp, \Delta_3, ?F_1} \perp \rightarrow \frac{h_2 \text{ :- } \Delta_3, ?F_1, ?F_1}{\bullet h_2 \text{ :- } \Delta_3, \perp, ?F_1, ?F_1} \text{ ax/ind} \perp$$

- Case rule \top

$$\frac{}{\bullet h_2 \text{ :- } \top, \Delta_3, ?F_1} \top \rightarrow \frac{}{\bullet h_2 \text{ :- } \Delta_3, \top, ?F_1, ?F_1} \top$$

- Case rule I

- Case rule \otimes

$$\frac{h_2 \text{ :- } \Delta_6, F_4, ?F_1 \quad h_2 \text{ :- } \Delta_3, F_5}{\bullet h_2 \text{ :- } (\Delta_6, ?F_1), \Delta_3, F_4 \otimes F_5} \otimes \rightarrow \frac{h_2 \text{ :- } \Delta_6, F_4, ?F_1}{\bullet h_2 \text{ :- } \Delta_3, \Delta_6, ?F_1, ?F_1, F_4 \otimes F_5} \text{ ax/ind} \quad \frac{h_2 \text{ :- } \Delta_3, F_5}{\bullet h_2 \text{ :- } \Delta_3, \Delta_6, ?F_1, ?F_1, F_4 \otimes F_5} \text{ ax} \otimes$$

$$\frac{h_2 \text{ :- } \Delta_3, F_4 \quad h_2 \text{ :- } \Delta_6, F_5, ?F_1}{\bullet h_2 \text{ :- } \Delta_3, (\Delta_6, ?F_1), F_4 \otimes F_5} \otimes \rightarrow \frac{h_2 \text{ :- } \Delta_3, F_4}{\bullet h_2 \text{ :- } \Delta_3, \Delta_6, ?F_1, ?F_1, F_4 \otimes F_5} \text{ ax} \quad \frac{h_2 \text{ :- } \Delta_6, F_5, ?F_1}{\bullet h_2 \text{ :- } \Delta_3, \Delta_6, ?F_1, ?F_1, F_4 \otimes F_5} \text{ ax/ind} \otimes$$

3.5 Status of $?:$: Non invertible

- Case rule 1

$$\frac{h_2 \text{ :- } F_3, ?\Upsilon_4, ?F_1}{\bullet h_2 \text{ :- } (? \Upsilon_4, ?F_1), !F_3} ! \rightarrow \frac{}{\bullet h_2 \text{ :- } F_1, ?\Upsilon_4, !F_3} \text{ fail}$$

- Case rule $?W$

$$\frac{h_2 \text{ :- } \Delta_4, ?F_1}{\bullet h_2 \text{ :- } (\Delta_4, ?F_1), ?F_3} ?W \rightarrow \frac{}{h_2 \text{ :- } \Delta_4, F_1} \text{ ax/ind} \frac{}{\bullet h_2 \text{ :- } \Delta_4, F_1, ?F_3} ?W$$

$$\frac{h_1 \text{ :- } \Delta_2}{\bullet h_1 \text{ :- } \Delta_2, ?F_3} ?W \rightarrow \frac{}{\bullet h_1 \text{ :- } \Delta_2, F_3} \text{ fail}$$

- Case rule $?C$

$$\frac{h_2 \text{ :- } \Delta_4, ?F_1, ?F_3, ?F_3}{\bullet h_2 \text{ :- } (\Delta_4, ?F_1), ?F_3} ?C \rightarrow \frac{}{h_2 \text{ :- } \Delta_4, F_1, ?F_3, ?F_3} \text{ ax/ind} \frac{}{\bullet h_2 \text{ :- } \Delta_4, F_1, ?F_3} ?C$$

$$\frac{h_1 \text{ :- } \Delta_2, ?F_3, ?F_3}{\bullet h_1 \text{ :- } \Delta_2, ?F_3} ?C \rightarrow \frac{}{\bullet h_1 \text{ :- } \Delta_2, F_3} \text{ fail}$$

- Case rule $?$

$$\frac{h_2 \text{ :- } \Delta_4, F_3, ?F_1}{\bullet h_2 \text{ :- } (\Delta_4, ?F_1), ?F_3} ? \rightarrow \frac{}{h_2 \text{ :- } \Delta_4, F_1, F_3} \text{ ax/ind} \frac{}{\bullet h_2 \text{ :- } \Delta_4, F_1, ?F_3} ?$$

$$\frac{h_1 \text{ :- } \Delta_2, F_3}{\bullet h_1 \text{ :- } \Delta_2, ?F_3} ? \rightarrow \frac{}{h_1 \text{ :- } \Delta_2, F_3} \text{ ax} \frac{}{\bullet h_1 \text{ :- } \Delta_2, F_3} \text{ H}$$

- Case rule $\$$

$$\frac{h_2 \text{ :- } \Delta_5, F_3, F_4, ?F_1}{\bullet h_2 \text{ :- } (\Delta_5, ?F_1), F_3 \$ F_4} \$ \rightarrow \frac{}{h_2 \text{ :- } \Delta_5, F_1, F_3, F_4} \text{ ax/ind} \frac{}{\bullet h_2 \text{ :- } \Delta_5, F_1, F_3 \$ F_4} \$$$

- Case rule $\&$

$$\frac{h_2 \text{ :- } \Delta_5, F_3, ?F_1}{\bullet h_2 \text{ :- } (\Delta_5, ?F_1), F_3 \& F_4} \frac{h_2 \text{ :- } \Delta_5, F_4, ?F_1}{\bullet h_2 \text{ :- } (\Delta_5, ?F_1), F_3 \& F_4} \& \rightarrow \frac{}{h_2 \text{ :- } \Delta_5, F_1, F_3} \text{ ax/ind} \frac{}{h_2 \text{ :- } \Delta_5, F_1, F_4} \text{ ax/ind} \frac{}{\bullet h_2 \text{ :- } \Delta_5, F_1, F_3 \& F_4} \&$$

- Case rule \oplus_B

$$\frac{h_2 \text{ :- } \Delta_5, F_4, ?F_1}{\bullet h_2 \text{ :- } (\Delta_5, ?F_1), F_3 \oplus F_4} \oplus_B \rightarrow \frac{}{h_2 \text{ :- } \Delta_5, F_1, F_4} \text{ ax/ind} \frac{}{\bullet h_2 \text{ :- } \Delta_5, F_1, F_3 \oplus F_4} \oplus_B$$

- Case rule \oplus_A

$$\frac{h_2 \vdash \Delta_5, F_3, ?F_1}{\bullet h_2 \vdash (\Delta_5, ?F_1), F_3 \oplus F_4} \oplus_A \rightarrow \frac{\overline{h_2 \vdash \Delta_5, F_1, F_3} \text{ ax/ind}}{\bullet h_2 \vdash \Delta_5, F_1, F_3 \oplus F_4} \oplus_A$$

- Case rule \perp

$$\frac{h_2 \vdash \Delta_3, ?F_1}{\bullet h_2 \vdash \perp, \Delta_3, ?F_1} \perp \rightarrow \frac{\overline{h_2 \vdash \Delta_3, F_1} \text{ ax/ind}}{\bullet h_2 \vdash \Delta_3, F_1, \perp} \perp$$

- Case rule \top

$$\frac{\overline{h_2 \vdash \top, \Delta_3, ?F_1} \top}{\bullet h_2 \vdash \top, \Delta_3, ?F_1} \top \rightarrow \frac{\overline{h_2 \vdash \Delta_3, F_1, \top} \top}{\bullet h_2 \vdash \Delta_3, F_1, \top} \top$$

- Case rule I

- Case rule \otimes

$$\frac{h_2 \vdash \Delta_6, F_4, ?F_1 \quad h_2 \vdash \Delta_3, F_5}{\bullet h_2 \vdash (\Delta_6, ?F_1), \Delta_3, F_4 \otimes F_5} \otimes \rightarrow \frac{\overline{h_2 \vdash \Delta_6, F_1, F_4} \text{ ax/ind} \quad \overline{h_2 \vdash \Delta_3, F_5} \text{ ax}}{\bullet h_2 \vdash \Delta_3, \Delta_6, F_1, F_4 \otimes F_5} \otimes$$

$$\frac{h_2 \vdash \Delta_3, F_4 \quad h_2 \vdash \Delta_6, F_5, ?F_1}{\bullet h_2 \vdash \Delta_3, (\Delta_6, ?F_1), F_4 \otimes F_5} \otimes \rightarrow \frac{\overline{h_2 \vdash \Delta_3, F_4} \text{ ax} \quad \overline{h_2 \vdash \Delta_6, F_1, F_5} \text{ ax/ind}}{\bullet h_2 \vdash \Delta_3, \Delta_6, F_1, F_4 \otimes F_5} \otimes$$

3.6 Status of $\$$: : Invertible

- Case rule 1

- Case rule !

- Case rule ?W

$$\frac{h_3 \vdash \Delta_5, F_1 \$F_2}{\bullet h_3 \vdash (\Delta_5, F_1 \$F_2), ?F_4} ?W \rightarrow \frac{\overline{h_3 \vdash \Delta_5, F_1, F_2} \text{ ax/ind}}{\bullet h_3 \vdash \Delta_5, F_1, F_2, ?F_4} ?W$$

- Case rule ?C

$$\frac{h_3 \vdash \Delta_5, ?F_4, ?F_4, F_1 \$F_2}{\bullet h_3 \vdash (\Delta_5, F_1 \$F_2), ?F_4} ?C \rightarrow \frac{\overline{h_3 \vdash \Delta_5, F_1, F_2, ?F_4, ?F_4} \text{ ax/ind}}{\bullet h_3 \vdash \Delta_5, F_1, F_2, ?F_4} ?C$$

- Case rule ?

$$\frac{h_3 \vdash \Delta_5, F_4, F_1 \$F_2}{\bullet h_3 \vdash (\Delta_5, F_1 \$F_2), ?F_4} ? \rightarrow \frac{\overline{h_3 \vdash \Delta_5, F_1, F_2, F_4} \text{ ax/ind}}{\bullet h_3 \vdash \Delta_5, F_1, F_2, ?F_4} ?$$

- Case rule $\$$

$$\frac{h_3 \text{ :- } \Delta_6, F_4, F_5, F_1 \text{ SF}_2}{\bullet h_3 \text{ :- } (\Delta_6, F_1 \text{ SF}_2), F_4 \text{ SF}_5} \$ \rightarrow \frac{h_3 \text{ :- } \Delta_6, F_1, F_2, F_4, F_5}{\bullet h_3 \text{ :- } \Delta_6, F_1, F_2, F_4 \text{ SF}_5} \text{ ax/ind } \$$$

$$\frac{h_1 \text{ :- } \Delta_2, F_3, F_4}{\bullet h_1 \text{ :- } \Delta_2, F_3 \text{ SF}_4} \$ \rightarrow \frac{h_1 \text{ :- } \Delta_2, F_3, F_4}{\bullet h_1 \text{ :- } \Delta_2, F_3, F_4} \text{ ax } H$$

- Case rule $\&$

$$\frac{h_3 \text{ :- } \Delta_6, F_4, F_1 \text{ SF}_2 \quad h_3 \text{ :- } \Delta_6, F_5, F_1 \text{ SF}_2}{\bullet h_3 \text{ :- } (\Delta_6, F_1 \text{ SF}_2), F_4 \& F_5} \& \rightarrow \frac{h_3 \text{ :- } \Delta_6, F_1, F_2, F_4}{\bullet h_3 \text{ :- } \Delta_6, F_1, F_2, F_4 \& F_5} \text{ ax/ind } \frac{h_3 \text{ :- } \Delta_6, F_1, F_2, F_5}{\bullet h_3 \text{ :- } \Delta_6, F_1, F_2, F_4 \& F_5} \text{ ax/ind } \&$$

- Case rule \oplus_B

$$\frac{h_3 \text{ :- } \Delta_6, F_5, F_1 \text{ SF}_2}{\bullet h_3 \text{ :- } (\Delta_6, F_1 \text{ SF}_2), F_4 \oplus F_5} \oplus_B \rightarrow \frac{h_3 \text{ :- } \Delta_6, F_1, F_2, F_5}{\bullet h_3 \text{ :- } \Delta_6, F_1, F_2, F_4 \oplus F_5} \text{ ax/ind } \oplus_B$$

- Case rule \oplus_A

$$\frac{h_3 \text{ :- } \Delta_6, F_4, F_1 \text{ SF}_2}{\bullet h_3 \text{ :- } (\Delta_6, F_1 \text{ SF}_2), F_4 \oplus F_5} \oplus_A \rightarrow \frac{h_3 \text{ :- } \Delta_6, F_1, F_2, F_4}{\bullet h_3 \text{ :- } \Delta_6, F_1, F_2, F_4 \oplus F_5} \text{ ax/ind } \oplus_A$$

- Case rule \perp

$$\frac{h_3 \text{ :- } \Delta_4, F_1 \text{ SF}_2}{\bullet h_3 \text{ :- } \perp, \Delta_4, F_1 \text{ SF}_2} \perp \rightarrow \frac{h_3 \text{ :- } \Delta_4, F_1, F_2}{\bullet h_3 \text{ :- } \Delta_4, F_1, F_2, \perp} \text{ ax/ind } \perp$$

- Case rule \top

$$\frac{h_3 \text{ :- } \top, \Delta_4, F_1 \text{ SF}_2}{\bullet h_3 \text{ :- } \top, \Delta_4, F_1, F_2, \top} \top \rightarrow \frac{h_3 \text{ :- } \Delta_4, F_1, F_2, \top}{\bullet h_3 \text{ :- } \Delta_4, F_1, F_2, \top} \top$$

- Case rule I

- Case rule \otimes

$$\frac{h_3 \text{ :- } \Delta_7, F_5, F_1 \text{ SF}_2 \quad h_3 \text{ :- } \Delta_4, F_6}{\bullet h_3 \text{ :- } (\Delta_7, F_1 \text{ SF}_2), \Delta_4, F_5 \otimes F_6} \otimes \rightarrow \frac{h_3 \text{ :- } \Delta_7, F_1, F_2, F_5}{\bullet h_3 \text{ :- } \Delta_4, \Delta_7, F_1, F_2, F_5 \otimes F_6} \text{ ax/ind } \frac{h_3 \text{ :- } \Delta_4, F_6}{\bullet h_3 \text{ :- } \Delta_4, \Delta_7, F_1, F_2, F_5 \otimes F_6} \text{ ax } \otimes$$

$$\frac{h_3 \text{ :- } \Delta_4, F_5 \quad h_3 \text{ :- } \Delta_7, F_6, F_1 \text{ SF}_2}{\bullet h_3 \text{ :- } \Delta_4, (\Delta_7, F_1 \text{ SF}_2), F_5 \otimes F_6} \otimes \rightarrow \frac{h_3 \text{ :- } \Delta_4, F_5}{\bullet h_3 \text{ :- } \Delta_4, \Delta_7, F_1, F_2, F_5 \otimes F_6} \text{ ax } \frac{h_3 \text{ :- } \Delta_7, F_1, F_2, F_6}{\bullet h_3 \text{ :- } \Delta_4, \Delta_7, F_1, F_2, F_5 \otimes F_6} \text{ ax/ind } \otimes$$

3.7 Status of $\&$: (Left Premise): Invertible

- Case rule $\mathbf{1}$
- Case rule $\mathbf{!}$
- Case rule $\mathbf{?W}$

$$\frac{h_3 : \vdash \Delta_5, F_1 \& F_2}{\bullet h_3 : \vdash (\Delta_5, F_1 \& F_2), ?F_4} ?W \rightarrow \frac{\overline{h_3 : \vdash \Delta_5, F_1} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_5, F_1, ?F_4} ?W$$

- Case rule $\mathbf{?C}$

$$\frac{h_3 : \vdash \Delta_5, ?F_4, ?F_4, F_1 \& F_2}{\bullet h_3 : \vdash (\Delta_5, F_1 \& F_2), ?F_4} ?C \rightarrow \frac{\overline{h_3 : \vdash \Delta_5, F_1, ?F_4, ?F_4} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_5, F_1, ?F_4} ?C$$

- Case rule $\mathbf{?}$

$$\frac{h_3 : \vdash \Delta_5, F_4, F_1 \& F_2}{\bullet h_3 : \vdash (\Delta_5, F_1 \& F_2), ?F_4} ? \rightarrow \frac{\overline{h_3 : \vdash \Delta_5, F_1, F_4} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_5, F_1, ?F_4} ?$$

- Case rule $\mathbf{\$}$

$$\frac{h_3 : \vdash \Delta_6, F_4, F_5, F_1 \& F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \& F_2), F_4 \$ F_5} \$ \rightarrow \frac{\overline{h_3 : \vdash \Delta_6, F_1, F_4, F_5} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_1, F_4 \$ F_5} \$$$

- Case rule $\mathbf{\&}$

$$\frac{h_3 : \vdash \Delta_6, F_4, F_1 \& F_2 \quad h_3 : \vdash \Delta_6, F_5, F_1 \& F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \& F_2), F_4 \& F_5} \& \rightarrow \frac{\overline{h_3 : \vdash \Delta_6, F_1, F_4} \text{ ax/ind} \quad \overline{h_3 : \vdash \Delta_6, F_1, F_5} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_1, F_4 \& F_5} \&$$

$$\frac{h_1 : \vdash \Delta_2, F_3 \quad h_1 : \vdash \Delta_2, F_4}{\bullet h_1 : \vdash \Delta_2, F_3 \& F_4} \& \rightarrow \frac{\overline{h_1 : \vdash \Delta_2, F_3} \text{ ax}}{\bullet h_1 : \vdash \Delta_2, F_3} \text{H}$$

- Case rule $\mathbf{\oplus_B}$

$$\frac{h_3 : \vdash \Delta_6, F_5, F_1 \& F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \& F_2), F_4 \oplus F_5} \oplus_B \rightarrow \frac{\overline{h_3 : \vdash \Delta_6, F_1, F_5} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_1, F_4 \oplus F_5} \oplus_B$$

- Case rule $\mathbf{\oplus_A}$

$$\frac{h_3 : \vdash \Delta_6, F_4, F_1 \& F_2}{\bullet h_3 : \vdash (\Delta_6, F_1 \& F_2), F_4 \oplus F_5} \oplus_A \rightarrow \frac{\overline{h_3 : \vdash \Delta_6, F_1, F_4} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_6, F_1, F_4 \oplus F_5} \oplus_A$$

- Case rule $\mathbf{\perp}$

$$\frac{h_3 : \vdash \Delta_4, F_1 \& F_2}{\bullet h_3 : \vdash \perp, \Delta_4, F_1 \& F_2} \perp \rightarrow \frac{\overline{h_3 : \vdash \Delta_4, F_1} \text{ ax/ind}}{\bullet h_3 : \vdash \Delta_4, F_1, \perp} \perp$$

- Case rule \top

$$\frac{}{\bullet h_3 \text{ :- } \top, \Delta_4, F_1 \& F_2} \top \rightarrow \frac{}{\bullet h_3 \text{ :- } \Delta_4, F_1, \top} \top$$

- Case rule I

- Case rule \otimes

$$\frac{h_3 \text{ :- } \Delta_7, F_5, F_1 \& F_2 \quad h_3 \text{ :- } \Delta_4, F_6}{\bullet h_3 \text{ :- } (\Delta_7, F_1 \& F_2), \Delta_4, F_5 \otimes F_6} \otimes \rightarrow \frac{\frac{}{h_3 \text{ :- } \Delta_7, F_1, F_5} \text{ ax/ind} \quad \frac{}{h_3 \text{ :- } \Delta_4, F_6} \text{ ax}}{\bullet h_3 \text{ :- } \Delta_4, \Delta_7, F_1, F_5 \otimes F_6} \otimes$$

$$\frac{h_3 \text{ :- } \Delta_4, F_5 \quad h_3 \text{ :- } \Delta_7, F_6, F_1 \& F_2}{\bullet h_3 \text{ :- } \Delta_4, (\Delta_7, F_1 \& F_2), F_5 \otimes F_6} \otimes \rightarrow \frac{\frac{}{h_3 \text{ :- } \Delta_4, F_5} \text{ ax} \quad \frac{}{h_3 \text{ :- } \Delta_7, F_1, F_6} \text{ ax/ind}}{\bullet h_3 \text{ :- } \Delta_4, \Delta_7, F_1, F_5 \otimes F_6} \otimes$$

3.8 Status of $\&$ (Right Premise): : Invertible

- Case rule 1

- Case rule $!$

- Case rule $?W$

$$\frac{h_3 \text{ :- } \Delta_5, F_1 \& F_2}{\bullet h_3 \text{ :- } (\Delta_5, F_1 \& F_2), ?F_4} ?W \rightarrow \frac{\frac{}{h_3 \text{ :- } \Delta_5, F_2} \text{ ax/ind}}{\bullet h_3 \text{ :- } \Delta_5, F_2, ?F_4} ?W$$

- Case rule $?C$

$$\frac{h_3 \text{ :- } \Delta_5, ?F_4, ?F_4, F_1 \& F_2}{\bullet h_3 \text{ :- } (\Delta_5, F_1 \& F_2), ?F_4} ?C \rightarrow \frac{\frac{}{h_3 \text{ :- } \Delta_5, F_2, ?F_4, ?F_4} \text{ ax/ind}}{\bullet h_3 \text{ :- } \Delta_5, F_2, ?F_4} ?C$$

- Case rule $?$

$$\frac{h_3 \text{ :- } \Delta_5, F_4, F_1 \& F_2}{\bullet h_3 \text{ :- } (\Delta_5, F_1 \& F_2), ?F_4} ? \rightarrow \frac{\frac{}{h_3 \text{ :- } \Delta_5, F_2, F_4} \text{ ax/ind}}{\bullet h_3 \text{ :- } \Delta_5, F_2, ?F_4} ?$$

- Case rule $\$$

$$\frac{h_3 \text{ :- } \Delta_6, F_4, F_5, F_1 \& F_2}{\bullet h_3 \text{ :- } (\Delta_6, F_1 \& F_2), F_4 \$ F_5} \$ \rightarrow \frac{\frac{}{h_3 \text{ :- } \Delta_6, F_2, F_4, F_5} \text{ ax/ind}}{\bullet h_3 \text{ :- } \Delta_6, F_2, F_4 \$ F_5} \$$$

- Case rule $\&$

$$\frac{h_3 \text{ :- } \Delta_6, F_4, F_1 \& F_2 \quad h_3 \text{ :- } \Delta_6, F_5, F_1 \& F_2}{\bullet h_3 \text{ :- } (\Delta_6, F_1 \& F_2), F_4 \& F_5} \& \rightarrow \frac{\frac{}{h_3 \text{ :- } \Delta_6, F_2, F_4} \text{ ax/ind} \quad \frac{}{h_3 \text{ :- } \Delta_6, F_2, F_5} \text{ ax/ind}}{\bullet h_3 \text{ :- } \Delta_6, F_2, F_4 \& F_5} \&$$

$$\frac{h_1 \text{ :- } \Delta_2, F_3 \quad h_1 \text{ :- } \Delta_2, F_4}{\bullet h_1 \text{ :- } \Delta_2, F_3 \& F_4} \& \rightarrow \frac{\frac{}{h_1 \text{ :- } \Delta_2, F_4} \text{ ax}}{\bullet h_1 \text{ :- } \Delta_2, F_4} \text{ H}$$

- Case rule \oplus_B

$$\frac{h_3 \text{ :- } \Delta_6, F_5, F_1 \& F_2}{\bullet h_3 \text{ :- } (\Delta_6, F_1 \& F_2), F_4 \oplus F_5} \oplus_B \rightarrow \frac{\overline{h_3 \text{ :- } \Delta_6, F_2, F_5} \text{ ax/ind}}{\bullet h_3 \text{ :- } \Delta_6, F_2, F_4 \oplus F_5} \oplus_B$$

- Case rule \oplus_A

$$\frac{h_3 \text{ :- } \Delta_6, F_4, F_1 \& F_2}{\bullet h_3 \text{ :- } (\Delta_6, F_1 \& F_2), F_4 \oplus F_5} \oplus_A \rightarrow \frac{\overline{h_3 \text{ :- } \Delta_6, F_2, F_4} \text{ ax/ind}}{\bullet h_3 \text{ :- } \Delta_6, F_2, F_4 \oplus F_5} \oplus_A$$

- Case rule \perp

$$\frac{h_3 \text{ :- } \Delta_4, F_1 \& F_2}{\bullet h_3 \text{ :- } \perp, \Delta_4, F_1 \& F_2} \perp \rightarrow \frac{\overline{h_3 \text{ :- } \Delta_4, F_2} \text{ ax/ind}}{\bullet h_3 \text{ :- } \Delta_4, F_2, \perp} \perp$$

- Case rule \top

$$\overline{\bullet h_3 \text{ :- } \top, \Delta_4, F_1 \& F_2} \top \rightarrow \overline{\bullet h_3 \text{ :- } \Delta_4, F_2, \top} \top$$

- Case rule I

- Case rule \otimes

$$\frac{h_3 \text{ :- } \Delta_7, F_5, F_1 \& F_2 \quad h_3 \text{ :- } \Delta_4, F_6}{\bullet h_3 \text{ :- } (\Delta_7, F_1 \& F_2), \Delta_4, F_5 \otimes F_6} \otimes \rightarrow \frac{\overline{h_3 \text{ :- } \Delta_7, F_2, F_5} \text{ ax/ind} \quad \overline{h_3 \text{ :- } \Delta_4, F_6} \text{ ax}}{\bullet h_3 \text{ :- } \Delta_4, \Delta_7, F_2, F_5 \otimes F_6} \otimes$$

$$\frac{h_3 \text{ :- } \Delta_4, F_5 \quad h_3 \text{ :- } \Delta_7, F_6, F_1 \& F_2}{\bullet h_3 \text{ :- } \Delta_4, (\Delta_7, F_1 \& F_2), F_5 \otimes F_6} \otimes \rightarrow \frac{\overline{h_3 \text{ :- } \Delta_4, F_5} \text{ ax} \quad \overline{h_3 \text{ :- } \Delta_7, F_2, F_6} \text{ ax/ind}}{\bullet h_3 \text{ :- } \Delta_4, \Delta_7, F_2, F_5 \otimes F_6} \otimes$$

3.9 Status of \oplus_B : : Non invertible

- Case rule $\mathbf{1}$

- Case rule $!$

- Case rule $?W$

$$\frac{h_3 \text{ :- } \Delta_5, F_1 \oplus F_2}{\bullet h_3 \text{ :- } (\Delta_5, F_1 \oplus F_2), ?F_4} ?W \rightarrow \frac{\overline{h_3 \text{ :- } \Delta_5, F_2} \text{ ax/ind}}{\bullet h_3 \text{ :- } \Delta_5, F_2, ?F_4} ?W$$

- Case rule $?C$

$$\frac{h_3 \text{ :- } \Delta_5, ?F_4, ?F_4, F_1 \oplus F_2}{\bullet h_3 \text{ :- } (\Delta_5, F_1 \oplus F_2), ?F_4} ?C \rightarrow \frac{\overline{h_3 \text{ :- } \Delta_5, F_2, ?F_4, ?F_4} \text{ ax/ind}}{\bullet h_3 \text{ :- } \Delta_5, F_2, ?F_4} ?C$$

- Case rule ?

$$\frac{h_3 \text{ :- } \Delta_5, F_4, F_1 \oplus F_2}{\bullet h_3 \text{ :- } (\Delta_5, F_1 \oplus F_2), ?F_4} ? \rightarrow \frac{\overline{h_3 \text{ :- } \Delta_5, F_2, F_4}}{\bullet h_3 \text{ :- } \Delta_5, F_2, ?F_4} \text{ ax/ind} ?$$

- Case rule \$

$$\frac{h_3 \text{ :- } \Delta_6, F_4, F_5, F_1 \oplus F_2}{\bullet h_3 \text{ :- } (\Delta_6, F_1 \oplus F_2), F_4 \$ F_5} \$ \rightarrow \frac{\overline{h_3 \text{ :- } \Delta_6, F_2, F_4, F_5}}{\bullet h_3 \text{ :- } \Delta_6, F_2, F_4 \$ F_5} \text{ ax/ind} \$$$

- Case rule &

$$\frac{h_3 \text{ :- } \Delta_6, F_4, F_1 \oplus F_2 \quad h_3 \text{ :- } \Delta_6, F_5, F_1 \oplus F_2}{\bullet h_3 \text{ :- } (\Delta_6, F_1 \oplus F_2), F_4 \& F_5} \& \rightarrow \frac{\overline{h_3 \text{ :- } \Delta_6, F_2, F_4} \text{ ax/ind} \quad \overline{h_3 \text{ :- } \Delta_6, F_2, F_5} \text{ ax/ind}}{\bullet h_3 \text{ :- } \Delta_6, F_2, F_4 \& F_5} \&$$

- Case rule \oplus_B

$$\frac{h_3 \text{ :- } \Delta_6, F_5, F_1 \oplus F_2}{\bullet h_3 \text{ :- } (\Delta_6, F_1 \oplus F_2), F_4 \oplus F_5} \oplus_B \rightarrow \frac{\overline{h_3 \text{ :- } \Delta_6, F_2, F_5} \text{ ax/ind}}{\bullet h_3 \text{ :- } \Delta_6, F_2, F_4 \oplus F_5} \oplus_B$$

$$\frac{h_1 \text{ :- } \Delta_2, F_4}{\bullet h_1 \text{ :- } \Delta_2, F_3 \oplus F_4} \oplus_B \rightarrow \frac{\overline{h_1 \text{ :- } \Delta_2, F_4} \text{ ax}}{\bullet h_1 \text{ :- } \Delta_2, F_4} \text{ H}$$

- Case rule \oplus_A

$$\frac{h_3 \text{ :- } \Delta_6, F_4, F_1 \oplus F_2}{\bullet h_3 \text{ :- } (\Delta_6, F_1 \oplus F_2), F_4 \oplus F_5} \oplus_A \rightarrow \frac{\overline{h_3 \text{ :- } \Delta_6, F_2, F_4} \text{ ax/ind}}{\bullet h_3 \text{ :- } \Delta_6, F_2, F_4 \oplus F_5} \oplus_A$$

$$\frac{h_1 \text{ :- } \Delta_2, F_3}{\bullet h_1 \text{ :- } \Delta_2, F_3 \oplus F_4} \oplus_A \rightarrow \bullet h_1 \text{ :- } \Delta_2, F_4 \text{ fail}$$

- Case rule \perp

$$\frac{h_3 \text{ :- } \Delta_4, F_1 \oplus F_2}{\bullet h_3 \text{ :- } \perp, \Delta_4, F_1 \oplus F_2} \perp \rightarrow \frac{\overline{h_3 \text{ :- } \Delta_4, F_2} \text{ ax/ind}}{\bullet h_3 \text{ :- } \Delta_4, F_2, \perp} \perp$$

- Case rule \top

$$\overline{\bullet h_3 \text{ :- } \top, \Delta_4, F_1 \oplus F_2} \top \rightarrow \overline{\bullet h_3 \text{ :- } \Delta_4, F_2, \top} \top$$

- Case rule I

- Case rule \otimes

$$\frac{h_3 \text{ :- } \Delta_7, F_5, F_1 \oplus F_2 \quad h_3 \text{ :- } \Delta_4, F_6}{\bullet h_3 \text{ :- } (\Delta_7, F_1 \oplus F_2), \Delta_4, F_5 \otimes F_6} \otimes \rightarrow \frac{\overline{h_3 \text{ :- } \Delta_7, F_2, F_5} \text{ ax/ind} \quad \overline{h_3 \text{ :- } \Delta_4, F_6} \text{ ax}}{\bullet h_3 \text{ :- } \Delta_4, \Delta_7, F_2, F_5 \otimes F_6} \otimes$$

$$\frac{h_3 \text{ :- } \Delta_4, F_5 \quad h_3 \text{ :- } \Delta_7, F_6, F_1 \oplus F_2}{\bullet h_3 \text{ :- } \Delta_4, (\Delta_7, F_1 \oplus F_2), F_5 \otimes F_6} \otimes \rightarrow \frac{\overline{h_3 \text{ :- } \Delta_4, F_5} \text{ ax} \quad \overline{h_3 \text{ :- } \Delta_7, F_2, F_6} \text{ ax/ind}}{\bullet h_3 \text{ :- } \Delta_4, \Delta_7, F_2, F_5 \otimes F_6} \otimes$$

3.10 Status of \oplus_A : : Non invertible

- Case rule 1
- Case rule !
- Case rule ?W

$$\frac{h_3 \text{ :- } \Delta_5, F_1 \oplus F_2}{\bullet h_3 \text{ :- } (\Delta_5, F_1 \oplus F_2), ?F_4} ?W \rightarrow \frac{\overline{h_3 \text{ :- } \Delta_5, F_1} \text{ ax/ind}}{\bullet h_3 \text{ :- } \Delta_5, F_1, ?F_4} ?W$$

- Case rule ?C

$$\frac{h_3 \text{ :- } \Delta_5, ?F_4, ?F_4, F_1 \oplus F_2}{\bullet h_3 \text{ :- } (\Delta_5, F_1 \oplus F_2), ?F_4} ?C \rightarrow \frac{\overline{h_3 \text{ :- } \Delta_5, F_1, ?F_4, ?F_4} \text{ ax/ind}}{\bullet h_3 \text{ :- } \Delta_5, F_1, ?F_4} ?C$$

- Case rule ?

$$\frac{h_3 \text{ :- } \Delta_5, F_4, F_1 \oplus F_2}{\bullet h_3 \text{ :- } (\Delta_5, F_1 \oplus F_2), ?F_4} ? \rightarrow \frac{\overline{h_3 \text{ :- } \Delta_5, F_1, F_4} \text{ ax/ind}}{\bullet h_3 \text{ :- } \Delta_5, F_1, ?F_4} ?$$

- Case rule \$

$$\frac{h_3 \text{ :- } \Delta_6, F_4, F_5, F_1 \oplus F_2}{\bullet h_3 \text{ :- } (\Delta_6, F_1 \oplus F_2), F_4 \$ F_5} \$ \rightarrow \frac{\overline{h_3 \text{ :- } \Delta_6, F_1, F_4, F_5} \text{ ax/ind}}{\bullet h_3 \text{ :- } \Delta_6, F_1, F_4 \$ F_5} \$$$

- Case rule &

$$\frac{h_3 \text{ :- } \Delta_6, F_4, F_1 \oplus F_2}{\bullet h_3 \text{ :- } (\Delta_6, F_1 \oplus F_2), F_4 \& F_5} \& \rightarrow \frac{\overline{h_3 \text{ :- } \Delta_6, F_1, F_4} \text{ ax/ind} \quad \overline{h_3 \text{ :- } \Delta_6, F_1, F_5} \text{ ax/ind}}{\bullet h_3 \text{ :- } \Delta_6, F_1, F_4 \& F_5} \&$$

- Case rule \oplus_B

$$\frac{h_3 \text{ :- } \Delta_6, F_5, F_1 \oplus F_2}{\bullet h_3 \text{ :- } (\Delta_6, F_1 \oplus F_2), F_4 \oplus F_5} \oplus_B \rightarrow \frac{\overline{h_3 \text{ :- } \Delta_6, F_1, F_5} \text{ ax/ind}}{\bullet h_3 \text{ :- } \Delta_6, F_1, F_4 \oplus F_5} \oplus_B$$

$$\frac{h_1 \text{ :- } \Delta_2, F_4}{\bullet h_1 \text{ :- } \Delta_2, F_3 \oplus F_4} \oplus_B \rightarrow \overline{h_1 \text{ :- } \Delta_2, F_3} \text{ fail}$$

- Case rule \oplus_A

$$\frac{h_3 \text{ :- } \Delta_6, F_4, F_1 \oplus F_2}{\bullet h_3 \text{ :- } (\Delta_6, F_1 \oplus F_2), F_4 \oplus F_5} \oplus_A \rightarrow \frac{\overline{h_3 \text{ :- } \Delta_6, F_1, F_4} \text{ ax/ind}}{\bullet h_3 \text{ :- } \Delta_6, F_1, F_4 \oplus F_5} \oplus_A$$

$$\frac{h_1 \text{ :- } \Delta_2, F_3}{\bullet h_1 \text{ :- } \Delta_2, F_3 \oplus F_4} \oplus_A \rightarrow \frac{\overline{h_1 \text{ :- } \Delta_2, F_3} \text{ ax}}{\bullet h_1 \text{ :- } \Delta_2, F_3} \text{ H}$$

- Case rule \perp

$$\frac{h_3 \vdash \Delta_4, F_1 \oplus F_2}{\bullet h_3 \vdash \perp, \Delta_4, F_1 \oplus F_2} \perp \rightarrow \frac{\overline{h_3 \vdash \Delta_4, F_1} \text{ ax/ind}}{\bullet h_3 \vdash \Delta_4, F_1, \perp} \perp$$

- Case rule \top

$$\frac{}{\bullet h_3 \vdash \top, \Delta_4, F_1 \oplus F_2} \top \rightarrow \frac{}{\bullet h_3 \vdash \Delta_4, F_1, \top} \top$$

- Case rule I

- Case rule \otimes

$$\frac{h_3 \vdash \Delta_7, F_5, F_1 \oplus F_2 \quad h_3 \vdash \Delta_4, F_6}{\bullet h_3 \vdash (\Delta_7, F_1 \oplus F_2), \Delta_4, F_5 \otimes F_6} \otimes \rightarrow \frac{\overline{h_3 \vdash \Delta_7, F_1, F_5} \text{ ax/ind} \quad \overline{h_3 \vdash \Delta_4, F_6} \text{ ax}}{\bullet h_3 \vdash \Delta_4, \Delta_7, F_1, F_5 \otimes F_6} \otimes$$

$$\frac{h_3 \vdash \Delta_4, F_5 \quad h_3 \vdash \Delta_7, F_6, F_1 \oplus F_2}{\bullet h_3 \vdash \Delta_4, (\Delta_7, F_1 \oplus F_2), F_5 \otimes F_6} \otimes \rightarrow \frac{\overline{h_3 \vdash \Delta_4, F_5} \text{ ax} \quad \overline{h_3 \vdash \Delta_7, F_1, F_6} \text{ ax/ind}}{\bullet h_3 \vdash \Delta_4, \Delta_7, F_1, F_5 \otimes F_6} \otimes$$

3.11 Status of \perp : Invertible

- Case rule 1

- Case rule $!$

- Case rule $?W$

$$\frac{h_1 \vdash \perp, \Delta_3}{\bullet h_1 \vdash (\perp, \Delta_3), ?F_2} ?W \rightarrow \frac{\overline{h_1 \vdash \Delta_3} \text{ ax/ind}}{\bullet h_1 \vdash \Delta_3, ?F_2} ?W$$

- Case rule $?C$

$$\frac{h_1 \vdash \perp, \Delta_3, ?F_2, ?F_2}{\bullet h_1 \vdash (\perp, \Delta_3), ?F_2} ?C \rightarrow \frac{\overline{h_1 \vdash \Delta_3, ?F_2, ?F_2} \text{ ax/ind}}{\bullet h_1 \vdash \Delta_3, ?F_2} ?C$$

- Case rule $?$

$$\frac{h_1 \vdash \perp, \Delta_3, F_2}{\bullet h_1 \vdash (\perp, \Delta_3), ?F_2} ? \rightarrow \frac{\overline{h_1 \vdash \Delta_3, F_2} \text{ ax/ind}}{\bullet h_1 \vdash \Delta_3, ?F_2} ?$$

- Case rule $\$$

$$\frac{h_1 \vdash \perp, \Delta_4, F_2, F_3}{\bullet h_1 \vdash (\perp, \Delta_4), F_2 \$F_3} \$ \rightarrow \frac{\overline{h_1 \vdash \Delta_4, F_2, F_3} \text{ ax/ind}}{\bullet h_1 \vdash \Delta_4, F_2 \$F_3} \$$$

- Case rule $\&$

$$\frac{\frac{h_1 : \vdash \perp, \Delta_4, F_2}{\bullet h_1 : \vdash (\perp, \Delta_4), F_2 \& F_3} \quad \frac{h_1 : \vdash \perp, \Delta_4, F_3}{\bullet h_1 : \vdash (\perp, \Delta_4), F_2 \& F_3}}{\bullet h_1 : \vdash (\perp, \Delta_4), F_2 \& F_3} \& \rightarrow \frac{\frac{h_1 : \vdash \Delta_4, F_2}{\bullet h_1 : \vdash \Delta_4, F_2 \& F_3} \text{ ax/ind} \quad \frac{h_1 : \vdash \Delta_4, F_3}{\bullet h_1 : \vdash \Delta_4, F_2 \& F_3} \text{ ax/ind}}{\bullet h_1 : \vdash \Delta_4, F_2 \& F_3} \&$$

- Case rule \oplus_B

$$\frac{\frac{h_1 : \vdash \perp, \Delta_4, F_3}{\bullet h_1 : \vdash (\perp, \Delta_4), F_2 \oplus F_3} \oplus_B}{\bullet h_1 : \vdash (\perp, \Delta_4), F_2 \oplus F_3} \oplus_B \rightarrow \frac{\frac{h_1 : \vdash \Delta_4, F_3}{\bullet h_1 : \vdash \Delta_4, F_2 \oplus F_3} \text{ ax/ind}}{\bullet h_1 : \vdash \Delta_4, F_2 \oplus F_3} \oplus_B$$

- Case rule \oplus_A

$$\frac{\frac{h_1 : \vdash \perp, \Delta_4, F_2}{\bullet h_1 : \vdash (\perp, \Delta_4), F_2 \oplus F_3} \oplus_A}{\bullet h_1 : \vdash (\perp, \Delta_4), F_2 \oplus F_3} \oplus_A \rightarrow \frac{\frac{h_1 : \vdash \Delta_4, F_2}{\bullet h_1 : \vdash \Delta_4, F_2 \oplus F_3} \text{ ax/ind}}{\bullet h_1 : \vdash \Delta_4, F_2 \oplus F_3} \oplus_A$$

- Case rule \perp

$$\frac{\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \rightarrow \frac{\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \Delta_2} \text{ ax}}{\bullet h_1 : \vdash \Delta_2} \text{ H}$$

- Case rule \top

$$\frac{\frac{h_1 : \vdash \top, \perp, \Delta_2}{\bullet h_1 : \vdash \top, \perp, \Delta_2} \top}{\bullet h_1 : \vdash \top, \perp, \Delta_2} \top \rightarrow \frac{\frac{h_1 : \vdash \Delta_2, \top}{\bullet h_1 : \vdash \Delta_2, \top} \top}{\bullet h_1 : \vdash \Delta_2, \top} \top$$

- Case rule I

- Case rule \otimes

$$\frac{\frac{h_1 : \vdash \perp, \Delta_5, F_3}{\bullet h_1 : \vdash (\perp, \Delta_5), \Delta_2, F_3 \otimes F_4} \otimes \quad \frac{h_1 : \vdash \Delta_2, F_4}{\bullet h_1 : \vdash \Delta_2, \Delta_5, F_3 \otimes F_4} \otimes}{\bullet h_1 : \vdash (\perp, \Delta_5), \Delta_2, F_3 \otimes F_4} \otimes \rightarrow \frac{\frac{h_1 : \vdash \Delta_5, F_3}{\bullet h_1 : \vdash \Delta_2, \Delta_5, F_3 \otimes F_4} \text{ ax/ind} \quad \frac{h_1 : \vdash \Delta_2, F_4}{\bullet h_1 : \vdash \Delta_2, \Delta_5, F_3 \otimes F_4} \text{ ax}}{\bullet h_1 : \vdash \Delta_2, \Delta_5, F_3 \otimes F_4} \otimes$$

$$\frac{\frac{h_1 : \vdash \Delta_2, F_3}{\bullet h_1 : \vdash \Delta_2, (\perp, \Delta_5), F_3 \otimes F_4} \otimes \quad \frac{h_1 : \vdash \perp, \Delta_5, F_4}{\bullet h_1 : \vdash \Delta_2, (\perp, \Delta_5), F_3 \otimes F_4} \otimes}{\bullet h_1 : \vdash \Delta_2, (\perp, \Delta_5), F_3 \otimes F_4} \otimes \rightarrow \frac{\frac{h_1 : \vdash \Delta_2, F_3}{\bullet h_1 : \vdash \Delta_2, \Delta_5, F_3 \otimes F_4} \text{ ax} \quad \frac{h_1 : \vdash \Delta_5, F_4}{\bullet h_1 : \vdash \Delta_2, \Delta_5, F_3 \otimes F_4} \text{ ax/ind}}{\bullet h_1 : \vdash \Delta_2, \Delta_5, F_3 \otimes F_4} \otimes$$

3.12 Status of \top : Invertible

- Case rule 1

- Case rule $!$

- Case rule $?W$

$$\frac{\frac{h_1 : \vdash \top, \Delta_3}{\bullet h_1 : \vdash (\top, \Delta_3), ?F_2} ?W}{\bullet h_1 : \vdash (\top, \Delta_3), ?F_2} ?W \rightarrow \text{trivial}$$

- Case rule $?C$

$$\frac{h_1 : \vdash \top, \Delta_3, ?F_2, ?F_2}{\bullet h_1 : \vdash (\top, \Delta_3), ?F_2} ?C \rightarrow \text{trivial}$$

- Case rule $?$

$$\frac{h_1 : \vdash \top, \Delta_3, F_2}{\bullet h_1 : \vdash (\top, \Delta_3), ?F_2} ? \rightarrow \text{trivial}$$

- Case rule $\$$

$$\frac{h_1 : \vdash \top, \Delta_4, F_2, F_3}{\bullet h_1 : \vdash (\top, \Delta_4), F_2 \$ F_3} \$ \rightarrow \text{trivial}$$

- Case rule $\&$

$$\frac{h_1 : \vdash \top, \Delta_4, F_2 \quad h_1 : \vdash \top, \Delta_4, F_3}{\bullet h_1 : \vdash (\top, \Delta_4), F_2 \& F_3} \& \rightarrow \text{trivial}$$

- Case rule \oplus_B

$$\frac{h_1 : \vdash \top, \Delta_4, F_3}{\bullet h_1 : \vdash (\top, \Delta_4), F_2 \oplus_B F_3} \oplus_B \rightarrow \text{trivial}$$

- Case rule \oplus_A

$$\frac{h_1 : \vdash \top, \Delta_4, F_2}{\bullet h_1 : \vdash (\top, \Delta_4), F_2 \oplus_A F_3} \oplus_A \rightarrow \text{trivial}$$

- Case rule \perp

$$\frac{h_1 : \vdash \top, \Delta_2}{\bullet h_1 : \vdash \perp, \top, \Delta_2} \perp \rightarrow \text{trivial}$$

