

System for Intuitionistic Linear Logic

May 11, 2021

Contents

1	Measure of derivations	3
2	Invertibility of Rules	5
2.1	Status of $!R$: : Non invertible	5
2.2	Status of $\mathbf{1}_R$: : Invertible	6
2.3	Status of \top : : Invertible	7
2.4	Status of $\&_R$: (Left Premise): Invertible	8
2.5	Status of $\&_R$: (Right Premise): : Invertible	10
2.6	Status of \multimap_R : : Invertible	11
2.7	Status of \oplus_{R_2} : : Non invertible	13
2.8	Status of \oplus_{R_1} : : Non invertible	14
2.9	Status of $\mathbf{1}_L$: : Invertible	16
2.10	Status of \otimes_R : (Left Premise): Non invertible	18
2.11	Status of \otimes_R : (Right Premise): : Non invertible	20
2.12	Status of W : : Non invertible	22
2.13	Status of C : : Non invertible	24
2.14	Status of $!L$: : Non invertible	26
2.15	Status of $\&_{L_2}$: : Non invertible	28
2.16	Status of $\&_{L_1}$: : Non invertible	30
2.17	Status of \otimes_L : : Invertible	32
2.18	Status of \oplus_L : (Left Premise): Invertible	34
2.19	Status of \oplus_L : (Right Premise): : Invertible	36
2.20	Status of \multimap_L : (Left Premise): Non invertible	38
2.21	Status of \multimap_L : (Right Premise): : Non invertible	40
2.22	Status of I : : Invertible	43
3	Identity-Expansion	44
4	Weakening on bang: $\Gamma \vdash !F$ implies $\Gamma \vdash F$.	45
5	Cut-Elimination	47
5.1	Status of $!R$: OK	47
5.2	Status of $\mathbf{1}_R$: OK	50
5.3	Status of \top : OK	53
5.4	Status of $\&_R$: OK	56
5.5	Status of \multimap_R : OK	59
5.6	Status of \oplus_{R_2} : OK	62
5.7	Status of \oplus_{R_1} : OK	65
5.8	Status of $\mathbf{1}_L$: OK	68

5.9	Status of \otimes_R : OK	72
5.10	Status of W : OK	75
5.11	Status of C : OK	79
5.12	Status of $!L$: OK	83
5.13	Status of $\&_{L2}$: OK	88
5.14	Status of $\&_{L1}$: OK	92
5.15	Status of \otimes_L : OK	96
5.16	Status of \oplus_L : OK	100
5.17	Status of $\neg\circ_L$: OK	104
5.18	Status of I : OK	109
6	Cut-Elimination	112
6.1	Status of $!R$: OK	112
6.2	Status of $\mathbf{1}_R$: OK	116
6.3	Status of \top : OK	117
6.4	Status of $\&_R$: OK	118
6.5	Status of $\neg\circ_R$: OK	119
6.6	Status of \oplus_{R2} : OK	120
6.7	Status of \oplus_{R1} : OK	121
6.8	Status of $\mathbf{1}_L$: OK	122
6.9	Status of \otimes_R : OK	123
6.10	Status of W : OK	124
6.11	Status of C : OK	128
6.12	Status of $!L$: OK	132
6.13	Status of $\&_{L2}$: OK	136
6.14	Status of $\&_{L1}$: OK	137
6.15	Status of \otimes_L : OK	138
6.16	Status of \oplus_L : OK	139
6.17	Status of $\neg\circ_L$: OK	140
6.18	Status of I : OK	141

1 Measure of derivations

- Case(s) rule $!R$

$$\frac{h_1 : !\Upsilon 2 \vdash F_3}{\bullet h_1 : !\Upsilon 2 \vdash !F_3} !R \rightarrow \frac{\frac{h_1 : !\Upsilon 2 \vdash F_3}{\bullet h_1 : !\Upsilon 2 \vdash F_3} \text{IH}}{\bullet \bullet h_1 : !\Upsilon 2 \vdash !F_3} \text{!R} \text{ ax}$$

- Case(s) rule $\mathbf{1}_R$

$$\frac{}{\bullet h_1 : * \vdash \mathbf{1}} \mathbf{1}_R \rightarrow \frac{}{\bullet \bullet h_1 : * \vdash \mathbf{1}} \mathbf{1}_R$$

- Case(s) rule \top

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \rightarrow \frac{}{\bullet \bullet h_1 : \Delta_2 \vdash \top} \top$$