- Case rule \top

$$\frac{}{\bullet h_1 : \vdash \top, \Delta_2} \top \rightarrow \text{trivial}$$

- Case rule I

- Case rule \otimes

$$\frac{h_1 : \vdash \top, \Delta_5, F_3 \quad h_1 : \vdash \Delta_2, F_4}{\bullet h_1 : \vdash (\top, \Delta_5), \Delta_2, F_3 \otimes F_4} \otimes \rightarrow \text{trivial}$$

$$\frac{h_1 : \vdash \Delta_2, F_3 \quad h_1 : \vdash \top, \Delta_5, F_4}{\bullet h_1 : \vdash \Delta_2, (\top, \Delta_5), F_3 \otimes F_4} \otimes \rightarrow \text{trivial}$$

3.13 Status of I : : Invertible

- Case rule $\mathbf{1}$
- Case rule $!$
- Case rule $?W$
- Case rule $?C$
- Case rule $?$
- Case rule $\$$
- Case rule $\&$
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I

$$\frac{}{\bullet h_1 : \vdash p(n_2), \neg(n_2)} I \rightarrow \text{trivial}$$

- Case rule \otimes

3.14 Status of \otimes : (Left Premise): Non invertible

- Case rule $\mathbf{1}$
- Case rule $!$
- Case rule $?W$

$$\frac{h_4 : \vdash \Delta_1, \Delta_6, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_6, F_2 \otimes F_3), ?F_5} ?W \rightarrow \frac{\frac{}{h_4 : \vdash \Delta_6, F_2} \text{ax/ind}}{\bullet h_4 : \vdash \Delta_6, F_2, ?F_5} ?W$$

$$\frac{h_4 : \vdash \Delta_1, \Delta_6, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_6, F_2 \otimes F_3), ?F_5} ?W \rightarrow \frac{\frac{}{h_4 : \vdash \Delta_1, F_2} \text{ax/ind}}{\bullet h_4 : \vdash \Delta_1, F_2} H$$

- Case rule $?C$

$$\frac{h_4 \vdash \Delta_1, \Delta_6, ?F_5, ?F_5, F_2 \otimes F_3}{\bullet h_4 \vdash (\Delta_1, \Delta_6, F_2 \otimes F_3), ?F_5} ?C \rightarrow \frac{\overline{h_4 \vdash \Delta_6, F_2}}{\bullet h_4 \vdash \Delta_6, F_2, ?F_5} \text{ax/ind} ?W$$

$$\frac{h_4 \vdash \Delta_1, \Delta_6, ?F_5, ?F_5, F_2 \otimes F_3}{\bullet h_4 \vdash (\Delta_1, \Delta_6, F_2 \otimes F_3), ?F_5} ?C \rightarrow \frac{\overline{h_4 \vdash \Delta_1, F_2}}{\bullet h_4 \vdash \Delta_1, F_2} \text{ax/ind} H$$

- Case rule $?$

$$\frac{h_4 \vdash \Delta_1, \Delta_6, F_5, F_2 \otimes F_3}{\bullet h_4 \vdash (\Delta_1, \Delta_6, F_2 \otimes F_3), ?F_5} ? \rightarrow \frac{\overline{h_4 \vdash \Delta_6, F_2}}{\bullet h_4 \vdash \Delta_6, F_2, ?F_5} \text{ax/ind} ?W$$

$$\frac{h_4 \vdash \Delta_1, \Delta_6, F_5, F_2 \otimes F_3}{\bullet h_4 \vdash (\Delta_1, \Delta_6, F_2 \otimes F_3), ?F_5} ? \rightarrow \frac{\overline{h_4 \vdash \Delta_1, F_2}}{\bullet h_4 \vdash \Delta_1, F_2} \text{ax/ind} H$$

- Case rule $\$$

$$\frac{h_4 \vdash \Delta_1, \Delta_7, F_5, F_6, F_2 \otimes F_3}{\bullet h_4 \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \$F_6} \$ \rightarrow \frac{\overline{h_4 \vdash \Delta_7, F_2, F_5, F_6}}{\bullet h_4 \vdash \Delta_7, F_2, F_5 \$F_6} \text{ax/ind} \$$$

$$\frac{h_4 \vdash \Delta_1, \Delta_7, F_5, F_6, F_2 \otimes F_3}{\bullet h_4 \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \$F_6} \$ \rightarrow \frac{\overline{h_4 \vdash \Delta_1, F_2}}{\bullet h_4 \vdash \Delta_1, F_2} \text{ax/ind} H$$

- Case rule $\&$

$$\frac{h_4 \vdash \Delta_1, \Delta_7, F_5, F_2 \otimes F_3 \quad h_4 \vdash \Delta_1, \Delta_7, F_6, F_2 \otimes F_3}{\bullet h_4 \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \&F_6} \& \rightarrow \frac{\overline{h_4 \vdash \Delta_7, F_2, F_5} \quad \overline{h_4 \vdash \Delta_7, F_2, F_6}}{\bullet h_4 \vdash \Delta_7, F_2, F_5 \&F_6} \text{ax/ind} \&$$

$$\frac{h_4 \vdash \Delta_1, \Delta_7, F_5, F_2 \otimes F_3 \quad h_4 \vdash \Delta_1, \Delta_7, F_6, F_2 \otimes F_3}{\bullet h_4 \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \&F_6} \& \rightarrow \frac{\overline{h_4 \vdash \Delta_1, F_2}}{\bullet h_4 \vdash \Delta_1, F_2} \text{ax/ind} H$$

- Case rule \oplus_B

$$\frac{h_4 \vdash \Delta_1, \Delta_7, F_6, F_2 \otimes F_3}{\bullet h_4 \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \oplus F_6} \oplus_B \rightarrow \frac{\overline{h_4 \vdash \Delta_7, F_2, F_6}}{\bullet h_4 \vdash \Delta_7, F_2, F_5 \oplus F_6} \text{ax/ind} \oplus_B$$

$$\frac{h_4 \vdash \Delta_1, \Delta_7, F_6, F_2 \otimes F_3}{\bullet h_4 \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \oplus F_6} \oplus_B \rightarrow \frac{\overline{h_4 \vdash \Delta_1, F_2}}{\bullet h_4 \vdash \Delta_1, F_2} \text{ax/ind} H$$

- Case rule \oplus_A

$$\frac{h_4 \vdash \Delta_1, \Delta_7, F_5, F_2 \otimes F_3}{\bullet h_4 \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \oplus F_6} \oplus_A \rightarrow \frac{\overline{h_4 \vdash \Delta_7, F_2, F_5}}{\bullet h_4 \vdash \Delta_7, F_2, F_5 \oplus F_6} \text{ax/ind} \oplus_A$$

$$\frac{h_4 \vdash \Delta_1, \Delta_7, F_5, F_2 \otimes F_3}{\bullet h_4 \vdash (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \oplus F_6} \oplus_A \rightarrow \frac{\overline{h_4 \vdash \Delta_1, F_2}}{\bullet h_4 \vdash \Delta_1, F_2} \text{ax/ind} H$$

- Case rule \perp

$$\frac{h_4 : \vdash \Delta_1, \Delta_5, F_2 \otimes F_3}{\bullet h_4 : \vdash \perp, \Delta_1, \Delta_5, F_2 \otimes F_3} \perp \rightarrow \frac{h_4 : \vdash \Delta_5, F_2}{\bullet h_4 : \vdash \Delta_5, F_2, \perp} \perp \text{ ax/ind}$$

$$\frac{h_4 : \vdash \Delta_1, \Delta_5, F_2 \otimes F_3}{\bullet h_4 : \vdash \perp, \Delta_1, \Delta_5, F_2 \otimes F_3} \perp \rightarrow \frac{h_4 : \vdash \Delta_1, F_2}{\bullet h_4 : \vdash \Delta_1, F_2} \text{ ax/ind}$$

- Case rule \top

$$\frac{}{\bullet h_4 : \vdash \top, \Delta_1, \Delta_5, F_2 \otimes F_3} \top \rightarrow \frac{}{\bullet h_4 : \vdash \Delta_5, F_2, \top} \top$$

$$\frac{}{\bullet h_4 : \vdash \top, \Delta_1, \Delta_5, F_2 \otimes F_3} \top \rightarrow \frac{}{\bullet h_4 : \vdash \Delta_1, F_2} \text{ fail}$$

- Case rule I

- Case rule \otimes

$$\frac{h_3 : \vdash \Delta_6, \Delta_7, F_4, F_1 \otimes F_2 \quad h_3 : \vdash \Delta_8, \Delta_9, F_5}{\bullet h_3 : \vdash (\Delta_6, \Delta_7, F_1 \otimes F_2), (\Delta_8, \Delta_9), F_4 \otimes F_5} \otimes \rightarrow \frac{}{\bullet h_3 : \vdash \Delta_6, \Delta_8, F_1, F_4 \otimes F_5} \text{ fail}$$

$$\frac{h_3 : \vdash \Delta_6, \Delta_7, F_4 \quad h_3 : \vdash \Delta_8, \Delta_9, F_5, F_1 \otimes F_2}{\bullet h_3 : \vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, F_1 \otimes F_2), F_4 \otimes F_5} \otimes \rightarrow \frac{}{\bullet h_3 : \vdash \Delta_6, \Delta_8, F_1, F_4 \otimes F_5} \text{ fail}$$

$$\frac{h_3 : \vdash \Delta_6, \Delta_7, F_4, F_1 \otimes F_2 \quad h_3 : \vdash \Delta_8, \Delta_9, F_5}{\bullet h_3 : \vdash (\Delta_6, \Delta_7, F_1 \otimes F_2), (\Delta_8, \Delta_9), F_4 \otimes F_5} \otimes \rightarrow \frac{}{\bullet h_3 : \vdash \Delta_6, \Delta_8, F_1} \text{ fail}$$

$$\frac{h_3 : \vdash \Delta_6, \Delta_7, F_4 \quad h_3 : \vdash \Delta_8, \Delta_9, F_5, F_1 \otimes F_2}{\bullet h_3 : \vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, F_1 \otimes F_2), F_4 \otimes F_5} \otimes \rightarrow \frac{}{\bullet h_3 : \vdash \Delta_6, \Delta_8, F_1} \text{ fail}$$

$$\frac{h_1 : \vdash \Delta_4, \Delta_5, F_2 \quad h_1 : \vdash \Delta_6, \Delta_7, F_3}{\bullet h_1 : \vdash (\Delta_4, \Delta_5), (\Delta_6, \Delta_7), F_2 \otimes F_3} \otimes \rightarrow \frac{}{\bullet h_1 : \vdash \Delta_4, \Delta_6, F_2} \text{ fail}$$

3.15 Status of \otimes (Right Premise): : Non invertible

- Case rule $\mathbf{1}$
- Case rule $!$
- Case rule $?W$

$$\frac{h_4 : \vdash \Delta_1, \Delta_6, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_6, F_2 \otimes F_3), ?F_5} ?W \rightarrow \frac{h_4 : \vdash \Delta_1, F_3}{\bullet h_4 : \vdash \Delta_1, F_3} \text{ ax/ind}$$

$$\frac{h_4 : \vdash \Delta_1, \Delta_6, F_2 \otimes F_3}{\bullet h_4 : \vdash (\Delta_1, \Delta_6, F_2 \otimes F_3), ?F_5} ?W \rightarrow \frac{h_4 : \vdash \Delta_6, F_3}{\bullet h_4 : \vdash \Delta_6, F_3, ?F_5} \text{ ax/ind}$$

- Case rule $?C$

$$\frac{h_4 \text{ :- } \Delta_1, \Delta_6, ?F_5, ?F_5, F_2 \otimes F_3}{\bullet h_4 \text{ :- } (\Delta_1, \Delta_6, F_2 \otimes F_3), ?F_5} ?C \rightarrow \frac{\frac{h_4 \text{ :- } \Delta_1, F_3}{\bullet h_4 \text{ :- } \Delta_1, F_3} \text{ ax/ind}}{H}$$

$$\frac{h_4 \text{ :- } \Delta_1, \Delta_6, ?F_5, ?F_5, F_2 \otimes F_3}{\bullet h_4 \text{ :- } (\Delta_1, \Delta_6, F_2 \otimes F_3), ?F_5} ?C \rightarrow \frac{\frac{h_4 \text{ :- } \Delta_6, F_3}{\bullet h_4 \text{ :- } \Delta_6, F_3, ?F_5} \text{ ax/ind}}{?W}$$

- Case rule $?$

$$\frac{h_4 \text{ :- } \Delta_1, \Delta_6, F_5, F_2 \otimes F_3}{\bullet h_4 \text{ :- } (\Delta_1, \Delta_6, F_2 \otimes F_3), ?F_5} ? \rightarrow \frac{\frac{h_4 \text{ :- } \Delta_1, F_3}{\bullet h_4 \text{ :- } \Delta_1, F_3} \text{ ax/ind}}{H}$$

$$\frac{h_4 \text{ :- } \Delta_1, \Delta_6, F_5, F_2 \otimes F_3}{\bullet h_4 \text{ :- } (\Delta_1, \Delta_6, F_2 \otimes F_3), ?F_5} ? \rightarrow \frac{\frac{h_4 \text{ :- } \Delta_6, F_3}{\bullet h_4 \text{ :- } \Delta_6, F_3, ?F_5} \text{ ax/ind}}{?W}$$

- Case rule $\$$

$$\frac{h_4 \text{ :- } \Delta_1, \Delta_7, F_5, F_6, F_2 \otimes F_3}{\bullet h_4 \text{ :- } (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \$F_6} \$ \rightarrow \frac{\frac{h_4 \text{ :- } \Delta_1, F_3}{\bullet h_4 \text{ :- } \Delta_1, F_3} \text{ ax/ind}}{H}$$

$$\frac{h_4 \text{ :- } \Delta_1, \Delta_7, F_5, F_6, F_2 \otimes F_3}{\bullet h_4 \text{ :- } (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \$F_6} \$ \rightarrow \frac{\frac{h_4 \text{ :- } \Delta_7, F_3, F_5, F_6}{\bullet h_4 \text{ :- } \Delta_7, F_3, F_5 \$F_6} \text{ ax/ind}}{\$}$$

- Case rule $\&$

$$\frac{h_4 \text{ :- } \Delta_1, \Delta_7, F_5, F_2 \otimes F_3 \quad h_4 \text{ :- } \Delta_1, \Delta_7, F_6, F_2 \otimes F_3}{\bullet h_4 \text{ :- } (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \&F_6} \& \rightarrow \frac{\frac{h_4 \text{ :- } \Delta_1, F_3}{\bullet h_4 \text{ :- } \Delta_1, F_3} \text{ ax/ind}}{H}$$

$$\frac{\frac{h_4 \text{ :- } \Delta_1, \Delta_7, F_5, F_2 \otimes F_3 \quad h_4 \text{ :- } \Delta_1, \Delta_7, F_6, F_2 \otimes F_3}{\bullet h_4 \text{ :- } (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \&F_6} \&}{\bullet h_4 \text{ :- } \Delta_7, F_3, F_5} \text{ ax/ind} \rightarrow \frac{\frac{h_4 \text{ :- } \Delta_7, F_3, F_5}{\bullet h_4 \text{ :- } \Delta_7, F_3, F_5 \&F_6} \text{ ax/ind}}{\&}$$

- Case rule \oplus_B

$$\frac{h_4 \text{ :- } \Delta_1, \Delta_7, F_6, F_2 \otimes F_3}{\bullet h_4 \text{ :- } (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \oplus F_6} \oplus_B \rightarrow \frac{\frac{h_4 \text{ :- } \Delta_1, F_3}{\bullet h_4 \text{ :- } \Delta_1, F_3} \text{ ax/ind}}{H}$$

$$\frac{h_4 \text{ :- } \Delta_1, \Delta_7, F_6, F_2 \otimes F_3}{\bullet h_4 \text{ :- } (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \oplus F_6} \oplus_B \rightarrow \frac{\frac{h_4 \text{ :- } \Delta_7, F_3, F_6}{\bullet h_4 \text{ :- } \Delta_7, F_3, F_5 \oplus F_6} \text{ ax/ind}}{\oplus_B}$$

- Case rule \oplus_A

$$\frac{h_4 \text{ :- } \Delta_1, \Delta_7, F_5, F_2 \otimes F_3}{\bullet h_4 \text{ :- } (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \oplus F_6} \oplus_A \rightarrow \frac{\frac{h_4 \text{ :- } \Delta_1, F_3}{\bullet h_4 \text{ :- } \Delta_1, F_3} \text{ ax/ind}}{H}$$

$$\frac{h_4 \text{ :- } \Delta_1, \Delta_7, F_5, F_2 \otimes F_3}{\bullet h_4 \text{ :- } (\Delta_1, \Delta_7, F_2 \otimes F_3), F_5 \oplus F_6} \oplus_A \rightarrow \frac{\frac{h_4 \text{ :- } \Delta_7, F_3, F_5}{\bullet h_4 \text{ :- } \Delta_7, F_3, F_5 \oplus F_6} \text{ ax/ind}}{\oplus_A}$$

- Case rule \perp

$$\frac{h_4 : \vdash \Delta_1, \Delta_5, F_2 \otimes F_3}{\bullet h_4 : \vdash \perp, \Delta_1, \Delta_5, F_2 \otimes F_3} \perp \rightarrow \frac{h_4 : \vdash \Delta_1, F_3}{\bullet h_4 : \vdash \Delta_1, F_3} \text{ax/ind} \text{ H}$$

$$\frac{h_4 : \vdash \Delta_1, \Delta_5, F_2 \otimes F_3}{\bullet h_4 : \vdash \perp, \Delta_1, \Delta_5, F_2 \otimes F_3} \perp \rightarrow \frac{h_4 : \vdash \Delta_5, F_3}{\bullet h_4 : \vdash \Delta_5, F_3, \perp} \text{ax/ind} \perp$$

- Case rule \top

$$\frac{}{\bullet h_4 : \vdash \top, \Delta_1, \Delta_5, F_2 \otimes F_3} \top \rightarrow \frac{}{\bullet h_4 : \vdash \Delta_1, F_3} \text{fail}$$

$$\frac{}{\bullet h_4 : \vdash \top, \Delta_1, \Delta_5, F_2 \otimes F_3} \top \rightarrow \frac{}{\bullet h_4 : \vdash \Delta_5, F_3, \top} \top$$

- Case rule I

- Case rule \otimes

$$\frac{h_3 : \vdash \Delta_6, \Delta_7, F_4, F_1 \otimes F_2 \quad h_3 : \vdash \Delta_8, \Delta_9, F_5}{\bullet h_3 : \vdash (\Delta_6, \Delta_7, F_1 \otimes F_2), (\Delta_8, \Delta_9), F_4 \otimes F_5} \otimes \rightarrow \frac{}{\bullet h_3 : \vdash \Delta_7, \Delta_9, F_2} \text{fail}$$

$$\frac{h_3 : \vdash \Delta_6, \Delta_7, F_4 \quad h_3 : \vdash \Delta_8, \Delta_9, F_5, F_1 \otimes F_2}{\bullet h_3 : \vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, F_1 \otimes F_2), F_4 \otimes F_5} \otimes \rightarrow \frac{}{\bullet h_3 : \vdash \Delta_7, \Delta_9, F_2} \text{fail}$$

$$\frac{h_3 : \vdash \Delta_6, \Delta_7, F_4, F_1 \otimes F_2 \quad h_3 : \vdash \Delta_8, \Delta_9, F_5}{\bullet h_3 : \vdash (\Delta_6, \Delta_7, F_1 \otimes F_2), (\Delta_8, \Delta_9), F_4 \otimes F_5} \otimes \rightarrow \frac{}{\bullet h_3 : \vdash \Delta_7, \Delta_9, F_2, F_4 \otimes F_5} \text{fail}$$

$$\frac{h_3 : \vdash \Delta_6, \Delta_7, F_4 \quad h_3 : \vdash \Delta_8, \Delta_9, F_5, F_1 \otimes F_2}{\bullet h_3 : \vdash (\Delta_6, \Delta_7), (\Delta_8, \Delta_9, F_1 \otimes F_2), F_4 \otimes F_5} \otimes \rightarrow \frac{}{\bullet h_3 : \vdash \Delta_7, \Delta_9, F_2, F_4 \otimes F_5} \text{fail}$$

$$\frac{h_1 : \vdash \Delta_4, \Delta_5, F_2 \quad h_1 : \vdash \Delta_6, \Delta_7, F_3}{\bullet h_1 : \vdash (\Delta_4, \Delta_5), (\Delta_6, \Delta_7), F_2 \otimes F_3} \otimes \rightarrow \frac{}{\bullet h_1 : \vdash \Delta_5, \Delta_7, F_3} \text{fail}$$

4 Identity-Expansion

$$\begin{array}{c}
\frac{}{- : \vdash \mathbf{0}, \top} \top \\
\\
\frac{\frac{}{- : \vdash \mathbf{1}} \mathbf{1}}{- : \vdash \mathbf{1}, \perp} \perp \\
\\
\frac{\frac{\frac{}{- : \vdash \mathbf{F}_0, dual(\mathbf{F}_0)} \text{IH}}{- : \vdash dual(\mathbf{F}_0), \mathbf{F}_0 \oplus \mathbf{F}_1} \oplus_A \quad \frac{\frac{}{- : \vdash \mathbf{F}_1, dual(\mathbf{F}_1)} \text{IH}}{- : \vdash dual(\mathbf{F}_1), \mathbf{F}_0 \oplus \mathbf{F}_1} \oplus_B}{- : \vdash dual(\mathbf{F}_0) \& dual(\mathbf{F}_1), \mathbf{F}_0 \oplus \mathbf{F}_1} \& \\
\\
\frac{\frac{\frac{}{- : \vdash \mathbf{F}_0, dual(\mathbf{F}_0)} \text{IH}}{- : \vdash dual(\mathbf{F}_0), dual(\mathbf{F}_1), \mathbf{F}_0 \otimes \mathbf{F}_1} \otimes \quad \frac{\frac{}{- : \vdash \mathbf{F}_1, dual(\mathbf{F}_1)} \text{IH}}{- : \vdash dual(\mathbf{F}_0), dual(\mathbf{F}_1), \mathbf{F}_0 \otimes \mathbf{F}_1} \otimes}{- : \vdash dual(\mathbf{F}_0) \$ dual(\mathbf{F}_1), \mathbf{F}_0 \otimes \mathbf{F}_1} \$ \\
\\
\frac{\frac{\frac{}{- : \vdash \mathbf{F}_0, dual(\mathbf{F}_0)} \text{IH}}{- : \vdash \mathbf{F}_0, dual(\mathbf{F}_0) \oplus dual(\mathbf{F}_1)} \oplus_A \quad \frac{\frac{}{- : \vdash \mathbf{F}_1, dual(\mathbf{F}_1)} \text{IH}}{- : \vdash \mathbf{F}_1, dual(\mathbf{F}_0) \oplus dual(\mathbf{F}_1)} \oplus_B}{- : \vdash \mathbf{F}_0 \& \mathbf{F}_1, dual(\mathbf{F}_0) \oplus dual(\mathbf{F}_1)} \& \\
\\
\frac{\frac{\frac{}{- : \vdash \mathbf{F}_0, dual(\mathbf{F}_0)} \text{IH}}{- : \vdash \mathbf{F}_0, \mathbf{F}_1, dual(\mathbf{F}_0) \otimes dual(\mathbf{F}_1)} \otimes \quad \frac{\frac{}{- : \vdash \mathbf{F}_1, dual(\mathbf{F}_1)} \text{IH}}{- : \vdash \mathbf{F}_0, \mathbf{F}_1, dual(\mathbf{F}_0) \otimes dual(\mathbf{F}_1)} \otimes}{- : \vdash \mathbf{F}_0 \$ \mathbf{F}_1, dual(\mathbf{F}_0) \otimes dual(\mathbf{F}_1)} \$ \\
\\
\frac{\frac{\frac{}{- : \vdash \mathbf{F}_0, dual(\mathbf{F}_0)} \text{IH}}{- : \vdash ?\mathbf{F}_0, dual(\mathbf{F}_0)} ?}{- : \vdash !dual(\mathbf{F}_0), ?\mathbf{F}_0} ! \\
\\
\frac{\frac{}{- : \vdash \mathbf{1}} \mathbf{1}}{- : \vdash \mathbf{1}, \perp} \perp \\
\\
\frac{}{- : \vdash \mathbf{0}, \top} \top \\
\\
\frac{\frac{\frac{}{- : \vdash \mathbf{F}_0, dual(\mathbf{F}_0)} \text{IH}}{- : \vdash \mathbf{F}_0, ?dual(\mathbf{F}_0)} ?}{- : \vdash !\mathbf{F}_0, ?dual(\mathbf{F}_0)} !
\end{array}$$

5 Cut-Elimination

5.1 Status of 1: OK

- Case rule 1
- Case rule !
- Case rule ?W

$$\frac{\frac{\frac{}{\bullet h_1 : \vdash \mathbf{1}, *}}{\bullet h_1 : \vdash \mathbf{1}, *}}{\mathbf{1}} \quad \frac{\frac{h_2 : \vdash \perp, \Delta_4}{\bullet h_2 : \vdash \mathit{dual}(\mathbf{1}), \Delta_4, ?F_3}}{?W}}{\text{Cut}}}{\vdash *, \Delta_4, ?F_3} \rightarrow$$

$$\frac{\frac{\frac{\bullet h_1 : \vdash \mathbf{1}}{\mathbf{1}} \quad \frac{ax}{h_2 : \vdash \Delta_4, \perp}}{hCut}}{\vdash \Delta_4} \quad W}{\vdash \Delta_4, ?F_3}$$

- Case rule ?C

$$\frac{\frac{\frac{}{\bullet h_1 : \vdash \mathbf{1}, *}}{\bullet h_1 : \vdash \mathbf{1}, *}}{\mathbf{1}} \quad \frac{\frac{h_2 : \vdash \perp, \Delta_4, ?F_3, ?F_3}{\bullet h_2 : \vdash \mathit{dual}(\mathbf{1}), \Delta_4, ?F_3}}{?C}}{\text{Cut}}}{\vdash *, \Delta_4, ?F_3} \rightarrow$$

$$\frac{\frac{\frac{\bullet h_1 : \vdash \mathbf{1}}{ax} \quad \frac{ax}{h_2 : \vdash \Delta_4, \perp, ?F_3, ?F_3}}{hCut}}{\vdash \Delta_4, ?F_3, ?F_3} \quad ?C}{\vdash \Delta_4, ?F_3}$$

- Case rule ?

$$\frac{\frac{\frac{}{\bullet h_1 : \vdash \mathbf{1}, *}}{\bullet h_1 : \vdash \mathbf{1}, *}}{\mathbf{1}} \quad \frac{\frac{h_2 : \vdash \perp, \Delta_4, F_3}{\bullet h_2 : \vdash \mathit{dual}(\mathbf{1}), \Delta_4, ?F_3}}{?}}{\text{Cut}}}{\vdash *, \Delta_4, ?F_3} \rightarrow$$

$$\frac{\frac{\frac{\bullet h_1 : \vdash \mathbf{1}}{ax} \quad \frac{ax}{h_2 : \vdash \Delta_4, F_3, \perp}}{hCut}}{\vdash \Delta_4, F_3} \quad ?}{\vdash \Delta_4, ?F_3}$$

- Case rule \$

$$\frac{\frac{\frac{}{\bullet h_1 : \vdash \mathbf{1}, *}}{\bullet h_1 : \vdash \mathbf{1}, *}}{\mathbf{1}} \quad \frac{\frac{h_2 : \vdash \perp, \Delta_5, F_3, F_4}{\bullet h_2 : \vdash \mathit{dual}(\mathbf{1}), \Delta_5, F_3 \$F_4}}{\$}}{\text{Cut}}}{\vdash *, \Delta_5, F_3 \$F_4} \rightarrow$$

$$\frac{\frac{\frac{\bullet h_1 : \vdash \mathbf{1}}{ax} \quad \frac{ax}{h_2 : \vdash \Delta_5, F_3, F_4, \perp}}{hCut}}{\vdash \Delta_5, F_3, F_4} \quad \$}{\vdash \Delta_5, F_3 \$F_4}$$

- Case rule &

$$\frac{\frac{\frac{}{\bullet h_1 : \vdash \mathbf{1}, *}}{\bullet h_1 : \vdash \mathbf{1}, *}}{\mathbf{1}} \quad \frac{\frac{h_2 : \vdash \perp, \Delta_5, F_3 \quad h_2 : \vdash \perp, \Delta_5, F_4}{\bullet h_2 : \vdash \mathit{dual}(\mathbf{1}), \Delta_5, F_3 \&F_4}}{\&}}{\text{Cut}}}{\vdash *, \Delta_5, F_3 \&F_4} \rightarrow$$

$$\frac{\frac{\frac{\bullet h_1 : \vdash \mathbf{1}}{\mathbf{1}} \quad \frac{ax}{h_2 : \vdash \Delta_5, F_3, \perp}}{hCut} \quad \frac{\frac{\bullet h_1 : \vdash \mathbf{1}}{\mathbf{1}} \quad \frac{ax}{h_2 : \vdash \Delta_5, F_4, \perp}}{hCut}}{\vdash \Delta_5, F_3} \quad \&}{\vdash \Delta_5, F_3 \&F_4}$$

- Case rule \oplus_B

$$\frac{\frac{\frac{}{\bullet h_1 : \vdash \mathbf{1}, *}}{\bullet h_1 : \vdash \mathbf{1}, *} \mathbf{1} \quad \frac{h_2 : \vdash \perp, \Delta_5, F_4}{\bullet h_2 : \vdash \mathit{dual}(\mathbf{1}), \Delta_5, F_3 \oplus F_4}}{- : \vdash *, \Delta_5, F_3 \oplus F_4} \text{Cut}}{\rightarrow} \oplus_B$$

$$\frac{\frac{\frac{}{\bullet h_1 : \vdash \mathbf{1}}{\bullet h_1 : \vdash \mathbf{1}} \text{ax} \quad \frac{h_2 : \vdash \Delta_5, F_4, \perp}{- : \vdash \Delta_5, F_4} \text{hCut}}{- : \vdash \Delta_5, F_3 \oplus F_4} \oplus_B}{- : \vdash \Delta_5, F_3 \oplus F_4} \oplus_B$$

- Case rule \oplus_A

$$\frac{\frac{\frac{}{\bullet h_1 : \vdash \mathbf{1}, *}}{\bullet h_1 : \vdash \mathbf{1}, *} \mathbf{1} \quad \frac{h_2 : \vdash \perp, \Delta_5, F_3}{\bullet h_2 : \vdash \mathit{dual}(\mathbf{1}), \Delta_5, F_3 \oplus F_4}}{- : \vdash *, \Delta_5, F_3 \oplus F_4} \text{Cut}}{\rightarrow} \oplus_A$$

$$\frac{\frac{\frac{}{\bullet h_1 : \vdash \mathbf{1}}{\bullet h_1 : \vdash \mathbf{1}} \text{ax} \quad \frac{h_2 : \vdash \Delta_5, F_3, \perp}{- : \vdash \Delta_5, F_3} \text{hCut}}{- : \vdash \Delta_5, F_3 \oplus F_4} \oplus_A}{- : \vdash \Delta_5, F_3 \oplus F_4} \oplus_A$$

- Case rule \perp

$$\frac{\frac{\frac{}{\bullet h_1 : \vdash \mathbf{1}, *}}{\bullet h_1 : \vdash \mathbf{1}, *} \mathbf{1} \quad \frac{h_2 : \vdash \Delta_3}{\bullet h_2 : \vdash \mathit{dual}(\mathbf{1}), \Delta_3}}{- : \vdash *, \Delta_3} \text{Cut}}{\rightarrow} \perp$$

$$\frac{}{- : \vdash \Delta_3} \text{ax}$$

- Case rule \top

$$\frac{\frac{\frac{}{\bullet h_1 : \vdash \mathbf{1}, *}}{\bullet h_1 : \vdash \mathbf{1}, *} \mathbf{1} \quad \frac{}{\bullet h_2 : \vdash \mathit{dual}(\mathbf{1}), \top, \Delta_3} \top}}{- : \vdash *, \top, \Delta_3} \text{Cut}}{\rightarrow} \top$$

$$\frac{}{- : \vdash \Delta_3, \top} \top$$

- Case rule I

- Case rule \otimes

$$\frac{\frac{\frac{}{\bullet h_1 : \vdash \mathbf{1}, *}}{\bullet h_1 : \vdash \mathbf{1}, *} \mathbf{1} \quad \frac{h_2 : \vdash \perp, \Delta_6, F_4 \quad h_2 : \vdash \Delta_3, F_5}{\bullet h_2 : \vdash \mathit{dual}(\mathbf{1}), \Delta_3, \Delta_6, F_4 \otimes F_5}}{- : \vdash *, \Delta_3, \Delta_6, F_4 \otimes F_5} \text{Cut}}{\rightarrow} \otimes$$

$$\frac{\frac{\frac{}{\bullet h_1 : \vdash \mathbf{1}}{\bullet h_1 : \vdash \mathbf{1}} \text{ax} \quad \frac{h_2 : \vdash \Delta_6, F_4, \perp}{- : \vdash \Delta_6, F_4} \text{hCut} \quad \frac{}{- : \vdash \Delta_3, F_5} \text{ax}}{- : \vdash \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes}{- : \vdash \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes$$

$$\frac{\frac{\frac{}{\bullet h_1 : \vdash \mathbf{1}, *}}{\bullet h_1 : \vdash \mathbf{1}, *} \mathbf{1} \quad \frac{h_2 : \vdash \Delta_3, F_4 \quad h_2 : \vdash \perp, \Delta_6, F_5}{\bullet h_2 : \vdash \mathit{dual}(\mathbf{1}), \Delta_3, \Delta_6, F_4 \otimes F_5}}{- : \vdash *, \Delta_3, \Delta_6, F_4 \otimes F_5} \text{Cut}}{\rightarrow} \otimes$$

$$\frac{\frac{}{- : \vdash \Delta_3, F_4} \text{ax} \quad \frac{\frac{}{\bullet h_1 : \vdash \mathbf{1}}{\bullet h_1 : \vdash \mathbf{1}} \text{ax} \quad \frac{h_2 : \vdash \Delta_6, F_5, \perp}{- : \vdash \Delta_6, F_5} \text{hCut}}{- : \vdash \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes}{- : \vdash \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes$$

5.2 Status of !: OK

- Case rule 1

- Case rule !

$$\begin{array}{c}
\frac{h_1 : \vdash F_4, ?\Upsilon 2}{\bullet h_1 : \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_5 : \vdash F_6, ?\Upsilon 7, ?dual(F_4)}{\bullet h_5 : \vdash dual(!F_4), ?\Upsilon 7, !F_6} !}{- : \vdash ?\Upsilon 2, ?\Upsilon 7, !F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash ?\Upsilon 2, !F_4}{- : \vdash ?\Upsilon 2, !F_4} \text{ax} \quad \frac{h_5 : \vdash ?\Upsilon 7, F_6, ?dual(F_4)}{h_5 : \vdash ?\Upsilon 7, F_6, ?dual(F_4)} \text{ax}}{- : \vdash ?\Upsilon 2, ?\Upsilon 7, F_6} \text{hCut}}{- : \vdash ?\Upsilon 2, ?\Upsilon 7, !F_6} !
\end{array}$$

- Case rule ?W

$$\begin{array}{c}
\frac{h_2 : \vdash F_3, ?\Upsilon 4, 5}{\bullet h_2 : \vdash 5, ?\Upsilon 4, !F_3} ! \quad \frac{h_6 : \vdash \Delta_8, dual(5)}{\bullet h_6 : \vdash dual(5), \Delta_8, ?F_7} ?W}{- : \vdash (?\Upsilon 4, !F_3), \Delta_8, ?F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : \vdash 5, ?\Upsilon 4, !F_3}{- : \vdash ?\Upsilon 4, \Delta_8, !F_3} \text{ax} \quad \frac{h_6 : \vdash \Delta_8, dual(5)}{h_6 : \vdash \Delta_8, dual(5)} \text{ax}}{- : \vdash ?\Upsilon 4, \Delta_8, !F_3, ?F_7} \text{hCut}}{- : \vdash ?\Upsilon 4, \Delta_8, !F_3, ?F_7} ?W
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \vdash F_4, ?\Upsilon 2}{\bullet h_1 : \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_5 : \vdash \Delta_7, ?dual(F_4)}{\bullet h_5 : \vdash dual(!F_4), \Delta_7, ?F_6} ?W}{- : \vdash ?\Upsilon 2, \Delta_7, ?F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash ?\Upsilon 2, !F_4}{- : \vdash ?\Upsilon 2, !F_4} \text{ax} \quad \frac{h_5 : \vdash \Delta_7, ?dual(F_4)}{h_5 : \vdash \Delta_7, ?dual(F_4)} \text{ax}}{- : \vdash ?\Upsilon 2, \Delta_7} \text{hCut}}{- : \vdash ?\Upsilon 2, \Delta_7, ?F_6} W
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \vdash F_4, ?\Upsilon 2}{\bullet h_1 : \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_5 : \vdash \Delta_6}{\bullet h_5 : \vdash dual(!F_4), \Delta_6} ?W}{- : \vdash ?\Upsilon 2, \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{- : \vdash \Delta_6}{- : \vdash ?\Upsilon 2, \Delta_6} \text{ax}}{- : \vdash ?\Upsilon 2, \Delta_6} W
\end{array}$$

- Case rule ?C

$$\begin{array}{c}
\frac{h_2 : \vdash F_3, ?\Upsilon 4, 5}{\bullet h_2 : \vdash 5, ?\Upsilon 4, !F_3} ! \quad \frac{h_6 : \vdash \Delta_8, ?F_7, ?F_7, dual(5)}{\bullet h_6 : \vdash dual(5), \Delta_8, ?F_7} ?C}{- : \vdash (?\Upsilon 4, !F_3), \Delta_8, ?F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : \vdash 5, ?\Upsilon 4, !F_3}{- : \vdash ?\Upsilon 4, \Delta_8, !F_3, ?F_7, ?F_7} \text{ax} \quad \frac{h_6 : \vdash \Delta_8, ?F_7, ?F_7, dual(5)}{h_6 : \vdash \Delta_8, ?F_7, ?F_7, dual(5)} \text{ax}}{- : \vdash ?\Upsilon 4, \Delta_8, !F_3, ?F_7} \text{hCut}}{- : \vdash ?\Upsilon 4, \Delta_8, !F_3, ?F_7} ?C
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \vdash F_4, ?\Upsilon 2}{\bullet h_1 : \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_5 : \vdash \Delta_7, ?F_6, ?F_6, ?dual(F_4)}{\bullet h_5 : \vdash dual(!F_4), \Delta_7, ?F_6} ?C}{- : \vdash ?\Upsilon 2, \Delta_7, ?F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash ?\Upsilon 2, !F_4}{- : \vdash ?\Upsilon 2, !F_4} \text{ax} \quad \frac{h_5 : \vdash \Delta_7, ?F_6, ?F_6, ?dual(F_4)}{h_5 : \vdash \Delta_7, ?F_6, ?F_6, ?dual(F_4)} \text{ax}}{- : \vdash ?\Upsilon 2, \Delta_7, ?F_6, ?F_6} \text{hCut}}{- : \vdash ?\Upsilon 2, \Delta_7, ?F_6} ?C
\end{array}$$

$$\begin{array}{c}
\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_5 \vdash \Delta_6, ?dual(F_4), ?dual(F_4)}{\bullet h_5 \vdash dual(!F_4), \Delta_6} ?C \\
\hline
- \vdash ?\Upsilon 2, \Delta_6 \\
\rightarrow \\
\frac{\bullet h_1 \vdash ?\Upsilon 2, !F_4}{- \vdash ?\Upsilon 2, \Delta_6} ax \quad \frac{h_5 \vdash \Delta_6, ?dual(F_4), ?dual(F_4)}{- \vdash ?\Upsilon 2, \Delta_6} ax \\
\hline
mCut
\end{array}$$

- Case rule ?

$$\begin{array}{c}
\frac{h_2 \vdash F_3, ?\Upsilon 4, 5}{\bullet h_2 \vdash 5, ?\Upsilon 4, !F_3} ! \quad \frac{h_6 \vdash \Delta_8, F_7, dual(5)}{\bullet h_6 \vdash dual(5), \Delta_8, ?F_7} ? \\
\hline
- \vdash (? \Upsilon 4, !F_3), \Delta_8, ?F_7 \\
\rightarrow \\
\frac{\bullet h_2 \vdash 5, ?\Upsilon 4, !F_3}{- \vdash ?\Upsilon 4, \Delta_8, F_7, !F_3} ax \quad \frac{h_6 \vdash \Delta_8, F_7, dual(5)}{- \vdash ?\Upsilon 4, \Delta_8, !F_3, ?F_7} ax \\
\hline
hCut \\
- \vdash ?\Upsilon 4, \Delta_8, !F_3, ?F_7 ? \\
\rightarrow \\
\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_5 \vdash \Delta_7, F_6, ?dual(F_4)}{\bullet h_5 \vdash dual(!F_4), \Delta_7, ?F_6} ? \\
\hline
- \vdash ?\Upsilon 2, \Delta_7, ?F_6 \\
\rightarrow \\
\frac{\bullet h_1 \vdash ?\Upsilon 2, !F_4}{- \vdash ?\Upsilon 2, \Delta_7, F_6} ax \quad \frac{h_5 \vdash \Delta_7, F_6, ?dual(F_4)}{- \vdash ?\Upsilon 2, \Delta_7, ?F_6} ax \\
\hline
hCut \\
- \vdash ?\Upsilon 2, \Delta_7, ?F_6 ? \\
\rightarrow \\
\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_5 \vdash \Delta_6, dual(F_4)}{\bullet h_5 \vdash dual(!F_4), \Delta_6} ? \\
\hline
- \vdash ?\Upsilon 2, \Delta_6 \\
\rightarrow \\
\frac{- \vdash ?\Upsilon 2, F_4}{- \vdash ?\Upsilon 2, \Delta_6} ax \quad \frac{- \vdash \Delta_6, dual(F_4)}{- \vdash ?\Upsilon 2, \Delta_6} ax \\
\hline
sCut
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{h_2 \vdash F_3, ?\Upsilon 4, 5}{\bullet h_2 \vdash 5, ?\Upsilon 4, !F_3} ! \quad \frac{h_6 \vdash \Delta_9, F_7, F_8, dual(5)}{\bullet h_6 \vdash dual(5), \Delta_9, F_7 \$F_8} \$ \\
\hline
- \vdash (? \Upsilon 4, !F_3), \Delta_9, F_7 \$F_8 \\
\rightarrow \\
\frac{\bullet h_2 \vdash 5, ?\Upsilon 4, !F_3}{- \vdash ?\Upsilon 4, \Delta_9, F_7, F_8, !F_3} ax \quad \frac{h_6 \vdash \Delta_9, F_7, F_8, dual(5)}{- \vdash ?\Upsilon 4, \Delta_9, !F_3, F_7 \$F_8} ax \\
\hline
hCut \\
- \vdash ?\Upsilon 4, \Delta_9, !F_3, F_7 \$F_8 \$ \\
\rightarrow \\
\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_5 \vdash \Delta_8, F_6, F_7, ?dual(F_4)}{\bullet h_5 \vdash dual(!F_4), \Delta_8, F_6 \$F_7} \$ \\
\hline
- \vdash ?\Upsilon 2, \Delta_8, F_6 \$F_7 \\
\rightarrow \\
\frac{\bullet h_1 \vdash ?\Upsilon 2, !F_4}{- \vdash ?\Upsilon 2, \Delta_8, F_6, F_7} ax \quad \frac{h_5 \vdash \Delta_8, F_6, F_7, ?dual(F_4)}{- \vdash ?\Upsilon 2, \Delta_8, F_6 \$F_7} ax \\
\hline
hCut \\
- \vdash ?\Upsilon 2, \Delta_8, F_6, F_7 \$ \\
- \vdash ?\Upsilon 2, \Delta_8, F_6 \$F_7 \$
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{h_2 \vdash F_3, ?\Upsilon 4, 5}{\bullet h_2 \vdash 5, ?\Upsilon 4, !F_3} ! \quad \frac{h_6 \vdash \Delta_9, F_7, dual(5)}{\bullet h_6 \vdash dual(5), \Delta_9, F_7 \&F_8} \& \quad \frac{h_6 \vdash \Delta_9, F_8, dual(5)}{\bullet h_6 \vdash dual(5), \Delta_9, F_7 \&F_8} \& \\
\hline
- \vdash (? \Upsilon 4, !F_3), \Delta_9, F_7 \&F_8 \\
\rightarrow \\
\frac{\bullet h_2 \vdash 5, ?\Upsilon 4, !F_3}{- \vdash ?\Upsilon 4, \Delta_9, F_7, !F_3} ax \quad \frac{h_6 \vdash \Delta_9, F_7, dual(5)}{- \vdash ?\Upsilon 4, \Delta_9, F_8, !F_3} ax \quad \frac{\bullet h_2 \vdash 5, ?\Upsilon 4, !F_3}{- \vdash ?\Upsilon 4, \Delta_9, F_7 \&F_8} ax \quad \frac{h_6 \vdash \Delta_9, F_8, dual(5)}{- \vdash ?\Upsilon 4, \Delta_9, F_8, !F_3} ax \\
\hline
hCut \quad hCut \quad hCut \quad hCut \\
- \vdash ?\Upsilon 4, \Delta_9, F_7 \&F_8 \&
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_5 \vdash \Delta_8, F_6, ?dual(F_4) \quad h_5 \vdash \Delta_8, F_7, ?dual(F_4)}{\bullet h_5 \vdash dual(!F_4), \Delta_8, F_6 \& F_7} \&}{- \vdash ?\Upsilon 2, \Delta_8, F_6 \& F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2, !F_4}{\bullet h_1 \vdash ?\Upsilon 2, !F_4} \text{ax} \quad \frac{h_5 \vdash \Delta_8, F_6, ?dual(F_4)}{\bullet h_5 \vdash \Delta_8, F_6} \text{ax}}{- \vdash ?\Upsilon 2, \Delta_8, F_6} \text{hCut} \quad \frac{\frac{h_1 \vdash F_4, ?\Upsilon 2, !F_4}{\bullet h_1 \vdash ?\Upsilon 2, !F_4} \text{ax} \quad \frac{h_5 \vdash \Delta_8, F_7, ?dual(F_4)}{\bullet h_5 \vdash \Delta_8, F_7} \text{ax}}{- \vdash ?\Upsilon 2, \Delta_8, F_7} \text{hCut}}{- \vdash ?\Upsilon 2, \Delta_8, F_6 \& F_7} \&
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{\frac{h_2 \vdash F_3, ?\Upsilon 4, 5}{\bullet h_2 \vdash 5, ?\Upsilon 4, !F_3} ! \quad \frac{h_6 \vdash \Delta_9, F_8, dual(5)}{\bullet h_6 \vdash dual(5), \Delta_9, F_7 \oplus F_8} \oplus_B}{- \vdash (?\Upsilon 4, !F_3), \Delta_9, F_7 \oplus F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_2 \vdash 5, ?\Upsilon 4, !F_3}{\bullet h_2 \vdash 5, ?\Upsilon 4, !F_3} \text{ax} \quad \frac{h_6 \vdash \Delta_9, F_8, dual(5)}{\bullet h_6 \vdash \Delta_9, F_8, dual(5)} \text{ax}}{- \vdash ?\Upsilon 4, \Delta_9, F_8, !F_3} \text{hCut}}{- \vdash ?\Upsilon 4, \Delta_9, !F_3, F_7 \oplus F_8} \oplus_B \\
\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_5 \vdash \Delta_8, F_7, ?dual(F_4)}{\bullet h_5 \vdash dual(!F_4), \Delta_8, F_6 \oplus F_7} \oplus_B}{- \vdash ?\Upsilon 2, \Delta_8, F_6 \oplus F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2, !F_4}{\bullet h_1 \vdash ?\Upsilon 2, !F_4} \text{ax} \quad \frac{h_5 \vdash \Delta_8, F_7, ?dual(F_4)}{\bullet h_5 \vdash \Delta_8, F_7, ?dual(F_4)} \text{ax}}{- \vdash ?\Upsilon 2, \Delta_8, F_7} \text{hCut}}{- \vdash ?\Upsilon 2, \Delta_8, F_6 \oplus F_7} \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{\frac{h_2 \vdash F_3, ?\Upsilon 4, 5}{\bullet h_2 \vdash 5, ?\Upsilon 4, !F_3} ! \quad \frac{h_6 \vdash \Delta_9, F_7, dual(5)}{\bullet h_6 \vdash dual(5), \Delta_9, F_7 \oplus F_8} \oplus_A}{- \vdash (?\Upsilon 4, !F_3), \Delta_9, F_7 \oplus F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_2 \vdash 5, ?\Upsilon 4, !F_3}{\bullet h_2 \vdash 5, ?\Upsilon 4, !F_3} \text{ax} \quad \frac{h_6 \vdash \Delta_9, F_7, dual(5)}{\bullet h_6 \vdash \Delta_9, F_7, dual(5)} \text{ax}}{- \vdash ?\Upsilon 4, \Delta_9, F_7, !F_3} \text{hCut}}{- \vdash ?\Upsilon 4, \Delta_9, !F_3, F_7 \oplus F_8} \oplus_A \\
\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_5 \vdash \Delta_8, F_6, ?dual(F_4)}{\bullet h_5 \vdash dual(!F_4), \Delta_8, F_6 \oplus F_7} \oplus_A}{- \vdash ?\Upsilon 2, \Delta_8, F_6 \oplus F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2, !F_4}{\bullet h_1 \vdash ?\Upsilon 2, !F_4} \text{ax} \quad \frac{h_5 \vdash \Delta_8, F_6, ?dual(F_4)}{\bullet h_5 \vdash \Delta_8, F_6, ?dual(F_4)} \text{ax}}{- \vdash ?\Upsilon 2, \Delta_8, F_6} \text{hCut}}{- \vdash ?\Upsilon 2, \Delta_8, F_6 \oplus F_7} \oplus_A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{\frac{h_2 \vdash F_3, ?\Upsilon 4, 5}{\bullet h_2 \vdash 5, ?\Upsilon 4, !F_3} ! \quad \frac{h_6 \vdash \Delta_7, dual(5)}{\bullet h_6 \vdash dual(5), \perp, \Delta_7} \perp}{- \vdash (?\Upsilon 4, !F_3), \perp, \Delta_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_2 \vdash 5, ?\Upsilon 4, !F_3}{\bullet h_2 \vdash 5, ?\Upsilon 4, !F_3} \text{ax} \quad \frac{h_6 \vdash \Delta_7, dual(5)}{\bullet h_6 \vdash \Delta_7, dual(5)} \text{ax}}{- \vdash ?\Upsilon 4, \Delta_7, !F_3} \text{hCut}}{- \vdash ?\Upsilon 4, \Delta_7, \perp, !F_3} \perp \\
\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_5 \vdash \Delta_6, ?dual(F_4)}{\bullet h_5 \vdash dual(!F_4), \perp, \Delta_6} \perp}{- \vdash ?\Upsilon 2, \perp, \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2, !F_4}{\bullet h_1 \vdash ?\Upsilon 2, !F_4} \text{ax} \quad \frac{h_5 \vdash \Delta_6, ?dual(F_4)}{\bullet h_5 \vdash \Delta_6, ?dual(F_4)} \text{ax}}{- \vdash ?\Upsilon 2, \Delta_6} \text{hCut}}{- \vdash ?\Upsilon 2, \Delta_6, \perp} \perp
\end{array}$$

- Case rule \top

$$\frac{\frac{h_2 \vdash F_3, ?\Upsilon 4, 5}{\bullet h_2 \vdash 5, ?\Upsilon 4, !F_3} ! \quad \frac{}{\bullet h_6 \vdash dual(5), \top, \Delta_7} \top}{- \vdash (? \Upsilon 4, !F_3), \top, \Delta_7} \text{Cut} \quad \top}{- \vdash ?\Upsilon 4, \Delta_7, \top, !F_3} \top$$

$$\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} ! \quad \frac{}{\bullet h_5 \vdash dual(!F_4), \top, \Delta_6} \top}{- \vdash ?\Upsilon 2, \top, \Delta_6} \text{Cut} \quad \top}{- \vdash ?\Upsilon 2, \Delta_6, \top} \top$$

- Case rule I

- Case rule \otimes

$$\frac{\frac{h_2 \vdash F_3, ?\Upsilon 4, 5}{\bullet h_2 \vdash 5, ?\Upsilon 4, !F_3} ! \quad \frac{h_6 \vdash \Delta_{10}, F_8, dual(5) \quad h_6 \vdash \Delta_7, F_9}{\bullet h_6 \vdash dual(5), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes}{- \vdash (? \Upsilon 4, !F_3), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \text{Cut} \quad \otimes}{- \vdash ?\Upsilon 4, \Delta_{10}, \Delta_7, !F_3, F_8 \otimes F_9} \otimes$$

$$\frac{\frac{\bullet h_2 \vdash 5, ?\Upsilon 4, !F_3}{- \vdash ?\Upsilon 4, \Delta_{10}, F_8, !F_3} \text{ax} \quad \frac{h_6 \vdash \Delta_{10}, F_8, dual(5)}{hCut} \text{ax}}{- \vdash ?\Upsilon 4, \Delta_{10}, \Delta_7, !F_3, F_8 \otimes F_9} \otimes \quad \frac{}{- \vdash \Delta_7, F_9} \text{ax}}{- \vdash ?\Upsilon 4, \Delta_{10}, \Delta_7, !F_3, F_8 \otimes F_9} \otimes$$

$$\frac{\frac{h_2 \vdash F_3, ?\Upsilon 4, 5}{\bullet h_2 \vdash 5, ?\Upsilon 4, !F_3} ! \quad \frac{h_6 \vdash \Delta_7, F_8 \quad h_6 \vdash \Delta_{10}, F_9, dual(5)}{\bullet h_6 \vdash dual(5), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes}{- \vdash (? \Upsilon 4, !F_3), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \text{Cut} \quad \otimes}{- \vdash ?\Upsilon 4, \Delta_{10}, \Delta_7, !F_3, F_8 \otimes F_9} \otimes$$

$$\frac{\frac{}{- \vdash \Delta_7, F_8} \text{ax} \quad \frac{\bullet h_2 \vdash 5, ?\Upsilon 4, !F_3}{- \vdash ?\Upsilon 4, \Delta_{10}, F_9, !F_3} \text{ax} \quad \frac{h_6 \vdash \Delta_{10}, F_9, dual(5)}{hCut} \text{ax}}{- \vdash ?\Upsilon 4, \Delta_{10}, \Delta_7, !F_3, F_8 \otimes F_9} \otimes \quad \frac{}{- \vdash \Delta_6, F_8} \text{ax}}{- \vdash ?\Upsilon 4, \Delta_{10}, \Delta_7, !F_3, F_8 \otimes F_9} \otimes$$

$$\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_5 \vdash \Delta_9, F_7, ?dual(F_4) \quad h_5 \vdash \Delta_6, F_8}{\bullet h_5 \vdash dual(!F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes}{- \vdash ?\Upsilon 2, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{Cut} \quad \otimes}{- \vdash ?\Upsilon 2, \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes$$

$$\frac{\frac{\bullet h_1 \vdash ?\Upsilon 2, !F_4}{- \vdash ?\Upsilon 2, \Delta_9, F_7} \text{ax} \quad \frac{h_5 \vdash \Delta_9, F_7, ?dual(F_4)}{hCut} \text{ax}}{- \vdash ?\Upsilon 2, \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes \quad \frac{}{- \vdash \Delta_6, F_8} \text{ax}}{- \vdash ?\Upsilon 2, \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes$$

$$\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_5 \vdash \Delta_6, F_7 \quad h_5 \vdash \Delta_9, F_8, ?dual(F_4)}{\bullet h_5 \vdash dual(!F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes}{- \vdash ?\Upsilon 2, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{Cut} \quad \otimes}{- \vdash ?\Upsilon 2, \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes$$

$$\frac{\frac{}{- \vdash \Delta_6, F_7} \text{ax} \quad \frac{\bullet h_1 \vdash ?\Upsilon 2, !F_4}{- \vdash ?\Upsilon 2, \Delta_9, F_8} \text{ax} \quad \frac{h_5 \vdash \Delta_9, F_8, ?dual(F_4)}{hCut} \text{ax}}{- \vdash ?\Upsilon 2, \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes \quad \frac{}{- \vdash \Delta_6, F_8} \text{ax}}{- \vdash ?\Upsilon 2, \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes$$

5.3 Status of $?W$: OK

- Case rule 1
- Case rule !

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash ?F_4, \Delta_2} \text{ ?W} \quad \frac{h_5 : \vdash ?\Upsilon 6, \text{dual}(F_4)}{\bullet h_5 : \vdash \text{dual}(?F_4), ?\Upsilon 6} ! \\
\hline
- : \vdash \Delta_2, ?\Upsilon 6 \\
\rightarrow \\
\frac{- : \vdash \Delta_2}{- : \vdash ?\Upsilon 6, \Delta_2} \text{ ax} \\
\text{W}
\end{array}
\text{ Cut}$$

- Case rule ?W

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_4, F_5}{\bullet h_2 : \vdash F_5, \Delta_4, ?F_3} \text{ ?W} \quad \frac{h_6 : \vdash \Delta_8, \text{dual}(F_5)}{\bullet h_6 : \vdash \text{dual}(F_5), \Delta_8, ?F_7} \text{ ?W} \\
\hline
- : \vdash (\Delta_4, ?F_3), \Delta_8, ?F_7 \\
\rightarrow \\
\frac{\frac{h_2 : \vdash \Delta_4, F_5}{- : \vdash \Delta_4, \Delta_8, ?F_7} \text{ ax} \quad \frac{\bullet h_6 : \vdash \Delta_8, ?F_7, \text{dual}(F_5)}{- : \vdash \Delta_4, \Delta_8, ?F_3, ?F_7} \text{ ax}}{- : \vdash \Delta_4, \Delta_8, ?F_3, ?F_7} \text{ hCut} \\
\text{ ?W} \\
\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash ?F_4, \Delta_2} \text{ ?W} \quad \frac{h_5 : \vdash \Delta_7, !\text{dual}(F_4)}{\bullet h_5 : \vdash \text{dual}(?F_4), \Delta_7, ?F_6} \text{ ?W} \\
\hline
- : \vdash \Delta_2, \Delta_7, ?F_6 \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, ?F_4}{- : \vdash \Delta_2, \Delta_7} \text{ ax} \quad \frac{h_5 : \vdash \Delta_7, !\text{dual}(F_4)}{- : \vdash \Delta_2, \Delta_7, ?F_6} \text{ ax}}{- : \vdash \Delta_2, \Delta_7, ?F_6} \text{ hCut} \\
\text{ ?W}
\end{array}
\text{ Cut}$$

- Case rule ?C

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_4, F_5}{\bullet h_2 : \vdash F_5, \Delta_4, ?F_3} \text{ ?W} \quad \frac{h_6 : \vdash \Delta_8, ?F_7, ?F_7, \text{dual}(F_5)}{\bullet h_6 : \vdash \text{dual}(F_5), \Delta_8, ?F_7} \text{ ?C} \\
\hline
- : \vdash (\Delta_4, ?F_3), \Delta_8, ?F_7 \\
\rightarrow \\
\frac{\frac{h_2 : \vdash \Delta_4, F_5}{- : \vdash \Delta_4, \Delta_8, ?F_7} \text{ ax} \quad \frac{\bullet h_6 : \vdash \Delta_8, ?F_7, \text{dual}(F_5)}{- : \vdash \Delta_4, \Delta_8, ?F_3, ?F_7} \text{ ax}}{- : \vdash \Delta_4, \Delta_8, ?F_3, ?F_7} \text{ hCut} \\
\text{ ?W} \\
\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash ?F_4, \Delta_2} \text{ ?W} \quad \frac{h_5 : \vdash \Delta_7, !\text{dual}(F_4), ?F_6, ?F_6}{\bullet h_5 : \vdash \text{dual}(?F_4), \Delta_7, ?F_6} \text{ ?C} \\
\hline
- : \vdash \Delta_2, \Delta_7, ?F_6 \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, ?F_4}{- : \vdash \Delta_2, \Delta_7, ?F_6, ?F_6} \text{ ax} \quad \frac{h_5 : \vdash \Delta_7, !\text{dual}(F_4), ?F_6, ?F_6}{- : \vdash \Delta_2, \Delta_7, ?F_6} \text{ ax}}{- : \vdash \Delta_2, \Delta_7, ?F_6} \text{ hCut} \\
\text{ ?C}
\end{array}
\text{ Cut}$$

- Case rule ?

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_4, F_5}{\bullet h_2 : \vdash F_5, \Delta_4, ?F_3} \text{ ?W} \quad \frac{h_6 : \vdash \Delta_8, F_7, \text{dual}(F_5)}{\bullet h_6 : \vdash \text{dual}(F_5), \Delta_8, ?F_7} \text{ ?} \\
\hline
- : \vdash (\Delta_4, ?F_3), \Delta_8, ?F_7 \\
\rightarrow \\
\frac{\frac{h_2 : \vdash \Delta_4, F_5}{- : \vdash \Delta_4, \Delta_8, ?F_7} \text{ ax} \quad \frac{\bullet h_6 : \vdash \Delta_8, ?F_7, \text{dual}(F_5)}{- : \vdash \Delta_4, \Delta_8, ?F_3, ?F_7} \text{ ax}}{- : \vdash \Delta_4, \Delta_8, ?F_3, ?F_7} \text{ hCut} \\
\text{ ?W} \\
\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash ?F_4, \Delta_2} \text{ ?W} \quad \frac{h_5 : \vdash \Delta_7, F_6, !\text{dual}(F_4)}{\bullet h_5 : \vdash \text{dual}(?F_4), \Delta_7, ?F_6} \text{ ?} \\
\hline
- : \vdash \Delta_2, \Delta_7, ?F_6 \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, ?F_4}{- : \vdash \Delta_2, \Delta_7, F_6} \text{ ax} \quad \frac{h_5 : \vdash \Delta_7, F_6, !\text{dual}(F_4)}{- : \vdash \Delta_2, \Delta_7, ?F_6} \text{ ax}}{- : \vdash \Delta_2, \Delta_7, ?F_6} \text{ hCut} \\
\text{ ?}
\end{array}
\text{ Cut}$$

- Case rule $\$$

$$\begin{array}{c}
\frac{h_2 \text{ :- } \Delta_4, F_5}{\bullet h_2 \text{ :- } F_5, \Delta_4, ?F_3} \text{ ?W} \quad \frac{h_6 \text{ :- } \Delta_9, F_7, F_8, \text{dual}(F_5)}{\bullet h_6 \text{ :- } \text{dual}(F_5), \Delta_9, F_7 \& F_8} \text{ \$} \\
\hline
- \text{ :- } (\Delta_4, ?F_3), \Delta_9, F_7 \& F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \text{ :- } \Delta_4, F_5}{- \text{ :- } \Delta_4, \Delta_9, F_7 \& F_8} \text{ ax} \quad \frac{\bullet h_6 \text{ :- } \Delta_9, \text{dual}(F_5), F_7 \& F_8}{- \text{ :- } \Delta_4, \Delta_9, ?F_3, F_7 \& F_8} \text{ ax}}{- \text{ :- } \Delta_4, \Delta_9, ?F_3, F_7 \& F_8} \text{ hCut} \\
\hline
\frac{h_1 \text{ :- } \Delta_2}{\bullet h_1 \text{ :- } ?F_4, \Delta_2} \text{ ?W} \quad \frac{h_5 \text{ :- } \Delta_8, F_6, F_7, \text{!dual}(F_4)}{\bullet h_5 \text{ :- } \text{dual}(?F_4), \Delta_8, F_6 \& F_7} \text{ \$} \\
\hline
- \text{ :- } \Delta_2, \Delta_8, F_6 \& F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \text{ :- } \Delta_2, ?F_4}{- \text{ :- } \Delta_2, \Delta_8, F_6, F_7} \text{ ax} \quad \frac{h_5 \text{ :- } \Delta_8, F_6, F_7, \text{!dual}(F_4)}{- \text{ :- } \Delta_2, \Delta_8, F_6 \& F_7} \text{ ax}}{- \text{ :- } \Delta_2, \Delta_8, F_6 \& F_7} \text{ hCut} \\
\hline
- \text{ :- } \Delta_2, \Delta_8, F_6 \& F_7 \quad \$
\end{array}$$

- Case rule $\&$

$$\begin{array}{c}
\frac{h_2 \text{ :- } \Delta_4, F_5}{\bullet h_2 \text{ :- } F_5, \Delta_4, ?F_3} \text{ ?W} \quad \frac{h_6 \text{ :- } \Delta_9, F_7, \text{dual}(F_5) \quad h_6 \text{ :- } \Delta_9, F_8, \text{dual}(F_5)}{\bullet h_6 \text{ :- } \text{dual}(F_5), \Delta_9, F_7 \& F_8} \text{ \$} \\
\hline
- \text{ :- } (\Delta_4, ?F_3), \Delta_9, F_7 \& F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \text{ :- } \Delta_4, F_5}{- \text{ :- } \Delta_4, \Delta_9, F_7 \& F_8} \text{ ax} \quad \frac{\bullet h_6 \text{ :- } \Delta_9, \text{dual}(F_5), F_7 \& F_8}{- \text{ :- } \Delta_4, \Delta_9, ?F_3, F_7 \& F_8} \text{ ax}}{- \text{ :- } \Delta_4, \Delta_9, ?F_3, F_7 \& F_8} \text{ hCut} \\
\hline
\frac{h_1 \text{ :- } \Delta_2}{\bullet h_1 \text{ :- } ?F_4, \Delta_2} \text{ ?W} \quad \frac{h_5 \text{ :- } \Delta_8, F_6, \text{!dual}(F_4) \quad h_5 \text{ :- } \Delta_8, F_7, \text{!dual}(F_4)}{\bullet h_5 \text{ :- } \text{dual}(?F_4), \Delta_8, F_6 \& F_7} \text{ \$} \\
\hline
- \text{ :- } \Delta_2, \Delta_8, F_6 \& F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \text{ :- } \Delta_2, ?F_4}{- \text{ :- } \Delta_2, \Delta_8, F_6} \text{ ax} \quad \frac{h_5 \text{ :- } \Delta_8, F_6, \text{!dual}(F_4)}{- \text{ :- } \Delta_2, \Delta_8, F_7} \text{ ax}}{- \text{ :- } \Delta_2, \Delta_8, F_6 \& F_7} \text{ hCut} \\
\hline
- \text{ :- } \Delta_2, \Delta_8, F_6 \& F_7 \quad \&
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{h_2 \text{ :- } \Delta_4, F_5}{\bullet h_2 \text{ :- } F_5, \Delta_4, ?F_3} \text{ ?W} \quad \frac{h_6 \text{ :- } \Delta_9, F_8, \text{dual}(F_5)}{\bullet h_6 \text{ :- } \text{dual}(F_5), \Delta_9, F_7 \oplus F_8} \text{ \$} \\
\hline
- \text{ :- } (\Delta_4, ?F_3), \Delta_9, F_7 \oplus F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \text{ :- } \Delta_4, F_5}{- \text{ :- } \Delta_4, \Delta_9, F_7 \oplus F_8} \text{ ax} \quad \frac{\bullet h_6 \text{ :- } \Delta_9, \text{dual}(F_5), F_7 \oplus F_8}{- \text{ :- } \Delta_4, \Delta_9, ?F_3, F_7 \oplus F_8} \text{ ax}}{- \text{ :- } \Delta_4, \Delta_9, ?F_3, F_7 \oplus F_8} \text{ hCut} \\
\hline
\frac{h_1 \text{ :- } \Delta_2}{\bullet h_1 \text{ :- } ?F_4, \Delta_2} \text{ ?W} \quad \frac{h_5 \text{ :- } \Delta_8, F_7, \text{!dual}(F_4)}{\bullet h_5 \text{ :- } \text{dual}(?F_4), \Delta_8, F_6 \oplus F_7} \text{ \$} \\
\hline
- \text{ :- } \Delta_2, \Delta_8, F_6 \oplus F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \text{ :- } \Delta_2, ?F_4}{- \text{ :- } \Delta_2, \Delta_8, F_7} \text{ ax} \quad \frac{h_5 \text{ :- } \Delta_8, F_7, \text{!dual}(F_4)}{- \text{ :- } \Delta_2, \Delta_8, F_6 \oplus F_7} \text{ ax}}{- \text{ :- } \Delta_2, \Delta_8, F_6 \oplus F_7} \text{ hCut} \\
\hline
- \text{ :- } \Delta_2, \Delta_8, F_6 \oplus F_7 \quad \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{\frac{h_2 \text{ :}\vdash \Delta_4, F_5}{\bullet h_2 \text{ :}\vdash F_5, \Delta_4, ?F_3} \text{ ?}W \quad \frac{h_6 \text{ :}\vdash \Delta_9, F_7, \text{dual}(F_5)}{\bullet h_6 \text{ :}\vdash \text{dual}(F_5), \Delta_9, F_7 \oplus F_8}}{- \text{ :}\vdash (\Delta_4, ?F_3), \Delta_9, F_7 \oplus F_8} \oplus_A \text{ Cut}}{\rightarrow} \\
\frac{\frac{h_2 \text{ :}\vdash \Delta_4, F_5}{- \text{ :}\vdash \Delta_4, \Delta_9, F_7 \oplus F_8} \text{ ax} \quad \frac{\bullet h_6 \text{ :}\vdash \Delta_9, \text{dual}(F_5), F_7 \oplus F_8}{- \text{ :}\vdash \Delta_4, \Delta_9, ?F_3, F_7 \oplus F_8} \text{ ax}}{- \text{ :}\vdash \Delta_4, \Delta_9, F_7 \oplus F_8} \text{ hCut}}{- \text{ :}\vdash \Delta_4, \Delta_9, ?F_3, F_7 \oplus F_8} W \\
\frac{\frac{h_1 \text{ :}\vdash \Delta_2}{\bullet h_1 \text{ :}\vdash ?F_4, \Delta_2} \text{ ?}W \quad \frac{h_5 \text{ :}\vdash \Delta_8, F_6, \text{!dual}(F_4)}{\bullet h_5 \text{ :}\vdash \text{dual}(?F_4), \Delta_8, F_6 \oplus F_7}}{- \text{ :}\vdash \Delta_2, \Delta_8, F_6 \oplus F_7} \oplus_A \text{ Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_1 \text{ :}\vdash \Delta_2, ?F_4}{- \text{ :}\vdash \Delta_2, \Delta_8, F_6} \text{ ax} \quad \frac{h_5 \text{ :}\vdash \Delta_8, F_6, \text{!dual}(F_4)}{- \text{ :}\vdash \Delta_2, \Delta_8, F_6 \oplus F_7} \text{ ax}}{- \text{ :}\vdash \Delta_2, \Delta_8, F_6 \oplus F_7} \oplus_A \text{ hCut}}{- \text{ :}\vdash \Delta_2, \Delta_8, F_6 \oplus F_7} \oplus_A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{\frac{h_2 \text{ :}\vdash \Delta_4, F_5}{\bullet h_2 \text{ :}\vdash F_5, \Delta_4, ?F_3} \text{ ?}W \quad \frac{h_6 \text{ :}\vdash \Delta_7, \text{dual}(F_5)}{\bullet h_6 \text{ :}\vdash \text{dual}(F_5), \perp, \Delta_7}}{- \text{ :}\vdash (\Delta_4, ?F_3), \perp, \Delta_7} \perp \text{ Cut}}{\rightarrow} \\
\frac{\frac{h_2 \text{ :}\vdash \Delta_4, F_5}{- \text{ :}\vdash \Delta_4, \Delta_7, \perp} \text{ ax} \quad \frac{\bullet h_6 \text{ :}\vdash \Delta_7, \perp, \text{dual}(F_5)}{- \text{ :}\vdash \Delta_4, \Delta_7, \perp, ?F_3} \text{ ax}}{- \text{ :}\vdash \Delta_4, \Delta_7, \perp, ?F_3} \text{ hCut}}{- \text{ :}\vdash \Delta_4, \Delta_7, \perp, ?F_3} W \\
\frac{\frac{h_1 \text{ :}\vdash \Delta_2}{\bullet h_1 \text{ :}\vdash ?F_4, \Delta_2} \text{ ?}W \quad \frac{h_5 \text{ :}\vdash \Delta_6, \text{!dual}(F_4)}{\bullet h_5 \text{ :}\vdash \text{dual}(?F_4), \perp, \Delta_6}}{- \text{ :}\vdash \Delta_2, \perp, \Delta_6} \perp \text{ Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_1 \text{ :}\vdash \Delta_2, ?F_4}{- \text{ :}\vdash \Delta_2, \Delta_6} \text{ ax} \quad \frac{h_5 \text{ :}\vdash \Delta_6, \text{!dual}(F_4)}{- \text{ :}\vdash \Delta_2, \Delta_6, \perp} \text{ ax}}{- \text{ :}\vdash \Delta_2, \Delta_6, \perp} \perp \text{ hCut}}{- \text{ :}\vdash \Delta_2, \Delta_6, \perp} \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{\frac{h_2 \text{ :}\vdash \Delta_4, F_5}{\bullet h_2 \text{ :}\vdash F_5, \Delta_4, ?F_3} \text{ ?}W \quad \frac{h_6 \text{ :}\vdash \text{dual}(F_5), \top, \Delta_7}{\bullet h_6 \text{ :}\vdash \text{dual}(F_5), \top, \Delta_7}}{- \text{ :}\vdash (\Delta_4, ?F_3), \top, \Delta_7} \top \text{ Cut}}{\rightarrow} \\
\frac{- \text{ :}\vdash \Delta_4, \Delta_7, \top, ?F_3}{- \text{ :}\vdash \Delta_4, \Delta_7, \top, ?F_3} \top \\
\frac{\frac{h_1 \text{ :}\vdash \Delta_2}{\bullet h_1 \text{ :}\vdash ?F_4, \Delta_2} \text{ ?}W \quad \frac{h_5 \text{ :}\vdash \text{dual}(?F_4), \top, \Delta_6}{\bullet h_5 \text{ :}\vdash \text{dual}(?F_4), \top, \Delta_6}}{- \text{ :}\vdash \Delta_2, \top, \Delta_6} \top \text{ Cut}}{\rightarrow} \\
\frac{- \text{ :}\vdash \Delta_2, \Delta_6, \top}{- \text{ :}\vdash \Delta_2, \Delta_6, \top} \top
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{\frac{h_2 \text{ :}\vdash \Delta_4, F_5}{\bullet h_2 \text{ :}\vdash F_5, \Delta_4, ?F_3} \text{ ?}W \quad \frac{h_6 \text{ :}\vdash \Delta_{10}, F_8, \text{dual}(F_5)}{\bullet h_6 \text{ :}\vdash \text{dual}(F_5), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \quad h_6 \text{ :}\vdash \Delta_7, F_9}{- \text{ :}\vdash (\Delta_4, ?F_3), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes \text{ Cut}}{\rightarrow} \\
\frac{\frac{h_2 \text{ :}\vdash \Delta_4, F_5}{- \text{ :}\vdash \Delta_{10}, \Delta_4, \Delta_7, F_8 \otimes F_9} \text{ ax} \quad \frac{\bullet h_6 \text{ :}\vdash \Delta_{10}, \Delta_7, \text{dual}(F_5), F_8 \otimes F_9}{- \text{ :}\vdash \Delta_{10}, \Delta_4, \Delta_7, ?F_3, F_8 \otimes F_9} \text{ ax}}{- \text{ :}\vdash \Delta_{10}, \Delta_4, \Delta_7, F_8 \otimes F_9} \text{ hCut}}{- \text{ :}\vdash \Delta_{10}, \Delta_4, \Delta_7, ?F_3, F_8 \otimes F_9} W
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \text{ : } \Delta_4, F_5}{\bullet h_2 \text{ : } F_5, \Delta_4, ?F_3} \text{ ?W} \quad \frac{h_6 \text{ : } \Delta_7, F_8 \quad h_6 \text{ : } \Delta_{10}, F_9, \text{dual}(F_5)}{\bullet h_6 \text{ : } \text{dual}(F_5), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes \\
\hline
- \text{ : } (\Delta_4, ?F_3), \Delta_7, \Delta_{10}, F_8 \otimes F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \text{ : } \Delta_4, F_5}{- \text{ : } \Delta_4, F_5} \text{ ax} \quad \frac{\bullet h_6 \text{ : } \Delta_{10}, \Delta_7, \text{dual}(F_5), F_8 \otimes F_9}{- \text{ : } \Delta_{10}, \Delta_4, \Delta_7, F_8 \otimes F_9} \text{ ax}}{- \text{ : } \Delta_{10}, \Delta_4, \Delta_7, ?F_3, F_8 \otimes F_9} \text{ hCut}}{- \text{ : } \Delta_{10}, \Delta_4, \Delta_7, ?F_3, F_8 \otimes F_9} \text{ ?W} \\
\\
\frac{h_1 \text{ : } \Delta_2}{\bullet h_1 \text{ : } ?F_4, \Delta_2} \text{ ?W} \quad \frac{h_5 \text{ : } \Delta_9, F_7, !\text{dual}(F_4) \quad h_5 \text{ : } \Delta_6, F_8}{\bullet h_5 \text{ : } \text{dual}(?F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes \\
\hline
- \text{ : } \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 \text{ : } \Delta_2, ?F_4}{\bullet h_1 \text{ : } \Delta_2, ?F_4} \text{ ax} \quad \frac{h_5 \text{ : } \Delta_9, F_7, !\text{dual}(F_4)}{h_5 \text{ : } \Delta_9, F_7, !\text{dual}(F_4)} \text{ ax}}{- \text{ : } \Delta_2, \Delta_9, F_7} \text{ hCut} \quad \frac{- \text{ : } \Delta_6, F_8}{- \text{ : } \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes \text{ ax}}{- \text{ : } \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes \\
\\
\frac{h_1 \text{ : } \Delta_2}{\bullet h_1 \text{ : } ?F_4, \Delta_2} \text{ ?W} \quad \frac{h_5 \text{ : } \Delta_6, F_7 \quad h_5 \text{ : } \Delta_9, F_8, !\text{dual}(F_4)}{\bullet h_5 \text{ : } \text{dual}(?F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes \\
\hline
- \text{ : } \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 \text{ : } \Delta_2, ?F_4}{\bullet h_1 \text{ : } \Delta_2, ?F_4} \text{ ax} \quad \frac{h_5 \text{ : } \Delta_9, F_8, !\text{dual}(F_4)}{h_5 \text{ : } \Delta_9, F_8, !\text{dual}(F_4)} \text{ ax}}{- \text{ : } \Delta_2, \Delta_9, F_7} \text{ hCut} \quad \frac{- \text{ : } \Delta_6, F_8}{- \text{ : } \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes \text{ ax}}{- \text{ : } \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes
\end{array}$$

5.4 Status of ?C: OK

- Case rule 1
- Case rule !

$$\begin{array}{c}
\frac{h_1 \text{ : } \Delta_2, ?F_4, ?F_4}{\bullet h_1 \text{ : } ?F_4, \Delta_2} \text{ ?C} \quad \frac{h_5 \text{ : } ?\Upsilon 6, \text{dual}(F_4)}{\bullet h_5 \text{ : } \text{dual}(?F_4), ?\Upsilon 6} ! \\
\hline
- \text{ : } \Delta_2, ?\Upsilon 6 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 \text{ : } \Delta_2, ?F_4, ?F_4}{\bullet h_1 \text{ : } \Delta_2, ?F_4, ?F_4} \text{ ax} \quad \frac{h_5 \text{ : } ?\Upsilon 6, \text{dual}(F_4)}{\bullet h_5 \text{ : } ?\Upsilon 6, \text{dual}(F_4)} !}{- \text{ : } ?\Upsilon 6, \Delta_2} \text{ mCut}
\end{array}$$

- Case rule ?W

$$\begin{array}{c}
\frac{h_2 \text{ : } \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 \text{ : } F_5, \Delta_4, ?F_3} \text{ ?C} \quad \frac{h_6 \text{ : } \Delta_8, \text{dual}(F_5)}{\bullet h_6 \text{ : } \text{dual}(F_5), \Delta_8, ?F_7} \text{ ?W} \\
\hline
- \text{ : } (\Delta_4, ?F_3), \Delta_8, ?F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \text{ : } \Delta_4, F_5, ?F_3}{\bullet h_2 \text{ : } \Delta_4, F_5, ?F_3} \text{ ax} \quad \frac{h_6 \text{ : } \Delta_8, \text{dual}(F_5)}{h_6 \text{ : } \Delta_8, \text{dual}(F_5)} \text{ ax}}{- \text{ : } \Delta_4, \Delta_8, ?F_3} \text{ hCut}}{- \text{ : } \Delta_4, \Delta_8, ?F_3, ?F_7} \text{ W} \\
\\
\frac{h_1 \text{ : } \Delta_2, ?F_4, ?F_4}{\bullet h_1 \text{ : } ?F_4, \Delta_2} \text{ ?C} \quad \frac{h_5 \text{ : } \Delta_7, !\text{dual}(F_4)}{\bullet h_5 \text{ : } \text{dual}(?F_4), \Delta_7, ?F_6} \text{ ?W} \\
\hline
- \text{ : } \Delta_2, \Delta_7, ?F_6 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \text{ : } \Delta_2, ?F_4}{\bullet h_1 \text{ : } \Delta_2, ?F_4} \text{ ax} \quad \frac{h_5 \text{ : } \Delta_7, !\text{dual}(F_4)}{h_5 \text{ : } \Delta_7, !\text{dual}(F_4)} \text{ ax}}{- \text{ : } \Delta_2, \Delta_7} \text{ hCut}}{- \text{ : } \Delta_2, \Delta_7, ?F_6} \text{ ?W}
\end{array}$$

- Case rule ?C

$$\begin{array}{c}
\frac{h_2 \vdash \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 \vdash F_5, \Delta_4, ?F_3} ?C \quad \frac{h_6 \vdash \Delta_8, ?F_7, ?F_7, dual(F_5)}{\bullet h_6 \vdash dual(F_5), \Delta_8, ?F_7} ?C}{- \vdash (\Delta_4, ?F_3), \Delta_8, ?F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \vdash \Delta_4, F_5, ?F_3, ?F_3}{- \vdash \Delta_4, \Delta_8, ?F_3, ?F_3, ?F_7} \text{ax} \quad \frac{\bullet h_6 \vdash \Delta_8, ?F_7, dual(F_5)}{- \vdash \Delta_4, \Delta_8, ?F_3, ?F_7} \text{ax}}{- \vdash \Delta_4, \Delta_8, ?F_3, ?F_7} \text{hCut} \\
- \vdash \Delta_4, \Delta_8, ?F_3, ?F_7} ?C
\end{array}$$

$$\begin{array}{c}
\frac{h_1 \vdash \Delta_2, ?F_4, ?F_4}{\bullet h_1 \vdash ?F_4, \Delta_2} ?C \quad \frac{h_5 \vdash \Delta_7, !dual(F_4), ?F_6, ?F_6}{\bullet h_5 \vdash dual(?F_4), \Delta_7, ?F_6} ?C}{- \vdash \Delta_2, \Delta_7, ?F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, ?F_4}{- \vdash \Delta_2, \Delta_7, ?F_6, ?F_6} \text{ax} \quad \frac{h_5 \vdash \Delta_7, !dual(F_4), ?F_6, ?F_6}{- \vdash \Delta_2, \Delta_7, ?F_6} \text{ax}}{- \vdash \Delta_2, \Delta_7, ?F_6} \text{hCut} \\
- \vdash \Delta_2, \Delta_7, ?F_6} ?C
\end{array}$$

• Case rule ?

$$\begin{array}{c}
\frac{h_2 \vdash \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 \vdash F_5, \Delta_4, ?F_3} ?C \quad \frac{h_6 \vdash \Delta_8, F_7, dual(F_5)}{\bullet h_6 \vdash dual(F_5), \Delta_8, ?F_7} ?}{- \vdash (\Delta_4, ?F_3), \Delta_8, ?F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \vdash \Delta_4, F_5, ?F_3, ?F_3}{- \vdash \Delta_4, \Delta_8, ?F_3, ?F_3, ?F_7} \text{ax} \quad \frac{\bullet h_6 \vdash \Delta_8, ?F_7, dual(F_5)}{- \vdash \Delta_4, \Delta_8, ?F_3, ?F_7} \text{ax}}{- \vdash \Delta_4, \Delta_8, ?F_3, ?F_7} \text{hCut} \\
- \vdash \Delta_4, \Delta_8, ?F_3, ?F_7} ?C
\end{array}$$

$$\begin{array}{c}
\frac{h_1 \vdash \Delta_2, ?F_4, ?F_4}{\bullet h_1 \vdash ?F_4, \Delta_2} ?C \quad \frac{h_5 \vdash \Delta_7, F_6, !dual(F_4)}{\bullet h_5 \vdash dual(?F_4), \Delta_7, ?F_6} ?}{- \vdash \Delta_2, \Delta_7, ?F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, ?F_4}{- \vdash \Delta_2, \Delta_7, F_6} \text{ax} \quad \frac{h_5 \vdash \Delta_7, F_6, !dual(F_4)}{- \vdash \Delta_2, \Delta_7, ?F_6} \text{ax}}{- \vdash \Delta_2, \Delta_7, ?F_6} \text{hCut} \\
- \vdash \Delta_2, \Delta_7, ?F_6} ?
\end{array}$$

• Case rule \$

$$\begin{array}{c}
\frac{h_2 \vdash \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 \vdash F_5, \Delta_4, ?F_3} ?C \quad \frac{h_6 \vdash \Delta_9, F_7, F_8, dual(F_5)}{\bullet h_6 \vdash dual(F_5), \Delta_9, F_7\$F_8} \$}{- \vdash (\Delta_4, ?F_3), \Delta_9, F_7\$F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \vdash \Delta_4, F_5, ?F_3, ?F_3}{- \vdash \Delta_4, \Delta_9, ?F_3, ?F_3, F_7\$F_8} \text{ax} \quad \frac{\bullet h_6 \vdash \Delta_9, dual(F_5), F_7\$F_8}{- \vdash \Delta_4, \Delta_9, ?F_3, F_7\$F_8} \text{ax}}{- \vdash \Delta_4, \Delta_9, ?F_3, F_7\$F_8} \text{hCut} \\
- \vdash \Delta_4, \Delta_9, ?F_3, F_7\$F_8} ?C
\end{array}$$

$$\begin{array}{c}
\frac{h_1 \vdash \Delta_2, ?F_4, ?F_4}{\bullet h_1 \vdash ?F_4, \Delta_2} ?C \quad \frac{h_5 \vdash \Delta_8, F_6, F_7, !dual(F_4)}{\bullet h_5 \vdash dual(?F_4), \Delta_8, F_6\$F_7} \$}{- \vdash \Delta_2, \Delta_8, F_6\$F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, ?F_4}{- \vdash \Delta_2, \Delta_8, F_6, F_7} \text{ax} \quad \frac{h_5 \vdash \Delta_8, F_6, F_7, !dual(F_4)}{- \vdash \Delta_2, \Delta_8, F_6\$F_7} \text{ax}}{- \vdash \Delta_2, \Delta_8, F_6, F_7} \text{hCut} \\
- \vdash \Delta_2, \Delta_8, F_6\$F_7} \$
\end{array}$$

• Case rule &

$$\begin{array}{c}
\frac{h_2 \vdash \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 \vdash F_5, \Delta_4, ?F_3} ?C \quad \frac{h_6 \vdash \Delta_9, F_7, dual(F_5)}{\bullet h_6 \vdash dual(F_5), \Delta_9, F_7\&F_8} \& \quad \frac{h_6 \vdash \Delta_9, F_8, dual(F_5)}{\bullet h_6 \vdash dual(F_5), \Delta_9, F_7\&F_8} \&}{- \vdash (\Delta_4, ?F_3), \Delta_9, F_7\&F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \vdash \Delta_4, F_5, ?F_3, ?F_3}{- \vdash \Delta_4, \Delta_9, ?F_3, ?F_3, F_7\&F_8} \text{ax} \quad \frac{\bullet h_6 \vdash \Delta_9, dual(F_5), F_7\&F_8}{- \vdash \Delta_4, \Delta_9, ?F_3, F_7\&F_8} \text{ax}}{- \vdash \Delta_4, \Delta_9, ?F_3, F_7\&F_8} \text{hCut} \\
- \vdash \Delta_4, \Delta_9, ?F_3, F_7\&F_8} ?C
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2, ?F_4, ?F_4}{\bullet h_1 : \vdash ?F_4, \Delta_2} ?C \quad \frac{h_5 : \vdash \Delta_8, F_6, !dual(F_4) \quad h_5 : \vdash \Delta_8, F_7, !dual(F_4)}{\bullet h_5 : \vdash dual(?F_4), \Delta_8, F_6 \& F_7} \& \\
\hline
- : \vdash \Delta_2, \Delta_8, F_6 \& F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{h_1 : \vdash \Delta_2, ?F_4, ?F_4}{\bullet h_1 : \vdash \Delta_2, ?F_4, ?F_4} \text{ax} \quad \frac{h_5 : \vdash \Delta_8, F_6, !dual(F_4)}{\bullet h_5 : \vdash \Delta_8, !dual(F_4), F_6 \& F_7} \text{ax} \\
\hline
- : \vdash \Delta_2, \Delta_8, F_6 \& F_7 \quad \text{mCut}
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 : \vdash F_5, \Delta_4, ?F_3} ?C \quad \frac{h_6 : \vdash \Delta_9, F_8, dual(F_5)}{\bullet h_6 : \vdash dual(F_5), \Delta_9, F_7 \oplus F_8} \oplus_B \\
\hline
- : \vdash (\Delta_4, ?F_3), \Delta_9, F_7 \oplus F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{h_2 : \vdash \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 : \vdash \Delta_4, \Delta_9, ?F_3, ?F_3, F_7 \oplus F_8} \text{ax} \quad \frac{h_6 : \vdash \Delta_9, dual(F_5), F_7 \oplus F_8}{\bullet h_6 : \vdash \Delta_9, ?F_3, ?F_3, F_7 \oplus F_8} \text{ax} \\
\hline
- : \vdash \Delta_4, \Delta_9, ?F_3, F_7 \oplus F_8 \quad \text{hCut} \\
\rightarrow \\
\frac{h_2 : \vdash \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 : \vdash \Delta_4, \Delta_9, ?F_3, ?F_3, F_7 \oplus F_8} ?C \\
\hline
- : \vdash \Delta_4, \Delta_9, ?F_3, F_7 \oplus F_8
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2, ?F_4, ?F_4}{\bullet h_1 : \vdash ?F_4, \Delta_2} ?C \quad \frac{h_5 : \vdash \Delta_8, F_7, !dual(F_4)}{\bullet h_5 : \vdash dual(?F_4), \Delta_8, F_6 \oplus F_7} \oplus_B \\
\hline
- : \vdash \Delta_2, \Delta_8, F_6 \oplus F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{h_1 : \vdash \Delta_2, ?F_4}{\bullet h_1 : \vdash \Delta_2, ?F_4} \text{ax} \quad \frac{h_5 : \vdash \Delta_8, F_7, !dual(F_4)}{\bullet h_5 : \vdash \Delta_8, F_6 \oplus F_7} \text{ax} \\
\hline
- : \vdash \Delta_2, \Delta_8, F_7 \quad \text{hCut} \\
\rightarrow \\
- : \vdash \Delta_2, \Delta_8, F_6 \oplus F_7 \quad \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 : \vdash F_5, \Delta_4, ?F_3} ?C \quad \frac{h_6 : \vdash \Delta_9, F_7, dual(F_5)}{\bullet h_6 : \vdash dual(F_5), \Delta_9, F_7 \oplus F_8} \oplus_A \\
\hline
- : \vdash (\Delta_4, ?F_3), \Delta_9, F_7 \oplus F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{h_2 : \vdash \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 : \vdash \Delta_4, \Delta_9, ?F_3, ?F_3, F_7 \oplus F_8} \text{ax} \quad \frac{h_6 : \vdash \Delta_9, dual(F_5), F_7 \oplus F_8}{\bullet h_6 : \vdash \Delta_9, ?F_3, ?F_3, F_7 \oplus F_8} \text{ax} \\
\hline
- : \vdash \Delta_4, \Delta_9, ?F_3, F_7 \oplus F_8 \quad \text{hCut} \\
\rightarrow \\
\frac{h_2 : \vdash \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 : \vdash \Delta_4, \Delta_9, ?F_3, ?F_3, F_7 \oplus F_8} ?C \\
\hline
- : \vdash \Delta_4, \Delta_9, ?F_3, F_7 \oplus F_8
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2, ?F_4, ?F_4}{\bullet h_1 : \vdash ?F_4, \Delta_2} ?C \quad \frac{h_5 : \vdash \Delta_8, F_6, !dual(F_4)}{\bullet h_5 : \vdash dual(?F_4), \Delta_8, F_6 \oplus F_7} \oplus_A \\
\hline
- : \vdash \Delta_2, \Delta_8, F_6 \oplus F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{h_1 : \vdash \Delta_2, ?F_4}{\bullet h_1 : \vdash \Delta_2, ?F_4} \text{ax} \quad \frac{h_5 : \vdash \Delta_8, F_6, !dual(F_4)}{\bullet h_5 : \vdash \Delta_8, F_6 \oplus F_7} \text{ax} \\
\hline
- : \vdash \Delta_2, \Delta_8, F_6 \quad \text{hCut} \\
\rightarrow \\
- : \vdash \Delta_2, \Delta_8, F_6 \oplus F_7 \quad \oplus_A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 : \vdash F_5, \Delta_4, ?F_3} ?C \quad \frac{h_6 : \vdash \Delta_7, dual(F_5)}{\bullet h_6 : \vdash dual(F_5), \perp, \Delta_7} \perp \\
\hline
- : \vdash (\Delta_4, ?F_3), \perp, \Delta_7 \quad \text{Cut} \\
\rightarrow \\
\frac{h_2 : \vdash \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 : \vdash \Delta_4, \Delta_7, \perp, ?F_3, ?F_3} \text{ax} \quad \frac{h_6 : \vdash \Delta_7, \perp, dual(F_5)}{\bullet h_6 : \vdash \Delta_7, \perp, ?F_3, ?F_3} \text{ax} \\
\hline
- : \vdash \Delta_4, \Delta_7, \perp, ?F_3, ?F_3 \quad \text{hCut} \\
\rightarrow \\
\frac{h_2 : \vdash \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 : \vdash \Delta_4, \Delta_7, \perp, ?F_3, ?F_3} ?C \\
\hline
- : \vdash \Delta_4, \Delta_7, \perp, ?F_3, ?F_3
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2, ?F_4, ?F_4}{\bullet h_1 : \vdash ?F_4, \Delta_2} ?C \quad \frac{h_5 : \vdash \Delta_6, !dual(F_4)}{\bullet h_5 : \vdash dual(?F_4), \perp, \Delta_6} \perp \\
\hline
- : \vdash \Delta_2, \perp, \Delta_6 \quad \text{Cut} \\
\rightarrow \\
\frac{h_1 : \vdash \Delta_2, ?F_4, ?F_4}{\bullet h_1 : \vdash \Delta_2, \Delta_6, \perp} \text{ax} \quad \frac{h_5 : \vdash \Delta_6, !dual(F_4)}{\bullet h_5 : \vdash \Delta_6, \perp, !dual(F_4)} \perp \\
\hline
- : \vdash \Delta_2, \Delta_6, \perp \quad \text{mCut}
\end{array}$$