- Case(s) rule $\&_R$

$$\frac{h_1 : \Delta_2 \vdash F_3 \quad h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \& F_4} \&_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3} \text{IH} \quad \frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_4} \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash F_3 \& F_4} \&_R \text{ ax}$$

- Case(s) rule $\neg\circ_R$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \neg\circ F_4} \neg\circ_R \rightarrow \frac{\frac{h_1 : \Delta_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_2, F_3 \vdash F_4} \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash F_3 \neg\circ F_4} \neg\circ_R \text{ ax}$$

- Case(s) rule \oplus_{R_2}

$$\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \oplus F_4} \oplus_{R_2} \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_4} \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash F_3 \oplus F_4} \oplus_{R_2} \text{ ax}$$

- Case(s) rule \oplus_{R_1}

$$\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3 \oplus F_4} \oplus_{R_1} \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3} \text{IH}}{\bullet \bullet h_1 : \Delta_2 \vdash F_3 \oplus F_4} \oplus_{R_1} \text{ ax}$$

- Case(s) rule $\mathbf{1}_L$

$$\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_3} \mathbf{1}_L \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3} \text{IH}}{\bullet \bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_3} \mathbf{1}_L \text{ ax}$$

- Case(s) rule \otimes_R

$$\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3 \vdash F_5}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_4 \otimes F_5} \otimes_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_4} \text{IH} \quad \frac{h_1 : \Delta_3 \vdash F_5}{\bullet h_1 : \Delta_3 \vdash F_5} \text{IH}}{\bullet \bullet h_1 : \Delta_2, \Delta_3 \vdash F_4 \otimes F_5} \otimes_R \text{ ax}$$

- Case(s) rule W

$$\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} W \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_4} \text{IH}}{\bullet \bullet h_1 : \Delta_2, !F_3 \vdash F_4} W \text{ ax}$$

- Case(s) rule C

$$\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_4}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} C}{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_4}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} IH} ax \quad C \rightarrow \frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_4}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} IH}{\bullet \bullet h_1 : \Delta_2, !F_3 \vdash F_4} C} ax$$

- Case(s) rule $!L$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} !L \rightarrow \frac{\frac{h_1 : \Delta_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_2, F_3 \vdash F_4} IH}{\bullet \bullet h_1 : \Delta_2, !F_3 \vdash F_4} !L} ax$$

- Case(s) rule $\&_{L2}$

$$\frac{h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_5} \&_{L2} \rightarrow \frac{\frac{h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_4 \vdash F_5} IH}{\bullet \bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_5} \&_{L2} ax$$

- Case(s) rule $\&_{L1}$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_5} \&_{L1} \rightarrow \frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \vdash F_5} IH}{\bullet \bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_5} \&_{L1} ax$$

- Case(s) rule \otimes_L

$$\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_5} \otimes_L \rightarrow \frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3, F_4 \vdash F_5} IH}{\bullet \bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_5} \otimes_L ax$$

- Case(s) rule \oplus_L

$$\frac{h_1 : \Delta_2, F_3 \vdash F_5 \quad h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_5} \oplus_L \rightarrow \frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \vdash F_5} IH \quad \frac{h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_4 \vdash F_5} IH}{\bullet \bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_5} \oplus_L} ax$$

- Case(s) rule $\neg\circ_L$

$$\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash F_6}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \neg\circ F_5 \vdash F_6} \neg\circ_L \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_4} IH \quad \frac{h_1 : \Delta_3, F_5 \vdash F_6}{\bullet h_1 : \Delta_3, F_5 \vdash F_6} IH}{\bullet \bullet h_1 : \Delta_2, \Delta_3, F_4 \neg\circ F_5 \vdash F_6} \neg\circ_L} ax$$

- Case(s) rule I

$$\frac{}{\bullet h_1 : p(n_2) \vdash p(n_2)} I \rightarrow \frac{}{\bullet \bullet h_1 : p(n_2) \vdash p(n_2)} I$$

2 Invertibility of Rules

2.1 Status of $!R$: : Non invertible

- Case rule $!R$

$$\frac{h_1 : !\Upsilon 2 \vdash F_3}{\bullet h_1 : !\Upsilon 2 \vdash !F_3} !R \quad \rightarrow \quad \frac{\overline{h_1 : !\Upsilon 2 \vdash F_3} \text{ ax}}{\bullet h_1 : !\Upsilon 2 \vdash F_3} H$$