- Case rule \top

$$\frac{\frac{h_2 \text{ :- } \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 \text{ :- } F_5, \Delta_4, ?F_3} ?C \quad \frac{}{\bullet h_6 \text{ :- } \text{dual}(F_5), \top, \Delta_7} \top}{\text{-- :- } (\Delta_4, ?F_3), \top, \Delta_7} \text{Cut}}{\text{-- :- } \Delta_4, \Delta_7, \top, ?F_3} \top$$

$$\frac{h_1 \text{ :- } \Delta_2, ?F_4, ?F_4}{\bullet h_1 \text{ :- } ?F_4, \Delta_2} ?C \quad \frac{}{\bullet h_5 \text{ :- } \text{dual}(?F_4), \top, \Delta_6} \top}{\text{-- :- } \Delta_2, \top, \Delta_6} \text{Cut}}{\text{-- :- } \Delta_2, \Delta_6, \top} \top$$

- Case rule I

- Case rule \otimes

$$\frac{\frac{h_2 \text{ :- } \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 \text{ :- } F_5, \Delta_4, ?F_3} ?C \quad \frac{h_6 \text{ :- } \Delta_{10}, F_8, \text{dual}(F_5) \quad h_6 \text{ :- } \Delta_7, F_9}{\bullet h_6 \text{ :- } \text{dual}(F_5), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes}{\text{-- :- } (\Delta_4, ?F_3), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \text{Cut}}{\text{-- :- } \Delta_{10}, \Delta_4, \Delta_7, ?F_3, ?F_3, F_8 \otimes F_9} \otimes} \text{ax}$$

$$\frac{\frac{h_2 \text{ :- } \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 \text{ :- } F_5, \Delta_4, ?F_3} ?C \quad \frac{h_6 \text{ :- } \Delta_{10}, \Delta_7, \text{dual}(F_5), F_8 \otimes F_9}{\bullet h_6 \text{ :- } \Delta_{10}, \Delta_7, ?F_3, ?F_3, F_8 \otimes F_9} \text{ax}}{\text{-- :- } \Delta_{10}, \Delta_4, \Delta_7, ?F_3, ?F_3, F_8 \otimes F_9} \text{hCut}}{\text{-- :- } \Delta_{10}, \Delta_4, \Delta_7, ?F_3, F_8 \otimes F_9} ?C$$

$$\frac{h_2 \text{ :- } \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 \text{ :- } F_5, \Delta_4, ?F_3} ?C \quad \frac{h_6 \text{ :- } \Delta_7, F_8 \quad h_6 \text{ :- } \Delta_{10}, F_9, \text{dual}(F_5)}{\bullet h_6 \text{ :- } \text{dual}(F_5), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes}{\text{-- :- } (\Delta_4, ?F_3), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \text{Cut}}{\text{-- :- } \Delta_{10}, \Delta_4, \Delta_7, ?F_3, ?F_3, F_8 \otimes F_9} \otimes} \text{ax}$$

$$\frac{\frac{h_2 \text{ :- } \Delta_4, F_5, ?F_3, ?F_3}{\bullet h_2 \text{ :- } F_5, \Delta_4, ?F_3} ?C \quad \frac{h_6 \text{ :- } \Delta_{10}, \Delta_7, \text{dual}(F_5), F_8 \otimes F_9}{\bullet h_6 \text{ :- } \Delta_{10}, \Delta_7, ?F_3, ?F_3, F_8 \otimes F_9} \text{ax}}{\text{-- :- } \Delta_{10}, \Delta_4, \Delta_7, ?F_3, ?F_3, F_8 \otimes F_9} \text{hCut}}{\text{-- :- } \Delta_{10}, \Delta_4, \Delta_7, ?F_3, F_8 \otimes F_9} ?C$$

$$\frac{h_1 \text{ :- } \Delta_2, ?F_4, ?F_4}{\bullet h_1 \text{ :- } ?F_4, \Delta_2} ?C \quad \frac{h_5 \text{ :- } \Delta_9, F_7, !\text{dual}(F_4) \quad h_5 \text{ :- } \Delta_6, F_8}{\bullet h_5 \text{ :- } \text{dual}(?F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes}{\text{-- :- } \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{Cut}}{\text{-- :- } \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes} \text{ax}$$

$$\frac{\frac{h_1 \text{ :- } \Delta_2, ?F_4, ?F_4}{\bullet h_1 \text{ :- } ?F_4, \Delta_2} ?C \quad \frac{h_5 \text{ :- } \Delta_9, F_7, !\text{dual}(F_4) \quad h_5 \text{ :- } \Delta_6, F_8}{\bullet h_5 \text{ :- } \text{dual}(?F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes}{\text{-- :- } \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{Cut}}{\text{-- :- } \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes} \text{mCut}$$

$$\frac{h_1 \text{ :- } \Delta_2, ?F_4, ?F_4}{\bullet h_1 \text{ :- } ?F_4, \Delta_2} ?C \quad \frac{h_5 \text{ :- } \Delta_6, F_7 \quad h_5 \text{ :- } \Delta_9, F_8, !\text{dual}(F_4)}{\bullet h_5 \text{ :- } \text{dual}(?F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes}{\text{-- :- } \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{Cut}}{\text{-- :- } \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes} \text{ax}$$

$$\frac{\frac{h_1 \text{ :- } \Delta_2, ?F_4, ?F_4}{\bullet h_1 \text{ :- } ?F_4, \Delta_2} ?C \quad \frac{h_5 \text{ :- } \Delta_6, F_7 \quad h_5 \text{ :- } \Delta_9, F_8, !\text{dual}(F_4)}{\bullet h_5 \text{ :- } \text{dual}(?F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes}{\text{-- :- } \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{Cut}}{\text{-- :- } \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes} \text{mCut}$$

5.5 Status of $?$: OK

- Case rule 1

- Case rule !

$$\begin{array}{c}
\frac{h_1 \vdash \Delta_2, F_4}{\bullet h_1 \vdash ?F_4, \Delta_2} ? \quad \frac{h_5 \vdash ?\Upsilon 6, \text{dual}(F_4)}{\bullet h_5 \vdash \text{dual}(?F_4), ?\Upsilon 6} !}{- \vdash \Delta_2, ?\Upsilon 6} \text{Cut} \\
\rightarrow \\
\frac{- \vdash \Delta_2, F_4}{- \vdash ?\Upsilon 6, \Delta_2} \text{ax} \quad \frac{- \vdash ?\Upsilon 6, \text{dual}(F_4)}{- \vdash ?\Upsilon 6, \Delta_2} \text{ax}}{- \vdash ?\Upsilon 6, \Delta_2} \text{sCut}
\end{array}$$

- Case rule ?W

$$\begin{array}{c}
\frac{h_2 \vdash \Delta_4, F_5, F_3}{\bullet h_2 \vdash F_5, \Delta_4, ?F_3} ? \quad \frac{h_6 \vdash \Delta_8, \text{dual}(F_5)}{\bullet h_6 \vdash \text{dual}(F_5), \Delta_8, ?F_7} ?W}{- \vdash (\Delta_4, ?F_3), \Delta_8, ?F_7} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 \vdash \Delta_4, F_5, ?F_3}{- \vdash \Delta_4, \Delta_8, ?F_3} \text{ax} \quad \frac{h_6 \vdash \Delta_8, \text{dual}(F_5)}{- \vdash \Delta_4, \Delta_8, ?F_3, ?F_7} \text{ax}}{- \vdash \Delta_4, \Delta_8, ?F_3, ?F_7} \text{hCut} \\
\frac{- \vdash \Delta_4, \Delta_8, ?F_3, ?F_7}{- \vdash \Delta_4, \Delta_8, ?F_3, ?F_7} ?W \\
\frac{h_1 \vdash \Delta_2, F_4}{\bullet h_1 \vdash ?F_4, \Delta_2} ? \quad \frac{h_5 \vdash \Delta_7, !\text{dual}(F_4)}{\bullet h_5 \vdash \text{dual}(?F_4), \Delta_7, ?F_6} ?W}{- \vdash \Delta_2, \Delta_7, ?F_6} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 \vdash \Delta_2, ?F_4}{- \vdash \Delta_2, \Delta_7} \text{ax} \quad \frac{h_5 \vdash \Delta_7, !\text{dual}(F_4)}{- \vdash \Delta_2, \Delta_7, ?F_6} \text{ax}}{- \vdash \Delta_2, \Delta_7, ?F_6} \text{hCut} \\
\frac{- \vdash \Delta_2, \Delta_7}{- \vdash \Delta_2, \Delta_7, ?F_6} W
\end{array}$$

- Case rule ?C

$$\begin{array}{c}
\frac{h_2 \vdash \Delta_4, F_5, F_3}{\bullet h_2 \vdash F_5, \Delta_4, ?F_3} ? \quad \frac{h_6 \vdash \Delta_8, ?F_7, ?F_7, \text{dual}(F_5)}{\bullet h_6 \vdash \text{dual}(F_5), \Delta_8, ?F_7} ?C}{- \vdash (\Delta_4, ?F_3), \Delta_8, ?F_7} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 \vdash \Delta_4, F_5, ?F_3}{- \vdash \Delta_4, \Delta_8, ?F_3, ?F_7, ?F_7} \text{ax} \quad \frac{h_6 \vdash \Delta_8, ?F_7, ?F_7, \text{dual}(F_5)}{- \vdash \Delta_4, \Delta_8, ?F_3, ?F_7, ?F_7} \text{ax}}{- \vdash \Delta_4, \Delta_8, ?F_3, ?F_7} \text{hCut} \\
\frac{- \vdash \Delta_4, \Delta_8, ?F_3, ?F_7, ?F_7}{- \vdash \Delta_4, \Delta_8, ?F_3, ?F_7} ?C \\
\frac{h_1 \vdash \Delta_2, F_4}{\bullet h_1 \vdash ?F_4, \Delta_2} ? \quad \frac{h_5 \vdash \Delta_7, !\text{dual}(F_4), ?F_6, ?F_6}{\bullet h_5 \vdash \text{dual}(?F_4), \Delta_7, ?F_6} ?C}{- \vdash \Delta_2, \Delta_7, ?F_6} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 \vdash \Delta_2, ?F_4}{- \vdash \Delta_2, \Delta_7, ?F_6, ?F_6} \text{ax} \quad \frac{h_5 \vdash \Delta_7, !\text{dual}(F_4), ?F_6, ?F_6}{- \vdash \Delta_2, \Delta_7, ?F_6, ?F_6} \text{ax}}{- \vdash \Delta_2, \Delta_7, ?F_6} \text{hCut} \\
\frac{- \vdash \Delta_2, \Delta_7, ?F_6, ?F_6}{- \vdash \Delta_2, \Delta_7, ?F_6} ?C
\end{array}$$

- Case rule ?

$$\begin{array}{c}
\frac{h_2 \vdash \Delta_4, F_5, F_3}{\bullet h_2 \vdash F_5, \Delta_4, ?F_3} ? \quad \frac{h_6 \vdash \Delta_8, F_7, \text{dual}(F_5)}{\bullet h_6 \vdash \text{dual}(F_5), \Delta_8, ?F_7} ?}{- \vdash (\Delta_4, ?F_3), \Delta_8, ?F_7} \text{Cut} \\
\rightarrow \\
\frac{h_2 \vdash \Delta_4, F_3, F_5}{- \vdash \Delta_4, \Delta_8, F_3, ?F_7} \text{ax} \quad \frac{\bullet h_6 \vdash \Delta_8, ?F_7, \text{dual}(F_5)}{- \vdash \Delta_4, \Delta_8, ?F_3, ?F_7} \text{ax}}{- \vdash \Delta_4, \Delta_8, F_3, ?F_7} \text{hCut} \\
\frac{- \vdash \Delta_4, \Delta_8, F_3, ?F_7}{- \vdash \Delta_4, \Delta_8, ?F_3, ?F_7} ? \\
\frac{h_1 \vdash \Delta_2, F_4}{\bullet h_1 \vdash ?F_4, \Delta_2} ? \quad \frac{h_5 \vdash \Delta_7, F_6, !\text{dual}(F_4)}{\bullet h_5 \vdash \text{dual}(?F_4), \Delta_7, ?F_6} ?}{- \vdash \Delta_2, \Delta_7, ?F_6} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 \vdash \Delta_2, ?F_4}{- \vdash \Delta_2, \Delta_7, F_6} \text{ax} \quad \frac{h_5 \vdash \Delta_7, F_6, !\text{dual}(F_4)}{- \vdash \Delta_2, \Delta_7, ?F_6} \text{ax}}{- \vdash \Delta_2, \Delta_7, ?F_6} \text{hCut} \\
\frac{- \vdash \Delta_2, \Delta_7, F_6}{- \vdash \Delta_2, \Delta_7, ?F_6} ?
\end{array}$$

- Case rule $\$$

$$\begin{array}{c}
\frac{h_2 \vdash \Delta_4, F_5, F_3}{\bullet h_2 \vdash F_5, \Delta_4, ?F_3} ? \quad \frac{h_6 \vdash \Delta_9, F_7, F_8, dual(F_5)}{\bullet h_6 \vdash dual(F_5), \Delta_9, F_7 \& F_8} \$}{- \vdash (\Delta_4, ?F_3), \Delta_9, F_7 \& F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \vdash \Delta_4, F_3, F_5}{ax} \quad \frac{\bullet h_6 \vdash \Delta_9, dual(F_5), F_7 \& F_8}{ax}}{- \vdash \Delta_4, \Delta_9, F_3, F_7 \& F_8} \text{hCut} \quad ?}{- \vdash \Delta_4, \Delta_9, ?F_3, F_7 \& F_8} \\
\\
\frac{h_1 \vdash \Delta_2, F_4}{\bullet h_1 \vdash ?F_4, \Delta_2} ? \quad \frac{h_5 \vdash \Delta_8, F_6, F_7, !dual(F_4)}{\bullet h_5 \vdash dual(?F_4), \Delta_8, F_6 \& F_7} \$}{- \vdash \Delta_2, \Delta_8, F_6 \& F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, ?F_4}{ax} \quad \frac{h_5 \vdash \Delta_8, F_6, F_7, !dual(F_4)}{ax}}{- \vdash \Delta_2, \Delta_8, F_6, F_7} \text{hCut} \quad \$}{- \vdash \Delta_2, \Delta_8, F_6 \& F_7}
\end{array}$$

- Case rule $\&$

$$\begin{array}{c}
\frac{h_2 \vdash \Delta_4, F_5, F_3}{\bullet h_2 \vdash F_5, \Delta_4, ?F_3} ? \quad \frac{h_6 \vdash \Delta_9, F_7, dual(F_5)}{\bullet h_6 \vdash dual(F_5), \Delta_9, F_7 \& F_8} \quad \frac{h_6 \vdash \Delta_9, F_8, dual(F_5)}{\bullet h_6 \vdash dual(F_5), \Delta_9, F_7 \& F_8} \&}{- \vdash (\Delta_4, ?F_3), \Delta_9, F_7 \& F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \vdash \Delta_4, F_3, F_5}{ax} \quad \frac{\bullet h_6 \vdash \Delta_9, dual(F_5), F_7 \& F_8}{ax}}{- \vdash \Delta_4, \Delta_9, F_3, F_7 \& F_8} \text{hCut} \quad ?}{- \vdash \Delta_4, \Delta_9, ?F_3, F_7 \& F_8} \\
\\
\frac{h_1 \vdash \Delta_2, F_4}{\bullet h_1 \vdash ?F_4, \Delta_2} ? \quad \frac{h_5 \vdash \Delta_8, F_6, !dual(F_4)}{\bullet h_5 \vdash dual(?F_4), \Delta_8, F_6 \& F_7} \quad \frac{h_5 \vdash \Delta_8, F_7, !dual(F_4)}{\bullet h_5 \vdash dual(?F_4), \Delta_8, F_6 \& F_7} \&}{- \vdash \Delta_2, \Delta_8, F_6 \& F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, ?F_4}{ax} \quad \frac{h_5 \vdash \Delta_8, F_6, !dual(F_4)}{ax}}{- \vdash \Delta_2, \Delta_8, F_6} \text{hCut} \quad \frac{\frac{\bullet h_1 \vdash \Delta_2, ?F_4}{ax} \quad \frac{h_5 \vdash \Delta_8, F_7, !dual(F_4)}{ax}}{- \vdash \Delta_2, \Delta_8, F_7} \text{hCut}}{- \vdash \Delta_2, \Delta_8, F_6 \& F_7} \&
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{h_2 \vdash \Delta_4, F_5, F_3}{\bullet h_2 \vdash F_5, \Delta_4, ?F_3} ? \quad \frac{h_6 \vdash \Delta_9, F_8, dual(F_5)}{\bullet h_6 \vdash dual(F_5), \Delta_9, F_7 \oplus F_8} \oplus_B}{- \vdash (\Delta_4, ?F_3), \Delta_9, F_7 \oplus F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \vdash \Delta_4, F_3, F_5}{ax} \quad \frac{\bullet h_6 \vdash \Delta_9, dual(F_5), F_7 \oplus F_8}{ax}}{- \vdash \Delta_4, \Delta_9, F_3, F_7 \oplus F_8} \text{hCut} \quad ?}{- \vdash \Delta_4, \Delta_9, ?F_3, F_7 \oplus F_8} \\
\\
\frac{h_1 \vdash \Delta_2, F_4}{\bullet h_1 \vdash ?F_4, \Delta_2} ? \quad \frac{h_5 \vdash \Delta_8, F_7, !dual(F_4)}{\bullet h_5 \vdash dual(?F_4), \Delta_8, F_6 \oplus F_7} \oplus_B}{- \vdash \Delta_2, \Delta_8, F_6 \oplus F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, ?F_4}{ax} \quad \frac{h_5 \vdash \Delta_8, F_7, !dual(F_4)}{ax}}{- \vdash \Delta_2, \Delta_8, F_7} \text{hCut}}{- \vdash \Delta_2, \Delta_8, F_6 \oplus F_7} \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{h_2 \text{ :- } \Delta_4, F_5, F_3}{\bullet h_2 \text{ :- } F_5, \Delta_4, ?F_3} \text{ ?} \quad \frac{h_6 \text{ :- } \Delta_9, F_7, \text{dual}(F_5)}{\bullet h_6 \text{ :- } \text{dual}(F_5), \Delta_9, F_7 \oplus F_8} \oplus_A}{- \text{ :- } (\Delta_4, ?F_3), \Delta_9, F_7 \oplus F_8} \text{ Cut} \\
\rightarrow \\
\frac{\frac{h_2 \text{ :- } \Delta_4, F_3, F_5}{\bullet h_2 \text{ :- } \Delta_4, F_3, F_5} \text{ ax} \quad \frac{h_6 \text{ :- } \Delta_9, \text{dual}(F_5), F_7 \oplus F_8}{\bullet h_6 \text{ :- } \Delta_9, \text{dual}(F_5), F_7 \oplus F_8} \text{ ax}}{- \text{ :- } \Delta_4, \Delta_9, F_3, F_7 \oplus F_8} \text{ hCut}}{- \text{ :- } \Delta_4, \Delta_9, ?F_3, F_7 \oplus F_8} \text{ ?} \\
\frac{h_1 \text{ :- } \Delta_2, F_4}{\bullet h_1 \text{ :- } ?F_4, \Delta_2} \text{ ?} \quad \frac{h_5 \text{ :- } \Delta_8, F_6, \text{!dual}(F_4)}{\bullet h_5 \text{ :- } \text{dual}(?F_4), \Delta_8, F_6 \oplus F_7} \oplus_A}{- \text{ :- } \Delta_2, \Delta_8, F_6 \oplus F_7} \text{ Cut} \\
\rightarrow \\
\frac{\frac{h_1 \text{ :- } \Delta_2, ?F_4}{\bullet h_1 \text{ :- } \Delta_2, ?F_4} \text{ ax} \quad \frac{h_5 \text{ :- } \Delta_8, F_6, \text{!dual}(F_4)}{\bullet h_5 \text{ :- } \Delta_8, F_6, \text{!dual}(F_4)} \text{ ax}}{- \text{ :- } \Delta_2, \Delta_8, F_6} \text{ hCut}}{- \text{ :- } \Delta_2, \Delta_8, F_6 \oplus F_7} \oplus_A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_2 \text{ :- } \Delta_4, F_5, F_3}{\bullet h_2 \text{ :- } F_5, \Delta_4, ?F_3} \text{ ?} \quad \frac{h_6 \text{ :- } \Delta_7, \text{dual}(F_5)}{\bullet h_6 \text{ :- } \text{dual}(F_5), \perp, \Delta_7} \perp}{- \text{ :- } (\Delta_4, ?F_3), \perp, \Delta_7} \text{ Cut} \\
\rightarrow \\
\frac{\frac{h_2 \text{ :- } \Delta_4, F_3, F_5}{\bullet h_2 \text{ :- } \Delta_4, F_3, F_5} \text{ ax} \quad \frac{h_6 \text{ :- } \Delta_7, \perp, \text{dual}(F_5)}{\bullet h_6 \text{ :- } \Delta_7, \perp, \text{dual}(F_5)} \text{ ax}}{- \text{ :- } \Delta_4, \Delta_7, F_3, \perp} \text{ hCut}}{- \text{ :- } \Delta_4, \Delta_7, \perp, ?F_3} \text{ ?} \\
\frac{h_1 \text{ :- } \Delta_2, F_4}{\bullet h_1 \text{ :- } ?F_4, \Delta_2} \text{ ?} \quad \frac{h_5 \text{ :- } \Delta_6, \text{!dual}(F_4)}{\bullet h_5 \text{ :- } \text{dual}(?F_4), \perp, \Delta_6} \perp}{- \text{ :- } \Delta_2, \perp, \Delta_6} \text{ Cut} \\
\rightarrow \\
\frac{\frac{h_1 \text{ :- } \Delta_2, ?F_4}{\bullet h_1 \text{ :- } \Delta_2, ?F_4} \text{ ax} \quad \frac{h_5 \text{ :- } \Delta_6, \text{!dual}(F_4)}{\bullet h_5 \text{ :- } \Delta_6, \text{!dual}(F_4)} \text{ ax}}{- \text{ :- } \Delta_2, \Delta_6} \text{ hCut}}{- \text{ :- } \Delta_2, \Delta_6, \perp} \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_2 \text{ :- } \Delta_4, F_5, F_3}{\bullet h_2 \text{ :- } F_5, \Delta_4, ?F_3} \text{ ?} \quad \frac{}{\bullet h_6 \text{ :- } \text{dual}(F_5), \top, \Delta_7} \top}{- \text{ :- } (\Delta_4, ?F_3), \top, \Delta_7} \text{ Cut} \\
\rightarrow \\
\frac{}{- \text{ :- } \Delta_4, \Delta_7, \top, ?F_3} \top \\
\frac{h_1 \text{ :- } \Delta_2, F_4}{\bullet h_1 \text{ :- } ?F_4, \Delta_2} \text{ ?} \quad \frac{}{\bullet h_5 \text{ :- } \text{dual}(?F_4), \top, \Delta_6} \top}{- \text{ :- } \Delta_2, \top, \Delta_6} \text{ Cut} \\
\rightarrow \\
\frac{}{- \text{ :- } \Delta_2, \Delta_6, \top} \top
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{h_2 \text{ :- } \Delta_4, F_5, F_3}{\bullet h_2 \text{ :- } F_5, \Delta_4, ?F_3} \text{ ?} \quad \frac{h_6 \text{ :- } \Delta_{10}, F_8, \text{dual}(F_5)}{\bullet h_6 \text{ :- } \text{dual}(F_5), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes \quad \frac{h_6 \text{ :- } \Delta_7, F_9}{\bullet h_6 \text{ :- } \Delta_7, F_9} \otimes}{- \text{ :- } (\Delta_4, ?F_3), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \text{ Cut} \\
\rightarrow \\
\frac{\frac{h_2 \text{ :- } \Delta_4, F_3, F_5}{\bullet h_2 \text{ :- } \Delta_4, F_3, F_5} \text{ ax} \quad \frac{h_6 \text{ :- } \Delta_{10}, \Delta_7, \text{dual}(F_5), F_8 \otimes F_9}{\bullet h_6 \text{ :- } \Delta_{10}, \Delta_7, \text{dual}(F_5), F_8 \otimes F_9} \text{ ax}}{- \text{ :- } \Delta_{10}, \Delta_4, \Delta_7, F_3, F_8 \otimes F_9} \text{ hCut}}{- \text{ :- } \Delta_{10}, \Delta_4, \Delta_7, ?F_3, F_8 \otimes F_9} \text{ ?}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_2 \text{ :- } \Delta_4, F_5, F_3}{\bullet h_2 \text{ :- } F_5, \Delta_4, ?F_3} ? \quad \frac{h_6 \text{ :- } \Delta_7, F_8 \quad h_6 \text{ :- } \Delta_{10}, F_9, \text{dual}(F_5)}{\bullet h_6 \text{ :- } \text{dual}(F_5), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes}{- \text{ :- } (\Delta_4, ?F_3), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \text{Cut}}{\rightarrow} \\
\frac{\frac{h_2 \text{ :- } \Delta_4, F_3, F_5}{- \text{ :- } \Delta_{10}, \Delta_4, \Delta_7, F_3, F_8 \otimes F_9} \text{ax} \quad \frac{\bullet h_6 \text{ :- } \Delta_{10}, \Delta_7, \text{dual}(F_5), F_8 \otimes F_9}{- \text{ :- } \Delta_{10}, \Delta_4, \Delta_7, ?F_3, F_8 \otimes F_9} \text{ax}}{- \text{ :- } \Delta_{10}, \Delta_4, \Delta_7, ?F_3, F_8 \otimes F_9} \text{hCut}}{- \text{ :- } \Delta_{10}, \Delta_4, \Delta_7, ?F_3, F_8 \otimes F_9} ? \\
\frac{\frac{h_1 \text{ :- } \Delta_2, F_4}{\bullet h_1 \text{ :- } ?F_4, \Delta_2} ? \quad \frac{h_5 \text{ :- } \Delta_9, F_7, !\text{dual}(F_4) \quad h_5 \text{ :- } \Delta_6, F_8}{\bullet h_5 \text{ :- } \text{dual}(?F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes}{- \text{ :- } \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_1 \text{ :- } \Delta_2, ?F_4}{- \text{ :- } \Delta_2, \Delta_9, F_7} \text{ax} \quad \frac{h_5 \text{ :- } \Delta_9, F_7, !\text{dual}(F_4)}{- \text{ :- } \Delta_6, F_8} \text{ax}}{- \text{ :- } \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{hCut}}{- \text{ :- } \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes \\
\frac{\frac{h_1 \text{ :- } \Delta_2, F_4}{\bullet h_1 \text{ :- } ?F_4, \Delta_2} ? \quad \frac{h_5 \text{ :- } \Delta_6, F_7 \quad h_5 \text{ :- } \Delta_9, F_8, !\text{dual}(F_4)}{\bullet h_5 \text{ :- } \text{dual}(?F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes}{- \text{ :- } \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{Cut}}{\rightarrow} \\
\frac{\frac{- \text{ :- } \Delta_6, F_7}{- \text{ :- } \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{ax} \quad \frac{\bullet h_1 \text{ :- } \Delta_2, ?F_4 \quad h_5 \text{ :- } \Delta_9, F_8, !\text{dual}(F_4)}{- \text{ :- } \Delta_2, \Delta_9, F_8} \text{ax}}{- \text{ :- } \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{hCut}}{- \text{ :- } \Delta_2, \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes
\end{array}$$

5.6 Status of \$: OK

- Case rule 1
- Case rule !
- Case rule ?W

$$\begin{array}{c}
\frac{\frac{h_2 \text{ :- } \Delta_5, F_6, F_3, F_4}{\bullet h_2 \text{ :- } F_6, \Delta_5, F_3 \$F_4} \$ \quad \frac{h_7 \text{ :- } \Delta_9, \text{dual}(F_6)}{\bullet h_7 \text{ :- } \text{dual}(F_6), \Delta_9, ?F_8} ?W}{- \text{ :- } (\Delta_5, F_3 \$F_4), \Delta_9, ?F_8} \text{Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_2 \text{ :- } \Delta_5, F_6, F_3 \$F_4}{- \text{ :- } \Delta_5, \Delta_9, F_3 \$F_4} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_9, \text{dual}(F_6)}{- \text{ :- } \Delta_9, \text{dual}(F_6)} \text{ax}}{- \text{ :- } \Delta_5, \Delta_9, F_3 \$F_4} \text{hCut}}{- \text{ :- } \Delta_5, \Delta_9, ?F_8, F_3 \$F_4} W \\
\frac{\frac{h_1 \text{ :- } \Delta_2, F_5, F_6}{\bullet h_1 \text{ :- } F_5 \$F_6, \Delta_2} \$ \quad \frac{h_7 \text{ :- } \Delta_9, \text{dual}(F_5) \otimes \text{dual}(F_6)}{\bullet h_7 \text{ :- } \text{dual}(F_5 \$F_6), \Delta_9, ?F_8} ?W}{- \text{ :- } \Delta_2, \Delta_9, ?F_8} \text{Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_1 \text{ :- } \Delta_2, F_5 \$F_6}{- \text{ :- } \Delta_2, \Delta_9} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_9, \text{dual}(F_5) \otimes \text{dual}(F_6)}{- \text{ :- } \Delta_2, \Delta_9} \text{ax}}{- \text{ :- } \Delta_2, \Delta_9, ?F_8} \text{hCut}}{- \text{ :- } \Delta_2, \Delta_9, ?F_8} ?W
\end{array}$$

- Case rule ?C

$$\begin{array}{c}
\frac{\frac{h_2 \text{ :- } \Delta_5, F_6, F_3, F_4}{\bullet h_2 \text{ :- } F_6, \Delta_5, F_3 \$F_4} \$ \quad \frac{h_7 \text{ :- } \Delta_9, ?F_8, ?F_8, \text{dual}(F_6)}{\bullet h_7 \text{ :- } \text{dual}(F_6), \Delta_9, ?F_8} ?C}{- \text{ :- } (\Delta_5, F_3 \$F_4), \Delta_9, ?F_8} \text{Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_2 \text{ :- } \Delta_5, F_6, F_3 \$F_4}{- \text{ :- } \Delta_5, \Delta_9, ?F_8, ?F_8, F_3 \$F_4} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_9, ?F_8, ?F_8, \text{dual}(F_6)}{- \text{ :- } \Delta_5, \Delta_9, ?F_8, ?F_8, F_3 \$F_4} \text{ax}}{- \text{ :- } \Delta_5, \Delta_9, ?F_8, ?F_8, F_3 \$F_4} \text{hCut}}{- \text{ :- } \Delta_5, \Delta_9, ?F_8, ?F_8, F_3 \$F_4} ?C
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{h_2 \text{ :- } \Delta_5, F_6, F_3, F_4 \quad \bullet h_2 \text{ :- } F_6, \Delta_5, F_3 \S F_4 \quad \S \quad \frac{h_7 \text{ :- } \Delta_{10}, F_9, \mathit{dual}(F_6)}{\bullet h_7 \text{ :- } \mathit{dual}(F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_B}{- \text{ :- } (\Delta_5, F_3 \S F_4), \Delta_{10}, F_8 \oplus F_9} \text{ Cut}}{\rightarrow} \\
\frac{\frac{h_2 \text{ :- } \Delta_5, F_3, F_4, F_6}{- \text{ :- } \Delta_{10}, \Delta_5, F_3, F_4, F_8 \oplus F_9} \text{ ax} \quad \frac{\bullet h_7 \text{ :- } \Delta_{10}, \mathit{dual}(F_6), F_8 \oplus F_9}{- \text{ :- } \Delta_{10}, \Delta_5, F_3 \S F_4, F_8 \oplus F_9} \text{ ax}}{- \text{ :- } \Delta_{10}, \Delta_5, F_3 \S F_4, F_8 \oplus F_9} \S \quad \text{hCut}}{\frac{h_1 \text{ :- } \Delta_2, F_5, F_6 \quad \bullet h_1 \text{ :- } F_5 \S F_6, \Delta_2 \quad \S \quad \frac{h_7 \text{ :- } \Delta_{10}, F_9, \mathit{dual}(F_5) \otimes \mathit{dual}(F_6)}{\bullet h_7 \text{ :- } \mathit{dual}(F_5 \S F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_B}{- \text{ :- } \Delta_2, \Delta_{10}, F_8 \oplus F_9} \text{ Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_1 \text{ :- } \Delta_2, F_5 \S F_6}{- \text{ :- } \Delta_{10}, \Delta_2, F_9} \text{ ax} \quad \frac{h_7 \text{ :- } \Delta_{10}, F_9, \mathit{dual}(F_5) \otimes \mathit{dual}(F_6)}{- \text{ :- } \Delta_{10}, \Delta_2, F_9} \text{ ax}}{- \text{ :- } \Delta_{10}, \Delta_2, F_8 \oplus F_9} \text{ hCut} \quad \oplus_B}{}
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{h_2 \text{ :- } \Delta_5, F_6, F_3, F_4 \quad \bullet h_2 \text{ :- } F_6, \Delta_5, F_3 \S F_4 \quad \S \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, \mathit{dual}(F_6)}{\bullet h_7 \text{ :- } \mathit{dual}(F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_A}{- \text{ :- } (\Delta_5, F_3 \S F_4), \Delta_{10}, F_8 \oplus F_9} \text{ Cut}}{\rightarrow} \\
\frac{\frac{h_2 \text{ :- } \Delta_5, F_3, F_4, F_6}{- \text{ :- } \Delta_{10}, \Delta_5, F_3, F_4, F_8 \oplus F_9} \text{ ax} \quad \frac{\bullet h_7 \text{ :- } \Delta_{10}, \mathit{dual}(F_6), F_8 \oplus F_9}{- \text{ :- } \Delta_{10}, \Delta_5, F_3 \S F_4, F_8 \oplus F_9} \text{ ax}}{- \text{ :- } \Delta_{10}, \Delta_5, F_3 \S F_4, F_8 \oplus F_9} \S \quad \text{hCut}}{\frac{h_1 \text{ :- } \Delta_2, F_5, F_6 \quad \bullet h_1 \text{ :- } F_5 \S F_6, \Delta_2 \quad \S \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, \mathit{dual}(F_5) \otimes \mathit{dual}(F_6)}{\bullet h_7 \text{ :- } \mathit{dual}(F_5 \S F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_A}{- \text{ :- } \Delta_2, \Delta_{10}, F_8 \oplus F_9} \text{ Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_1 \text{ :- } \Delta_2, F_5 \S F_6}{- \text{ :- } \Delta_{10}, \Delta_2, F_8} \text{ ax} \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, \mathit{dual}(F_5) \otimes \mathit{dual}(F_6)}{- \text{ :- } \Delta_{10}, \Delta_2, F_8} \text{ ax}}{- \text{ :- } \Delta_{10}, \Delta_2, F_8 \oplus F_9} \text{ hCut} \quad \oplus_A}{}
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_2 \text{ :- } \Delta_5, F_6, F_3, F_4 \quad \bullet h_2 \text{ :- } F_6, \Delta_5, F_3 \S F_4 \quad \S \quad \frac{h_7 \text{ :- } \Delta_8, \mathit{dual}(F_6)}{\bullet h_7 \text{ :- } \mathit{dual}(F_6), \perp, \Delta_8} \perp}{- \text{ :- } (\Delta_5, F_3 \S F_4), \perp, \Delta_8} \text{ Cut}}{\rightarrow} \\
\frac{\frac{h_2 \text{ :- } \Delta_5, F_3, F_4, F_6}{- \text{ :- } \Delta_5, \Delta_8, F_3, F_4, \perp} \text{ ax} \quad \frac{\bullet h_7 \text{ :- } \Delta_8, \perp, \mathit{dual}(F_6)}{- \text{ :- } \Delta_5, \Delta_8, \perp, F_3 \S F_4} \text{ ax}}{- \text{ :- } \Delta_5, \Delta_8, \perp, F_3 \S F_4} \S \quad \text{hCut}}{\frac{h_1 \text{ :- } \Delta_2, F_5, F_6 \quad \bullet h_1 \text{ :- } F_5 \S F_6, \Delta_2 \quad \S \quad \frac{h_7 \text{ :- } \Delta_8, \mathit{dual}(F_5) \otimes \mathit{dual}(F_6)}{\bullet h_7 \text{ :- } \mathit{dual}(F_5 \S F_6), \perp, \Delta_8} \perp}{- \text{ :- } \Delta_2, \perp, \Delta_8} \text{ Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_1 \text{ :- } \Delta_2, F_5 \S F_6}{- \text{ :- } \Delta_2, \Delta_8} \text{ ax} \quad \frac{h_7 \text{ :- } \Delta_8, \mathit{dual}(F_5) \otimes \mathit{dual}(F_6)}{- \text{ :- } \Delta_2, \Delta_8} \text{ ax}}{- \text{ :- } \Delta_2, \Delta_8, \perp} \text{ hCut} \quad \perp}{}
\end{array}$$

- Case rule \top

$$\frac{\frac{h_2 \vdash \Delta_5, F_6, F_3, F_4}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \S F_4} \S \frac{}{\bullet h_7 \vdash \text{dual}(F_6), \top, \Delta_8} \top}{- \vdash (\Delta_5, F_3 \S F_4), \top, \Delta_8} \text{Cut}}{- \vdash \Delta_5, \Delta_8, \top, F_3 \S F_4} \top$$

$$\frac{\frac{h_1 \vdash \Delta_2, F_5, F_6}{\bullet h_1 \vdash F_5 \S F_6, \Delta_2} \S \frac{}{\bullet h_7 \vdash \text{dual}(F_5 \S F_6), \top, \Delta_8} \top}{- \vdash \Delta_2, \top, \Delta_8} \text{Cut}}{- \vdash \Delta_2, \Delta_8, \top} \top$$

• Case rule I

• Case rule \otimes

$$\frac{\frac{h_2 \vdash \Delta_5, F_6, F_3, F_4}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \S F_4} \S \frac{h_7 \vdash \Delta_{11}, F_9, \text{dual}(F_6) \quad h_7 \vdash \Delta_8, F_{10}}{\bullet h_7 \vdash \text{dual}(F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- \vdash (\Delta_5, F_3 \S F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut}}{- \vdash \Delta_{11}, \Delta_5, \Delta_8, F_3, F_4, F_9 \otimes F_{10}} \text{ax} \quad \frac{}{\bullet h_7 \vdash \Delta_{11}, \Delta_8, \text{dual}(F_6), F_9 \otimes F_{10}} \text{ax}}{- \vdash \Delta_{11}, \Delta_5, \Delta_8, F_3 \S F_4, F_9 \otimes F_{10}} \S \quad \text{hCut}$$

$$\frac{\frac{h_2 \vdash \Delta_5, F_6, F_3, F_4}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \S F_4} \S \frac{h_7 \vdash \Delta_8, F_9 \quad h_7 \vdash \Delta_{11}, F_{10}, \text{dual}(F_6)}{\bullet h_7 \vdash \text{dual}(F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- \vdash (\Delta_5, F_3 \S F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut}}{- \vdash \Delta_{11}, \Delta_5, \Delta_8, F_3, F_4, F_9 \otimes F_{10}} \text{ax} \quad \frac{}{\bullet h_7 \vdash \Delta_{11}, \Delta_8, \text{dual}(F_6), F_9 \otimes F_{10}} \text{ax}}{- \vdash \Delta_{11}, \Delta_5, \Delta_8, F_3 \S F_4, F_9 \otimes F_{10}} \S \quad \text{hCut}$$

$$\frac{\frac{h_1 \vdash \Delta_2, F_5, F_6}{\bullet h_1 \vdash F_5 \S F_6, \Delta_2} \S \frac{h_7 \vdash \Delta_{11}, F_9, \text{dual}(F_5) \otimes \text{dual}(F_6) \quad h_7 \vdash \Delta_8, F_{10}}{\bullet h_7 \vdash \text{dual}(F_5 \S F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- \vdash \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut}}{- \vdash \Delta_{11}, \Delta_2, F_9} \text{ax} \quad \frac{}{\bullet h_7 \vdash \Delta_{11}, F_9, \text{dual}(F_5) \otimes \text{dual}(F_6)} \text{ax}}{- \vdash \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \text{hCut} \quad \frac{}{- \vdash \Delta_8, F_{10}} \text{ax} \quad \otimes$$

$$\frac{\frac{h_1 \vdash \Delta_2, F_5, F_6}{\bullet h_1 \vdash F_5 \S F_6, \Delta_2} \S \frac{h_7 \vdash \Delta_8, F_9 \quad h_7 \vdash \Delta_{11}, F_{10}, \text{dual}(F_5) \otimes \text{dual}(F_6)}{\bullet h_7 \vdash \text{dual}(F_5 \S F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- \vdash \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut}}{- \vdash \Delta_8, F_9} \text{ax} \quad \frac{}{\bullet h_1 \vdash \Delta_2, F_5 \S F_6} \text{ax} \quad \frac{}{\bullet h_7 \vdash \Delta_{11}, F_{10}, \text{dual}(F_5) \otimes \text{dual}(F_6)} \text{ax}}{- \vdash \Delta_{11}, \Delta_2, F_{10}} \text{hCut} \quad \otimes$$

$$\frac{\frac{h_1 \vdash \Delta_2, F_5, F_6}{\bullet h_1 \vdash F_5 \S F_6, \Delta_2} \S \frac{h_7 \vdash \Delta_8, \text{dual}(F_5) \quad h_7 \vdash \Delta_9, \text{dual}(F_6)}{\bullet h_7 \vdash \text{dual}(F_5 \S F_6), \Delta_8, \Delta_9} \otimes}{- \vdash \Delta_2, \Delta_8, \Delta_9} \text{Cut}}{- \vdash \Delta_2, F_5, F_6} \text{ax} \quad \frac{}{- \vdash \Delta_9, \text{dual}(F_6)} \text{ax}}{- \vdash \Delta_2, \Delta_9, F_5} \text{sCut} \quad \frac{}{- \vdash \Delta_8, \text{dual}(F_5)} \text{ax} \quad \text{sCut}}{- \vdash \Delta_2, \Delta_8, \Delta_9}$$

5.7 Status of $\&$: OK

• Case rule 1

- Case rule !

- Case rule ?W

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_5, F_6, F_3 \quad h_2 : \vdash \Delta_5, F_6, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \& F_4} \& \frac{h_7 : \vdash \Delta_9, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_9, ?F_8} \text{?W} \\
\frac{}{- : \vdash (\Delta_5, F_3 \& F_4), \Delta_9, ?F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : \vdash \Delta_5, F_6, F_3 \& F_4}{- : \vdash \Delta_5, \Delta_9, F_3 \& F_4} \text{ax} \quad \frac{h_7 : \vdash \Delta_9, dual(F_6)}{- : \vdash \Delta_5, \Delta_9, ?F_8, F_3 \& F_4} \text{ax}}{- : \vdash \Delta_5, \Delta_9, ?F_8, F_3 \& F_4} \text{hCut} \text{?W} \\
\frac{h_1 : \vdash \Delta_2, F_5 \quad h_1 : \vdash \Delta_2, F_6}{\bullet h_1 : \vdash F_5 \& F_6, \Delta_2} \& \frac{h_7 : \vdash \Delta_9, dual(F_5) \oplus dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \& F_6), \Delta_9, ?F_8} \text{?W} \\
\frac{}{- : \vdash \Delta_2, \Delta_9, ?F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \& F_6}{- : \vdash \Delta_2, \Delta_9} \text{ax} \quad \frac{h_7 : \vdash \Delta_9, dual(F_5) \oplus dual(F_6)}{- : \vdash \Delta_2, \Delta_9, ?F_8} \text{ax}}{- : \vdash \Delta_2, \Delta_9, ?F_8} \text{hCut} \text{W}
\end{array}$$

- Case rule ?C

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_5, F_6, F_3 \quad h_2 : \vdash \Delta_5, F_6, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \& F_4} \& \frac{h_7 : \vdash \Delta_9, ?F_8, ?F_8, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_9, ?F_8} \text{?C} \\
\frac{}{- : \vdash (\Delta_5, F_3 \& F_4), \Delta_9, ?F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : \vdash \Delta_5, F_6, F_3 \& F_4}{- : \vdash \Delta_5, \Delta_9, ?F_8, ?F_8, F_3 \& F_4} \text{ax} \quad \frac{h_7 : \vdash \Delta_9, ?F_8, ?F_8, dual(F_6)}{- : \vdash \Delta_5, \Delta_9, ?F_8, F_3 \& F_4} \text{ax}}{- : \vdash \Delta_5, \Delta_9, ?F_8, F_3 \& F_4} \text{hCut} \text{?C} \\
\frac{h_1 : \vdash \Delta_2, F_5 \quad h_1 : \vdash \Delta_2, F_6}{\bullet h_1 : \vdash F_5 \& F_6, \Delta_2} \& \frac{h_7 : \vdash \Delta_9, ?F_8, ?F_8, dual(F_5) \oplus dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \& F_6), \Delta_9, ?F_8} \text{?C} \\
\frac{}{- : \vdash \Delta_2, \Delta_9, ?F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \& F_6}{- : \vdash \Delta_2, \Delta_9, ?F_8, ?F_8} \text{ax} \quad \frac{h_7 : \vdash \Delta_9, ?F_8, ?F_8, dual(F_5) \oplus dual(F_6)}{- : \vdash \Delta_2, \Delta_9, ?F_8, ?F_8} \text{ax}}{- : \vdash \Delta_2, \Delta_9, ?F_8} \text{hCut} \text{?C}
\end{array}$$

- Case rule ?

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_5, F_6, F_3 \quad h_2 : \vdash \Delta_5, F_6, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \& F_4} \& \frac{h_7 : \vdash \Delta_9, F_8, dual(F_6)}{\bullet h_7 : \vdash dual(F_6), \Delta_9, ?F_8} \text{?} \\
\frac{}{- : \vdash (\Delta_5, F_3 \& F_4), \Delta_9, ?F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : \vdash \Delta_5, F_6, F_3 \& F_4}{- : \vdash \Delta_5, \Delta_9, F_8, F_3 \& F_4} \text{ax} \quad \frac{h_7 : \vdash \Delta_9, F_8, dual(F_6)}{- : \vdash \Delta_5, \Delta_9, ?F_8, F_3 \& F_4} \text{ax}}{- : \vdash \Delta_5, \Delta_9, ?F_8, F_3 \& F_4} \text{hCut} \text{?} \\
\frac{h_1 : \vdash \Delta_2, F_5 \quad h_1 : \vdash \Delta_2, F_6}{\bullet h_1 : \vdash F_5 \& F_6, \Delta_2} \& \frac{h_7 : \vdash \Delta_9, F_8, dual(F_5) \oplus dual(F_6)}{\bullet h_7 : \vdash dual(F_5 \& F_6), \Delta_9, ?F_8} \text{?} \\
\frac{}{- : \vdash \Delta_2, \Delta_9, ?F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \& F_6}{- : \vdash \Delta_2, \Delta_9, F_8} \text{ax} \quad \frac{h_7 : \vdash \Delta_9, F_8, dual(F_5) \oplus dual(F_6)}{- : \vdash \Delta_2, \Delta_9, ?F_8} \text{ax}}{- : \vdash \Delta_2, \Delta_9, ?F_8} \text{hCut} \text{?}
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_5, F_6, F_3 \quad h_2 : \vdash \Delta_5, F_6, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \& F_4} \& \quad \frac{h_7 : \vdash \Delta_{10}, F_8, F_9, \text{dual}(F_6)}{\bullet h_7 : \vdash \text{dual}(F_6), \Delta_{10}, F_8 \& F_9} \S}{\vdash (\Delta_5, F_3 \& F_4), \Delta_{10}, F_8 \& F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : \vdash \Delta_5, F_6, F_3 \& F_4}{\vdash \Delta_{10}, \Delta_5, F_8, F_9, F_3 \& F_4} \text{ax} \quad \frac{h_7 : \vdash \Delta_{10}, F_8, F_9, \text{dual}(F_6)}{\vdash \Delta_{10}, \Delta_5, F_8 \& F_9, F_3 \& F_4} \text{ax}}{\vdash \Delta_{10}, \Delta_5, F_8 \& F_9, F_3 \& F_4} \text{hCut} \quad \S \\
\vdash \Delta_{10}, \Delta_5, F_8 \& F_9, F_3 \& F_4} \S \\
\frac{h_1 : \vdash \Delta_2, F_5 \quad h_1 : \vdash \Delta_2, F_6}{\bullet h_1 : \vdash F_5 \& F_6, \Delta_2} \& \quad \frac{h_7 : \vdash \Delta_{10}, F_8, F_9, \text{dual}(F_5) \oplus \text{dual}(F_6)}{\bullet h_7 : \vdash \text{dual}(F_5 \& F_6), \Delta_{10}, F_8 \& F_9} \S}{\vdash \Delta_2, \Delta_{10}, F_8 \& F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \& F_6}{\vdash \Delta_{10}, \Delta_2, F_8, F_9} \text{ax} \quad \frac{h_7 : \vdash \Delta_{10}, F_8, F_9, \text{dual}(F_5) \oplus \text{dual}(F_6)}{\vdash \Delta_{10}, \Delta_2, F_8 \& F_9} \text{ax}}{\vdash \Delta_{10}, \Delta_2, F_8 \& F_9} \text{hCut} \quad \S \\
\vdash \Delta_{10}, \Delta_2, F_8 \& F_9} \S
\end{array}$$

• Case rule $\&$

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_5, F_6, F_3 \quad h_2 : \vdash \Delta_5, F_6, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \& F_4} \& \quad \frac{h_7 : \vdash \Delta_{10}, F_8, \text{dual}(F_6) \quad h_7 : \vdash \Delta_{10}, F_9, \text{dual}(F_6)}{\bullet h_7 : \vdash \text{dual}(F_6), \Delta_{10}, F_8 \& F_9} \& \text{Cut}}{\vdash (\Delta_5, F_3 \& F_4), \Delta_{10}, F_8 \& F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \vdash \Delta_5, F_3, F_6}{\vdash \Delta_{10}, \Delta_5, F_3, F_8 \& F_9} \text{ax} \quad \frac{\bullet h_7 : \vdash \Delta_{10}, \text{dual}(F_6), F_8 \& F_9}{\vdash \Delta_{10}, \Delta_5, F_3 \& F_4, F_8 \& F_9} \text{ax}}{\vdash \Delta_{10}, \Delta_5, F_3 \& F_4, F_8 \& F_9} \text{hCut} \quad \frac{\frac{h_2 : \vdash \Delta_5, F_4, F_6}{\vdash \Delta_{10}, \Delta_5, F_4, F_8 \& F_9} \text{ax} \quad \frac{\bullet h_7 : \vdash \Delta_{10}, \text{dual}(F_6), F_8 \& F_9}{\vdash \Delta_{10}, \Delta_5, F_4, F_8 \& F_9} \text{ax}}{\vdash \Delta_{10}, \Delta_5, F_4, F_8 \& F_9} \text{hCut} \quad \&}{\vdash \Delta_{10}, \Delta_5, F_3 \& F_4, F_8 \& F_9} \& \\
\frac{h_1 : \vdash \Delta_2, F_5 \quad h_1 : \vdash \Delta_2, F_6}{\bullet h_1 : \vdash F_5 \& F_6, \Delta_2} \& \quad \frac{h_7 : \vdash \Delta_{10}, F_8, \text{dual}(F_5) \oplus \text{dual}(F_6) \quad h_7 : \vdash \Delta_{10}, F_9, \text{dual}(F_5) \oplus \text{dual}(F_6)}{\bullet h_7 : \vdash \text{dual}(F_5 \& F_6), \Delta_{10}, F_8 \& F_9} \& \text{Cut}}{\vdash \Delta_2, \Delta_{10}, F_8 \& F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \& F_6}{\vdash \Delta_{10}, \Delta_2, F_8} \text{ax} \quad \frac{h_7 : \vdash \Delta_{10}, F_8, \text{dual}(F_5) \oplus \text{dual}(F_6)}{\vdash \Delta_{10}, \Delta_2, F_8} \text{ax}}{\vdash \Delta_{10}, \Delta_2, F_8} \text{hCut} \quad \frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \& F_6}{\vdash \Delta_{10}, \Delta_2, F_9} \text{ax} \quad \frac{h_7 : \vdash \Delta_{10}, F_9, \text{dual}(F_5) \oplus \text{dual}(F_6)}{\vdash \Delta_{10}, \Delta_2, F_9} \text{ax}}{\vdash \Delta_{10}, \Delta_2, F_8 \& F_9} \text{hCut} \quad \&}{\vdash \Delta_{10}, \Delta_2, F_8 \& F_9} \&
\end{array}$$

• Case rule \oplus_B

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_5, F_6, F_3 \quad h_2 : \vdash \Delta_5, F_6, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \& F_4} \& \quad \frac{h_7 : \vdash \Delta_{10}, F_9, \text{dual}(F_6)}{\bullet h_7 : \vdash \text{dual}(F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_B \text{Cut}}{\vdash (\Delta_5, F_3 \& F_4), \Delta_{10}, F_8 \oplus F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : \vdash \Delta_5, F_6, F_3 \& F_4}{\vdash \Delta_{10}, \Delta_5, F_9, F_3 \& F_4} \text{ax} \quad \frac{h_7 : \vdash \Delta_{10}, F_9, \text{dual}(F_6)}{\vdash \Delta_{10}, \Delta_5, F_3 \& F_4, F_8 \oplus F_9} \text{ax}}{\vdash \Delta_{10}, \Delta_5, F_9, F_3 \& F_4} \text{hCut} \quad \oplus_B \\
\vdash \Delta_{10}, \Delta_5, F_3 \& F_4, F_8 \oplus F_9} \oplus_B \\
\frac{h_1 : \vdash \Delta_2, F_5 \quad h_1 : \vdash \Delta_2, F_6}{\bullet h_1 : \vdash F_5 \& F_6, \Delta_2} \& \quad \frac{h_7 : \vdash \Delta_{10}, F_9, \text{dual}(F_5) \oplus \text{dual}(F_6)}{\bullet h_7 : \vdash \text{dual}(F_5 \& F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_B \text{Cut}}{\vdash \Delta_2, \Delta_{10}, F_8 \oplus F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \& F_6}{\vdash \Delta_{10}, \Delta_2, F_9} \text{ax} \quad \frac{h_7 : \vdash \Delta_{10}, F_9, \text{dual}(F_5) \oplus \text{dual}(F_6)}{\vdash \Delta_{10}, \Delta_2, F_9} \text{ax}}{\vdash \Delta_{10}, \Delta_2, F_9} \text{hCut} \quad \oplus_B \\
\vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9} \oplus_B \\
\frac{h_1 : \vdash \Delta_2, F_5 \quad h_1 : \vdash \Delta_2, F_6}{\bullet h_1 : \vdash F_5 \& F_6, \Delta_2} \& \quad \frac{h_7 : \vdash \Delta_8, \text{dual}(F_6)}{\bullet h_7 : \vdash \text{dual}(F_5 \& F_6), \Delta_8} \oplus_B \text{Cut}}{\vdash \Delta_2, \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\vdash \Delta_2, F_6}{\vdash \Delta_2, \Delta_8} \text{ax} \quad \frac{\vdash \Delta_8, \text{dual}(F_6)}{\vdash \Delta_2, \Delta_8} \text{ax}}{\vdash \Delta_2, \Delta_8} \text{sCut} \\
\vdash \Delta_2, \Delta_8} \text{sCut}
\end{array}$$

• Case rule \oplus_A

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \Delta_5, F_6, F_3 \quad h_2 : \vdash \Delta_5, F_6, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \& F_4} \quad \& \quad \frac{h_7 : \vdash \Delta_{10}, F_8, \mathit{dual}(F_6)}{\bullet h_7 : \vdash \mathit{dual}(F_6), \Delta_{10}, F_8 \oplus F_9}}{\vdash (\Delta_5, F_3 \& F_4), \Delta_{10}, F_8 \oplus F_9} \oplus^A \text{Cut}}{\vdash (\Delta_5, F_3 \& F_4), \Delta_{10}, F_8 \oplus F_9} \rightarrow \\
\frac{\frac{\bullet h_2 : \vdash \Delta_5, F_6, F_3 \& F_4}{\vdash \Delta_{10}, \Delta_5, F_8, F_3 \& F_4} \text{ ax} \quad \frac{h_7 : \vdash \Delta_{10}, F_8, \mathit{dual}(F_6)}{\vdash \Delta_{10}, \Delta_5, F_8, F_3 \& F_4} \text{ ax}}{\vdash \Delta_{10}, \Delta_5, F_8, F_3 \& F_4} \text{ hCut}}{\vdash \Delta_{10}, \Delta_5, F_3 \& F_4, F_8 \oplus F_9} \oplus^A \\
\\
\frac{\frac{h_1 : \vdash \Delta_2, F_5 \quad h_1 : \vdash \Delta_2, F_6}{\bullet h_1 : \vdash F_5 \& F_6, \Delta_2} \quad \& \quad \frac{h_7 : \vdash \Delta_{10}, F_8, \mathit{dual}(F_5) \oplus \mathit{dual}(F_6)}{\bullet h_7 : \vdash \mathit{dual}(F_5 \& F_6), \Delta_{10}, F_8 \oplus F_9}}{\vdash \Delta_2, \Delta_{10}, F_8 \oplus F_9} \oplus^A \text{Cut}}{\vdash \Delta_2, \Delta_{10}, F_8 \oplus F_9} \rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \& F_6}{\vdash \Delta_{10}, \Delta_2, F_8} \text{ ax} \quad \frac{h_7 : \vdash \Delta_{10}, F_8, \mathit{dual}(F_5) \oplus \mathit{dual}(F_6)}{\vdash \Delta_{10}, \Delta_2, F_8} \text{ ax}}{\vdash \Delta_{10}, \Delta_2, F_8} \text{ hCut}}{\vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9} \oplus^A \\
\\
\frac{\frac{h_1 : \vdash \Delta_2, F_5 \quad h_1 : \vdash \Delta_2, F_6}{\bullet h_1 : \vdash F_5 \& F_6, \Delta_2} \quad \& \quad \frac{h_7 : \vdash \Delta_8, \mathit{dual}(F_5)}{\bullet h_7 : \vdash \mathit{dual}(F_5 \& F_6), \Delta_8}}{\vdash \Delta_2, \Delta_8} \oplus^A \text{Cut}}{\vdash \Delta_2, \Delta_8} \rightarrow \\
\frac{\frac{\vdash \Delta_2, F_5}{\vdash \Delta_2, \Delta_8} \text{ ax} \quad \frac{\vdash \Delta_8, \mathit{dual}(F_5)}{\vdash \Delta_2, \Delta_8} \text{ ax}}{\vdash \Delta_2, \Delta_8} \text{ sCut}
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \Delta_5, F_6, F_3 \quad h_2 : \vdash \Delta_5, F_6, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \& F_4} \quad \& \quad \frac{h_7 : \vdash \Delta_8, \mathit{dual}(F_6)}{\bullet h_7 : \vdash \mathit{dual}(F_6), \perp, \Delta_8}}{\vdash (\Delta_5, F_3 \& F_4), \perp, \Delta_8} \perp \text{Cut}}{\vdash (\Delta_5, F_3 \& F_4), \perp, \Delta_8} \rightarrow \\
\frac{\frac{\bullet h_2 : \vdash \Delta_5, F_6, F_3 \& F_4}{\vdash \Delta_5, \Delta_8, F_3 \& F_4} \text{ ax} \quad \frac{h_7 : \vdash \Delta_8, \mathit{dual}(F_6)}{\vdash \Delta_5, \Delta_8, F_3 \& F_4} \text{ ax}}{\vdash \Delta_5, \Delta_8, F_3 \& F_4} \text{ hCut}}{\vdash \Delta_5, \Delta_8, \perp, F_3 \& F_4} \perp \\
\\
\frac{\frac{h_1 : \vdash \Delta_2, F_5 \quad h_1 : \vdash \Delta_2, F_6}{\bullet h_1 : \vdash F_5 \& F_6, \Delta_2} \quad \& \quad \frac{h_7 : \vdash \Delta_8, \mathit{dual}(F_5) \oplus \mathit{dual}(F_6)}{\bullet h_7 : \vdash \mathit{dual}(F_5 \& F_6), \perp, \Delta_8}}{\vdash \Delta_2, \perp, \Delta_8} \perp \text{Cut}}{\vdash \Delta_2, \perp, \Delta_8} \rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \Delta_2, F_5 \& F_6}{\vdash \Delta_2, \Delta_8} \text{ ax} \quad \frac{h_7 : \vdash \Delta_8, \mathit{dual}(F_5) \oplus \mathit{dual}(F_6)}{\vdash \Delta_2, \Delta_8} \text{ ax}}{\vdash \Delta_2, \Delta_8} \text{ hCut}}{\vdash \Delta_2, \Delta_8, \perp} \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{\frac{h_2 : \vdash \Delta_5, F_6, F_3 \quad h_2 : \vdash \Delta_5, F_6, F_4}{\bullet h_2 : \vdash F_6, \Delta_5, F_3 \& F_4} \quad \& \quad \frac{\vdash \Delta_8, \mathit{dual}(F_6), \top, \Delta_8}}{\bullet h_7 : \vdash \mathit{dual}(F_6), \top, \Delta_8} \top \text{Cut}}{\vdash (\Delta_5, F_3 \& F_4), \top, \Delta_8} \top \text{Cut}}{\vdash (\Delta_5, F_3 \& F_4), \top, \Delta_8} \rightarrow \\
\frac{\vdash \Delta_5, \Delta_8, \top, F_3 \& F_4}{\vdash \Delta_5, \Delta_8, \top, F_3 \& F_4} \top \\
\\
\frac{\frac{h_1 : \vdash \Delta_2, F_5 \quad h_1 : \vdash \Delta_2, F_6}{\bullet h_1 : \vdash F_5 \& F_6, \Delta_2} \quad \& \quad \frac{\vdash \Delta_8, \mathit{dual}(F_5 \& F_6), \top, \Delta_8}}{\bullet h_7 : \vdash \mathit{dual}(F_5 \& F_6), \top, \Delta_8} \top \text{Cut}}{\vdash \Delta_2, \top, \Delta_8} \top \text{Cut}}{\vdash \Delta_2, \top, \Delta_8} \rightarrow \\
\frac{\vdash \Delta_2, \Delta_8, \top}{\vdash \Delta_2, \Delta_8, \top} \top
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{h_2 \text{ :- } \Delta_5, F_6, F_3 \quad h_2 \text{ :- } \Delta_5, F_6, F_4}{\bullet h_2 \text{ :- } F_6, \Delta_5, F_3 \& F_4} \& \frac{h_7 \text{ :- } \Delta_{11}, F_9, \mathit{dual}(F_6) \quad h_7 \text{ :- } \Delta_8, F_{10}}{\bullet h_7 \text{ :- } \mathit{dual}(F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\frac{}{- \text{ :- } (\Delta_5, F_3 \& F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \text{ :- } \Delta_5, F_6, F_3 \& F_4}{- \text{ :- } \Delta_{11}, \Delta_5, F_9, F_3 \& F_4} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_{11}, F_9, \mathit{dual}(F_6)}{h\text{Cut}} \text{ax}}{- \text{ :- } \Delta_{11}, \Delta_5, \Delta_8, F_3 \& F_4, F_9 \otimes F_{10}} \otimes \\
\frac{h_2 \text{ :- } \Delta_5, F_6, F_3 \quad h_2 \text{ :- } \Delta_5, F_6, F_4}{\bullet h_2 \text{ :- } F_6, \Delta_5, F_3 \& F_4} \& \frac{h_7 \text{ :- } \Delta_8, F_9 \quad h_7 \text{ :- } \Delta_{11}, F_{10}, \mathit{dual}(F_6)}{\bullet h_7 \text{ :- } \mathit{dual}(F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\frac{}{- \text{ :- } (\Delta_5, F_3 \& F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{- \text{ :- } \Delta_8, F_9} \text{ax} \quad \frac{\bullet h_2 \text{ :- } \Delta_5, F_6, F_3 \& F_4 \quad h_7 \text{ :- } \Delta_{11}, F_{10}, \mathit{dual}(F_6)}{- \text{ :- } \Delta_{11}, \Delta_5, F_{10}, F_3 \& F_4} \text{ax}}{- \text{ :- } \Delta_{11}, \Delta_5, \Delta_8, F_3 \& F_4, F_9 \otimes F_{10}} \otimes \\
\frac{h_1 \text{ :- } \Delta_2, F_5 \quad h_1 \text{ :- } \Delta_2, F_6}{\bullet h_1 \text{ :- } F_5 \& F_6, \Delta_2} \& \frac{h_7 \text{ :- } \Delta_{11}, F_9, \mathit{dual}(F_5) \oplus \mathit{dual}(F_6) \quad h_7 \text{ :- } \Delta_8, F_{10}}{\bullet h_7 \text{ :- } \mathit{dual}(F_5 \& F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\frac{}{- \text{ :- } \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \text{ :- } \Delta_2, F_5 \& F_6}{- \text{ :- } \Delta_{11}, \Delta_2, F_9} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_{11}, F_9, \mathit{dual}(F_5) \oplus \mathit{dual}(F_6)}{h\text{Cut}} \text{ax}}{- \text{ :- } \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \otimes \\
\frac{h_1 \text{ :- } \Delta_2, F_5 \quad h_1 \text{ :- } \Delta_2, F_6}{\bullet h_1 \text{ :- } F_5 \& F_6, \Delta_2} \& \frac{h_7 \text{ :- } \Delta_8, F_9 \quad h_7 \text{ :- } \Delta_{11}, F_{10}, \mathit{dual}(F_5) \oplus \mathit{dual}(F_6)}{\bullet h_7 \text{ :- } \mathit{dual}(F_5 \& F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\frac{}{- \text{ :- } \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{- \text{ :- } \Delta_8, F_9} \text{ax} \quad \frac{\bullet h_1 \text{ :- } \Delta_2, F_5 \& F_6 \quad h_7 \text{ :- } \Delta_{11}, F_{10}, \mathit{dual}(F_5) \oplus \mathit{dual}(F_6)}{- \text{ :- } \Delta_{11}, \Delta_2, F_{10}} \text{ax}}{- \text{ :- } \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \otimes
\end{array}$$

5.8 Status of \oplus_B : OK

- Case rule 1
- Case rule !
- Case rule ?W

$$\begin{array}{c}
\frac{h_2 \text{ :- } \Delta_5, F_6, F_4}{\bullet h_2 \text{ :- } F_6, \Delta_5, F_3 \oplus F_4} \oplus_B \frac{h_7 \text{ :- } \Delta_9, \mathit{dual}(F_6)}{\bullet h_7 \text{ :- } \mathit{dual}(F_6), \Delta_9, ?F_8} ?W \\
\frac{}{- \text{ :- } (\Delta_5, F_3 \oplus F_4), \Delta_9, ?F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \text{ :- } \Delta_5, F_6, F_3 \oplus F_4}{- \text{ :- } \Delta_5, \Delta_9, F_3 \oplus F_4} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_9, \mathit{dual}(F_6)}{h\text{Cut}} \text{ax}}{- \text{ :- } \Delta_5, \Delta_9, ?F_8, F_3 \oplus F_4} W \\
\frac{h_1 \text{ :- } \Delta_2, F_6}{\bullet h_1 \text{ :- } F_5 \oplus F_6, \Delta_2} \oplus_B \frac{h_7 \text{ :- } \Delta_9, \mathit{dual}(F_5) \& \mathit{dual}(F_6)}{\bullet h_7 \text{ :- } \mathit{dual}(F_5 \oplus F_6), \Delta_9, ?F_8} ?W \\
\frac{}{- \text{ :- } \Delta_2, \Delta_9, ?F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \text{ :- } \Delta_2, F_5 \oplus F_6}{- \text{ :- } \Delta_2, \Delta_9} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_9, \mathit{dual}(F_5) \& \mathit{dual}(F_6)}{h\text{Cut}} \text{ax}}{- \text{ :- } \Delta_2, \Delta_9, ?F_8} W
\end{array}$$

- Case rule ?C

$$\begin{array}{c}
\frac{h_2 \text{ :- } \Delta_5, F_6, F_4}{\bullet h_2 \text{ :- } F_6, \Delta_5, F_3 \oplus F_4} \oplus_B \quad \frac{h_7 \text{ :- } \Delta_9, ?F_8, ?F_8, \mathit{dual}(F_6)}{\bullet h_7 \text{ :- } \mathit{dual}(F_6), \Delta_9, ?F_8} ?C}{- \text{ :- } (\Delta_5, F_3 \oplus F_4), \Delta_9, ?F_8} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 \text{ :- } \Delta_5, F_6, F_3 \oplus F_4}{- \text{ :- } \Delta_5, \Delta_9, ?F_8, ?F_8, F_3 \oplus F_4} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_9, ?F_8, ?F_8, \mathit{dual}(F_6)}{- \text{ :- } \Delta_5, \Delta_9, ?F_8, F_3 \oplus F_4} \text{ax}}{- \text{ :- } \Delta_5, \Delta_9, ?F_8, F_3 \oplus F_4} \text{hCut} \\
\text{?C}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 \text{ :- } \Delta_2, F_6}{\bullet h_1 \text{ :- } F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{h_7 \text{ :- } \Delta_9, ?F_8, ?F_8, \mathit{dual}(F_5) \& \mathit{dual}(F_6)}{\bullet h_7 \text{ :- } \mathit{dual}(F_5 \oplus F_6), \Delta_9, ?F_8} ?C}{- \text{ :- } \Delta_2, \Delta_9, ?F_8} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 \text{ :- } \Delta_2, F_5 \oplus F_6}{- \text{ :- } \Delta_2, \Delta_9, ?F_8, ?F_8} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_9, ?F_8, ?F_8, \mathit{dual}(F_5) \& \mathit{dual}(F_6)}{- \text{ :- } \Delta_2, \Delta_9, ?F_8, ?F_8} \text{ax}}{- \text{ :- } \Delta_2, \Delta_9, ?F_8} \text{hCut} \\
\text{?C}
\end{array}$$

• Case rule ?

$$\begin{array}{c}
\frac{h_2 \text{ :- } \Delta_5, F_6, F_4}{\bullet h_2 \text{ :- } F_6, \Delta_5, F_3 \oplus F_4} \oplus_B \quad \frac{h_7 \text{ :- } \Delta_9, F_8, \mathit{dual}(F_6)}{\bullet h_7 \text{ :- } \mathit{dual}(F_6), \Delta_9, ?F_8} ?}{- \text{ :- } (\Delta_5, F_3 \oplus F_4), \Delta_9, ?F_8} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 \text{ :- } \Delta_5, F_6, F_3 \oplus F_4}{- \text{ :- } \Delta_5, \Delta_9, F_8, F_3 \oplus F_4} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_9, F_8, \mathit{dual}(F_6)}{- \text{ :- } \Delta_5, \Delta_9, ?F_8, F_3 \oplus F_4} \text{ax}}{- \text{ :- } \Delta_5, \Delta_9, ?F_8, F_3 \oplus F_4} \text{hCut} \\
?
\end{array}$$

$$\begin{array}{c}
\frac{h_1 \text{ :- } \Delta_2, F_6}{\bullet h_1 \text{ :- } F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{h_7 \text{ :- } \Delta_9, F_8, \mathit{dual}(F_5) \& \mathit{dual}(F_6)}{\bullet h_7 \text{ :- } \mathit{dual}(F_5 \oplus F_6), \Delta_9, ?F_8} ?}{- \text{ :- } \Delta_2, \Delta_9, ?F_8} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 \text{ :- } \Delta_2, F_5 \oplus F_6}{- \text{ :- } \Delta_2, \Delta_9, F_8} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_9, F_8, \mathit{dual}(F_5) \& \mathit{dual}(F_6)}{- \text{ :- } \Delta_2, \Delta_9, ?F_8} \text{ax}}{- \text{ :- } \Delta_2, \Delta_9, ?F_8} \text{hCut} \\
?
\end{array}$$

• Case rule \$

$$\begin{array}{c}
\frac{h_2 \text{ :- } \Delta_5, F_6, F_4}{\bullet h_2 \text{ :- } F_6, \Delta_5, F_3 \oplus F_4} \oplus_B \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, F_9, \mathit{dual}(F_6)}{\bullet h_7 \text{ :- } \mathit{dual}(F_6), \Delta_{10}, F_8 \& F_9} \$}{- \text{ :- } (\Delta_5, F_3 \oplus F_4), \Delta_{10}, F_8 \& F_9} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 \text{ :- } \Delta_5, F_6, F_3 \oplus F_4}{- \text{ :- } \Delta_{10}, \Delta_5, F_8, F_9, F_3 \oplus F_4} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, F_9, \mathit{dual}(F_6)}{- \text{ :- } \Delta_{10}, \Delta_5, F_8 \& F_9, F_3 \oplus F_4} \text{ax}}{- \text{ :- } \Delta_{10}, \Delta_5, F_8 \& F_9, F_3 \oplus F_4} \text{hCut} \\
\$
\end{array}$$

$$\begin{array}{c}
\frac{h_1 \text{ :- } \Delta_2, F_6}{\bullet h_1 \text{ :- } F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, F_9, \mathit{dual}(F_5) \& \mathit{dual}(F_6)}{\bullet h_7 \text{ :- } \mathit{dual}(F_5 \oplus F_6), \Delta_{10}, F_8 \& F_9} \$}{- \text{ :- } \Delta_2, \Delta_{10}, F_8 \& F_9} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 \text{ :- } \Delta_2, F_5 \oplus F_6}{- \text{ :- } \Delta_{10}, \Delta_2, F_8, F_9} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, F_9, \mathit{dual}(F_5) \& \mathit{dual}(F_6)}{- \text{ :- } \Delta_{10}, \Delta_2, F_8 \& F_9} \text{ax}}{- \text{ :- } \Delta_{10}, \Delta_2, F_8 \& F_9} \text{hCut} \\
\$
\end{array}$$

• Case rule &

$$\begin{array}{c}
\frac{h_2 \text{ :- } \Delta_5, F_6, F_4}{\bullet h_2 \text{ :- } F_6, \Delta_5, F_3 \oplus F_4} \oplus_B \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, \mathit{dual}(F_6)}{\bullet h_7 \text{ :- } \mathit{dual}(F_6), \Delta_{10}, F_8 \& F_9} \quad \frac{h_7 \text{ :- } \Delta_{10}, F_9, \mathit{dual}(F_6)}{\bullet h_7 \text{ :- } \mathit{dual}(F_6), \Delta_{10}, F_8 \& F_9} \&}{- \text{ :- } (\Delta_5, F_3 \oplus F_4), \Delta_{10}, F_8 \& F_9} \text{Cut} \\
\rightarrow \\
\frac{h_2 \text{ :- } \Delta_5, F_4, F_6}{- \text{ :- } \Delta_{10}, \Delta_5, F_4, F_8 \& F_9} \text{ax} \quad \frac{\bullet h_7 \text{ :- } \Delta_{10}, \mathit{dual}(F_6), F_8 \& F_9}{- \text{ :- } \Delta_{10}, \Delta_5, F_8 \& F_9, F_3 \oplus F_4} \text{ax}}{- \text{ :- } \Delta_{10}, \Delta_5, F_8 \& F_9, F_3 \oplus F_4} \text{hCut} \\
\oplus_B
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 \vdash \Delta_2, F_6}{\bullet h_1 \vdash F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{h_7 \vdash \Delta_{10}, F_8, \text{dual}(F_5) \& \text{dual}(F_6) \quad h_7 \vdash \Delta_{10}, F_9, \text{dual}(F_5) \& \text{dual}(F_6)}{\bullet h_7 \vdash \text{dual}(F_5 \oplus F_6), \Delta_{10}, F_8 \& F_9} \&}{- \vdash \Delta_2, \Delta_{10}, F_8 \& F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 \vdash \Delta_2, F_5 \oplus F_6}{- \vdash \Delta_{10}, \Delta_2, F_8} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_8, \text{dual}(F_5) \& \text{dual}(F_6)}{\text{hCut}} \text{ax}}{- \vdash \Delta_{10}, \Delta_2, F_8} \oplus_B \quad \frac{\frac{h_7 \vdash \Delta_{10}, F_9, \text{dual}(F_5) \& \text{dual}(F_6)}{- \vdash \Delta_{10}, \Delta_2, F_9} \text{ax}}{- \vdash \Delta_{10}, \Delta_2, F_8 \& F_9} \&}{- \vdash \Delta_{10}, \Delta_2, F_8 \& F_9} \text{hCut} \\
\& \\
\frac{\frac{h_1 \vdash \Delta_2, F_6}{\bullet h_1 \vdash F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{h_7 \vdash \Delta_8, \text{dual}(F_5) \quad h_7 \vdash \Delta_8, \text{dual}(F_6)}{\bullet h_7 \vdash \text{dual}(F_5 \oplus F_6), \Delta_8} \&}{- \vdash \Delta_2, \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 \vdash \Delta_2, F_6}{- \vdash \Delta_2, F_6} \text{ax} \quad \frac{h_7 \vdash \Delta_8, \text{dual}(F_6)}{- \vdash \Delta_8, \text{dual}(F_6)} \text{ax}}{- \vdash \Delta_2, \Delta_8} \text{sCut}
\end{array}$$

• Case rule \oplus_B

$$\begin{array}{c}
\frac{\frac{h_2 \vdash \Delta_5, F_6, F_4}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_B \quad \frac{h_7 \vdash \Delta_{10}, F_9, \text{dual}(F_6)}{\bullet h_7 \vdash \text{dual}(F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_B}{- \vdash (\Delta_5, F_3 \oplus F_4), \Delta_{10}, F_8 \oplus F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_2 \vdash \Delta_5, F_4, F_6}{- \vdash \Delta_{10}, \Delta_5, F_4, F_8 \oplus F_9} \text{ax} \quad \frac{\bullet h_7 \vdash \Delta_{10}, \text{dual}(F_6), F_8 \oplus F_9}{\text{hCut}} \text{ax}}{- \vdash \Delta_{10}, \Delta_5, F_3 \oplus F_4, F_8 \oplus F_9} \oplus_B}{- \vdash \Delta_{10}, \Delta_5, F_3 \oplus F_4, F_8 \oplus F_9} \oplus_B \\
\& \\
\frac{\frac{h_1 \vdash \Delta_2, F_6}{\bullet h_1 \vdash F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{h_7 \vdash \Delta_{10}, F_9, \text{dual}(F_5) \& \text{dual}(F_6)}{\bullet h_7 \vdash \text{dual}(F_5 \oplus F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_B}{- \vdash \Delta_2, \Delta_{10}, F_8 \oplus F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, F_5 \oplus F_6}{- \vdash \Delta_{10}, \Delta_2, F_9} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_9, \text{dual}(F_5) \& \text{dual}(F_6)}{\text{hCut}} \text{ax}}{- \vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9} \oplus_B
\end{array}$$

• Case rule \oplus_A

$$\begin{array}{c}
\frac{\frac{h_2 \vdash \Delta_5, F_6, F_4}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_B \quad \frac{h_7 \vdash \Delta_{10}, F_8, \text{dual}(F_6)}{\bullet h_7 \vdash \text{dual}(F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_A}{- \vdash (\Delta_5, F_3 \oplus F_4), \Delta_{10}, F_8 \oplus F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_2 \vdash \Delta_5, F_4, F_6}{- \vdash \Delta_{10}, \Delta_5, F_4, F_8 \oplus F_9} \text{ax} \quad \frac{\bullet h_7 \vdash \Delta_{10}, \text{dual}(F_6), F_8 \oplus F_9}{\text{hCut}} \text{ax}}{- \vdash \Delta_{10}, \Delta_5, F_3 \oplus F_4, F_8 \oplus F_9} \oplus_B}{- \vdash \Delta_{10}, \Delta_5, F_3 \oplus F_4, F_8 \oplus F_9} \oplus_B \\
\& \\
\frac{\frac{h_1 \vdash \Delta_2, F_6}{\bullet h_1 \vdash F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{h_7 \vdash \Delta_{10}, F_8, \text{dual}(F_5) \& \text{dual}(F_6)}{\bullet h_7 \vdash \text{dual}(F_5 \oplus F_6), \Delta_{10}, F_8 \oplus F_9} \oplus_A}{- \vdash \Delta_2, \Delta_{10}, F_8 \oplus F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \Delta_2, F_5 \oplus F_6}{- \vdash \Delta_{10}, \Delta_2, F_8} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_8, \text{dual}(F_5) \& \text{dual}(F_6)}{\text{hCut}} \text{ax}}{- \vdash \Delta_{10}, \Delta_2, F_8 \oplus F_9} \oplus_A
\end{array}$$

• Case rule \perp

$$\begin{array}{c}
\frac{\frac{h_2 \vdash \Delta_5, F_6, F_4}{\bullet h_2 \vdash F_6, \Delta_5, F_3 \oplus F_4} \oplus_B \quad \frac{h_7 \vdash \Delta_8, \text{dual}(F_6)}{\bullet h_7 \vdash \text{dual}(F_6), \perp, \Delta_8} \perp}{- \vdash (\Delta_5, F_3 \oplus F_4), \perp, \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_2 \vdash \Delta_5, F_4, F_6}{- \vdash \Delta_5, \Delta_8, F_4, \perp} \text{ax} \quad \frac{\bullet h_7 \vdash \Delta_8, \perp, \text{dual}(F_6)}{\text{hCut}} \text{ax}}{- \vdash \Delta_5, \Delta_8, F_4, \perp} \oplus_B}{- \vdash \Delta_5, \Delta_8, \perp, F_3 \oplus F_4} \oplus_B
\end{array}$$

$$\begin{array}{c}
\frac{h_1 \text{ :- } \Delta_2, F_6}{\bullet h_1 \text{ :- } F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{h_7 \text{ :- } \Delta_8, \text{dual}(F_5) \& \text{dual}(F_6)}{\bullet h_7 \text{ :- } \text{dual}(F_5 \oplus F_6), \perp, \Delta_8} \perp}{- \text{ :- } \Delta_2, \perp, \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 \text{ :- } \Delta_2, F_5 \oplus F_6 \quad \text{ax} \quad \frac{h_7 \text{ :- } \Delta_8, \text{dual}(F_5) \& \text{dual}(F_6)}{- \text{ :- } \Delta_2, \Delta_8} \text{ax}}{- \text{ :- } \Delta_2, \Delta_8, \perp} \text{hCut} \\
\perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_2 \text{ :- } \Delta_5, F_6, F_4}{\bullet h_2 \text{ :- } F_6, \Delta_5, F_3 \oplus F_4} \oplus_B \quad \frac{}{\bullet h_7 \text{ :- } \text{dual}(F_6), \top, \Delta_8} \top}{- \text{ :- } (\Delta_5, F_3 \oplus F_4), \top, \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{}{- \text{ :- } \Delta_5, \Delta_8, \top, F_3 \oplus F_4} \top \\
\frac{h_1 \text{ :- } \Delta_2, F_6}{\bullet h_1 \text{ :- } F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{}{\bullet h_7 \text{ :- } \text{dual}(F_5 \oplus F_6), \top, \Delta_8} \top}{- \text{ :- } \Delta_2, \top, \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{}{- \text{ :- } \Delta_2, \Delta_8, \top} \top
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{h_2 \text{ :- } \Delta_5, F_6, F_4}{\bullet h_2 \text{ :- } F_6, \Delta_5, F_3 \oplus F_4} \oplus_B \quad \frac{h_7 \text{ :- } \Delta_{11}, F_9, \text{dual}(F_6)}{\bullet h_7 \text{ :- } \text{dual}(F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \quad \frac{h_7 \text{ :- } \Delta_8, F_{10}}{} \otimes}{- \text{ :- } (\Delta_5, F_3 \oplus F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{h_2 \text{ :- } \Delta_5, F_4, F_6 \quad \text{ax} \quad \frac{h_7 \text{ :- } \Delta_{11}, \Delta_8, \text{dual}(F_6), F_9 \otimes F_{10}}{- \text{ :- } \Delta_{11}, \Delta_5, \Delta_8, F_4, F_9 \otimes F_{10}} \text{ax}}{- \text{ :- } \Delta_{11}, \Delta_5, \Delta_8, F_9 \otimes F_{10}, F_3 \oplus F_4} \oplus_B \quad \text{hCut}}{} \\
\frac{h_2 \text{ :- } \Delta_5, F_6, F_4}{\bullet h_2 \text{ :- } F_6, \Delta_5, F_3 \oplus F_4} \oplus_B \quad \frac{h_7 \text{ :- } \Delta_8, F_9 \quad h_7 \text{ :- } \Delta_{11}, F_{10}, \text{dual}(F_6)}{\bullet h_7 \text{ :- } \text{dual}(F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \quad \otimes}{- \text{ :- } (\Delta_5, F_3 \oplus F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{h_2 \text{ :- } \Delta_5, F_4, F_6 \quad \text{ax} \quad \frac{h_7 \text{ :- } \Delta_{11}, \Delta_8, \text{dual}(F_6), F_9 \otimes F_{10}}{- \text{ :- } \Delta_{11}, \Delta_5, \Delta_8, F_4, F_9 \otimes F_{10}} \text{ax}}{- \text{ :- } \Delta_{11}, \Delta_5, \Delta_8, F_9 \otimes F_{10}, F_3 \oplus F_4} \oplus_B \quad \text{hCut}}{} \\
\frac{h_1 \text{ :- } \Delta_2, F_6}{\bullet h_1 \text{ :- } F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{h_7 \text{ :- } \Delta_{11}, F_9, \text{dual}(F_5) \& \text{dual}(F_6)}{\bullet h_7 \text{ :- } \text{dual}(F_5 \oplus F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \quad \frac{h_7 \text{ :- } \Delta_8, F_{10}}{} \otimes}{- \text{ :- } \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 \text{ :- } \Delta_2, F_5 \oplus F_6 \quad \text{ax} \quad \frac{h_7 \text{ :- } \Delta_{11}, F_9, \text{dual}(F_5) \& \text{dual}(F_6)}{- \text{ :- } \Delta_{11}, \Delta_2, F_9} \text{ax}}{- \text{ :- } \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \text{hCut} \quad \frac{}{- \text{ :- } \Delta_8, F_{10}} \text{ax}}{- \text{ :- } \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \otimes \\
\frac{h_1 \text{ :- } \Delta_2, F_6}{\bullet h_1 \text{ :- } F_5 \oplus F_6, \Delta_2} \oplus_B \quad \frac{h_7 \text{ :- } \Delta_8, F_9 \quad h_7 \text{ :- } \Delta_{11}, F_{10}, \text{dual}(F_5) \& \text{dual}(F_6)}{\bullet h_7 \text{ :- } \text{dual}(F_5 \oplus F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \quad \otimes}{- \text{ :- } \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{}{- \text{ :- } \Delta_8, F_9} \text{ax} \quad \frac{\bullet h_1 \text{ :- } \Delta_2, F_5 \oplus F_6 \quad \text{ax} \quad \frac{h_7 \text{ :- } \Delta_{11}, F_{10}, \text{dual}(F_5) \& \text{dual}(F_6)}{- \text{ :- } \Delta_{11}, \Delta_2, F_{10}} \text{ax}}{- \text{ :- } \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \text{hCut}}{- \text{ :- } \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \otimes
\end{array}$$

5.9 Status of \oplus_A : OK

- Case rule 1
- Case rule !
- Case rule ?W

$$\begin{array}{c}
\frac{\frac{h_2 \text{ :- } \Delta_5, F_6, F_3}{\bullet h_2 \text{ :- } F_6, \Delta_5, F_3 \oplus F_4} \oplus_A \quad \frac{h_7 \text{ :- } \Delta_9, \mathit{dual}(F_6)}{\bullet h_7 \text{ :- } \mathit{dual}(F_6), \Delta_9, ?F_8} ?W}{- \text{ :- } (\Delta_5, F_3 \oplus F_4), \Delta_9, ?F_8} \text{Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_2 \text{ :- } \Delta_5, F_6, F_3 \oplus F_4}{- \text{ :- } \Delta_5, \Delta_9, F_3 \oplus F_4} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_9, \mathit{dual}(F_6)}{h_7 \text{ :- } \Delta_9, \mathit{dual}(F_6)} \text{ax}}{- \text{ :- } \Delta_5, \Delta_9, ?F_8, F_3 \oplus F_4} \text{hCut}}{- \text{ :- } \Delta_5, \Delta_9, ?F_8, F_3 \oplus F_4} W \\
\frac{\frac{h_1 \text{ :- } \Delta_2, F_5}{\bullet h_1 \text{ :- } F_5 \oplus F_6, \Delta_2} \oplus_A \quad \frac{h_7 \text{ :- } \Delta_9, \mathit{dual}(F_5) \& \mathit{dual}(F_6)}{\bullet h_7 \text{ :- } \mathit{dual}(F_5 \oplus F_6), \Delta_9, ?F_8} ?W}{- \text{ :- } \Delta_2, \Delta_9, ?F_8} \text{Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_1 \text{ :- } \Delta_2, F_5 \oplus F_6}{- \text{ :- } \Delta_2, \Delta_9} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_9, \mathit{dual}(F_5) \& \mathit{dual}(F_6)}{h_7 \text{ :- } \Delta_9, \mathit{dual}(F_5) \& \mathit{dual}(F_6)} \text{ax}}{- \text{ :- } \Delta_2, \Delta_9, ?F_8} \text{hCut}}{- \text{ :- } \Delta_2, \Delta_9, ?F_8} ?W
\end{array}$$

- Case rule ?C

$$\begin{array}{c}
\frac{\frac{h_2 \text{ :- } \Delta_5, F_6, F_3}{\bullet h_2 \text{ :- } F_6, \Delta_5, F_3 \oplus F_4} \oplus_A \quad \frac{h_7 \text{ :- } \Delta_9, ?F_8, ?F_8, \mathit{dual}(F_6)}{\bullet h_7 \text{ :- } \mathit{dual}(F_6), \Delta_9, ?F_8} ?C}{- \text{ :- } (\Delta_5, F_3 \oplus F_4), \Delta_9, ?F_8} \text{Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_2 \text{ :- } \Delta_5, F_6, F_3 \oplus F_4}{- \text{ :- } \Delta_5, \Delta_9, ?F_8, ?F_8, F_3 \oplus F_4} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_9, ?F_8, ?F_8, \mathit{dual}(F_6)}{h_7 \text{ :- } \Delta_9, ?F_8, ?F_8, \mathit{dual}(F_6)} \text{ax}}{- \text{ :- } \Delta_5, \Delta_9, ?F_8, ?F_8, F_3 \oplus F_4} \text{hCut}}{- \text{ :- } \Delta_5, \Delta_9, ?F_8, F_3 \oplus F_4} ?C \\
\frac{\frac{h_1 \text{ :- } \Delta_2, F_5}{\bullet h_1 \text{ :- } F_5 \oplus F_6, \Delta_2} \oplus_A \quad \frac{h_7 \text{ :- } \Delta_9, ?F_8, ?F_8, \mathit{dual}(F_5) \& \mathit{dual}(F_6)}{\bullet h_7 \text{ :- } \mathit{dual}(F_5 \oplus F_6), \Delta_9, ?F_8} ?C}{- \text{ :- } \Delta_2, \Delta_9, ?F_8} \text{Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_1 \text{ :- } \Delta_2, F_5 \oplus F_6}{- \text{ :- } \Delta_2, \Delta_9, ?F_8, ?F_8} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_9, ?F_8, ?F_8, \mathit{dual}(F_5) \& \mathit{dual}(F_6)}{h_7 \text{ :- } \Delta_9, ?F_8, ?F_8, \mathit{dual}(F_5) \& \mathit{dual}(F_6)} \text{ax}}{- \text{ :- } \Delta_2, \Delta_9, ?F_8, ?F_8} \text{hCut}}{- \text{ :- } \Delta_2, \Delta_9, ?F_8} ?C
\end{array}$$

- Case rule ?