- Case rule $\mathbf{1}_R$

- Case rule \top

- Case rule $\&_R$

- Case rule $\neg\circ_R$

- Case rule \oplus_{R_2}

- Case rule \oplus_{R_1}

- Case rule $\mathbf{1}_L$

- Case rule \otimes_R

- Case rule W

$$\frac{h_2 : !\Upsilon 3 \vdash !F_1}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_1} W \quad \rightarrow \quad \frac{\overline{h_2 : !\Upsilon 3 \vdash F_1} \text{ ax/ind}}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash F_1} W$$

- Case rule C

$$\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_1}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_1} C \quad \rightarrow \quad \frac{\overline{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash F_1} \text{ ax/ind}}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash F_1} C$$

- Case rule $!L$

$$\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_1}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_1} !L \quad \rightarrow \quad \frac{\overline{\bullet h_2 : !\Upsilon 3, !F_4 \vdash F_1} \text{ fail}}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash F_1} \text{ fail}$$

- Case rule $\&_{L_2}$

- Case rule $\&_{L_1}$

- Case rule \otimes_L
- Case rule \oplus_L
- Case rule $\neg\circ_L$
- Case rule I

2.2 Status of $\mathbf{1}_R$: : Invertible

- Case rule $!R$
- Case rule $\mathbf{1}_R$

$$\frac{}{\bullet h_1 : * \vdash \mathbf{1}} \mathbf{1}_R \rightarrow \text{trivial}$$

- Case rule \top
- Case rule $\&_R$
- Case rule $\neg\circ_R$
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L_2}$
- Case rule $\&_{L_1}$

- Case rule \otimes_L
- Case rule \oplus_L
- Case rule $\neg\circ_L$
- Case rule I

2.3 Status of \top : : Invertible

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule \top

$$\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \rightarrow \text{trivial}$$

- Case rule $\&_R$
- Case rule $\neg\circ_R$
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule $\mathbf{1}_L$

$$\frac{h_1 : \Delta_2 \vdash \top}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash \top} \mathbf{1}_L \rightarrow \text{trivial}$$

- Case rule \otimes_R
- Case rule W

$$\frac{h_1 : \Delta_2 \vdash \top}{\bullet h_1 : \Delta_2, !F_3 \vdash \top} W \rightarrow \text{trivial}$$

- Case rule C

$$\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash \top}{\bullet h_1 : \Delta_2, !F_3 \vdash \top} C \rightarrow \text{trivial}$$

- Case rule $!L$

$$\frac{h_1 : \Delta_2, F_3 \vdash \top}{\bullet h_1 : \Delta_2, !F_3 \vdash \top} !L \rightarrow \text{trivial}$$

- Case rule $\&_{L2}$

$$\frac{h_1 : \Delta_2, F_4 \vdash \top}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash \top} \&_{L2} \rightarrow \text{trivial}$$

- Case rule $\&_{L1}$

$$\frac{h_1 : \Delta_2, F_3 \vdash \top}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash \top} \&_{L1} \rightarrow \text{trivial}$$

- Case rule \otimes_L

$$\frac{h_1 : \Delta_2, F_3, F_4 \vdash \top}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash \top} \otimes_L \rightarrow \text{trivial}$$

- Case rule \oplus_L

$$\frac{h_1 : \Delta_2, F_3 \vdash \top \quad h_1 : \Delta_2, F_4 \vdash \top}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash \top} \oplus_L \rightarrow \text{trivial}$$

- Case rule $\neg\circ_L$

$$\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash \top}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \neg\circ F_5 \vdash \top} \neg\circ_L \rightarrow \text{trivial}$$

- Case rule I

2.4 Status of $\&_R$: (Left Premise): Invertible

- Case rule $!R$

- Case rule $\mathbf{1}_R$

- Case rule \top

- Case rule $\&_R$

$$\frac{h_1 : \Delta_2 \vdash F_3 \quad h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \& F_4} \&_R \rightarrow \frac{\overline{h_1 : \Delta_2 \vdash F_3}^{\text{ax}}}{\bullet h_1 : \Delta_2 \vdash F_3}^{\text{H}}$$

- Case rule $\neg\circ_R$

- Case rule \oplus_{R_2}

- Case rule \oplus_{R_1}

- Case rule $\mathbf{1}_L$

$$\frac{h_3 : \Delta_4 \vdash F_1 \& F_2}{\bullet h_3 : \mathbf{1}, \Delta_4 \vdash F_1 \& F_2} \mathbf{1}_L \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \mathbf{1}, \Delta_4 \vdash F_1} \mathbf{1}_L$$

- Case rule \otimes_R

- Case rule W

$$\frac{h_3 : \Delta_4 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1 \& F_2} W \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1} W$$

- Case rule C

$$\frac{h_3 : \Delta_4, !F_5, !F_5 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1 \& F_2} C \rightarrow \frac{\overline{h_3 : \Delta_4, !F_5, !F_5 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1} C$$

- Case rule $!L$

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1 \& F_2} !L \rightarrow \frac{\overline{h_3 : \Delta_4, F_5 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1} !L$$

- Case rule $\&_{L2}$

$$\frac{h_3 : \Delta_4, F_6 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_1 \& F_2} \&_{L2} \rightarrow \frac{\overline{h_3 : \Delta_4, F_6 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_1} \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_1 \& F_2} \&_{L1} \rightarrow \frac{\overline{h_3 : \Delta_4, F_5 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_1} \&_{L1}$$

- Case rule \otimes_L

$$\frac{h_3 : \Delta_4, F_5, F_6 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, F_5 \otimes F_6 \vdash F_1 \& F_2} \otimes_L \rightarrow \frac{\overline{h_3 : \Delta_4, F_5, F_6 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_5 \otimes F_6 \vdash F_1} \otimes_L$$

- Case rule \oplus_L

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \& F_2 \quad h_3 : \Delta_4, F_6 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, F_5 \oplus F_6 \vdash F_1 \& F_2} \oplus_L \rightarrow \frac{\overline{h_3 : \Delta_4, F_5 \vdash F_1} \text{ ax/ind} \quad \overline{h_3 : \Delta_4, F_6 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_5 \oplus F_6 \vdash F_1} \oplus_L$$

- Case rule $\neg\circ_L$

$$\frac{h_3 : \Delta_4 \vdash F_6 \quad h_3 : \Delta_5, F_7 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, \Delta_5, F_6 \neg\circ F_7 \vdash F_1 \& F_2} \neg\circ_L \rightarrow \frac{\frac{h_3 : \Delta_4 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_5, F_6} \text{ ax} \quad \frac{h_3 : \Delta_5, F_7 \vdash F_1}{\bullet h_3 : \Delta_4, \Delta_5, F_6 \neg\circ F_7 \vdash F_1} \text{ ax/ind}}{\bullet h_3 : \Delta_4, \Delta_5, F_6 \neg\circ F_7 \vdash F_1} \neg\circ_L$$

- Case rule I

2.5 Status of $\&_R$ (Right Premise): : Invertible

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule \top
- Case rule $\&_R$

$$\frac{h_1 : \Delta_2 \vdash F_3 \quad h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \& F_4} \&_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_4} \text{ ax}}{h_1 : \Delta_2 \vdash F_4} \text{ H}$$

- Case rule $\neg\circ_R$
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule $\mathbf{1}_L$

$$\frac{h_3 : \Delta_4 \vdash F_1 \& F_2}{\bullet h_3 : \mathbf{1}, \Delta_4 \vdash F_1 \& F_2} \mathbf{1}_L \rightarrow \frac{\frac{h_3 : \Delta_4 \vdash F_2}{\bullet h_3 : \mathbf{1}, \Delta_4 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \mathbf{1}, \Delta_4 \vdash F_2} \mathbf{1}_L$$

- Case rule \otimes_R
- Case rule W

$$\frac{h_3 : \Delta_4 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1 \& F_2} W \rightarrow \frac{\frac{h_3 : \Delta_4 \vdash F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, !F_5 \vdash F_2} W$$

- Case rule C

$$\frac{h_3 : \Delta_4, !F_5, !F_5 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1 \& F_2} C \rightarrow \frac{\frac{h_3 : \Delta_4, !F_5, !F_5 \vdash F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, !F_5 \vdash F_2} C$$

- Case rule $!L$

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1 \& F_2} !L \rightarrow \frac{\frac{h_3 : \Delta_4, F_5 \vdash F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_2} \text{ax/ind}}{!L}$$

- Case rule $\&L_2$

$$\frac{h_3 : \Delta_4, F_6 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_1 \& F_2} \&L_2 \rightarrow \frac{\frac{h_3 : \Delta_4, F_6 \vdash F_2}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_2} \text{ax/ind}}{\&L_2}$$