$$\begin{array}{c}
\frac{\frac{h_2 \text{ :- } \Delta_5, F_6, F_3}{\bullet h_2 \text{ :- } F_6, \Delta_5, F_3 \oplus F_4} \oplus_A \quad \frac{h_7 \text{ :- } \Delta_9, F_8, \mathit{dual}(F_6)}{\bullet h_7 \text{ :- } \mathit{dual}(F_6), \Delta_9, ?F_8} ?}{- \text{ :- } (\Delta_5, F_3 \oplus F_4), \Delta_9, ?F_8} \text{Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_2 \text{ :- } \Delta_5, F_6, F_3 \oplus F_4}{- \text{ :- } \Delta_5, \Delta_9, F_8, F_3 \oplus F_4} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_9, F_8, \mathit{dual}(F_6)}{h_7 \text{ :- } \Delta_9, F_8, \mathit{dual}(F_6)} \text{ax}}{- \text{ :- } \Delta_5, \Delta_9, ?F_8, F_3 \oplus F_4} \text{hCut}}{- \text{ :- } \Delta_5, \Delta_9, ?F_8, F_3 \oplus F_4} ? \\
\frac{\frac{h_1 \text{ :- } \Delta_2, F_5}{\bullet h_1 \text{ :- } F_5 \oplus F_6, \Delta_2} \oplus_A \quad \frac{h_7 \text{ :- } \Delta_9, F_8, \mathit{dual}(F_5) \& \mathit{dual}(F_6)}{\bullet h_7 \text{ :- } \mathit{dual}(F_5 \oplus F_6), \Delta_9, ?F_8} ?}{- \text{ :- } \Delta_2, \Delta_9, ?F_8} \text{Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_1 \text{ :- } \Delta_2, F_5 \oplus F_6}{- \text{ :- } \Delta_2, \Delta_9, F_8} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_9, F_8, \mathit{dual}(F_5) \& \mathit{dual}(F_6)}{h_7 \text{ :- } \Delta_9, F_8, \mathit{dual}(F_5) \& \mathit{dual}(F_6)} \text{ax}}{- \text{ :- } \Delta_2, \Delta_9, F_8} \text{hCut}}{- \text{ :- } \Delta_2, \Delta_9, ?F_8} ?
\end{array}$$

- Case rule $\$$

$$\begin{array}{c}
\frac{h_2 \text{ :- } \Delta_5, F_6, F_3}{\bullet h_2 \text{ :- } F_6, \Delta_5, F_3 \oplus F_4} \oplus A \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, F_9, \text{dual}(F_6)}{\bullet h_7 \text{ :- } \text{dual}(F_6), \Delta_{10}, F_8 \& F_9} \$}{- \text{ :- } (\Delta_5, F_3 \oplus F_4), \Delta_{10}, F_8 \& F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \text{ :- } \Delta_5, F_6, F_3 \oplus F_4}{\bullet h_2 \text{ :- } \Delta_5, F_6, F_3 \oplus F_4} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, F_9, \text{dual}(F_6)}{h_7 \text{ :- } \Delta_{10}, F_8, F_9, \text{dual}(F_6)} \text{ax}}{- \text{ :- } \Delta_{10}, \Delta_5, F_8, F_9, F_3 \oplus F_4} \text{hCut}}{- \text{ :- } \Delta_{10}, \Delta_5, F_8 \& F_9, F_3 \oplus F_4} \$ \\
\\
\frac{h_1 \text{ :- } \Delta_2, F_5}{\bullet h_1 \text{ :- } F_5 \oplus F_6, \Delta_2} \oplus A \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, F_9, \text{dual}(F_5) \& \text{dual}(F_6)}{\bullet h_7 \text{ :- } \text{dual}(F_5 \oplus F_6), \Delta_{10}, F_8 \& F_9} \$}{- \text{ :- } \Delta_2, \Delta_{10}, F_8 \& F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 \text{ :- } \Delta_2, F_5 \oplus F_6}{\bullet h_1 \text{ :- } \Delta_2, F_5 \oplus F_6} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, F_9, \text{dual}(F_5) \& \text{dual}(F_6)}{h_7 \text{ :- } \Delta_{10}, F_8, F_9, \text{dual}(F_5) \& \text{dual}(F_6)} \text{ax}}{- \text{ :- } \Delta_{10}, \Delta_2, F_8, F_9} \text{hCut}}{- \text{ :- } \Delta_{10}, \Delta_2, F_8 \& F_9} \$
\end{array}$$

- Case rule $\&$

$$\begin{array}{c}
\frac{h_2 \text{ :- } \Delta_5, F_6, F_3}{\bullet h_2 \text{ :- } F_6, \Delta_5, F_3 \oplus F_4} \oplus A \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, \text{dual}(F_6) \quad h_7 \text{ :- } \Delta_{10}, F_9, \text{dual}(F_6)}{\bullet h_7 \text{ :- } \text{dual}(F_6), \Delta_{10}, F_8 \& F_9} \&}{- \text{ :- } (\Delta_5, F_3 \oplus F_4), \Delta_{10}, F_8 \& F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \text{ :- } \Delta_5, F_3, F_6}{\bullet h_2 \text{ :- } \Delta_5, F_3, F_6} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_{10}, \text{dual}(F_6), F_8 \& F_9}{\bullet h_7 \text{ :- } \Delta_{10}, \text{dual}(F_6), F_8 \& F_9} \text{ax}}{- \text{ :- } \Delta_{10}, \Delta_5, F_3, F_8 \& F_9} \text{hCut}}{- \text{ :- } \Delta_{10}, \Delta_5, F_8 \& F_9, F_3 \oplus F_4} \oplus A \\
\\
\frac{h_1 \text{ :- } \Delta_2, F_5}{\bullet h_1 \text{ :- } F_5 \oplus F_6, \Delta_2} \oplus A \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, \text{dual}(F_5) \& \text{dual}(F_6) \quad h_7 \text{ :- } \Delta_{10}, F_9, \text{dual}(F_5) \& \text{dual}(F_6)}{\bullet h_7 \text{ :- } \text{dual}(F_5 \oplus F_6), \Delta_{10}, F_8 \& F_9} \&}{- \text{ :- } \Delta_2, \Delta_{10}, F_8 \& F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 \text{ :- } \Delta_2, F_5 \oplus F_6}{\bullet h_1 \text{ :- } \Delta_2, F_5 \oplus F_6} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, \text{dual}(F_5) \& \text{dual}(F_6)}{h_7 \text{ :- } \Delta_{10}, F_8, \text{dual}(F_5) \& \text{dual}(F_6)} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_{10}, F_9, \text{dual}(F_5) \& \text{dual}(F_6)}{h_7 \text{ :- } \Delta_{10}, F_9, \text{dual}(F_5) \& \text{dual}(F_6)} \text{ax}}{- \text{ :- } \Delta_{10}, \Delta_2, F_8} \text{hCut}}{- \text{ :- } \Delta_{10}, \Delta_2, F_8 \& F_9} \& \\
\\
\frac{h_1 \text{ :- } \Delta_2, F_5}{\bullet h_1 \text{ :- } F_5 \oplus F_6, \Delta_2} \oplus A \quad \frac{h_7 \text{ :- } \Delta_8, \text{dual}(F_5) \quad h_7 \text{ :- } \Delta_8, \text{dual}(F_6)}{\bullet h_7 \text{ :- } \text{dual}(F_5 \oplus F_6), \Delta_8} \&}{- \text{ :- } \Delta_2, \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 \text{ :- } \Delta_2, F_5}{\bullet h_1 \text{ :- } \Delta_2, F_5} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_8, \text{dual}(F_5)}{h_7 \text{ :- } \Delta_8, \text{dual}(F_5)} \text{ax}}{- \text{ :- } \Delta_2, \Delta_8} \text{sCut}
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{h_2 \text{ :- } \Delta_5, F_6, F_3}{\bullet h_2 \text{ :- } F_6, \Delta_5, F_3 \oplus F_4} \oplus A \quad \frac{h_7 \text{ :- } \Delta_{10}, F_9, \text{dual}(F_6)}{\bullet h_7 \text{ :- } \text{dual}(F_6), \Delta_{10}, F_8 \oplus F_9} \oplus B}{- \text{ :- } (\Delta_5, F_3 \oplus F_4), \Delta_{10}, F_8 \oplus F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \text{ :- } \Delta_5, F_6, F_3 \oplus F_4}{\bullet h_2 \text{ :- } \Delta_5, F_6, F_3 \oplus F_4} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_{10}, F_9, \text{dual}(F_6)}{h_7 \text{ :- } \Delta_{10}, F_9, \text{dual}(F_6)} \text{ax}}{- \text{ :- } \Delta_{10}, \Delta_5, F_9, F_3 \oplus F_4} \text{hCut}}{- \text{ :- } \Delta_{10}, \Delta_5, F_3 \oplus F_4, F_8 \oplus F_9} \oplus B \\
\\
\frac{h_1 \text{ :- } \Delta_2, F_5}{\bullet h_1 \text{ :- } F_5 \oplus F_6, \Delta_2} \oplus A \quad \frac{h_7 \text{ :- } \Delta_{10}, F_9, \text{dual}(F_5) \& \text{dual}(F_6)}{\bullet h_7 \text{ :- } \text{dual}(F_5 \oplus F_6), \Delta_{10}, F_8 \oplus F_9} \oplus B}{- \text{ :- } \Delta_2, \Delta_{10}, F_8 \oplus F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 \text{ :- } \Delta_2, F_5 \oplus F_6}{\bullet h_1 \text{ :- } \Delta_2, F_5 \oplus F_6} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_{10}, F_9, \text{dual}(F_5) \& \text{dual}(F_6)}{h_7 \text{ :- } \Delta_{10}, F_9, \text{dual}(F_5) \& \text{dual}(F_6)} \text{ax}}{- \text{ :- } \Delta_{10}, \Delta_2, F_9} \text{hCut}}{- \text{ :- } \Delta_{10}, \Delta_2, F_8 \oplus F_9} \oplus B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{\frac{h_2 \text{ :- } \Delta_5, F_6, F_3}{\bullet h_2 \text{ :- } F_6, \Delta_5, F_3 \oplus F_4} \oplus_A \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, \text{dual}(F_6)}{\bullet h_7 \text{ :- } \text{dual}(F_6), \Delta_{10}, F_8 \oplus F_9}}{- \text{ :- } (\Delta_5, F_3 \oplus F_4), \Delta_{10}, F_8 \oplus F_9} \text{Cut}}{\rightarrow} \oplus_A \\
\frac{\frac{h_2 \text{ :- } \Delta_5, F_3, F_6}{- \text{ :- } \Delta_{10}, \Delta_5, F_3, F_8 \oplus F_9} \text{ax} \quad \frac{\bullet h_7 \text{ :- } \Delta_{10}, \text{dual}(F_6), F_8 \oplus F_9}{- \text{ :- } \Delta_{10}, \Delta_5, F_3 \oplus F_4, F_8 \oplus F_9} \text{hCut}}{- \text{ :- } \Delta_{10}, \Delta_5, F_3 \oplus F_4, F_8 \oplus F_9} \oplus_A}{} \\
\frac{\frac{h_1 \text{ :- } \Delta_2, F_5}{\bullet h_1 \text{ :- } F_5 \oplus F_6, \Delta_2} \oplus_A \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, \text{dual}(F_5) \& \text{dual}(F_6)}{\bullet h_7 \text{ :- } \text{dual}(F_5 \oplus F_6), \Delta_{10}, F_8 \oplus F_9}}{- \text{ :- } \Delta_2, \Delta_{10}, F_8 \oplus F_9} \text{Cut}}{\rightarrow} \oplus_A \\
\frac{\frac{\bullet h_1 \text{ :- } \Delta_2, F_5 \oplus F_6}{- \text{ :- } \Delta_{10}, \Delta_2, F_8} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, \text{dual}(F_5) \& \text{dual}(F_6)}{- \text{ :- } \Delta_{10}, \Delta_2, F_8 \oplus F_9} \text{hCut}}{- \text{ :- } \Delta_{10}, \Delta_2, F_8 \oplus F_9} \oplus_A}{}
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{\frac{h_2 \text{ :- } \Delta_5, F_6, F_3}{\bullet h_2 \text{ :- } F_6, \Delta_5, F_3 \oplus F_4} \oplus_A \quad \frac{h_7 \text{ :- } \Delta_8, \text{dual}(F_6)}{\bullet h_7 \text{ :- } \text{dual}(F_6), \perp, \Delta_8}}{- \text{ :- } (\Delta_5, F_3 \oplus F_4), \perp, \Delta_8} \text{Cut}}{\rightarrow} \perp \\
\frac{\frac{h_2 \text{ :- } \Delta_5, F_3, F_6}{- \text{ :- } \Delta_5, \Delta_8, F_3, \perp} \text{ax} \quad \frac{\bullet h_7 \text{ :- } \Delta_8, \perp, \text{dual}(F_6)}{- \text{ :- } \Delta_5, \Delta_8, \perp, F_3 \oplus F_4} \text{hCut}}{- \text{ :- } \Delta_5, \Delta_8, \perp, F_3 \oplus F_4} \oplus_A}{} \\
\frac{\frac{h_1 \text{ :- } \Delta_2, F_5}{\bullet h_1 \text{ :- } F_5 \oplus F_6, \Delta_2} \oplus_A \quad \frac{h_7 \text{ :- } \Delta_8, \text{dual}(F_5) \& \text{dual}(F_6)}{\bullet h_7 \text{ :- } \text{dual}(F_5 \oplus F_6), \perp, \Delta_8}}{- \text{ :- } \Delta_2, \perp, \Delta_8} \text{Cut}}{\rightarrow} \perp \\
\frac{\frac{\bullet h_1 \text{ :- } \Delta_2, F_5 \oplus F_6}{- \text{ :- } \Delta_2, \Delta_8} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_8, \text{dual}(F_5) \& \text{dual}(F_6)}{- \text{ :- } \Delta_2, \Delta_8, \perp} \text{hCut}}{- \text{ :- } \Delta_2, \Delta_8, \perp} \perp}{}
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{\frac{h_2 \text{ :- } \Delta_5, F_6, F_3}{\bullet h_2 \text{ :- } F_6, \Delta_5, F_3 \oplus F_4} \oplus_A \quad \frac{}{\bullet h_7 \text{ :- } \text{dual}(F_6), \top, \Delta_8} \top}{- \text{ :- } (\Delta_5, F_3 \oplus F_4), \top, \Delta_8} \text{Cut}}{\rightarrow} \top \\
\frac{}{- \text{ :- } \Delta_5, \Delta_8, \top, F_3 \oplus F_4} \top}{} \\
\frac{\frac{h_1 \text{ :- } \Delta_2, F_5}{\bullet h_1 \text{ :- } F_5 \oplus F_6, \Delta_2} \oplus_A \quad \frac{}{\bullet h_7 \text{ :- } \text{dual}(F_5 \oplus F_6), \top, \Delta_8} \top}{- \text{ :- } \Delta_2, \top, \Delta_8} \text{Cut}}{\rightarrow} \top \\
\frac{}{- \text{ :- } \Delta_2, \Delta_8, \top} \top}{}
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{\frac{h_2 \text{ :- } \Delta_5, F_6, F_3}{\bullet h_2 \text{ :- } F_6, \Delta_5, F_3 \oplus F_4} \oplus A \quad \frac{h_7 \text{ :- } \Delta_{11}, F_9, \mathit{dual}(F_6)}{\bullet h_7 \text{ :- } \mathit{dual}(F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \quad h_7 \text{ :- } \Delta_8, F_{10}}{\text{Cut}}}{\text{-- :- } (\Delta_5, F_3 \oplus F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \rightarrow \\
\frac{\frac{h_2 \text{ :- } \Delta_5, F_3, F_6}{\text{-- :- } \Delta_{11}, \Delta_5, \Delta_8, F_3, F_9 \otimes F_{10}} \text{ ax} \quad \frac{\bullet h_7 \text{ :- } \Delta_{11}, \Delta_8, \mathit{dual}(F_6), F_9 \otimes F_{10}}{\text{-- :- } \Delta_{11}, \Delta_5, \Delta_8, F_9 \otimes F_{10}, F_3 \oplus F_4} \text{ ax}}{\text{-- :- } \Delta_{11}, \Delta_5, \Delta_8, F_9 \otimes F_{10}, F_3 \oplus F_4} \text{ hCut} \oplus A \\
\frac{h_2 \text{ :- } \Delta_5, F_6, F_3}{\bullet h_2 \text{ :- } F_6, \Delta_5, F_3 \oplus F_4} \oplus A \quad \frac{h_7 \text{ :- } \Delta_8, F_9 \quad h_7 \text{ :- } \Delta_{11}, F_{10}, \mathit{dual}(F_6)}{\bullet h_7 \text{ :- } \mathit{dual}(F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{\text{Cut}}}{\text{-- :- } (\Delta_5, F_3 \oplus F_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \rightarrow \\
\frac{h_2 \text{ :- } \Delta_5, F_3, F_6}{\text{-- :- } \Delta_{11}, \Delta_5, \Delta_8, F_3, F_9 \otimes F_{10}} \text{ ax} \quad \frac{\bullet h_7 \text{ :- } \Delta_{11}, \Delta_8, \mathit{dual}(F_6), F_9 \otimes F_{10}}{\text{-- :- } \Delta_{11}, \Delta_5, \Delta_8, F_9 \otimes F_{10}, F_3 \oplus F_4} \text{ ax}}{\text{-- :- } \Delta_{11}, \Delta_5, \Delta_8, F_9 \otimes F_{10}, F_3 \oplus F_4} \text{ hCut} \oplus A \\
\frac{h_1 \text{ :- } \Delta_2, F_5}{\bullet h_1 \text{ :- } F_5 \oplus F_6, \Delta_2} \oplus A \quad \frac{h_7 \text{ :- } \Delta_{11}, F_9, \mathit{dual}(F_5) \& \mathit{dual}(F_6)}{\bullet h_7 \text{ :- } \mathit{dual}(F_5 \oplus F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \quad h_7 \text{ :- } \Delta_8, F_{10}}{\text{Cut}}}{\text{-- :- } \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \rightarrow \\
\frac{\bullet h_1 \text{ :- } \Delta_2, F_5 \oplus F_6}{\text{-- :- } \Delta_{11}, \Delta_2, F_9} \text{ ax} \quad \frac{h_7 \text{ :- } \Delta_{11}, F_9, \mathit{dual}(F_5) \& \mathit{dual}(F_6)}{\text{-- :- } \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \text{ hCut} \quad \frac{h_7 \text{ :- } \Delta_8, F_{10}}{\text{-- :- } \Delta_8, F_{10}} \text{ ax}}{\text{-- :- } \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \otimes \\
\frac{h_1 \text{ :- } \Delta_2, F_5}{\bullet h_1 \text{ :- } F_5 \oplus F_6, \Delta_2} \oplus A \quad \frac{h_7 \text{ :- } \Delta_8, F_9 \quad h_7 \text{ :- } \Delta_{11}, F_{10}, \mathit{dual}(F_5) \& \mathit{dual}(F_6)}{\bullet h_7 \text{ :- } \mathit{dual}(F_5 \oplus F_6), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{\text{Cut}}}{\text{-- :- } \Delta_2, \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \rightarrow \\
\frac{\text{-- :- } \Delta_8, F_9}{\text{-- :- } \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \text{ ax} \quad \frac{\bullet h_1 \text{ :- } \Delta_2, F_5 \oplus F_6}{\text{-- :- } \Delta_{11}, \Delta_2, F_9} \text{ ax} \quad \frac{h_7 \text{ :- } \Delta_{11}, F_{10}, \mathit{dual}(F_5) \& \mathit{dual}(F_6)}{\text{-- :- } \Delta_{11}, \Delta_2, F_{10}} \text{ ax}}{\text{-- :- } \Delta_{11}, \Delta_2, \Delta_8, F_9 \otimes F_{10}} \text{ hCut} \otimes
\end{array}$$

5.10 Status of \perp : OK

- Case rule 1

$$\frac{\frac{h_1 \text{ :- } \Delta_2}{\bullet h_1 \text{ :- } \perp, \Delta_2} \perp \quad \frac{\bullet h_3 \text{ :- } \mathit{dual}(\perp), *}{\text{-- :- } \Delta_2, *} \mathbf{1}}{\text{-- :- } \Delta_2, *} \text{Cut}}{\text{-- :- } \Delta_2} \text{ ax}$$

- Case rule !

- Case rule ?W

$$\frac{\frac{h_1 \text{ :- } \Delta_2}{\bullet h_1 \text{ :- } \perp, \Delta_2} \perp \quad \frac{h_3 \text{ :- } \mathbf{1}, \Delta_5}{\bullet h_3 \text{ :- } \mathit{dual}(\perp), \Delta_5, ?F_4} ?W}{\text{-- :- } \Delta_2, \Delta_5, ?F_4} \text{Cut}}{\text{-- :- } \Delta_2, \Delta_5, ?F_4} \rightarrow \\
\frac{\frac{\bullet h_1 \text{ :- } \Delta_2, \perp}{\text{-- :- } \Delta_2, \Delta_5} \text{ ax} \quad \frac{h_3 \text{ :- } \mathbf{1}, \Delta_5}{\text{-- :- } \Delta_2, \Delta_5, ?F_4} \text{ ax}}{\text{-- :- } \Delta_2, \Delta_5, ?F_4} \text{ hCut} \quad W \\
\frac{h_2 \text{ :- } \Delta_3, F_4}{\bullet h_2 \text{ :- } F_4, \perp, \Delta_3} \perp \quad \frac{h_5 \text{ :- } \Delta_7, \mathit{dual}(F_4)}{\bullet h_5 \text{ :- } \mathit{dual}(F_4), \Delta_7, ?F_6} ?W}{\text{-- :- } (\perp, \Delta_3), \Delta_7, ?F_6} \text{Cut}}{\text{-- :- } (\perp, \Delta_3), \Delta_7, ?F_6} \rightarrow \\
\frac{\frac{\bullet h_2 \text{ :- } \Delta_3, F_4, \perp}{\text{-- :- } \Delta_3, \Delta_7, \perp} \text{ ax} \quad \frac{h_5 \text{ :- } \Delta_7, \mathit{dual}(F_4)}{\text{-- :- } \Delta_3, \Delta_7, \perp, ?F_6} \text{ ax}}{\text{-- :- } \Delta_3, \Delta_7, \perp, ?F_6} \text{ hCut} \quad W$$

- Case rule ?C

$$\begin{array}{c}
\frac{h_1 \text{ : } \Delta_2}{\bullet h_1 \text{ : } \perp, \Delta_2} \perp \quad \frac{h_3 \text{ : } \mathbf{1}, \Delta_5, ?F_4, ?F_4}{\bullet h_3 \text{ : } \mathit{dual}(\perp), \Delta_5, ?F_4} ?C}{- \text{ : } \Delta_2, \Delta_5, ?F_4} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \text{ : } \Delta_2, \perp}{- \text{ : } \Delta_2, \Delta_5, ?F_4, ?F_4} \text{ax} \quad \frac{h_3 \text{ : } \mathbf{1}, \Delta_5, ?F_4, ?F_4}{- \text{ : } \Delta_2, \Delta_5, ?F_4} \text{ax}}{- \text{ : } \Delta_2, \Delta_5, ?F_4} \text{hCut}}{- \text{ : } \Delta_2, \Delta_5, ?F_4} ?C \\
\\
\frac{h_2 \text{ : } \Delta_3, F_4}{\bullet h_2 \text{ : } F_4, \perp, \Delta_3} \perp \quad \frac{h_5 \text{ : } \Delta_7, ?F_6, ?F_6, \mathit{dual}(F_4)}{\bullet h_5 \text{ : } \mathit{dual}(F_4), \Delta_7, ?F_6} ?C}{- \text{ : } (\perp, \Delta_3), \Delta_7, ?F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \text{ : } \Delta_3, F_4, \perp}{- \text{ : } \Delta_3, \Delta_7, \perp, ?F_6, ?F_6} \text{ax} \quad \frac{h_5 \text{ : } \Delta_7, ?F_6, ?F_6, \mathit{dual}(F_4)}{- \text{ : } \Delta_3, \Delta_7, \perp, ?F_6} \text{ax}}{- \text{ : } \Delta_3, \Delta_7, \perp, ?F_6} \text{hCut}}{- \text{ : } \Delta_3, \Delta_7, \perp, ?F_6} ?C
\end{array}$$

- Case rule ?

$$\begin{array}{c}
\frac{h_1 \text{ : } \Delta_2}{\bullet h_1 \text{ : } \perp, \Delta_2} \perp \quad \frac{h_3 \text{ : } \mathbf{1}, \Delta_5, F_4}{\bullet h_3 \text{ : } \mathit{dual}(\perp), \Delta_5, ?F_4} ?}{- \text{ : } \Delta_2, \Delta_5, ?F_4} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \text{ : } \Delta_2, \perp}{- \text{ : } \Delta_2, \Delta_5, F_4} \text{ax} \quad \frac{h_3 \text{ : } \mathbf{1}, \Delta_5, F_4}{- \text{ : } \Delta_2, \Delta_5, ?F_4} \text{ax}}{- \text{ : } \Delta_2, \Delta_5, ?F_4} \text{hCut}}{- \text{ : } \Delta_2, \Delta_5, ?F_4} ? \\
\\
\frac{h_2 \text{ : } \Delta_3, F_4}{\bullet h_2 \text{ : } F_4, \perp, \Delta_3} \perp \quad \frac{h_5 \text{ : } \Delta_7, F_6, \mathit{dual}(F_4)}{\bullet h_5 \text{ : } \mathit{dual}(F_4), \Delta_7, ?F_6} ?}{- \text{ : } (\perp, \Delta_3), \Delta_7, ?F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \text{ : } \Delta_3, F_4, \perp}{- \text{ : } \Delta_3, \Delta_7, F_6, \perp} \text{ax} \quad \frac{h_5 \text{ : } \Delta_7, F_6, \mathit{dual}(F_4)}{- \text{ : } \Delta_3, \Delta_7, \perp, ?F_6} \text{ax}}{- \text{ : } \Delta_3, \Delta_7, \perp, ?F_6} \text{hCut}}{- \text{ : } \Delta_3, \Delta_7, \perp, ?F_6} ?
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{h_1 \text{ : } \Delta_2}{\bullet h_1 \text{ : } \perp, \Delta_2} \perp \quad \frac{h_3 \text{ : } \mathbf{1}, \Delta_6, F_4, F_5}{\bullet h_3 \text{ : } \mathit{dual}(\perp), \Delta_6, F_4 \$F_5} \$}{- \text{ : } \Delta_2, \Delta_6, F_4 \$F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \text{ : } \Delta_2, \perp}{- \text{ : } \Delta_2, \Delta_6, F_4, F_5} \text{ax} \quad \frac{h_3 \text{ : } \mathbf{1}, \Delta_6, F_4, F_5}{- \text{ : } \Delta_2, \Delta_6, F_4 \$F_5} \text{ax}}{- \text{ : } \Delta_2, \Delta_6, F_4 \$F_5} \text{hCut}}{- \text{ : } \Delta_2, \Delta_6, F_4 \$F_5} \$ \\
\\
\frac{h_2 \text{ : } \Delta_3, F_4}{\bullet h_2 \text{ : } F_4, \perp, \Delta_3} \perp \quad \frac{h_5 \text{ : } \Delta_8, F_6, F_7, \mathit{dual}(F_4)}{\bullet h_5 \text{ : } \mathit{dual}(F_4), \Delta_8, F_6 \$F_7} \$}{- \text{ : } (\perp, \Delta_3), \Delta_8, F_6 \$F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \text{ : } \Delta_3, F_4, \perp}{- \text{ : } \Delta_3, \Delta_8, F_6, F_7, \perp} \text{ax} \quad \frac{h_5 \text{ : } \Delta_8, F_6, F_7, \mathit{dual}(F_4)}{- \text{ : } \Delta_3, \Delta_8, \perp, F_6 \$F_7} \text{ax}}{- \text{ : } \Delta_3, \Delta_8, \perp, F_6 \$F_7} \text{hCut}}{- \text{ : } \Delta_3, \Delta_8, \perp, F_6 \$F_7} \$
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_6, F_4 \quad h_3 : \vdash \mathbf{1}, \Delta_6, F_5}{\bullet h_3 : \vdash dual(\perp), \Delta_6, F_4 \& F_5} \&}{- : \vdash \Delta_2, \Delta_6, F_4 \& F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \vdash \Delta_2, \perp}{- : \vdash \Delta_2, \Delta_6, F_4} \text{ax} \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_6, F_4}{h\text{Cut}} \text{ax}}{- : \vdash \Delta_2, \Delta_6, F_4 \& F_5} \& \quad \frac{\frac{h_1 : \vdash \Delta_2, \perp}{- : \vdash \Delta_2, \Delta_6, F_5} \text{ax} \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_6, F_5}{h\text{Cut}} \text{ax}}{- : \vdash \Delta_2, \Delta_6, F_4 \& F_5} \&}{- : \vdash \Delta_2, \Delta_6, F_4 \& F_5} \& \\
\frac{\frac{h_2 : \vdash \Delta_3, F_4}{\bullet h_2 : \vdash F_4, \perp, \Delta_3} \perp \quad \frac{h_5 : \vdash \Delta_8, F_6, dual(F_4) \quad h_5 : \vdash \Delta_8, F_7, dual(F_4)}{\bullet h_5 : \vdash dual(F_4), \Delta_8, F_6 \& F_7} \&}{- : \vdash (\perp, \Delta_3), \Delta_8, F_6 \& F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_2 : \vdash \Delta_3, F_4}{- : \vdash \Delta_3, \Delta_8, F_6 \& F_7} \text{ax} \quad \frac{\bullet h_5 : \vdash \Delta_8, dual(F_4), F_6 \& F_7}{- : \vdash \Delta_3, \Delta_8, F_6 \& F_7} \text{ax}}{- : \vdash \Delta_3, \Delta_8, \perp, F_6 \& F_7} \perp}{- : \vdash \Delta_3, \Delta_8, \perp, F_6 \& F_7} \perp
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_6, F_5}{\bullet h_3 : \vdash dual(\perp), \Delta_6, F_4 \oplus F_5} \oplus_B}{- : \vdash \Delta_2, \Delta_6, F_4 \oplus F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \vdash \Delta_2, \perp}{- : \vdash \Delta_2, \Delta_6, F_5} \text{ax} \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_6, F_5}{h\text{Cut}} \text{ax}}{- : \vdash \Delta_2, \Delta_6, F_4 \oplus F_5} \oplus_B}{- : \vdash \Delta_2, \Delta_6, F_4 \oplus F_5} \oplus_B \\
\frac{\frac{h_2 : \vdash \Delta_3, F_4}{\bullet h_2 : \vdash F_4, \perp, \Delta_3} \perp \quad \frac{h_5 : \vdash \Delta_8, F_7, dual(F_4)}{\bullet h_5 : \vdash dual(F_4), \Delta_8, F_6 \oplus F_7} \oplus_B}{- : \vdash (\perp, \Delta_3), \Delta_8, F_6 \oplus F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_2 : \vdash \Delta_3, F_4, \perp}{- : \vdash \Delta_3, \Delta_8, F_7, \perp} \text{ax} \quad \frac{h_5 : \vdash \Delta_8, F_7, dual(F_4)}{h\text{Cut}} \text{ax}}{- : \vdash \Delta_3, \Delta_8, \perp, F_6 \oplus F_7} \oplus_B}{- : \vdash \Delta_3, \Delta_8, \perp, F_6 \oplus F_7} \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_6, F_4}{\bullet h_3 : \vdash dual(\perp), \Delta_6, F_4 \oplus F_5} \oplus_A}{- : \vdash \Delta_2, \Delta_6, F_4 \oplus F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \vdash \Delta_2, \perp}{- : \vdash \Delta_2, \Delta_6, F_4} \text{ax} \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_6, F_4}{h\text{Cut}} \text{ax}}{- : \vdash \Delta_2, \Delta_6, F_4 \oplus F_5} \oplus_A}{- : \vdash \Delta_2, \Delta_6, F_4 \oplus F_5} \oplus_A \\
\frac{\frac{h_2 : \vdash \Delta_3, F_4}{\bullet h_2 : \vdash F_4, \perp, \Delta_3} \perp \quad \frac{h_5 : \vdash \Delta_8, F_6, dual(F_4)}{\bullet h_5 : \vdash dual(F_4), \Delta_8, F_6 \oplus F_7} \oplus_A}{- : \vdash (\perp, \Delta_3), \Delta_8, F_6 \oplus F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_2 : \vdash \Delta_3, F_4, \perp}{- : \vdash \Delta_3, \Delta_8, F_6, \perp} \text{ax} \quad \frac{h_5 : \vdash \Delta_8, F_6, dual(F_4)}{h\text{Cut}} \text{ax}}{- : \vdash \Delta_3, \Delta_8, \perp, F_6 \oplus F_7} \oplus_A}{- : \vdash \Delta_3, \Delta_8, \perp, F_6 \oplus F_7} \oplus_A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_4}{\bullet h_3 : \vdash dual(\perp), \perp, \Delta_4} \perp}{- : \vdash \Delta_2, \perp, \Delta_4} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \vdash \Delta_2, \perp}{- : \vdash \Delta_2, \Delta_4} \text{ax} \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_4}{h\text{Cut}} \text{ax}}{- : \vdash \Delta_2, \Delta_4, \perp} \perp}{- : \vdash \Delta_2, \Delta_4, \perp} \perp
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_3, F_4}{\bullet h_2 : \vdash F_4, \perp, \Delta_3} \perp \quad \frac{h_5 : \vdash \Delta_6, dual(F_4)}{\bullet h_5 : \vdash dual(F_4), \perp, \Delta_6} \perp}{- : \vdash (\perp, \Delta_3), \perp, \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \vdash \Delta_3, F_4}{- : \vdash \Delta_3, \Delta_6, \perp} \text{ax} \quad \frac{h_5 : \vdash \Delta_6, \perp, dual(F_4)}{- : \vdash \Delta_3, \Delta_6, \perp} \text{ax}}{- : \vdash \Delta_3, \Delta_6, \perp, \perp} \text{hCut} \\
- : \vdash \Delta_3, \Delta_6, \perp, \perp \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \quad \frac{}{\bullet h_3 : \vdash dual(\perp), \top, \Delta_4} \top}{- : \vdash \Delta_2, \top, \Delta_4} \text{Cut} \\
\rightarrow \\
- : \vdash \Delta_2, \Delta_4, \top \top \\
\frac{h_2 : \vdash \Delta_3, F_4}{\bullet h_2 : \vdash F_4, \perp, \Delta_3} \perp \quad \frac{}{\bullet h_5 : \vdash dual(F_4), \top, \Delta_6} \top}{- : \vdash (\perp, \Delta_3), \top, \Delta_6} \text{Cut} \\
\rightarrow \\
- : \vdash \Delta_3, \Delta_6, \perp, \top \top
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_7, F_5 \quad h_3 : \vdash \Delta_4, F_6}{\bullet h_3 : \vdash dual(\perp), \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes}{- : \vdash \Delta_2, \Delta_4, \Delta_7, F_5 \otimes F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \vdash \Delta_2, \perp}{- : \vdash \Delta_2, \Delta_7, F_5} \text{ax} \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_7, F_5}{- : \vdash \Delta_4, F_6} \text{ax}}{- : \vdash \Delta_2, \Delta_4, \Delta_7, F_5 \otimes F_6} \text{hCut} \\
- : \vdash \Delta_2, \Delta_4, \Delta_7, F_5 \otimes F_6 \otimes
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \vdash \Delta_2}{\bullet h_1 : \vdash \perp, \Delta_2} \perp \quad \frac{h_3 : \vdash \Delta_4, F_5 \quad h_3 : \vdash \mathbf{1}, \Delta_7, F_6}{\bullet h_3 : \vdash dual(\perp), \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes}{- : \vdash \Delta_2, \Delta_4, \Delta_7, F_5 \otimes F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \vdash \Delta_2, \perp}{- : \vdash \Delta_4, F_5} \text{ax} \quad \frac{h_3 : \vdash \mathbf{1}, \Delta_7, F_6}{- : \vdash \Delta_2, \Delta_7, F_6} \text{ax}}{- : \vdash \Delta_2, \Delta_4, \Delta_7, F_5 \otimes F_6} \text{hCut} \\
- : \vdash \Delta_2, \Delta_4, \Delta_7, F_5 \otimes F_6 \otimes
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_3, F_4}{\bullet h_2 : \vdash F_4, \perp, \Delta_3} \perp \quad \frac{h_5 : \vdash \Delta_9, F_7, dual(F_4) \quad h_5 : \vdash \Delta_6, F_8}{\bullet h_5 : \vdash dual(F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes}{- : \vdash (\perp, \Delta_3), \Delta_6, \Delta_9, F_7 \otimes F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \vdash \Delta_3, F_4}{- : \vdash \Delta_3, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{ax} \quad \frac{h_5 : \vdash \Delta_6, \Delta_9, dual(F_4), F_7 \otimes F_8}{- : \vdash \Delta_3, \Delta_6, \Delta_9, \perp, F_7 \otimes F_8} \text{ax}}{- : \vdash \Delta_3, \Delta_6, \Delta_9, \perp, F_7 \otimes F_8} \text{hCut} \\
- : \vdash \Delta_3, \Delta_6, \Delta_9, \perp, F_7 \otimes F_8 \perp
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \vdash \Delta_3, F_4}{\bullet h_2 : \vdash F_4, \perp, \Delta_3} \perp \quad \frac{h_5 : \vdash \Delta_6, F_7 \quad h_5 : \vdash \Delta_9, F_8, dual(F_4)}{\bullet h_5 : \vdash dual(F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes}{- : \vdash (\perp, \Delta_3), \Delta_6, \Delta_9, F_7 \otimes F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : \vdash \Delta_3, F_4}{- : \vdash \Delta_3, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{ax} \quad \frac{h_5 : \vdash \Delta_6, \Delta_9, dual(F_4), F_7 \otimes F_8}{- : \vdash \Delta_3, \Delta_6, \Delta_9, \perp, F_7 \otimes F_8} \text{ax}}{- : \vdash \Delta_3, \Delta_6, \Delta_9, \perp, F_7 \otimes F_8} \text{hCut} \\
- : \vdash \Delta_3, \Delta_6, \Delta_9, \perp, F_7 \otimes F_8 \perp
\end{array}$$

5.11 Status of \top : OK

- Case rule 1
- Case rule !
- Case rule ?W

$$\frac{\frac{\frac{}{\bullet h_1 \vdash \top, \Delta_2} \top \quad \frac{h_3 \vdash \mathbf{0}, \Delta_5}{\bullet h_3 \vdash \text{dual}(\top), \Delta_5, ?F_4} ?W}{- \vdash \Delta_2, \Delta_5, ?F_4} \text{Cut}}{\frac{\frac{\bullet h_1 \vdash \Delta_2, \top, ?F_4}{- \vdash \Delta_2, \Delta_5, ?F_4} \top \quad \frac{h_3 \vdash \mathbf{0}, \Delta_5}{h_3 \vdash \mathbf{0}, \Delta_5} \text{ax}}{- \vdash \Delta_2, \Delta_5, ?F_4} \text{hCut}}{\frac{\frac{}{\bullet h_2 \vdash F_4, \top, \Delta_3} \top \quad \frac{h_5 \vdash \Delta_7, \text{dual}(F_4)}{\bullet h_5 \vdash \text{dual}(F_4), \Delta_7, ?F_6} ?W}{- \vdash (\top, \Delta_3), \Delta_7, ?F_6} \text{Cut}}{- \vdash \Delta_3, \Delta_7, \top, ?F_6} \top} \text{Cut}$$

- Case rule ?C

$$\frac{\frac{\frac{}{\bullet h_1 \vdash \top, \Delta_2} \top \quad \frac{h_3 \vdash \mathbf{0}, \Delta_5, ?F_4, ?F_4}{\bullet h_3 \vdash \text{dual}(\top), \Delta_5, ?F_4} ?C}{- \vdash \Delta_2, \Delta_5, ?F_4} \text{Cut}}{\frac{\frac{\bullet h_1 \vdash \Delta_2, \top}{- \vdash \Delta_2, \Delta_5, ?F_4, ?F_4} \top \quad \frac{h_3 \vdash \mathbf{0}, \Delta_5, ?F_4, ?F_4}{h_3 \vdash \mathbf{0}, \Delta_5, ?F_4, ?F_4} \text{ax}}{- \vdash \Delta_2, \Delta_5, ?F_4} \text{hCut}}{- \vdash \Delta_2, \Delta_5, ?F_4} ?C}{\frac{\frac{}{\bullet h_2 \vdash F_4, \top, \Delta_3} \top \quad \frac{h_5 \vdash \Delta_7, ?F_6, ?F_6, \text{dual}(F_4)}{\bullet h_5 \vdash \text{dual}(F_4), \Delta_7, ?F_6} ?C}{- \vdash (\top, \Delta_3), \Delta_7, ?F_6} \text{Cut}}{- \vdash \Delta_3, \Delta_7, \top, ?F_6} \top} \text{Cut}$$

- Case rule ?

$$\frac{\frac{\frac{}{\bullet h_1 \vdash \top, \Delta_2} \top \quad \frac{h_3 \vdash \mathbf{0}, \Delta_5, F_4}{\bullet h_3 \vdash \text{dual}(\top), \Delta_5, ?F_4} ?}{- \vdash \Delta_2, \Delta_5, ?F_4} \text{Cut}}{\frac{\frac{\bullet h_1 \vdash \Delta_2, \top}{- \vdash \Delta_2, \Delta_5, F_4} \top \quad \frac{h_3 \vdash \mathbf{0}, \Delta_5, F_4}{h_3 \vdash \mathbf{0}, \Delta_5, F_4} \text{ax}}{- \vdash \Delta_2, \Delta_5, ?F_4} \text{hCut}}{- \vdash \Delta_2, \Delta_5, ?F_4} ?}{\frac{\frac{}{\bullet h_2 \vdash F_4, \top, \Delta_3} \top \quad \frac{h_5 \vdash \Delta_7, F_6, \text{dual}(F_4)}{\bullet h_5 \vdash \text{dual}(F_4), \Delta_7, ?F_6} ?}{- \vdash (\top, \Delta_3), \Delta_7, ?F_6} \text{Cut}}{- \vdash \Delta_3, \Delta_7, \top, ?F_6} \top} \text{Cut}$$

- Case rule \$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \top, \Delta_2} \top \quad \frac{h_3 : \top, \Delta_6, F_4, F_5}{\bullet h_3 : \top, \text{dual}(\top), \Delta_6, F_4 \& F_5} \S \\
\hline
- : \top, \Delta_2, \Delta_6, F_4 \& F_5 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : \Delta_2, \top} \top \quad \frac{h_3 : \top, \Delta_6, F_4, F_5}{\bullet h_3 : \top, \text{dual}(\top), \Delta_6, F_4 \& F_5} \text{ax}}{- : \Delta_2, \Delta_6, F_4, F_5} \text{hCut}}{- : \Delta_2, \Delta_6, F_4 \& F_5} \S \\
\\
\frac{}{\bullet h_2 : F_4, \top, \Delta_3} \top \quad \frac{h_5 : \top, \Delta_8, F_6, F_7, \text{dual}(F_4)}{\bullet h_5 : \top, \text{dual}(F_4), \Delta_8, F_6 \& F_7} \S \\
\hline
- : (\top, \Delta_3), \Delta_8, F_6 \& F_7 \quad \text{Cut} \\
\rightarrow \\
- : \Delta_3, \Delta_8, \top, F_6 \& F_7 \quad \top
\end{array}$$

• Case rule $\&$

$$\begin{array}{c}
\frac{}{\bullet h_1 : \top, \Delta_2} \top \quad \frac{h_3 : \top, \Delta_6, F_4 \quad h_3 : \top, \Delta_6, F_5}{\bullet h_3 : \top, \text{dual}(\top), \Delta_6, F_4 \& F_5} \& \\
\hline
- : \top, \Delta_2, \Delta_6, F_4 \& F_5 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{}{\bullet h_1 : \Delta_2, \top} \text{ax}}{- : \Delta_2, \Delta_6, F_4} \quad \frac{h_3 : \top, \Delta_6, F_4}{\bullet h_3 : \top, \text{dual}(\top), \Delta_6, F_4 \& F_5} \text{ax}}{- : \Delta_2, \Delta_6, F_4 \& F_5} \text{hCut}}{\frac{\frac{}{\bullet h_1 : \Delta_2, \top} \text{ax}}{- : \Delta_2, \Delta_6, F_4} \quad \frac{h_3 : \top, \Delta_6, F_5}{\bullet h_3 : \top, \text{dual}(\top), \Delta_6, F_4 \& F_5} \text{ax}}{- : \Delta_2, \Delta_6, F_4 \& F_5} \&} \\
\\
\frac{}{\bullet h_2 : F_4, \top, \Delta_3} \top \quad \frac{h_5 : \top, \Delta_8, F_6, \text{dual}(F_4) \quad h_5 : \top, \Delta_8, F_7, \text{dual}(F_4)}{\bullet h_5 : \top, \text{dual}(F_4), \Delta_8, F_6 \& F_7} \& \\
\hline
- : (\top, \Delta_3), \Delta_8, F_6 \& F_7 \quad \text{Cut} \\
\rightarrow \\
- : \Delta_3, \Delta_8, \top, F_6 \& F_7 \quad \top
\end{array}$$

• Case rule \oplus_B

$$\begin{array}{c}
\frac{}{\bullet h_1 : \top, \Delta_2} \top \quad \frac{h_3 : \top, \Delta_6, F_5}{\bullet h_3 : \top, \text{dual}(\top), \Delta_6, F_4 \oplus F_5} \oplus_B \\
\hline
- : \top, \Delta_2, \Delta_6, F_4 \oplus F_5 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : \Delta_2, \top} \top \quad \frac{h_3 : \top, \Delta_6, F_5}{\bullet h_3 : \top, \text{dual}(\top), \Delta_6, F_4 \oplus F_5} \text{ax}}{- : \Delta_2, \Delta_6, F_5} \text{hCut}}{- : \Delta_2, \Delta_6, F_4 \oplus F_5} \oplus_B \\
\\
\frac{}{\bullet h_2 : F_4, \top, \Delta_3} \top \quad \frac{h_5 : \top, \Delta_8, F_7, \text{dual}(F_4)}{\bullet h_5 : \top, \text{dual}(F_4), \Delta_8, F_6 \oplus F_7} \oplus_B \\
\hline
- : (\top, \Delta_3), \Delta_8, F_6 \oplus F_7 \quad \text{Cut} \\
\rightarrow \\
- : \Delta_3, \Delta_8, \top, F_6 \oplus F_7 \quad \top
\end{array}$$