- Case rule $\&L_1$

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_1 \& F_2} \&L_1 \rightarrow \frac{\frac{h_3 : \Delta_4, F_5 \vdash F_2}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_2} \text{ax/ind}}{\&L_1}$$

- Case rule \otimes_L

$$\frac{h_3 : \Delta_4, F_5, F_6 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, F_5 \otimes F_6 \vdash F_1 \& F_2} \otimes_L \rightarrow \frac{\frac{h_3 : \Delta_4, F_5, F_6 \vdash F_2}{\bullet h_3 : \Delta_4, F_5 \otimes F_6 \vdash F_2} \text{ax/ind}}{\otimes_L}$$

- Case rule \oplus_L

$$\frac{\frac{h_3 : \Delta_4, F_5 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, F_5 \oplus F_6 \vdash F_1 \& F_2} \oplus_L \quad \frac{h_3 : \Delta_4, F_6 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, F_5 \oplus F_6 \vdash F_1 \& F_2} \oplus_L}{\frac{\frac{h_3 : \Delta_4, F_5 \vdash F_2}{\bullet h_3 : \Delta_4, F_5 \oplus F_6 \vdash F_2} \text{ax/ind} \quad \frac{h_3 : \Delta_4, F_6 \vdash F_2}{\bullet h_3 : \Delta_4, F_5 \oplus F_6 \vdash F_2} \text{ax/ind}}{\oplus_L}}$$

- Case rule $\neg\circ_L$

$$\frac{\frac{h_3 : \Delta_4 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_5, F_6 \neg\circ F_7 \vdash F_1 \& F_2} \neg\circ_L \quad \frac{h_3 : \Delta_5, F_7 \vdash F_1 \& F_2}{\bullet h_3 : \Delta_4, \Delta_5, F_6 \neg\circ F_7 \vdash F_1 \& F_2} \neg\circ_L}{\frac{\frac{h_3 : \Delta_4 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_5, F_6 \neg\circ F_7 \vdash F_2} \text{ax} \quad \frac{h_3 : \Delta_5, F_7 \vdash F_2}{\bullet h_3 : \Delta_4, \Delta_5, F_6 \neg\circ F_7 \vdash F_2} \text{ax/ind}}{\neg\circ_L}}$$

- Case rule I

2.6 Status of $\neg\circ_R$: : Invertible

- Case rule $!R$

- Case rule $\mathbf{1}_R$

- Case rule \top

- Case rule $\&_R$

- Case rule $\neg\circ_R$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \neg\circ F_4} \neg\circ_R \rightarrow \frac{\frac{h_1 : \Delta_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_2, F_3 \vdash F_4} \text{ax}}{\text{H}}$$

- Case rule \oplus_{R2}

- Case rule \oplus_{R1}

- Case rule 1_L

$$\frac{h_3 : \Delta_4 \vdash F_1 \multimap F_2}{\bullet h_3 : 1, \Delta_4 \vdash F_1 \multimap F_2} 1_L \rightarrow \frac{\overline{h_3 : \Delta_4, F_1 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : 1, \Delta_4, F_1 \vdash F_2} 1_L$$

- Case rule \otimes_R

- Case rule W

$$\frac{h_3 : \Delta_4 \vdash F_1 \multimap F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1 \multimap F_2} W \rightarrow \frac{\overline{h_3 : \Delta_4, F_1 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_1, !F_5 \vdash F_2} W$$

- Case rule C

$$\frac{h_3 : \Delta_4, !F_5, !F_5 \vdash F_1 \multimap F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1 \multimap F_2} C \rightarrow \frac{\overline{h_3 : \Delta_4, F_1, !F_5, !F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_1, !F_5 \vdash F_2} C$$

- Case rule $!L$

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \multimap F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1 \multimap F_2} !L \rightarrow \frac{\overline{h_3 : \Delta_4, F_1, F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_1, !F_5 \vdash F_2} !L$$

- Case rule $\&_{L2}$

$$\frac{h_3 : \Delta_4, F_6 \vdash F_1 \multimap F_2}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_1 \multimap F_2} \&_{L2} \rightarrow \frac{\overline{h_3 : \Delta_4, F_1, F_6 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_1, F_5 \& F_6 \vdash F_2} \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \multimap F_2}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_1 \multimap F_2} \&_{L1} \rightarrow \frac{\overline{h_3 : \Delta_4, F_1, F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_1, F_5 \& F_6 \vdash F_2} \&_{L1}$$