• Case rule \oplus_A

$$\begin{array}{c}
\frac{}{\bullet h_1 : \top, \Delta_2} \top \quad \frac{h_3 : \top, \Delta_6, F_4}{\bullet h_3 : \top, \text{dual}(\top), \Delta_6, F_4 \oplus F_5} \oplus_A \\
\hline
- : \top, \Delta_2, \Delta_6, F_4 \oplus F_5 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : \Delta_2, \top} \top \quad \frac{h_3 : \top, \Delta_6, F_4}{\bullet h_3 : \top, \text{dual}(\top), \Delta_6, F_4 \oplus F_5} \text{ax}}{- : \Delta_2, \Delta_6, F_4} \text{hCut}}{- : \Delta_2, \Delta_6, F_4 \oplus F_5} \oplus_A \\
\\
\frac{}{\bullet h_2 : F_4, \top, \Delta_3} \top \quad \frac{h_5 : \top, \Delta_8, F_6, \text{dual}(F_4)}{\bullet h_5 : \top, \text{dual}(F_4), \Delta_8, F_6 \oplus F_7} \oplus_A \\
\hline
- : (\top, \Delta_3), \Delta_8, F_6 \oplus F_7 \quad \text{Cut} \\
\rightarrow \\
- : \Delta_3, \Delta_8, \top, F_6 \oplus F_7 \quad \top
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \vdash \top, \Delta_2} \top \quad \frac{h_3 : \vdash \mathbf{0}, \Delta_4}{\bullet h_3 : \vdash \text{dual}(\top), \perp, \Delta_4} \perp}{- : \vdash \Delta_2, \perp, \Delta_4} \text{Cut}}{\frac{\frac{\bullet h_1 : \vdash \Delta_2, \perp, \top}{- : \vdash \Delta_2, \Delta_4, \perp} \top \quad \frac{h_3 : \vdash \mathbf{0}, \Delta_4}{\text{hCut}} \text{ax}}{- : \vdash \Delta_2, \Delta_4, \perp} \text{hCut}}{\frac{\frac{}{\bullet h_2 : \vdash F_4, \top, \Delta_3} \top \quad \frac{h_5 : \vdash \Delta_6, \text{dual}(F_4)}{\bullet h_5 : \vdash \text{dual}(F_4), \perp, \Delta_6} \perp}{- : \vdash (\top, \Delta_3), \perp, \Delta_6} \text{Cut}}{- : \vdash \Delta_3, \Delta_6, \perp, \top} \top} \perp}
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \vdash \top, \Delta_2} \top \quad \frac{\bullet h_3 : \vdash \text{dual}(\top), \top, \Delta_4}{- : \vdash \Delta_2, \top, \Delta_4} \top}{- : \vdash \Delta_2, \Delta_4, \top} \top}{\frac{\frac{}{\bullet h_2 : \vdash F_4, \top, \Delta_3} \top \quad \frac{\bullet h_5 : \vdash \text{dual}(F_4), \top, \Delta_6}{- : \vdash (\top, \Delta_3), \top, \Delta_6} \top}{- : \vdash \Delta_3, \Delta_6, \top, \top} \top} \top} \top}
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \vdash \top, \Delta_2} \top \quad \frac{h_3 : \vdash \mathbf{0}, \Delta_7, F_5 \quad h_3 : \vdash \Delta_4, F_6}{\bullet h_3 : \vdash \text{dual}(\top), \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes}{- : \vdash \Delta_2, \Delta_4, \Delta_7, F_5 \otimes F_6} \text{Cut}}{\frac{\frac{\bullet h_1 : \vdash \Delta_2, \top}{- : \vdash \Delta_2, \Delta_7, F_5} \top \quad \frac{h_3 : \vdash \mathbf{0}, \Delta_7, F_5}{\text{hCut}} \text{ax}}{- : \vdash \Delta_2, \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes} \otimes} \otimes}
\end{array}$$

$$\frac{\frac{}{\bullet h_1 : \vdash \top, \Delta_2} \top \quad \frac{h_3 : \vdash \Delta_4, F_5 \quad h_3 : \vdash \mathbf{0}, \Delta_7, F_6}{\bullet h_3 : \vdash \text{dual}(\top), \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes}{- : \vdash \Delta_2, \Delta_4, \Delta_7, F_5 \otimes F_6} \text{Cut}}{\frac{\frac{}{- : \vdash \Delta_4, F_5} \text{ax} \quad \frac{\bullet h_1 : \vdash \Delta_2, \top}{- : \vdash \Delta_2, \Delta_7, F_6} \top \quad \frac{h_3 : \vdash \mathbf{0}, \Delta_7, F_6}{\text{hCut}} \text{ax}}{- : \vdash \Delta_2, \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes} \otimes} \otimes}$$

$$\frac{\frac{}{\bullet h_2 : \vdash F_4, \top, \Delta_3} \top \quad \frac{h_5 : \vdash \Delta_9, F_7, \text{dual}(F_4) \quad h_5 : \vdash \Delta_6, F_8}{\bullet h_5 : \vdash \text{dual}(F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes}{- : \vdash (\top, \Delta_3), \Delta_6, \Delta_9, F_7 \otimes F_8} \text{Cut}}{\frac{}{- : \vdash \Delta_3, \Delta_6, \Delta_9, \top, F_7 \otimes F_8} \top} \top} \otimes}$$

$$\frac{\frac{}{\bullet h_2 : \vdash F_4, \top, \Delta_3} \top \quad \frac{h_5 : \vdash \Delta_6, F_7 \quad h_5 : \vdash \Delta_9, F_8, \text{dual}(F_4)}{\bullet h_5 : \vdash \text{dual}(F_4), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes}{- : \vdash (\top, \Delta_3), \Delta_6, \Delta_9, F_7 \otimes F_8} \text{Cut}}{\frac{}{- : \vdash \Delta_3, \Delta_6, \Delta_9, \top, F_7 \otimes F_8} \top} \top} \otimes}$$

5.12 Status of I : OK

- Case rule 1
- Case rule !
- Case rule ? W

$$\frac{\frac{\bullet \mathbf{h}_1 \vdash p(\mathbf{n}_3), \hat{\wedge}(\mathbf{n}_3)}{\bullet \mathbf{h}_4 \vdash p(\mathbf{n}_3), \hat{\wedge}(\mathbf{n}_3)} I \quad \frac{\mathbf{h}_4 \vdash \Delta_6, \hat{\wedge}(\mathbf{n}_3)}{\bullet \mathbf{h}_4 \vdash dual(p(\mathbf{n}_3)), \Delta_6, ?F_5} ?W}{\frac{}{- \vdash \hat{\wedge}(\mathbf{n}_3), \Delta_6, ?F_5} \text{Cut}}{\frac{}{- \vdash \Delta_6, \hat{\wedge}(\mathbf{n}_3)} \text{ax}}{\frac{}{- \vdash \Delta_6, ?F_5, \hat{\wedge}(\mathbf{n}_3)} ?W} \rightarrow$$

$$\frac{\frac{\bullet \mathbf{h}_1 \vdash \hat{\wedge}(\mathbf{n}_3), p(\mathbf{n}_3)}{\bullet \mathbf{h}_4 \vdash dual(\hat{\wedge}(\mathbf{n}_3)), \Delta_6, ?F_5} I \quad \frac{\mathbf{h}_4 \vdash \Delta_6, p(\mathbf{n}_3)}{\bullet \mathbf{h}_4 \vdash dual(\hat{\wedge}(\mathbf{n}_3)), \Delta_6, ?F_5} ?W}{\frac{}{- \vdash p(\mathbf{n}_3), \Delta_6, ?F_5} \text{Cut}}{\frac{}{- \vdash \Delta_6, p(\mathbf{n}_3)} \text{ax}}{\frac{}{- \vdash \Delta_6, ?F_5, p(\mathbf{n}_3)} W} \rightarrow$$

- Case rule ? C

$$\frac{\frac{\bullet \mathbf{h}_1 \vdash p(\mathbf{n}_3), \hat{\wedge}(\mathbf{n}_3)}{\bullet \mathbf{h}_4 \vdash p(\mathbf{n}_3), \hat{\wedge}(\mathbf{n}_3)} I \quad \frac{\mathbf{h}_4 \vdash \Delta_6, ?F_5, ?F_5, \hat{\wedge}(\mathbf{n}_3)}{\bullet \mathbf{h}_4 \vdash dual(p(\mathbf{n}_3)), \Delta_6, ?F_5} ?C}{\frac{}{- \vdash \hat{\wedge}(\mathbf{n}_3), \Delta_6, ?F_5} \text{Cut}}{\frac{}{- \vdash \Delta_6, ?F_5, ?F_5, \hat{\wedge}(\mathbf{n}_3)} \text{ax}}{\frac{}{- \vdash \Delta_6, ?F_5, \hat{\wedge}(\mathbf{n}_3)} ?C} \rightarrow$$

$$\frac{\frac{\bullet \mathbf{h}_1 \vdash \hat{\wedge}(\mathbf{n}_3), p(\mathbf{n}_3)}{\bullet \mathbf{h}_4 \vdash p(\mathbf{n}_3), \hat{\wedge}(\mathbf{n}_3)} I \quad \frac{\mathbf{h}_4 \vdash \Delta_6, ?F_5, ?F_5, p(\mathbf{n}_3)}{\bullet \mathbf{h}_4 \vdash dual(\hat{\wedge}(\mathbf{n}_3)), \Delta_6, ?F_5} ?C}{\frac{}{- \vdash p(\mathbf{n}_3), \Delta_6, ?F_5} \text{Cut}}{\frac{}{- \vdash \Delta_6, ?F_5, ?F_5, p(\mathbf{n}_3)} \text{ax}}{\frac{}{- \vdash \Delta_6, ?F_5, p(\mathbf{n}_3)} ?C} \rightarrow$$

- Case rule ?

$$\frac{\frac{\bullet \mathbf{h}_1 \vdash p(\mathbf{n}_3), \hat{\wedge}(\mathbf{n}_3)}{\bullet \mathbf{h}_4 \vdash p(\mathbf{n}_3), \hat{\wedge}(\mathbf{n}_3)} I \quad \frac{\mathbf{h}_4 \vdash \Delta_6, F_5, \hat{\wedge}(\mathbf{n}_3)}{\bullet \mathbf{h}_4 \vdash dual(p(\mathbf{n}_3)), \Delta_6, ?F_5} ?}{\frac{}{- \vdash \hat{\wedge}(\mathbf{n}_3), \Delta_6, ?F_5} \text{Cut}}{\frac{}{- \vdash \Delta_6, F_5, \hat{\wedge}(\mathbf{n}_3)} \text{ax}}{\frac{}{- \vdash \Delta_6, ?F_5, \hat{\wedge}(\mathbf{n}_3)} ?} \rightarrow$$

$$\frac{\frac{\bullet \mathbf{h}_1 \vdash \hat{\wedge}(\mathbf{n}_3), p(\mathbf{n}_3)}{\bullet \mathbf{h}_4 \vdash p(\mathbf{n}_3), \hat{\wedge}(\mathbf{n}_3)} I \quad \frac{\mathbf{h}_4 \vdash \Delta_6, F_5, p(\mathbf{n}_3)}{\bullet \mathbf{h}_4 \vdash dual(\hat{\wedge}(\mathbf{n}_3)), \Delta_6, ?F_5} ?}{\frac{}{- \vdash p(\mathbf{n}_3), \Delta_6, ?F_5} \text{Cut}}{\frac{}{- \vdash \Delta_6, F_5, p(\mathbf{n}_3)} \text{ax}}{\frac{}{- \vdash \Delta_6, ?F_5, p(\mathbf{n}_3)} ?} \rightarrow$$

- Case rule \$

- Case rule \perp

$$\frac{\frac{\frac{}{\bullet \mathbf{h}_1 \vdash p(\mathbf{n}_3), \hat{\mathbf{n}}_3} I \quad \frac{\mathbf{h}_4 \vdash \Delta_5, \hat{\mathbf{n}}_3}{\bullet \mathbf{h}_4 \vdash \text{dual}(p(\mathbf{n}_3)), \perp, \Delta_5} \perp}{- \vdash \hat{\mathbf{n}}_3, \perp, \Delta_5} \text{Cut}}{\rightarrow} \frac{\frac{}{- \vdash \Delta_5, \hat{\mathbf{n}}_3} \text{ax}}{- \vdash \Delta_5, \perp, \hat{\mathbf{n}}_3} \perp}{\perp}$$

$$\frac{\frac{\frac{}{\bullet \mathbf{h}_1 \vdash \hat{\mathbf{n}}_3, p(\mathbf{n}_3)} I \quad \frac{\mathbf{h}_4 \vdash \Delta_5, p(\mathbf{n}_3)}{\bullet \mathbf{h}_4 \vdash \text{dual}(\hat{\mathbf{n}}_3), \perp, \Delta_5} \perp}{- \vdash p(\mathbf{n}_3), \perp, \Delta_5} \text{Cut}}{\rightarrow} \frac{\frac{}{- \vdash \Delta_5, p(\mathbf{n}_3)} \text{ax}}{- \vdash \Delta_5, \perp, p(\mathbf{n}_3)} \perp}{\perp}$$

- Case rule \top

$$\frac{\frac{\frac{}{\bullet \mathbf{h}_1 \vdash p(\mathbf{n}_3), \hat{\mathbf{n}}_3} I \quad \frac{\mathbf{h}_4 \vdash \text{dual}(p(\mathbf{n}_3)), \top, \Delta_5}{\bullet \mathbf{h}_4 \vdash \text{dual}(p(\mathbf{n}_3)), \top, \Delta_5} \top}{- \vdash \hat{\mathbf{n}}_3, \top, \Delta_5} \text{Cut}}{\rightarrow} \frac{\frac{}{- \vdash \Delta_5, \top, \hat{\mathbf{n}}_3} \top}{- \vdash \Delta_5, \top, \hat{\mathbf{n}}_3} \top}{\top}$$

$$\frac{\frac{\frac{}{\bullet \mathbf{h}_1 \vdash \hat{\mathbf{n}}_3, p(\mathbf{n}_3)} I \quad \frac{\mathbf{h}_4 \vdash \text{dual}(\hat{\mathbf{n}}_3), \top, \Delta_5}{\bullet \mathbf{h}_4 \vdash \text{dual}(\hat{\mathbf{n}}_3), \top, \Delta_5} \top}{- \vdash p(\mathbf{n}_3), \top, \Delta_5} \text{Cut}}{\rightarrow} \frac{\frac{}{- \vdash \Delta_5, \top, p(\mathbf{n}_3)} \top}{- \vdash \Delta_5, \top, p(\mathbf{n}_3)} \top}{\top}$$

- Case rule I

$$\frac{\frac{\frac{}{\bullet \mathbf{h}_1 \vdash p(\mathbf{n}_4), \hat{\mathbf{n}}_4} I \quad \frac{\mathbf{h}_3 \vdash \text{dual}(p(\mathbf{n}_4)), p(\mathbf{n}_4)}{\bullet \mathbf{h}_3 \vdash \text{dual}(p(\mathbf{n}_4)), p(\mathbf{n}_4)} I}{- \vdash \hat{\mathbf{n}}_4, p(\mathbf{n}_4)} \text{Cut}}{\rightarrow} \frac{\frac{}{- \vdash p(\mathbf{n}_4), \hat{\mathbf{n}}_4} \text{ax}}{- \vdash p(\mathbf{n}_4), \hat{\mathbf{n}}_4} \text{ax}}{\text{ax}}$$

$$\frac{\frac{\frac{}{\bullet \mathbf{h}_1 \vdash \hat{\mathbf{n}}_4, p(\mathbf{n}_4)} I \quad \frac{\mathbf{h}_3 \vdash \text{dual}(\hat{\mathbf{n}}_4), \hat{\mathbf{n}}_4}{\bullet \mathbf{h}_3 \vdash \text{dual}(\hat{\mathbf{n}}_4), \hat{\mathbf{n}}_4} I}{- \vdash p(\mathbf{n}_4), \hat{\mathbf{n}}_4} \text{Cut}}{\rightarrow} \frac{\frac{}{- \vdash p(\mathbf{n}_4), \hat{\mathbf{n}}_4} I}{- \vdash p(\mathbf{n}_4), \hat{\mathbf{n}}_4} I}{I}$$

- Case rule \otimes

$$\frac{\frac{\frac{}{\bullet \mathbf{h}_1 \vdash p(\mathbf{n}_3), \hat{\mathbf{n}}_3} I \quad \frac{\mathbf{h}_4 \vdash \Delta_8, \mathbf{F}_6, \hat{\mathbf{n}}_3 \quad \mathbf{h}_4 \vdash \Delta_5, \mathbf{F}_7}{\bullet \mathbf{h}_4 \vdash \text{dual}(p(\mathbf{n}_3)), \Delta_5, \Delta_8, \mathbf{F}_6 \otimes \mathbf{F}_7} \otimes}{- \vdash \hat{\mathbf{n}}_3, \Delta_5, \Delta_8, \mathbf{F}_6 \otimes \mathbf{F}_7} \text{Cut}}{\rightarrow} \frac{\frac{\frac{}{- \vdash \Delta_8, \mathbf{F}_6, \hat{\mathbf{n}}_3} \text{ax} \quad \frac{}{- \vdash \Delta_5, \mathbf{F}_7} \text{ax}}{- \vdash \Delta_5, \Delta_8, \hat{\mathbf{n}}_3, \mathbf{F}_6 \otimes \mathbf{F}_7} \otimes}{- \vdash \Delta_5, \Delta_8, \hat{\mathbf{n}}_3, \mathbf{F}_6 \otimes \mathbf{F}_7} \otimes}{\otimes}$$

$$\frac{\frac{\frac{}{\bullet \mathbf{h}_1 \vdash p(\mathbf{n}_3), \hat{\mathbf{n}}_3} I \quad \frac{\mathbf{h}_4 \vdash \Delta_5, \mathbf{F}_6 \quad \mathbf{h}_4 \vdash \Delta_8, \mathbf{F}_7, \hat{\mathbf{n}}_3}{\bullet \mathbf{h}_4 \vdash \text{dual}(p(\mathbf{n}_3)), \Delta_5, \Delta_8, \mathbf{F}_6 \otimes \mathbf{F}_7} \otimes}{- \vdash \hat{\mathbf{n}}_3, \Delta_5, \Delta_8, \mathbf{F}_6 \otimes \mathbf{F}_7} \text{Cut}}{\rightarrow} \frac{\frac{\frac{}{- \vdash \Delta_5, \mathbf{F}_6} \text{ax} \quad \frac{}{- \vdash \Delta_8, \mathbf{F}_7, \hat{\mathbf{n}}_3} \text{ax}}{- \vdash \Delta_5, \Delta_8, \hat{\mathbf{n}}_3, \mathbf{F}_6 \otimes \mathbf{F}_7} \otimes}{- \vdash \Delta_5, \Delta_8, \hat{\mathbf{n}}_3, \mathbf{F}_6 \otimes \mathbf{F}_7} \otimes}{\otimes}$$

$$\begin{array}{c}
\frac{\bullet \mathbf{h}_1 \vdash \wedge(\mathbf{n}_3), p(\mathbf{n}_3)}{\vdash p(\mathbf{n}_3), \Delta_5, \Delta_8, F_6 \otimes F_7} I \quad \frac{\mathbf{h}_4 \vdash \Delta_8, F_6, p(\mathbf{n}_3) \quad \mathbf{h}_4 \vdash \Delta_5, F_7}{\bullet \mathbf{h}_4 \vdash \mathit{dual}(\wedge(\mathbf{n}_3)), \Delta_5, \Delta_8, F_6 \otimes F_7} \otimes \\
\text{Cut} \\
\rightarrow \\
\frac{\frac{\vdash \Delta_8, F_6, p(\mathbf{n}_3)}{\vdash \Delta_5, \Delta_8, p(\mathbf{n}_3), F_6 \otimes F_7} \text{ax} \quad \frac{\vdash \Delta_5, F_7}{\vdash \Delta_5, \Delta_8, p(\mathbf{n}_3), F_6 \otimes F_7} \text{ax}}{\vdash \Delta_5, \Delta_8, p(\mathbf{n}_3), F_6 \otimes F_7} \otimes \\
\frac{\bullet \mathbf{h}_1 \vdash \wedge(\mathbf{n}_3), p(\mathbf{n}_3)}{\vdash p(\mathbf{n}_3), \Delta_5, \Delta_8, F_6 \otimes F_7} I \quad \frac{\mathbf{h}_4 \vdash \Delta_5, F_6 \quad \mathbf{h}_4 \vdash \Delta_8, F_7, p(\mathbf{n}_3)}{\bullet \mathbf{h}_4 \vdash \mathit{dual}(\wedge(\mathbf{n}_3)), \Delta_5, \Delta_8, F_6 \otimes F_7} \otimes \\
\text{Cut} \\
\rightarrow \\
\frac{\frac{\vdash \Delta_5, F_6}{\vdash \Delta_5, \Delta_8, p(\mathbf{n}_3), F_6 \otimes F_7} \text{ax} \quad \frac{\vdash \Delta_8, F_7, p(\mathbf{n}_3)}{\vdash \Delta_5, \Delta_8, p(\mathbf{n}_3), F_6 \otimes F_7} \text{ax}}{\vdash \Delta_5, \Delta_8, p(\mathbf{n}_3), F_6 \otimes F_7} \otimes
\end{array}$$

5.13 Status of \otimes : OK

- Case rule 1
- Case rule !
- Case rule ?W

$$\begin{array}{c}
\frac{\mathbf{h}_2 \vdash \Delta_6, F_7, F_4 \quad \mathbf{h}_2 \vdash \Delta_3, F_5}{\bullet \mathbf{h}_2 \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \frac{\mathbf{h}_8 \vdash \Delta_{10}, \mathit{dual}(F_7)}{\bullet \mathbf{h}_8 \vdash \mathit{dual}(F_7), \Delta_{10}, ?F_9} ?W \\
\text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet \mathbf{h}_2 \vdash \Delta_3, \Delta_6, F_7, F_4 \otimes F_5}{\vdash \Delta_{10}, \Delta_3, \Delta_6, F_4 \otimes F_5} \text{ax} \quad \frac{\mathbf{h}_8 \vdash \Delta_{10}, \mathit{dual}(F_7)}{\mathbf{h}_8 \vdash \Delta_{10}, \mathit{dual}(F_7)} \text{ax}}{\vdash \Delta_{10}, \Delta_3, \Delta_6, ?F_9, F_4 \otimes F_5} \text{hCut} \\
\text{W} \\
\frac{\mathbf{h}_2 \vdash \Delta_3, F_4 \quad \mathbf{h}_2 \vdash \Delta_6, F_7, F_5}{\bullet \mathbf{h}_2 \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \frac{\mathbf{h}_8 \vdash \Delta_{10}, \mathit{dual}(F_7)}{\bullet \mathbf{h}_8 \vdash \mathit{dual}(F_7), \Delta_{10}, ?F_9} ?W \\
\text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet \mathbf{h}_2 \vdash \Delta_3, \Delta_6, F_7, F_4 \otimes F_5}{\vdash \Delta_{10}, \Delta_3, \Delta_6, F_4 \otimes F_5} \text{ax} \quad \frac{\mathbf{h}_8 \vdash \Delta_{10}, \mathit{dual}(F_7)}{\mathbf{h}_8 \vdash \Delta_{10}, \mathit{dual}(F_7)} \text{ax}}{\vdash \Delta_{10}, \Delta_3, \Delta_6, ?F_9, F_4 \otimes F_5} \text{hCut} \\
?W \\
\frac{\mathbf{h}_1 \vdash \Delta_2, F_6 \quad \mathbf{h}_1 \vdash \Delta_3, F_7}{\bullet \mathbf{h}_1 \vdash F_6 \otimes F_7, \Delta_2, \Delta_3} \otimes \quad \frac{\mathbf{h}_8 \vdash \Delta_{10}, \mathit{dual}(F_6) \mathit{dual}(F_7)}{\bullet \mathbf{h}_8 \vdash \mathit{dual}(F_6 \otimes F_7), \Delta_{10}, ?F_9} ?W \\
\text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet \mathbf{h}_1 \vdash \Delta_2, \Delta_3, F_6 \otimes F_7}{\vdash \Delta_{10}, \Delta_2, \Delta_3} \text{ax} \quad \frac{\mathbf{h}_8 \vdash \Delta_{10}, \mathit{dual}(F_6) \mathit{dual}(F_7)}{\mathbf{h}_8 \vdash \Delta_{10}, \mathit{dual}(F_6) \mathit{dual}(F_7)} \text{ax}}{\vdash \Delta_{10}, \Delta_2, \Delta_3, ?F_9} \text{hCut} \\
\text{W}
\end{array}$$

- Case rule ?C

$$\begin{array}{c}
\frac{\mathbf{h}_2 \vdash \Delta_6, F_7, F_4 \quad \mathbf{h}_2 \vdash \Delta_3, F_5}{\bullet \mathbf{h}_2 \vdash F_7, \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes \quad \frac{\mathbf{h}_8 \vdash \Delta_{10}, ?F_9, ?F_9, \mathit{dual}(F_7)}{\bullet \mathbf{h}_8 \vdash \mathit{dual}(F_7), \Delta_{10}, ?F_9} ?C \\
\text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet \mathbf{h}_2 \vdash \Delta_3, \Delta_6, F_7, F_4 \otimes F_5}{\vdash \Delta_{10}, \Delta_3, \Delta_6, ?F_9, ?F_9, F_4 \otimes F_5} \text{ax} \quad \frac{\mathbf{h}_8 \vdash \Delta_{10}, ?F_9, ?F_9, \mathit{dual}(F_7)}{\mathbf{h}_8 \vdash \Delta_{10}, ?F_9, ?F_9, \mathit{dual}(F_7)} \text{ax}}{\vdash \Delta_{10}, \Delta_3, \Delta_6, ?F_9, F_4 \otimes F_5} \text{hCut} \\
?C
\end{array}$$

$$\begin{array}{c}
\frac{h_1 \text{ :- } \Delta_2, F_6 \quad h_1 \text{ :- } \Delta_3, F_7}{\bullet h_1 \text{ :- } F_6 \otimes F_7, \Delta_2, \Delta_3} \otimes \quad \frac{h_8 \text{ :- } \Delta_{12}, F_{10}, \mathit{dual}(F_6)\$\mathit{dual}(F_7) \quad h_8 \text{ :- } \Delta_9, F_{11}}{\bullet h_8 \text{ :- } \mathit{dual}(F_6 \otimes F_7), \Delta_9, \Delta_{12}, F_{10} \otimes F_{11}} \otimes \\
\frac{}{- \text{ :- } (\Delta_2, \Delta_3), \Delta_9, \Delta_{12}, F_{10} \otimes F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \text{ :- } \Delta_2, \Delta_3, F_6 \otimes F_7}{- \text{ :- } \Delta_{12}, \Delta_2, \Delta_3, F_{10}} \text{ax} \quad \frac{h_8 \text{ :- } \Delta_{12}, F_{10}, \mathit{dual}(F_6)\$\mathit{dual}(F_7)}{- \text{ :- } \Delta_{12}, \Delta_2, \Delta_3, \Delta_9, F_{10} \otimes F_{11}} \text{ax}}{- \text{ :- } \Delta_{12}, \Delta_2, \Delta_3, F_{10}} \text{hCut} \quad \frac{}{- \text{ :- } \Delta_9, F_{11}} \text{ax}}{- \text{ :- } \Delta_{12}, \Delta_2, \Delta_3, \Delta_9, F_{10} \otimes F_{11}} \otimes \\
\frac{h_1 \text{ :- } \Delta_2, F_6 \quad h_1 \text{ :- } \Delta_3, F_7}{\bullet h_1 \text{ :- } F_6 \otimes F_7, \Delta_2, \Delta_3} \otimes \quad \frac{h_8 \text{ :- } \Delta_9, F_{10} \quad h_8 \text{ :- } \Delta_{12}, F_{11}, \mathit{dual}(F_6)\$\mathit{dual}(F_7)}{\bullet h_8 \text{ :- } \mathit{dual}(F_6 \otimes F_7), \Delta_9, \Delta_{12}, F_{10} \otimes F_{11}} \otimes \\
\frac{}{- \text{ :- } (\Delta_2, \Delta_3), \Delta_9, \Delta_{12}, F_{10} \otimes F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{- \text{ :- } \Delta_9, F_{10}} \text{ax} \quad \frac{\bullet h_1 \text{ :- } \Delta_2, \Delta_3, F_6 \otimes F_7}{- \text{ :- } \Delta_{12}, \Delta_2, \Delta_3, F_{11}} \text{ax} \quad \frac{h_8 \text{ :- } \Delta_{12}, F_{11}, \mathit{dual}(F_6)\$\mathit{dual}(F_7)}{- \text{ :- } \Delta_{12}, \Delta_2, \Delta_3, F_{11}} \text{ax}}{- \text{ :- } \Delta_{12}, \Delta_2, \Delta_3, \Delta_9, F_{10} \otimes F_{11}} \text{hCut}}{- \text{ :- } \Delta_{12}, \Delta_2, \Delta_3, \Delta_9, F_{10} \otimes F_{11}} \otimes
\end{array}$$

6 Cut-Elimination

6.1 Status of !: OK

- Case rule **1**
- Case rule **!**
- Case rule $?W$
- Case rule $?C$
- Case rule $?$
- Case rule $\$$
- Case rule $\&$
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I
- Case rule \otimes

6.2 Status of !: OK

- Case rule **1**
- Case rule **!**

$$\begin{array}{c}
 \frac{\frac{h_1 \vdash F_4, ?Y_2}{\bullet h_1 \vdash !F_4, ?Y_2} ! \quad \frac{h_6 \vdash F_7, ?Y_8, ?dual(F_4), contract(n_5, ?dual(F_4))}{\bullet h_6 \vdash contract(sn_5, ?dual(F_4)), ?Y_8, !F_7} !}{\frac{\quad}{\vdash ?Y_2, ?Y_8, !F_7} \text{Cut}} \text{Cut} \\
 \rightarrow \\
 \frac{\frac{\bullet h_1 \vdash ?Y_2, !F_4}{\vdash ?Y_2, ?Y_8, F_7, ?dual(F_4), contract(n_5, ?dual(F_4))} \text{ax} \quad \frac{\quad}{\vdash ?Y_2, ?Y_8, F_7} \text{ax}}{\frac{\quad}{\vdash ?Y_2, ?Y_8, !F_7} !} \text{hCut}
 \end{array}$$

- Case rule ?W

$$\begin{array}{c}
\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_6 \vdash \Delta_8, ?dual(F_4), contract(n_5, ?dual(F_4))}{\bullet h_6 \vdash contract(sn_5, ?dual(F_4)), \Delta_8, ?F_7} ?W}{- \vdash ?\Upsilon 2, \Delta_8, ?F_7} \text{Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_1 \vdash ?\Upsilon 2, !F_4}{ax} \quad \frac{h_6 \vdash \Delta_8, ?dual(F_4), contract(n_5, ?dual(F_4))}{- \vdash ?\Upsilon 2, \Delta_8} ?W}{- \vdash ?\Upsilon 2, \Delta_8, ?F_7} \text{hCut}}{\rightarrow} \\
\frac{\frac{h_1 \vdash F_5, ?\Upsilon 2}{\bullet h_1 \vdash !F_5, ?\Upsilon 2} ! \quad \frac{h_7 \vdash \Delta_4, contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_4} ?W}{- \vdash ?\Upsilon 2, \Delta_4} \text{Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_1 \vdash ?\Upsilon 2, !F_5}{ax} \quad \frac{h_7 \vdash \Delta_4, contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 2, \Delta_4} \text{hCut}}{\rightarrow}
\end{array}$$

- Case rule ?C

$$\begin{array}{c}
\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_6 \vdash \Delta_8, ?F_7, ?F_7, ?dual(F_4), contract(n_5, ?dual(F_4))}{\bullet h_6 \vdash contract(sn_5, ?dual(F_4)), \Delta_8, ?F_7} ?C}{- \vdash ?\Upsilon 2, \Delta_8, ?F_7} \text{Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_1 \vdash ?\Upsilon 2, !F_4}{ax} \quad \frac{h_6 \vdash \Delta_8, ?F_7, ?F_7, ?dual(F_4), contract(n_5, ?dual(F_4))}{- \vdash ?\Upsilon 2, \Delta_8, ?F_7, ?F_7} ?C}{- \vdash ?\Upsilon 2, \Delta_8, ?F_7} \text{hCut}}{\rightarrow} \\
\frac{\frac{h_1 \vdash F_5, ?\Upsilon 2}{\bullet h_1 \vdash !F_5, ?\Upsilon 2} ! \quad \frac{h_7 \vdash \Delta_4, ?dual(F_5), ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_4} ?C}{- \vdash ?\Upsilon 2, \Delta_4} \text{Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_1 \vdash ?\Upsilon 2, !F_5}{ax} \quad \frac{h_7 \vdash \Delta_4, ?dual(F_5), ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 2, \Delta_4} \text{hCut}}{\rightarrow}
\end{array}$$

- Case rule ?

$$\begin{array}{c}
\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_6 \vdash \Delta_8, F_7, ?dual(F_4), contract(n_5, ?dual(F_4))}{\bullet h_6 \vdash contract(sn_5, ?dual(F_4)), \Delta_8, ?F_7} ?}{- \vdash ?\Upsilon 2, \Delta_8, ?F_7} \text{Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_1 \vdash ?\Upsilon 2, !F_4}{ax} \quad \frac{h_6 \vdash \Delta_8, F_7, ?dual(F_4), contract(n_5, ?dual(F_4))}{- \vdash ?\Upsilon 2, \Delta_8, F_7} ?}{- \vdash ?\Upsilon 2, \Delta_8, ?F_7} \text{hCut}}{\rightarrow} \\
\frac{\frac{h_1 \vdash F_5, ?\Upsilon 2}{\bullet h_1 \vdash !F_5, ?\Upsilon 2} ! \quad \frac{h_7 \vdash \Delta_4, dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_4} ?}{- \vdash ?\Upsilon 2, \Delta_4} \text{Cut}}{\rightarrow} \\
\frac{\frac{- \vdash ?\Upsilon 2, F_5}{ax} \quad \frac{\bullet h_1 \vdash ?\Upsilon 2, !F_5}{ax} \quad \frac{h_7 \vdash \Delta_4, dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 2, \Delta_4, dual(F_5)} \text{hCut}}{- \vdash ?\Upsilon 2, \Delta_4} \text{mCut}}{\rightarrow}
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} ! \quad \frac{h_6 \vdash \Delta_9, F_7, F_8, ?dual(F_4), contract(n_5, ?dual(F_4))}{\bullet h_6 \vdash contract(sn_5, ?dual(F_4)), \Delta_9, F_7 \$F_8} \$}{- \vdash ?\Upsilon 2, \Delta_9, F_7 \$F_8} \text{Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_1 \vdash ?\Upsilon 2, !F_4}{ax} \quad \frac{h_6 \vdash \Delta_9, F_7, F_8, ?dual(F_4), contract(n_5, ?dual(F_4))}{- \vdash ?\Upsilon 2, \Delta_9, F_7, F_8} \$}{- \vdash ?\Upsilon 2, \Delta_9, F_7 \$F_8} \text{hCut}}{\rightarrow}
\end{array}$$

- Case rule $\&$

$$\begin{array}{c}
\frac{h_1 \vdash F_4, ?\Upsilon 2 \quad \bullet h_1 \vdash !F_4, ?\Upsilon 2 \quad ! \quad \frac{h_6 \vdash \Delta_9, F_7, ?dual(F_4), contract(n_5, ?dual(F_4)) \quad h_6 \vdash \Delta_9, F_8, ?dual(F_4), contract(n_5, ?dual(F_4))}{\bullet h_6 \vdash contract(sn_5, ?dual(F_4)), \Delta_9, F_7 \& F_8} \text{Cut}}{- :!?\Upsilon 2, \Delta_9, F_7 \& F_8}}{\bullet h_1 \vdash ?\Upsilon 2, !F_4} \text{ax} \quad \frac{h_6 \vdash \Delta_9, F_7, ?dual(F_4), contract(n_5, ?dual(F_4))}{- :!?\Upsilon 2, \Delta_9, F_7} \text{ax} \quad \frac{h_6 \vdash \Delta_9, F_8, ?dual(F_4), contract(n_5, ?dual(F_4))}{- :!?\Upsilon 2, \Delta_9, F_8} \text{ax}}{- :!?\Upsilon 2, \Delta_9, F_7 \& F_8} \text{hCut} \quad \& \\
\rightarrow \\
\frac{\bullet h_1 \vdash ?\Upsilon 2, !F_4}{- :!?\Upsilon 2, \Delta_9, F_7} \text{ax} \quad \frac{h_6 \vdash \Delta_9, F_7, ?dual(F_4), contract(n_5, ?dual(F_4))}{- :!?\Upsilon 2, \Delta_9, F_7 \& F_8} \text{hCut} \quad \frac{\bullet h_1 \vdash ?\Upsilon 2, !F_4}{- :!?\Upsilon 2, \Delta_9, F_8} \text{ax} \quad \frac{h_6 \vdash \Delta_9, F_8, ?dual(F_4), contract(n_5, ?dual(F_4))}{- :!?\Upsilon 2, \Delta_9, F_8} \text{ax}}{- :!?\Upsilon 2, \Delta_9, F_7 \& F_8} \text{hCut} \quad \&
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{h_1 \vdash F_4, ?\Upsilon 2 \quad \bullet h_1 \vdash !F_4, ?\Upsilon 2 \quad ! \quad \frac{h_6 \vdash \Delta_9, F_8, ?dual(F_4), contract(n_5, ?dual(F_4))}{\bullet h_6 \vdash contract(sn_5, ?dual(F_4)), \Delta_9, F_7 \oplus F_8} \oplus_B}{- :!?\Upsilon 2, \Delta_9, F_7 \oplus F_8} \text{Cut}}{\bullet h_1 \vdash ?\Upsilon 2, !F_4} \text{ax} \quad \frac{h_6 \vdash \Delta_9, F_8, ?dual(F_4), contract(n_5, ?dual(F_4))}{- :!?\Upsilon 2, \Delta_9, F_8} \text{ax}}{- :!?\Upsilon 2, \Delta_9, F_7 \oplus F_8} \text{hCut} \quad \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{h_1 \vdash F_4, ?\Upsilon 2 \quad \bullet h_1 \vdash !F_4, ?\Upsilon 2 \quad ! \quad \frac{h_6 \vdash \Delta_9, F_7, ?dual(F_4), contract(n_5, ?dual(F_4))}{\bullet h_6 \vdash contract(sn_5, ?dual(F_4)), \Delta_9, F_7 \oplus F_8} \oplus_A}{- :!?\Upsilon 2, \Delta_9, F_7 \oplus F_8} \text{Cut}}{\bullet h_1 \vdash ?\Upsilon 2, !F_4} \text{ax} \quad \frac{h_6 \vdash \Delta_9, F_7, ?dual(F_4), contract(n_5, ?dual(F_4))}{- :!?\Upsilon 2, \Delta_9, F_7} \text{ax}}{- :!?\Upsilon 2, \Delta_9, F_7 \oplus F_8} \text{hCut} \quad \oplus_A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_1 \vdash F_4, ?\Upsilon 2 \quad \bullet h_1 \vdash !F_4, ?\Upsilon 2 \quad ! \quad \frac{h_6 \vdash \Delta_7, ?dual(F_4), contract(n_5, ?dual(F_4))}{\bullet h_6 \vdash contract(sn_5, ?dual(F_4)), \perp, \Delta_7} \perp}{- :!?\Upsilon 2, \perp, \Delta_7} \text{Cut}}{\bullet h_1 \vdash ?\Upsilon 2, !F_4} \text{ax} \quad \frac{h_6 \vdash \Delta_7, ?dual(F_4), contract(n_5, ?dual(F_4))}{- :!?\Upsilon 2, \Delta_7} \text{ax}}{- :!?\Upsilon 2, \Delta_7, \perp} \text{hCut} \quad \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_1 \vdash F_4, ?\Upsilon 2 \quad \bullet h_1 \vdash !F_4, ?\Upsilon 2 \quad ! \quad \frac{h_6 \vdash \Delta_7, ?dual(F_4), contract(n_5, ?dual(F_4))}{\bullet h_6 \vdash contract(sn_5, ?dual(F_4)), \top, \Delta_7} \top}{- :!?\Upsilon 2, \top, \Delta_7} \text{Cut}}{\bullet h_1 \vdash ?\Upsilon 2, !F_4} \text{ax} \quad \frac{h_6 \vdash \Delta_7, ?dual(F_4), contract(n_5, ?dual(F_4))}{- :!?\Upsilon 2, \Delta_7, \top} \text{ax}}{- :!?\Upsilon 2, \Delta_7, \top} \text{hCut} \quad \top
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} \quad \frac{h_6 \vdash \Delta_{10}, F_8, ?dual(F_4), contract(n_5, ?dual(F_4)) \quad h_6 \vdash \Delta_7, F_9}{\bullet h_6 \vdash contract(sn_5, ?dual(F_4)), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \text{Cut}}{- \vdash ?\Upsilon 2, \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes}{\rightarrow} \\
\frac{\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} \text{ ax} \quad \frac{h_6 \vdash \Delta_{10}, F_8, ?dual(F_4), contract(n_5, ?dual(F_4))}{- \vdash ?\Upsilon 2, \Delta_{10}, F_8} \text{ ax}}{- \vdash ?\Upsilon 2, \Delta_{10}, \Delta_7, F_8 \otimes F_9} \text{ hCut}}{- \vdash \Delta_7, F_9} \text{ ax}}{\otimes} \\
\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} \quad \frac{h_6 \vdash \Delta_9, F_7, contract(n_5, ?dual(F_4)) \quad h_6 \vdash \Delta_{10}, F_8, ?dual(F_4)}{\bullet h_6 \vdash contract(sn_5, ?dual(F_4)), \Delta_9, \Delta_{10}, F_7 \otimes F_8} \text{Cut}}{- \vdash ?\Upsilon 2, \Delta_9, \Delta_{10}, F_7 \otimes F_8} \otimes}{\rightarrow} \\
\frac{\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} \text{ ax} \quad \frac{h_6 \vdash \Delta_9, F_7, contract(n_5, ?dual(F_4))}{- \vdash ?\Upsilon 2, \Delta_9, F_7} \text{ ax}}{- \vdash ?\Upsilon 2, ?\Upsilon 2, \Delta_{10}, \Delta_9, F_7 \otimes F_8} \text{ hCut}}{- \vdash ?\Upsilon 2, \Delta_{10}, F_8, ?dual(F_4)} \text{ ax}}{\otimes} \\
\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} \quad \frac{h_6 \vdash \Delta_9, F_7, ?dual(F_4) \quad h_6 \vdash \Delta_{10}, F_8, contract(n_5, ?dual(F_4))}{\bullet h_6 \vdash contract(sn_5, ?dual(F_4)), \Delta_9, \Delta_{10}, F_7 \otimes F_8} \text{Cut}}{- \vdash ?\Upsilon 2, \Delta_9, \Delta_{10}, F_7 \otimes F_8} \otimes}{\rightarrow} \\
\frac{\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} \text{ ax} \quad \frac{h_6 \vdash \Delta_9, F_7, ?dual(F_4)}{- \vdash ?\Upsilon 2, \Delta_9, F_7} \text{ ax}}{- \vdash ?\Upsilon 2, ?\Upsilon 2, \Delta_{10}, \Delta_9, F_7 \otimes F_8} \text{ hCut}}{- \vdash ?\Upsilon 2, \Delta_{10}, F_8, contract(n_5, ?dual(F_4))} \text{ ax}}{\otimes} \\
\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} \quad \frac{h_6 \vdash \Delta_9, F_7, ?dual(F_4) \quad h_6 \vdash \Delta_{10}, F_8, contract(n_5, ?dual(F_4))}{\bullet h_6 \vdash contract(sn_5, ?dual(F_4)), \Delta_9, \Delta_{10}, F_7 \otimes F_8} \text{Cut}}{- \vdash ?\Upsilon 2, \Delta_9, \Delta_{10}, F_7 \otimes F_8} \otimes}{\rightarrow} \\
\frac{\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} \text{ ax} \quad \frac{h_6 \vdash \Delta_9, F_7, ?dual(F_4)}{- \vdash ?\Upsilon 2, \Delta_9, F_7} \text{ ax}}{- \vdash ?\Upsilon 2, ?\Upsilon 2, \Delta_{10}, \Delta_9, F_7 \otimes F_8} \text{ hCut}}{- \vdash ?\Upsilon 2, \Delta_{10}, F_8, contract(n_5, ?dual(F_4))} \text{ ax}}{\otimes} \\
\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} \quad \frac{h_6 \vdash \Delta_7, F_8 \quad h_6 \vdash \Delta_{10}, F_9, ?dual(F_4), contract(n_5, ?dual(F_4))}{\bullet h_6 \vdash contract(sn_5, ?dual(F_4)), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \text{Cut}}{- \vdash ?\Upsilon 2, \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes}{\rightarrow} \\
\frac{\frac{\frac{h_1 \vdash F_4, ?\Upsilon 2}{\bullet h_1 \vdash !F_4, ?\Upsilon 2} \text{ ax} \quad \frac{h_6 \vdash \Delta_{10}, F_9, ?dual(F_4), contract(n_5, ?dual(F_4))}{- \vdash ?\Upsilon 2, \Delta_{10}, F_9} \text{ ax}}{- \vdash \Delta_7, F_8} \text{ ax}}{- \vdash ?\Upsilon 2, \Delta_{10}, \Delta_7, F_8 \otimes F_9} \text{ hCut}}{\otimes}
\end{array}$$

6.3 Status of ?W: OK

- Case rule 1
- Case rule !

$$\begin{array}{c}
\frac{\frac{h_2 \vdash ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} \text{ ?W} \quad \frac{h_7 \vdash F_8, ?\Upsilon 9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), ?\Upsilon 9, !F_8} \text{ !}}{- \vdash (?\Upsilon 4, ?F_3), ?\Upsilon 9, !F_8} \text{Cut}}{\rightarrow} \\
\frac{\frac{h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} \text{ ax} \quad \frac{h_7 \vdash ?\Upsilon 9, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, ?\Upsilon 9, F_8, ?F_3} \text{ ax}}{- \vdash ?\Upsilon 4, ?\Upsilon 9, !F_8, ?F_3} \text{ !}}{\text{hCut}}
\end{array}$$

- Case rule ?W

$$\begin{array}{c}
\frac{\frac{h_2 \vdash ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} \text{ ?W} \quad \frac{h_7 \vdash \Delta_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_9, ?F_8} \text{ ?W}}{- \vdash (?\Upsilon 4, ?F_3), \Delta_9, ?F_8} \text{Cut}}{\rightarrow} \\
\frac{\frac{h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} \text{ ax} \quad \frac{h_7 \vdash \Delta_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_9, ?F_3} \text{ ax}}{- \vdash ?\Upsilon 4, \Delta_9, ?F_3, ?F_8} \text{ ?W}}{\text{hCut}}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \text{ :- ?}\Upsilon 4, !F_6}{\bullet h_2 \text{ :- !}F_6, ?\Upsilon 4, ?F_3} \text{ ?W} \quad \frac{h_8 \text{ :- } \Delta_5, \text{contract}(n_7, ?dual(F_6))}{\bullet h_8 \text{ :- contract}(sn_7, ?dual(F_6)), \Delta_5} \text{ ?W} \\
\hline
- \text{ :- } (? \Upsilon 4, ?F_3), \Delta_5 \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_6, ?F_3 \text{ ax}}{- \text{ :- ?}\Upsilon 4, \Delta_5, ?F_3} \quad \frac{h_8 \text{ :- } \Delta_5, \text{contract}(n_7, ?dual(F_6)) \text{ ax}}{- \text{ :- ?}\Upsilon 4, \Delta_5, ?F_3} \text{ hCut}
\end{array}$$