- Case rule \otimes_L

$$\frac{h_3 : \Delta_4, F_5, F_6 \vdash F_1 \multimap F_2}{\bullet h_3 : \Delta_4, F_5 \otimes F_6 \vdash F_1 \multimap F_2} \otimes_L \rightarrow \frac{\overline{h_3 : \Delta_4, F_1, F_5, F_6 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_1, F_5 \otimes F_6 \vdash F_2} \otimes_L$$

- Case rule \oplus_L

$$\frac{\frac{h_3 : \Delta_4, F_5 \vdash F_1 \multimap F_2}{\bullet h_3 : \Delta_4, F_5 \oplus F_6 \vdash F_1 \multimap F_2} \oplus_L \quad \frac{h_3 : \Delta_4, F_6 \vdash F_1 \multimap F_2}{\bullet h_3 : \Delta_4, F_5 \oplus F_6 \vdash F_1 \multimap F_2} \oplus_L}{\bullet h_3 : \Delta_4, F_1, F_5 \oplus F_6 \vdash F_2} \oplus_L \rightarrow \frac{\overline{h_3 : \Delta_4, F_1, F_5 \vdash F_2} \text{ ax/ind} \quad \overline{h_3 : \Delta_4, F_1, F_6 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_1, F_5 \oplus F_6 \vdash F_2} \oplus_L$$

- Case rule $\neg_O L$

$$\frac{\frac{h_3 : \Delta_4 \vdash F_6 \quad h_3 : \Delta_5, F_7 \vdash F_1 \neg_O F_2}{\bullet h_3 : \Delta_4, \Delta_5, F_6 \neg_O F_7 \vdash F_1 \neg_O F_2} \neg_O L}{\frac{h_3 : \Delta_4 \vdash F_6 \quad \text{ax} \quad \frac{h_3 : \Delta_5, F_1, F_7 \vdash F_2}{\bullet h_3 : \Delta_4, \Delta_5, F_1, F_6 \neg_O F_7 \vdash F_2} \neg_O L}{\text{ax/ind}} \neg_O L} \rightarrow$$

- Case rule I

2.7 Status of \oplus_{R_2} : : Non invertible

- Case rule $!R$

- Case rule 1_R

- Case rule \top

- Case rule $\&_R$

- Case rule $\neg_O R$

- Case rule \oplus_{R_2}

$$\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \oplus F_4} \oplus_{R_2} \rightarrow \frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_4} \begin{matrix} \text{ax} \\ \text{H} \end{matrix}$$

- Case rule \oplus_{R_1}

$$\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3 \oplus F_4} \oplus_{R_1} \rightarrow \frac{}{\bullet h_1 : \Delta_2 \vdash F_4} \text{fail}$$

- Case rule 1_L

$$\frac{h_3 : \Delta_4 \vdash F_1 \oplus F_2}{\bullet h_3 : 1, \Delta_4 \vdash F_1 \oplus F_2} 1_L \rightarrow \frac{h_3 : \Delta_4 \vdash F_2}{\bullet h_3 : 1, \Delta_4 \vdash F_2} \begin{matrix} \text{ax/ind} \\ 1_L \end{matrix}$$

- Case rule \otimes_R

- Case rule W

$$\frac{h_3 : \Delta_4 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1 \oplus F_2} W \rightarrow \frac{h_3 : \Delta_4 \vdash F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_2} \begin{matrix} \text{ax/ind} \\ W \end{matrix}$$

- Case rule C

$$\frac{h_3 : \Delta_4, !F_5, !F_5 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1 \oplus F_2} C \rightarrow \frac{h_3 : \Delta_4, !F_5, !F_5 \vdash F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_2} \begin{matrix} \text{ax/ind} \\ C \end{matrix}$$

- Case rule $!L$

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1 \oplus F_2} !L \rightarrow \frac{\overline{h_3 : \Delta_4, F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, !F_5 \vdash F_2} !L$$

- Case rule $\&_{L2}$

$$\frac{h_3 : \Delta_4, F_6 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_1 \oplus F_2} \&_{L2} \rightarrow \frac{\overline{h_3 : \Delta_4, F_6 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_2} \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_1 \oplus F_2} \&_{L1} \rightarrow \frac{\overline{h_3 : \Delta_4, F_5 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_2} \&_{L1}$$

- Case rule \otimes_L

$$\frac{h_3 : \Delta_4, F_5, F_6 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, F_5 \otimes F_6 \vdash F_1 \oplus F_2} \otimes_L \rightarrow \frac{\overline{h_3 : \Delta_4, F_5, F_6 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_5 \otimes F_6 \vdash F_2} \otimes_L$$

- Case rule \oplus_L

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \oplus F_2 \quad h_3 : \Delta_4, F_6 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, F_5 \oplus F_6 \vdash F_1 \oplus F_2} \oplus_L \rightarrow \frac{\overline{h_3 : \Delta_4, F_5 \vdash F_2} \text{ ax/ind} \quad \overline{h_3 : \Delta_4, F_6 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, F_5 \oplus F_6 \vdash F_2} \oplus_L$$

- Case rule $\neg\circ_L$

$$\frac{h_3 : \Delta_4 \vdash F_6 \quad h_3 : \Delta_5, F_7 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, \Delta_5, F_6 \neg\circ F_7 \vdash F_1 \oplus F_2} \neg\circ_L \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash F_6} \text{ ax} \quad \overline{h_3 : \Delta_5, F_7 \vdash F_2} \text{ ax/ind}}{\bullet h_3 : \Delta_4, \Delta_5, F_6 \neg\circ F_7 \vdash F_2} \neg\circ_L$$

- Case rule I

2.8 Status of \oplus_{R1} : : Non invertible

- Case rule $!R$

- Case rule $\mathbf{1}_R$

- Case rule \top

- Case rule $\&_R$

- Case rule $\neg\circ_R$

- Case rule \oplus_{R2}

$$\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2 \vdash F_3 \oplus F_4} \oplus_{R2} \rightarrow \frac{\text{fail}}{\bullet h_1 : \Delta_2 \vdash F_3} \text{fail}$$

- Case rule \oplus_{R1}

$$\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3 \oplus F_4} \oplus_{R1} \rightarrow \frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3} \begin{array}{l} \text{ax} \\ \text{H} \end{array}$$

- Case rule 1_L

$$\frac{h_3 : \Delta_4 \vdash F_1 \oplus F_2}{\bullet h_3 : 1, \Delta_4 \vdash F_1 \oplus F_2} 1_L \rightarrow \frac{h_3 : \Delta_4 \vdash F_1}{\bullet h_3 : 1, \Delta_4 \vdash F_1} \begin{array}{l} \text{ax/ind} \\ 1_L \end{array}$$

- Case rule \otimes_R

- Case rule W

$$\frac{h_3 : \Delta_4 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1 \oplus F_2} W \rightarrow \frac{h_3 : \Delta_4 \vdash F_1}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1} \begin{array}{l} \text{ax/ind} \\ W \end{array}$$

- Case rule C

$$\frac{h_3 : \Delta_4, !F_5, !F_5 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1 \oplus F_2} C \rightarrow \frac{h_3 : \Delta_4, !F_5 \vdash F_1}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1} \begin{array}{l} \text{ax/ind} \\ C \end{array}$$

- Case rule $!L$

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1 \oplus F_2} !L \rightarrow \frac{h_3 : \Delta_4, F_5 \vdash F_1}{\bullet h_3 : \Delta_4, !F_5 \vdash F_1} \begin{array}{l} \text{ax/ind} \\ !L \end{array}$$

- Case rule $\&_{L2}$

$$\frac{h_3 : \Delta_4, F_6 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_1 \oplus F_2} \&_{L2} \rightarrow \frac{h_3 : \Delta_4, F_6 \vdash F_1}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_1} \begin{array}{l} \text{ax/ind} \\ \&_{L2} \end{array}$$

- Case rule $\&_{L1}$

$$\frac{h_3 : \Delta_4, F_5 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_1 \oplus F_2} \&_{L1} \rightarrow \frac{h_3 : \Delta_4, F_5 \vdash F_1}{\bullet h_3 : \Delta_4, F_5 \& F_6 \vdash F_1} \begin{array}{l} \text{ax/ind} \\ \&_{L1} \end{array}$$

- Case rule \otimes_L

$$\frac{h_3 : \Delta_4, F_5, F_6 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, F_5 \otimes F_6 \vdash F_1 \oplus F_2} \otimes_L \rightarrow \frac{h_3 : \Delta_4, F_5, F_6 \vdash F_1}{\bullet h_3 : \Delta_4, F_5 \otimes F_6 \vdash F_1} \begin{array}{l} \text{ax/ind} \\ \otimes_L \end{array}$$