- Case rule ?C

$$\begin{array}{c}
\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ?W} \quad \frac{h_7 \text{ :- } \Delta_9, ?F_8, ?F_8, ?dual(F_5), \text{contract}(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- contract}(sn_6, ?dual(F_5)), \Delta_9, ?F_8} \text{ ?C} \\
\hline
- \text{ :- } (? \Upsilon 4, ?F_3), \Delta_9, ?F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3 \text{ ax}}{- \text{ :- ?}\Upsilon 4, \Delta_9, ?F_3, ?F_8} \quad \frac{h_7 \text{ :- } \Delta_9, ?F_8, ?F_8, ?dual(F_5), \text{contract}(n_6, ?dual(F_5)) \text{ ax}}{- \text{ :- ?}\Upsilon 4, \Delta_9, ?F_3, ?F_8} \text{ hCut} \\
\rightarrow \\
- \text{ :- ?}\Upsilon 4, \Delta_9, ?F_3, ?F_8 \quad ?C \\
\rightarrow \\
- \text{ :- ?}\Upsilon 4, \Delta_9, ?F_3, ?F_8
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \text{ :- ?}\Upsilon 4, !F_6}{\bullet h_2 \text{ :- !}F_6, ?\Upsilon 4, ?F_3} \text{ ?W} \quad \frac{h_8 \text{ :- } \Delta_5, ?dual(F_6), ?dual(F_6), \text{contract}(n_7, ?dual(F_6))}{\bullet h_8 \text{ :- contract}(sn_7, ?dual(F_6)), \Delta_5} \text{ ?C} \\
\hline
- \text{ :- } (? \Upsilon 4, ?F_3), \Delta_5 \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_6, ?F_3 \text{ ax}}{- \text{ :- ?}\Upsilon 4, \Delta_5, ?F_3} \quad \frac{h_8 \text{ :- } \Delta_5, ?dual(F_6), ?dual(F_6), \text{contract}(n_7, ?dual(F_6)) \text{ ax}}{- \text{ :- ?}\Upsilon 4, \Delta_5, ?F_3} \text{ hCut}
\end{array}$$

- Case rule ?

$$\begin{array}{c}
\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ?W} \quad \frac{h_7 \text{ :- } \Delta_9, F_8, ?dual(F_5), \text{contract}(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- contract}(sn_6, ?dual(F_5)), \Delta_9, ?F_8} \text{ ?} \\
\hline
- \text{ :- } (? \Upsilon 4, ?F_3), \Delta_9, ?F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3 \text{ ax}}{- \text{ :- ?}\Upsilon 4, \Delta_9, F_8, ?F_3} \quad \frac{h_7 \text{ :- } \Delta_9, F_8, ?dual(F_5), \text{contract}(n_6, ?dual(F_5)) \text{ ax}}{- \text{ :- ?}\Upsilon 4, \Delta_9, ?F_3, ?F_8} \text{ hCut} \\
\rightarrow \\
- \text{ :- ?}\Upsilon 4, \Delta_9, ?F_3, ?F_8 \quad ?
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \text{ :- ?}\Upsilon 4, !F_6}{\bullet h_2 \text{ :- !}F_6, ?\Upsilon 4, ?F_3} \text{ ?W} \quad \frac{h_8 \text{ :- } \Delta_5, dual(F_6), \text{contract}(n_7, ?dual(F_6))}{\bullet h_8 \text{ :- contract}(sn_7, ?dual(F_6)), \Delta_5} \text{ ?} \\
\hline
- \text{ :- } (? \Upsilon 4, ?F_3), \Delta_5 \quad \text{Cut} \\
\rightarrow \\
\frac{h_8 \text{ :- } \Delta_5, dual(F_6), \text{contract}(n_7, ?dual(F_6)) \text{ ax}}{\bullet h_8 \text{ :- } \Delta_5, ?dual(F_6), \text{contract}(n_7, ?dual(F_6)) \text{ ?}} \text{ ax} \\
\frac{h_2 \text{ :- ?}\Upsilon 4, !F_6 \text{ ax}}{- \text{ :- ?}\Upsilon 4, \Delta_5} \text{ ?} \quad \frac{\bullet h_8 \text{ :- } \Delta_5, ?dual(F_6), \text{contract}(n_7, ?dual(F_6)) \text{ ?}}{- \text{ :- ?}\Upsilon 4, \Delta_5, ?F_3} \text{ hCut} \\
\rightarrow \\
- \text{ :- ?}\Upsilon 4, \Delta_5, ?F_3 \quad ?W
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ?W} \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, F_9, ?dual(F_5), \text{contract}(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- contract}(sn_6, ?dual(F_5)), \Delta_{10}, F_8 \$F_9} \text{ \$} \\
\hline
- \text{ :- } (? \Upsilon 4, ?F_3), \Delta_{10}, F_8 \$F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3 \text{ ax}}{- \text{ :- ?}\Upsilon 4, \Delta_{10}, F_8, F_9, ?F_3} \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, F_9, ?dual(F_5), \text{contract}(n_6, ?dual(F_5)) \text{ ax}}{- \text{ :- ?}\Upsilon 4, \Delta_{10}, F_8, F_9, ?F_3} \text{ hCut} \\
\rightarrow \\
- \text{ :- ?}\Upsilon 4, \Delta_{10}, ?F_3, F_8 \$F_9 \quad \$
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ?}W \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, ?dual(F_5), contract(n_6, ?dual(F_5)) \quad h_7 \text{ :- } \Delta_{10}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- } contract(sn_6, ?dual(F_5)), \Delta_{10}, F_8 \& F_9} \text{ Cut} \quad \& \\
\frac{}{- \text{ :- } (? \Upsilon 4, ?F_3), \Delta_{10}, F_8 \& F_9} \rightarrow \\
\frac{\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5}{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3} \text{ ax} \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, ?dual(F_5), contract(n_6, ?dual(F_5)) \quad h_7 \text{ :- } \Delta_{10}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- } \Delta_{10}, ?dual(F_5), F_8 \& F_9, contract(n_6, ?dual(F_5))} \text{ ax}}{- \text{ :- ?}\Upsilon 4, \Delta_{10}, F_8 \& F_9} \text{ hCut} \quad \& \\
\frac{}{- \text{ :- ?}\Upsilon 4, \Delta_{10}, ?F_3, F_8 \& F_9} \text{ ?}W
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ?}W \quad \frac{h_7 \text{ :- } \Delta_{10}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- } contract(sn_6, ?dual(F_5)), \Delta_{10}, F_8 \oplus F_9} \text{ } \oplus_B \\
\frac{}{- \text{ :- } (? \Upsilon 4, ?F_3), \Delta_{10}, F_8 \oplus F_9} \text{ Cut} \\
\rightarrow \\
\frac{\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3}{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3} \text{ ax} \quad \frac{h_7 \text{ :- } \Delta_{10}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \text{ :- ?}\Upsilon 4, \Delta_{10}, F_9, ?F_3} \text{ ax}}{- \text{ :- ?}\Upsilon 4, \Delta_{10}, ?F_3, F_8 \oplus F_9} \text{ hCut} \quad \oplus_B \\
\frac{}{- \text{ :- ?}\Upsilon 4, \Delta_{10}, ?F_3, F_8 \oplus F_9} \text{ } \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ?}W \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- } contract(sn_6, ?dual(F_5)), \Delta_{10}, F_8 \oplus F_9} \text{ } \oplus_A \\
\frac{}{- \text{ :- } (? \Upsilon 4, ?F_3), \Delta_{10}, F_8 \oplus F_9} \text{ Cut} \\
\rightarrow \\
\frac{\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3}{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3} \text{ ax} \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \text{ :- ?}\Upsilon 4, \Delta_{10}, F_8, ?F_3} \text{ ax}}{- \text{ :- ?}\Upsilon 4, \Delta_{10}, ?F_3, F_8 \oplus F_9} \text{ hCut} \quad \oplus_A \\
\frac{}{- \text{ :- ?}\Upsilon 4, \Delta_{10}, ?F_3, F_8 \oplus F_9} \text{ } \oplus_A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ?}W \quad \frac{h_7 \text{ :- } \Delta_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- } contract(sn_6, ?dual(F_5)), \perp, \Delta_8} \text{ } \perp \\
\frac{}{- \text{ :- } (? \Upsilon 4, ?F_3), \perp, \Delta_8} \text{ Cut} \\
\rightarrow \\
\frac{\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3}{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3} \text{ ax} \quad \frac{h_7 \text{ :- } \Delta_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \text{ :- ?}\Upsilon 4, \Delta_8, ?F_3} \text{ ax}}{- \text{ :- ?}\Upsilon 4, \Delta_8, \perp, ?F_3} \text{ hCut} \quad \perp \\
\frac{}{- \text{ :- ?}\Upsilon 4, \Delta_8, \perp, ?F_3} \text{ } \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ?}W \quad \frac{}{\bullet h_7 \text{ :- } contract(sn_6, ?dual(F_5)), \top, \Delta_8} \text{ } \top \\
\frac{}{- \text{ :- } (? \Upsilon 4, ?F_3), \top, \Delta_8} \text{ Cut} \\
\rightarrow \\
\frac{}{- \text{ :- ?}\Upsilon 4, \Delta_8, \top, ?F_3} \text{ } \top
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ?}W \quad \frac{h_7 \text{ :- } \Delta_{11}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5)) \quad h_7 \text{ :- } \Delta_8, F_{10}}{\bullet h_7 \text{ :- } contract(sn_6, ?dual(F_5)), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{ } \otimes \\
\frac{}{- \text{ :- } (? \Upsilon 4, ?F_3), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{ Cut} \\
\rightarrow \\
\frac{\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3}{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3} \text{ ax} \quad \frac{h_7 \text{ :- } \Delta_{11}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \text{ :- ?}\Upsilon 4, \Delta_{11}, F_9, ?F_3} \text{ hCut} \quad \frac{}{- \text{ :- } \Delta_8, F_{10}} \text{ ax}}{- \text{ :- ?}\Upsilon 4, \Delta_{11}, \Delta_8, ?F_3, F_9 \otimes F_{10}} \text{ } \otimes \\
\frac{}{- \text{ :- ?}\Upsilon 4, \Delta_{11}, \Delta_8, ?F_3, F_9 \otimes F_{10}} \text{ } \otimes
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ?}W \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, \text{contract}(n_6, ?dual(F_5)) \quad h_7 \text{ :- } \Delta_{11}, F_9, ?dual(F_5)}{\bullet h_7 \text{ :- contract}(sn_6, ?dual(F_5)), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9} \otimes}{- \text{ :- } (? \Upsilon 4, ?F_3), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9} \text{ Cut}}{\rightarrow} \\
\frac{\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ax} \quad \frac{\frac{h_7 \text{ :- } \Delta_{10}, F_8, \text{contract}(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- } \Delta_{10}, \Delta_{11}, ?dual(F_5), F_8 \otimes F_9, \text{contract}(n_6, ?dual(F_5))} \text{ ax} \quad \frac{h_7 \text{ :- } \Delta_{11}, F_9, ?dual(F_5)}{\bullet h_7 \text{ :- } \Delta_{11}, F_9, ?dual(F_5)} \text{ ax}}{- \text{ :- ?}\Upsilon 4, \Delta_{10}, \Delta_{11}, F_8 \otimes F_9} \text{ hCut}}{- \text{ :- ?}\Upsilon 4, \Delta_{10}, \Delta_{11}, ?F_3, F_8 \otimes F_9} \text{ ?}W}}{\rightarrow} \\
\frac{\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ?}W \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, ?dual(F_5) \quad h_7 \text{ :- } \Delta_{11}, F_9, \text{contract}(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- contract}(sn_6, ?dual(F_5)), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9} \otimes}{- \text{ :- } (? \Upsilon 4, ?F_3), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9} \text{ Cut}}{\rightarrow} \\
\frac{\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ax} \quad \frac{\frac{h_7 \text{ :- } \Delta_{10}, F_8, ?dual(F_5)}{\bullet h_7 \text{ :- } \Delta_{10}, \Delta_{11}, ?dual(F_5), F_8 \otimes F_9, \text{contract}(n_6, ?dual(F_5))} \text{ ax} \quad \frac{h_7 \text{ :- } \Delta_{11}, F_9, \text{contract}(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- } \Delta_{11}, F_9, \text{contract}(n_6, ?dual(F_5))} \text{ ax}}{- \text{ :- ?}\Upsilon 4, \Delta_{10}, \Delta_{11}, F_8 \otimes F_9} \text{ hCut}}{- \text{ :- ?}\Upsilon 4, \Delta_{10}, \Delta_{11}, ?F_3, F_8 \otimes F_9} \text{ ?}W}}{\rightarrow} \\
\frac{\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ?}W \quad \frac{h_7 \text{ :- } \Delta_8, F_9 \quad h_7 \text{ :- } \Delta_{11}, F_{10}, ?dual(F_5), \text{contract}(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- contract}(sn_6, ?dual(F_5)), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- \text{ :- } (? \Upsilon 4, ?F_3), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{ Cut}}{\rightarrow} \\
\frac{\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ax} \quad \frac{\frac{h_7 \text{ :- } \Delta_{11}, F_{10}, ?dual(F_5), \text{contract}(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- } \Delta_{11}, F_{10}, ?dual(F_5), \text{contract}(n_6, ?dual(F_5))} \text{ ax} \quad \frac{h_7 \text{ :- } \Delta_{11}, F_{10}, ?dual(F_5)}{\bullet h_7 \text{ :- } \Delta_{11}, F_{10}, ?dual(F_5)} \text{ ax}}{- \text{ :- ?}\Upsilon 4, \Delta_{11}, F_{10}, ?F_3} \text{ hCut}}{- \text{ :- } \Delta_8, F_9} \text{ ax}}{- \text{ :- ?}\Upsilon 4, \Delta_{11}, \Delta_8, ?F_3, F_9 \otimes F_{10}} \otimes}
\end{array}$$

6.4 Status of ?C: OK

- Case rule 1

- Case rule !

$$\begin{array}{c}
\frac{\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ?}C \quad \frac{h_7 \text{ :- } F_8, ?\Upsilon 9, ?dual(F_5), \text{contract}(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- contract}(sn_6, ?dual(F_5)), ?\Upsilon 9, !F_8} \text{ !}}{- \text{ :- } (? \Upsilon 4, ?F_3), ?\Upsilon 9, !F_8} \text{ Cut}}{\rightarrow} \\
\frac{\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ax} \quad \frac{h_7 \text{ :- ?}\Upsilon 9, F_8, ?dual(F_5), \text{contract}(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- ?}\Upsilon 9, F_8, ?F_3} \text{ ax}}{- \text{ :- ?}\Upsilon 4, ?\Upsilon 9, F_8, ?F_3} \text{ hCut}}{- \text{ :- ?}\Upsilon 4, ?\Upsilon 9, !F_8, ?F_3} \text{ !}}
\end{array}$$

- Case rule ?W

$$\begin{array}{c}
\frac{\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ?}C \quad \frac{h_7 \text{ :- } \Delta_9, ?dual(F_5), \text{contract}(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- contract}(sn_6, ?dual(F_5)), \Delta_9, ?F_8} \text{ ?}W}{- \text{ :- } (? \Upsilon 4, ?F_3), \Delta_9, ?F_8} \text{ Cut}}{\rightarrow} \\
\frac{\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ax} \quad \frac{h_7 \text{ :- } \Delta_9, ?dual(F_5), \text{contract}(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- } \Delta_9, ?F_3} \text{ ax}}{- \text{ :- ?}\Upsilon 4, \Delta_9, ?F_3} \text{ hCut}}{- \text{ :- ?}\Upsilon 4, \Delta_9, ?F_3, ?F_8} \text{ ?}W}}{\rightarrow} \\
\frac{\frac{h_2 \text{ :- ?}\Upsilon 4, !F_6, ?F_3, ?F_3}{\bullet h_2 \text{ :- !}F_6, ?\Upsilon 4, ?F_3} \text{ ?}C \quad \frac{h_8 \text{ :- } \Delta_5, \text{contract}(n_7, ?dual(F_6))}{\bullet h_8 \text{ :- contract}(sn_7, ?dual(F_6)), \Delta_5} \text{ ?}W}{- \text{ :- } (? \Upsilon 4, ?F_3), \Delta_5} \text{ Cut}}{\rightarrow} \\
\frac{\frac{h_2 \text{ :- ?}\Upsilon 4, !F_6, ?F_3}{\bullet h_2 \text{ :- !}F_6, ?\Upsilon 4, ?F_3} \text{ ax} \quad \frac{h_8 \text{ :- } \Delta_5, \text{contract}(n_7, ?dual(F_6))}{\bullet h_8 \text{ :- } \Delta_5, ?F_3} \text{ ax}}{- \text{ :- ?}\Upsilon 4, \Delta_5, ?F_3} \text{ hCut}}{\rightarrow}
\end{array}$$

- Case rule ?C

$$\begin{array}{c}
\frac{\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} ?C \quad \frac{h_7 \text{ :- } \Delta_9, ?F_8, ?F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- } contract(sn_6, ?dual(F_5)), \Delta_9, ?F_8} ?C}{- \text{ :- } (? \Upsilon 4, ?F_3), \Delta_9, ?F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3}{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_9, ?F_8, ?F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \text{ :- ?}\Upsilon 4, \Delta_9, ?F_3, ?F_8} \text{ax}}{- \text{ :- ?}\Upsilon 4, \Delta_9, ?F_3, ?F_8} \text{hCut} \\
\frac{- \text{ :- ?}\Upsilon 4, \Delta_9, ?F_3, ?F_8}{} ?C \\
\frac{h_2 \text{ :- ?}\Upsilon 4, !F_6, ?F_3, ?F_3}{\bullet h_2 \text{ :- !}F_6, ?\Upsilon 4, ?F_3} ?C \quad \frac{h_8 \text{ :- } \Delta_5, ?dual(F_6), ?dual(F_6), contract(n_7, ?dual(F_6))}{\bullet h_8 \text{ :- } contract(sn_7, ?dual(F_6)), \Delta_5} ?C}{- \text{ :- } (? \Upsilon 4, ?F_3), \Delta_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_6, ?F_3}{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_6, ?F_3} \text{ax} \quad \frac{h_8 \text{ :- } \Delta_5, ?dual(F_6), ?dual(F_6), contract(n_7, ?dual(F_6))}{- \text{ :- ?}\Upsilon 4, \Delta_5, ?F_3} \text{ax}}{- \text{ :- ?}\Upsilon 4, \Delta_5, ?F_3} \text{hCut}
\end{array}$$

- Case rule ?

$$\begin{array}{c}
\frac{\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} ?C \quad \frac{h_7 \text{ :- } \Delta_9, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- } contract(sn_6, ?dual(F_5)), \Delta_9, ?F_8} ?}{- \text{ :- } (? \Upsilon 4, ?F_3), \Delta_9, ?F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3}{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_9, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \text{ :- ?}\Upsilon 4, \Delta_9, F_8, ?F_3} \text{ax}}{- \text{ :- ?}\Upsilon 4, \Delta_9, ?F_3, ?F_8} \text{hCut} \\
\frac{- \text{ :- ?}\Upsilon 4, \Delta_9, ?F_3, ?F_8}{} ? \\
\frac{h_2 \text{ :- ?}\Upsilon 4, !F_6, ?F_3, ?F_3}{\bullet h_2 \text{ :- !}F_6, ?\Upsilon 4, ?F_3} ?C \quad \frac{h_8 \text{ :- } \Delta_5, dual(F_6), contract(n_7, ?dual(F_6))}{\bullet h_8 \text{ :- } contract(sn_7, ?dual(F_6)), \Delta_5} ?}{- \text{ :- } (? \Upsilon 4, ?F_3), \Delta_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \text{ :- ?}\Upsilon 4, !F_6, ?F_3, ?F_3}{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_6, ?F_3, ?F_3} \text{ax} \quad \frac{h_8 \text{ :- } \Delta_5, dual(F_6), contract(n_7, ?dual(F_6))}{\bullet h_8 \text{ :- } \Delta_5, ?dual(F_6), contract(n_7, ?dual(F_6))} \text{ax}}{- \text{ :- ?}\Upsilon 4, \Delta_5, ?F_3, ?F_3} \text{hCut} \\
\frac{- \text{ :- ?}\Upsilon 4, \Delta_5, ?F_3, ?F_3}{- \text{ :- ?}\Upsilon 4, \Delta_5, ?F_3} ?C
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} ?C \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- } contract(sn_6, ?dual(F_5)), \Delta_{10}, F_8 \$ F_9} \$}{- \text{ :- } (? \Upsilon 4, ?F_3), \Delta_{10}, F_8 \$ F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3}{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \text{ :- ?}\Upsilon 4, \Delta_{10}, F_8, F_9, ?F_3} \text{ax}}{- \text{ :- ?}\Upsilon 4, \Delta_{10}, ?F_3, F_8 \$ F_9} \text{hCut} \\
\frac{- \text{ :- ?}\Upsilon 4, \Delta_{10}, ?F_3, F_8 \$ F_9}{- \text{ :- ?}\Upsilon 4, \Delta_{10}, ?F_3, F_8 \$ F_9} \$
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} ?C \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- } contract(sn_6, ?dual(F_5)), \Delta_{10}, F_8 \& F_9} \& \quad \frac{h_7 \text{ :- } \Delta_{10}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- } contract(sn_6, ?dual(F_5)), \Delta_{10}, F_8 \& F_9} \&}{- \text{ :- } (? \Upsilon 4, ?F_3), \Delta_{10}, F_8 \& F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3, ?F_3} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- } \Delta_{10}, ?dual(F_5), F_8 \& F_9, contract(n_6, ?dual(F_5))} \text{ax} \quad \frac{h_7 \text{ :- } \Delta_{10}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- } \Delta_{10}, ?dual(F_5), F_8 \& F_9, contract(n_6, ?dual(F_5))} \text{ax}}{- \text{ :- ?}\Upsilon 4, \Delta_{10}, ?F_3, ?F_3, F_8 \& F_9} \text{hCut} \\
\frac{- \text{ :- ?}\Upsilon 4, \Delta_{10}, ?F_3, F_8 \& F_9}{- \text{ :- ?}\Upsilon 4, \Delta_{10}, ?F_3, F_8 \& F_9} ?C
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ?C} \quad \frac{h_7 \text{ :- } \Delta_{10}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- } contract(sn_6, ?dual(F_5)), \Delta_{10}, F_8 \oplus F_9} \oplus_B}{- \text{ :- } (? \Upsilon 4, ?F_3), \Delta_{10}, F_8 \oplus F_9} \text{ Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3}{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3} \text{ ax} \quad \frac{h_7 \text{ :- } \Delta_{10}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \text{ :- ?}\Upsilon 4, \Delta_{10}, F_9, ?F_3} \text{ ax}}{- \text{ :- ?}\Upsilon 4, \Delta_{10}, ?F_3, F_8 \oplus F_9} \oplus_B \text{ hCut}
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ?C} \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- } contract(sn_6, ?dual(F_5)), \Delta_{10}, F_8 \oplus F_9} \oplus_A}{- \text{ :- } (? \Upsilon 4, ?F_3), \Delta_{10}, F_8 \oplus F_9} \text{ Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3}{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3} \text{ ax} \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \text{ :- ?}\Upsilon 4, \Delta_{10}, F_8, ?F_3} \text{ ax}}{- \text{ :- ?}\Upsilon 4, \Delta_{10}, ?F_3, F_8 \oplus F_9} \oplus_A \text{ hCut}
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ?C} \quad \frac{h_7 \text{ :- } \Delta_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- } contract(sn_6, ?dual(F_5)), \perp, \Delta_8} \perp}{- \text{ :- } (? \Upsilon 4, ?F_3), \perp, \Delta_8} \text{ Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3}{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3} \text{ ax} \quad \frac{h_7 \text{ :- } \Delta_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \text{ :- ?}\Upsilon 4, \Delta_8, ?F_3} \text{ ax}}{- \text{ :- ?}\Upsilon 4, \Delta_8, \perp, ?F_3} \perp \text{ hCut}
\end{array}$$

- Case rule \top

$$\frac{\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ?C} \quad \frac{\bullet h_7 \text{ :- } contract(sn_6, ?dual(F_5)), \top, \Delta_8}{\bullet h_7 \text{ :- } contract(sn_6, ?dual(F_5)), \top, \Delta_8} \top}{- \text{ :- } (? \Upsilon 4, ?F_3), \top, \Delta_8} \text{ Cut} \\
\rightarrow \\
\frac{}{- \text{ :- ?}\Upsilon 4, \Delta_8, \top, ?F_3} \top$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ?C} \quad \frac{h_7 \text{ :- } \Delta_{11}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5)) \quad h_7 \text{ :- } \Delta_8, F_{10}}{\bullet h_7 \text{ :- } contract(sn_6, ?dual(F_5)), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- \text{ :- } (? \Upsilon 4, ?F_3), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{ Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3}{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3} \text{ ax} \quad \frac{h_7 \text{ :- } \Delta_{11}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \text{ :- ?}\Upsilon 4, \Delta_{11}, F_9, ?F_3} \text{ ax}}{- \text{ :- ?}\Upsilon 4, \Delta_{11}, \Delta_8, ?F_3, F_9 \otimes F_{10}} \otimes \text{ hCut} \\
\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ?C} \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, contract(n_6, ?dual(F_5)) \quad h_7 \text{ :- } \Delta_{11}, F_9, ?dual(F_5)}{\bullet h_7 \text{ :- } contract(sn_6, ?dual(F_5)), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9} \otimes}{- \text{ :- } (? \Upsilon 4, ?F_3), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9} \text{ Cut} \\
\rightarrow \\
\frac{\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3, ?F_3} \text{ ax} \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, contract(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- } \Delta_{10}, \Delta_{11}, ?dual(F_5), F_8 \otimes F_9, contract(n_6, ?dual(F_5))} \otimes}{- \text{ :- ?}\Upsilon 4, \Delta_{10}, \Delta_{11}, ?F_3, ?F_3, F_8 \otimes F_9} \otimes \text{ hCut} \\
\frac{}{- \text{ :- ?}\Upsilon 4, \Delta_{10}, \Delta_{11}, ?F_3, F_8 \otimes F_9} \text{ ?C}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ?C} \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, ?dual(F_5) \quad h_7 \text{ :- } \Delta_{11}, F_9, contract(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- } contract(sn_6, ?dual(F_5)), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9} \text{ ?C} \quad \otimes \\
\hline
- \text{ :- } (? \Upsilon 4, ?F_3), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9 \\
\rightarrow \\
\frac{\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ax} \quad \frac{h_7 \text{ :- } \Delta_{10}, F_8, ?dual(F_5)}{\bullet h_7 \text{ :- } \Delta_{10}, \Delta_{11}, ?dual(F_5), F_8 \otimes F_9, contract(n_6, ?dual(F_5))} \text{ ax}}{- \text{ :- ?}\Upsilon 4, \Delta_{10}, \Delta_{11}, ?F_3, ?F_3, F_8 \otimes F_9} \text{ hCut} \quad \otimes \\
\hline
- \text{ :- ?}\Upsilon 4, \Delta_{10}, \Delta_{11}, ?F_3, F_8 \otimes F_9 \quad \text{?C} \\
\rightarrow \\
\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3, ?F_3}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ?C} \quad \frac{h_7 \text{ :- } \Delta_8, F_9 \quad h_7 \text{ :- } \Delta_{11}, F_{10}, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- } contract(sn_6, ?dual(F_5)), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{ ?C} \quad \otimes \\
\hline
- \text{ :- } (? \Upsilon 4, ?F_3), \Delta_8, \Delta_{11}, F_9 \otimes F_{10} \\
\rightarrow \\
\frac{\frac{h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ax} \quad \frac{h_7 \text{ :- } \Delta_{11}, F_{10}, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \text{ :- ?}\Upsilon 4, \Delta_{11}, F_{10}, ?F_3} \text{ ax}}{- \text{ :- } \Delta_8, F_9} \text{ ax} \quad \otimes \\
\hline
- \text{ :- } \Delta_8, F_9 \quad \text{hCut} \\
\rightarrow \\
- \text{ :- ?}\Upsilon 4, \Delta_{11}, \Delta_8, ?F_3, F_9 \otimes F_{10} \quad \otimes
\end{array}$$

6.5 Status of ? : OK

- Case rule 1

- Case rule !

$$\begin{array}{c}
\frac{h_2 \text{ :- } F_3, ?\Upsilon 4, !F_5}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ?} \quad \frac{h_7 \text{ :- } F_8, ?\Upsilon 9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- } contract(sn_6, ?dual(F_5)), ?\Upsilon 9, !F_8} \text{ !} \\
\hline
- \text{ :- } (? \Upsilon 4, ?F_3), ?\Upsilon 9, !F_8 \\
\rightarrow \\
\frac{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3}{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3} \text{ ax} \quad \frac{h_7 \text{ :- ?}\Upsilon 9, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \text{ :- ?}\Upsilon 4, ?\Upsilon 9, F_8, ?F_3} \text{ ax} \\
\hline
- \text{ :- ?}\Upsilon 4, ?\Upsilon 9, F_8, ?F_3 \quad \text{hCut} \\
\rightarrow \\
- \text{ :- ?}\Upsilon 4, ?\Upsilon 9, !F_8, ?F_3 \quad \text{!}
\end{array}$$

- Case rule ?W

$$\begin{array}{c}
\frac{h_2 \text{ :- } F_3, ?\Upsilon 4, !F_5}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ?} \quad \frac{h_7 \text{ :- } \Delta_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- } contract(sn_6, ?dual(F_5)), \Delta_9, ?F_8} \text{ ?W} \\
\hline
- \text{ :- } (? \Upsilon 4, ?F_3), \Delta_9, ?F_8 \\
\rightarrow \\
\frac{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3}{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3} \text{ ax} \quad \frac{h_7 \text{ :- } \Delta_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \text{ :- ?}\Upsilon 4, \Delta_9, ?F_3} \text{ ax} \\
\hline
- \text{ :- ?}\Upsilon 4, \Delta_9, ?F_3 \quad \text{hCut} \\
\rightarrow \\
- \text{ :- ?}\Upsilon 4, \Delta_9, ?F_3, ?F_8 \quad \text{?W} \\
\rightarrow \\
\frac{h_2 \text{ :- } F_3, ?\Upsilon 4, !F_6}{\bullet h_2 \text{ :- !}F_6, ?\Upsilon 4, ?F_3} \text{ ?} \quad \frac{h_8 \text{ :- } \Delta_5, contract(n_7, ?dual(F_6))}{\bullet h_8 \text{ :- } contract(sn_7, ?dual(F_6)), \Delta_5} \text{ ?W} \\
\hline
- \text{ :- } (? \Upsilon 4, ?F_3), \Delta_5 \\
\rightarrow \\
\frac{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_6, ?F_3}{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_6, ?F_3} \text{ ax} \quad \frac{h_8 \text{ :- } \Delta_5, contract(n_7, ?dual(F_6))}{- \text{ :- ?}\Upsilon 4, \Delta_5, ?F_3} \text{ ax} \\
\hline
- \text{ :- ?}\Upsilon 4, \Delta_5, ?F_3 \quad \text{hCut}
\end{array}$$

- Case rule ?C

$$\begin{array}{c}
\frac{h_2 \text{ :- } F_3, ?\Upsilon 4, !F_5}{\bullet h_2 \text{ :- !}F_5, ?\Upsilon 4, ?F_3} \text{ ?} \quad \frac{h_7 \text{ :- } \Delta_9, ?F_8, ?F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \text{ :- } contract(sn_6, ?dual(F_5)), \Delta_9, ?F_8} \text{ ?C} \\
\hline
- \text{ :- } (? \Upsilon 4, ?F_3), \Delta_9, ?F_8 \\
\rightarrow \\
\frac{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3}{\bullet h_2 \text{ :- ?}\Upsilon 4, !F_5, ?F_3} \text{ ax} \quad \frac{h_7 \text{ :- } \Delta_9, ?F_8, ?F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \text{ :- ?}\Upsilon 4, \Delta_9, ?F_3, ?F_8, ?F_8} \text{ ax} \\
\hline
- \text{ :- ?}\Upsilon 4, \Delta_9, ?F_3, ?F_8, ?F_8 \quad \text{hCut} \\
\rightarrow \\
- \text{ :- ?}\Upsilon 4, \Delta_9, ?F_3, ?F_8 \quad \text{?C}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash F_3, ?\Upsilon 4, !F_6}{\bullet h_2 \vdash !F_6, ?\Upsilon 4, ?F_3} ? \quad \frac{h_8 \vdash \Delta_5, ?dual(F_6), ?dual(F_6), contract(n_7, ?dual(F_6))}{\bullet h_8 \vdash contract(sn_7, ?dual(F_6)), \Delta_5} ?C}{- \vdash (?\Upsilon 4, ?F_3), \Delta_5} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_6, ?F_3}{- \vdash ?\Upsilon 4, \Delta_5, ?F_3} \text{ax} \quad \frac{h_8 \vdash \Delta_5, ?dual(F_6), ?dual(F_6), contract(n_7, ?dual(F_6))}{- \vdash ?\Upsilon 4, \Delta_5, ?F_3} \text{ax}}{- \vdash ?\Upsilon 4, \Delta_5, ?F_3} \text{hCut}
\end{array}$$

- Case rule ?

$$\begin{array}{c}
\frac{h_2 \vdash F_3, ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ? \quad \frac{h_7 \vdash \Delta_9, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_9, ?F_8} ?}{- \vdash (?\Upsilon 4, ?F_3), \Delta_9, ?F_8} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{- \vdash ?\Upsilon 4, \Delta_9, ?F_3, ?F_8} \text{ax} \quad \frac{h_7 \vdash \Delta_9, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_9, F_8, ?F_3} \text{ax}}{- \vdash ?\Upsilon 4, \Delta_9, ?F_3, ?F_8} \text{hCut} \\
\frac{h_2 \vdash F_3, ?\Upsilon 4, !F_6}{\bullet h_2 \vdash !F_6, ?\Upsilon 4, ?F_3} ? \quad \frac{h_8 \vdash \Delta_5, dual(F_6), contract(n_7, ?dual(F_6))}{\bullet h_8 \vdash contract(sn_7, ?dual(F_6)), \Delta_5} ?}{- \vdash (?\Upsilon 4, ?F_3), \Delta_5} \text{Cut} \\
\rightarrow \\
\frac{- \vdash ?\Upsilon 4, !F_6, ?F_3}{- \vdash ?\Upsilon 4, F_6, ?F_3} \text{ax} \quad \frac{\bullet h_2 \vdash ?\Upsilon 4, !F_6, ?F_3}{- \vdash ?\Upsilon 4, \Delta_5, ?F_3} \text{ax} \quad \frac{h_8 \vdash \Delta_5, dual(F_6), contract(n_7, ?dual(F_6))}{- \vdash ?\Upsilon 4, \Delta_5, ?F_3, dual(F_6)} \text{ax}}{- \vdash ?\Upsilon 4, \Delta_5, ?F_3} \text{bInv} \quad \frac{- \vdash ?\Upsilon 4, \Delta_5, ?F_3, dual(F_6)}{- \vdash ?\Upsilon 4, \Delta_5, ?F_3} \text{mCut} \quad \text{hCut}
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{h_2 \vdash F_3, ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ? \quad \frac{h_7 \vdash \Delta_{10}, F_8, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_{10}, F_8 \$F_9} \$}{- \vdash (?\Upsilon 4, ?F_3), \Delta_{10}, F_8 \$F_9} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{- \vdash ?\Upsilon 4, \Delta_{10}, F_8, F_9, ?F_3} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_8, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_{10}, F_8, F_9, ?F_3} \text{ax}}{- \vdash ?\Upsilon 4, \Delta_{10}, ?F_3, F_8 \$F_9} \text{hCut} \\
\frac{- \vdash ?\Upsilon 4, \Delta_{10}, ?F_3, F_8 \$F_9}{- \vdash ?\Upsilon 4, \Delta_{10}, ?F_3, F_8 \$F_9} \$
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{h_2 \vdash F_3, ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ? \quad \frac{h_7 \vdash \Delta_{10}, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_{10}, F_8 \&F_9} \& \quad \frac{h_7 \vdash \Delta_{10}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_{10}, F_8 \&F_9} \&}{- \vdash (?\Upsilon 4, ?F_3), \Delta_{10}, F_8 \&F_9} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{- \vdash ?\Upsilon 4, \Delta_{10}, F_8, ?F_3} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_{10}, F_8, ?F_3} \text{ax}}{- \vdash ?\Upsilon 4, \Delta_{10}, ?F_3, F_8 \&F_9} \text{hCut} \quad \frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{- \vdash ?\Upsilon 4, \Delta_{10}, F_9, ?F_3} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_{10}, F_9, ?F_3} \&}{- \vdash ?\Upsilon 4, \Delta_{10}, ?F_3, F_8 \&F_9} \&
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{h_2 \vdash F_3, ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ? \quad \frac{h_7 \vdash \Delta_{10}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_{10}, F_8 \oplus F_9} \oplus_B}{- \vdash (?\Upsilon 4, ?F_3), \Delta_{10}, F_8 \oplus F_9} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{- \vdash ?\Upsilon 4, \Delta_{10}, F_9, ?F_3} \text{ax} \quad \frac{h_7 \vdash \Delta_{10}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_{10}, F_9, ?F_3} \text{ax}}{- \vdash ?\Upsilon 4, \Delta_{10}, ?F_3, F_8 \oplus F_9} \text{hCut} \\
\frac{- \vdash ?\Upsilon 4, \Delta_{10}, ?F_3, F_8 \oplus F_9}{- \vdash ?\Upsilon 4, \Delta_{10}, ?F_3, F_8 \oplus F_9} \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{h_2 \vdash F_3, ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ? \quad \frac{h_7 \vdash \Delta_{10}, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_{10}, F_8 \oplus F_9} \oplus^A}{- \vdash (? \Upsilon 4, ?F_3), \Delta_{10}, F_8 \oplus F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{- \vdash ?\Upsilon 4, \Delta_{10}, ?F_3, F_8 \oplus F_9} \oplus^A \quad \frac{h_7 \vdash \Delta_{10}, F_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_{10}, F_8, ?F_3} \oplus^A}{- \vdash ?\Upsilon 4, \Delta_{10}, ?F_3, F_8 \oplus F_9} \oplus^A} \text{hCut}
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_2 \vdash F_3, ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ? \quad \frac{h_7 \vdash \Delta_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \perp, \Delta_8} \perp}{- \vdash (? \Upsilon 4, ?F_3), \perp, \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{- \vdash ?\Upsilon 4, \Delta_8, ?F_3} \perp \quad \frac{h_7 \vdash \Delta_8, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_8, \perp, ?F_3} \perp}{- \vdash ?\Upsilon 4, \Delta_8, \perp, ?F_3} \perp} \text{hCut}
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_2 \vdash F_3, ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ? \quad \frac{h_7 \vdash contract(sn_6, ?dual(F_5)), \top, \Delta_8}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \top, \Delta_8} \top}{- \vdash (? \Upsilon 4, ?F_3), \top, \Delta_8} \text{Cut} \\
\rightarrow \\
\frac{- \vdash ?\Upsilon 4, \Delta_8, \top, ?F_3}{- \vdash ?\Upsilon 4, \Delta_8, \top, ?F_3} \top
\end{array}$$

- Case rule I

- Case rule \otimes

$$\begin{array}{c}
\frac{h_2 \vdash F_3, ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ? \quad \frac{h_7 \vdash \Delta_{11}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \quad \frac{h_7 \vdash \Delta_8, F_{10}}{h_7 \vdash \Delta_8, F_{10}} \otimes}{- \vdash (? \Upsilon 4, ?F_3), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{- \vdash ?\Upsilon 4, \Delta_{11}, F_9, ?F_3} \otimes \quad \frac{h_7 \vdash \Delta_{11}, F_9, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_{11}, F_9, ?F_3} \otimes}{- \vdash ?\Upsilon 4, \Delta_{11}, \Delta_8, ?F_3, F_9 \otimes F_{10}} \otimes} \text{hCut} \quad \frac{- \vdash \Delta_8, F_{10}}{- \vdash \Delta_8, F_{10}} \otimes \\
\rightarrow \\
\frac{h_2 \vdash F_3, ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ? \quad \frac{h_7 \vdash \Delta_{10}, F_8, contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9} \otimes \quad \frac{h_7 \vdash \Delta_{11}, F_9, ?dual(F_5)}{h_7 \vdash \Delta_{11}, F_9, ?dual(F_5)} \otimes}{- \vdash (? \Upsilon 4, ?F_3), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{- \vdash ?\Upsilon 4, \Delta_{10}, F_8, ?F_3} \otimes \quad \frac{h_7 \vdash \Delta_{10}, F_8, contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_{10}, F_8, ?F_3} \otimes \quad \frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{- \vdash ?\Upsilon 4, \Delta_{11}, F_9, ?F_3} \otimes \quad \frac{h_7 \vdash \Delta_{11}, F_9, ?dual(F_5)}{h_7 \vdash \Delta_{11}, F_9, ?dual(F_5)} \otimes}{- \vdash ?\Upsilon 4, ?\Upsilon 4, \Delta_{10}, \Delta_{11}, ?F_3, ?F_3, F_8 \otimes F_9} \otimes \quad \frac{- \vdash ?\Upsilon 4, \Delta_{11}, F_9, ?F_3}{- \vdash ?\Upsilon 4, \Delta_{11}, F_9, ?F_3} \otimes}{- \vdash ?\Upsilon 4, \Delta_{10}, \Delta_{11}, ?F_3, F_8 \otimes F_9} \otimes} \text{hCut} \quad \text{hCut} \\
\rightarrow \\
\frac{h_2 \vdash F_3, ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ? \quad \frac{h_7 \vdash \Delta_{10}, F_8, ?dual(F_5)}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9} \otimes \quad \frac{h_7 \vdash \Delta_{11}, F_9, contract(n_6, ?dual(F_5))}{h_7 \vdash \Delta_{11}, F_9, contract(n_6, ?dual(F_5))} \otimes}{- \vdash (? \Upsilon 4, ?F_3), \Delta_{10}, \Delta_{11}, F_8 \otimes F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{- \vdash ?\Upsilon 4, \Delta_{10}, F_8, ?F_3} \otimes \quad \frac{h_7 \vdash \Delta_{10}, F_8, ?dual(F_5)}{- \vdash ?\Upsilon 4, \Delta_{10}, F_8, ?F_3} \otimes \quad \frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3}{- \vdash ?\Upsilon 4, \Delta_{11}, F_9, ?F_3} \otimes \quad \frac{h_7 \vdash \Delta_{11}, F_9, contract(n_6, ?dual(F_5))}{h_7 \vdash \Delta_{11}, F_9, contract(n_6, ?dual(F_5))} \otimes}{- \vdash ?\Upsilon 4, ?\Upsilon 4, \Delta_{10}, \Delta_{11}, ?F_3, ?F_3, F_8 \otimes F_9} \otimes \quad \frac{- \vdash ?\Upsilon 4, \Delta_{11}, F_9, ?F_3}{- \vdash ?\Upsilon 4, \Delta_{11}, F_9, ?F_3} \otimes}{- \vdash ?\Upsilon 4, \Delta_{10}, \Delta_{11}, ?F_3, F_8 \otimes F_9} \otimes} \text{hCut} \quad \text{hCut} \\
\rightarrow \\
\frac{- \vdash ?\Upsilon 4, ?\Upsilon 4, \Delta_{10}, \Delta_{11}, ?F_3, ?F_3, F_8 \otimes F_9}{- \vdash ?\Upsilon 4, \Delta_{10}, \Delta_{11}, ?F_3, F_8 \otimes F_9} \otimes
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_2 \vdash F_3, ?\Upsilon 4, !F_5}{\bullet h_2 \vdash !F_5, ?\Upsilon 4, ?F_3} ? \quad \frac{h_7 \vdash \Delta_8, F_9 \quad h_7 \vdash \Delta_{11}, F_{10}, ?dual(F_5), contract(n_6, ?dual(F_5))}{\bullet h_7 \vdash contract(sn_6, ?dual(F_5)), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- \vdash (? \Upsilon 4, ?F_3), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut}}{\rightarrow} \\
\frac{- \vdash \Delta_8, F_9 \quad \text{ax} \quad \frac{\bullet h_2 \vdash ?\Upsilon 4, !F_5, ?F_3 \quad \text{ax} \quad \frac{h_7 \vdash \Delta_{11}, F_{10}, ?dual(F_5), contract(n_6, ?dual(F_5))}{- \vdash ?\Upsilon 4, \Delta_{11}, F_{10}, ?F_3} \otimes}{- \vdash ?\Upsilon 4, \Delta_{11}, \Delta_8, ?F_3, F_9 \otimes F_{10}} \text{hCut}}{\rightarrow}
\end{array}$$

6.6 Status of \$: OK

- Case rule 1
- Case rule !
- Case rule ?W
- Case rule ?C
- Case rule ?
- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I
- Case rule \otimes

6.7 Status of &: OK

- Case rule 1
- Case rule !

- Case rule $?W$
- Case rule $?C$
- Case rule $?$
- Case rule $\$$
- Case rule $\&$
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I
- Case rule \otimes

6.8 Status of \oplus_B : OK

- Case rule $\mathbf{1}$
- Case rule $!$
- Case rule $?W$
- Case rule $?C$
- Case rule $?$
- Case rule $\$$
- Case rule $\&$
- Case rule \oplus_B

- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I
- Case rule \otimes

6.9 Status of \oplus_A : OK

- Case rule **1**
- Case rule **!**
- Case rule **?W**
- Case rule **?C**
- Case rule **?**
- Case rule **\$**
- Case rule **&**
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I
- Case rule \otimes

6.10 Status of \perp : OK

- Case rule **1**
- Case rule **!**
- Case rule **?W**
- Case rule **?C**
- Case rule **?**
- Case rule **\$**
- Case rule **&**
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule **I**
- Case rule \otimes

6.11 Status of \top : OK

- Case rule **1**
- Case rule **!**
- Case rule **?W**
- Case rule **?C**
- Case rule **?**

- Case rule $\$$
- Case rule $\&$
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I
- Case rule \otimes

6.12 Status of I : OK

- Case rule $\mathbf{1}$
- Case rule $!$
- Case rule $?W$
- Case rule $?C$
- Case rule $?$
- Case rule $\$$
- Case rule $\&$
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top

- Case rule I
- Case rule \otimes

6.13 Status of \otimes : OK

- Case rule $\mathbf{1}$
- Case rule $!$
- Case rule $?W$
- Case rule $?C$
- Case rule $?$
- Case rule $\$$
- Case rule $\&$
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I
- Case rule \otimes