- Case rule \oplus_L

$$\frac{\frac{h_3 : \Delta_4, F_5 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, F_5 \oplus F_6 \vdash F_1 \oplus F_2} \oplus_L \quad \frac{h_3 : \Delta_4, F_6 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, F_5 \oplus F_6 \vdash F_1 \oplus F_2} \oplus_L}{\frac{h_3 : \Delta_4, F_5 \vdash F_1 \quad h_3 : \Delta_4, F_6 \vdash F_1}{\bullet h_3 : \Delta_4, F_5 \oplus F_6 \vdash F_1} \oplus_L} \rightarrow \frac{\frac{h_3 : \Delta_4, F_5 \vdash F_1}{\bullet h_3 : \Delta_4, F_5 \oplus F_6 \vdash F_1} \text{ax/ind} \quad \frac{h_3 : \Delta_4, F_6 \vdash F_1}{\bullet h_3 : \Delta_4, F_5 \oplus F_6 \vdash F_1} \text{ax/ind}}{\bullet h_3 : \Delta_4, F_5 \oplus F_6 \vdash F_1} \oplus_L$$

- Case rule \neg_O_L

$$\frac{\frac{h_3 : \Delta_4 \vdash F_6 \quad h_3 : \Delta_5, F_7 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, \Delta_5, F_6 \neg_O F_7 \vdash F_1 \oplus F_2} \neg_O_L}{\bullet h_3 : \Delta_4, \Delta_5, F_6 \neg_O F_7 \vdash F_1 \oplus F_2} \neg_O_L \rightarrow \frac{\frac{h_3 : \Delta_4 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_5, F_6 \neg_O F_7 \vdash F_1 \oplus F_2} \text{ax} \quad \frac{h_3 : \Delta_5, F_7 \vdash F_1 \oplus F_2}{\bullet h_3 : \Delta_4, \Delta_5, F_6 \neg_O F_7 \vdash F_1 \oplus F_2} \text{ax/ind}}{\bullet h_3 : \Delta_4, \Delta_5, F_6 \neg_O F_7 \vdash F_1 \oplus F_2} \neg_O_L$$

- Case rule I

2.9 Status of $\mathbf{1}_L$: : Invertible

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule \top

$$\frac{}{\bullet h_2 : \mathbf{1}, \Delta_1 \vdash \top} \top \rightarrow \frac{}{\bullet h_2 : \Delta_1 \vdash \top} \top$$

- Case rule $\&R$

$$\frac{\frac{h_2 : \mathbf{1}, \Delta_1 \vdash F_3 \quad h_2 : \mathbf{1}, \Delta_1 \vdash F_4}{\bullet h_2 : \mathbf{1}, \Delta_1 \vdash F_3 \& F_4} \&R}{\bullet h_2 : \mathbf{1}, \Delta_1 \vdash F_3 \& F_4} \&R \rightarrow \frac{\frac{h_2 : \Delta_1 \vdash F_3}{\bullet h_2 : \Delta_1 \vdash F_3 \& F_4} \text{ax/ind} \quad \frac{h_2 : \Delta_1 \vdash F_4}{\bullet h_2 : \Delta_1 \vdash F_3 \& F_4} \text{ax/ind}}{\bullet h_2 : \Delta_1 \vdash F_3 \& F_4} \&R$$

- Case rule \neg_O_R

$$\frac{\frac{h_2 : \mathbf{1}, \Delta_1, F_3 \vdash F_4}{\bullet h_2 : \mathbf{1}, \Delta_1 \vdash F_3 \neg_O F_4} \neg_O_R}{\bullet h_2 : \mathbf{1}, \Delta_1 \vdash F_3 \neg_O F_4} \neg_O_R \rightarrow \frac{\frac{h_2 : \Delta_1, F_3 \vdash F_4}{\bullet h_2 : \Delta_1 \vdash F_3 \neg_O F_4} \text{ax/ind}}{\bullet h_2 : \Delta_1 \vdash F_3 \neg_O F_4} \neg_O_R$$

- Case rule \oplus_{R_2}

$$\frac{\frac{h_2 : \mathbf{1}, \Delta_1 \vdash F_4}{\bullet h_2 : \mathbf{1}, \Delta_1 \vdash F_3 \oplus F_4} \oplus_{R_2}}{\bullet h_2 : \mathbf{1}, \Delta_1 \vdash F_3 \oplus F_4} \oplus_{R_2} \rightarrow \frac{\frac{h_2 : \Delta_1 \vdash F_4}{\bullet h_2 : \Delta_1 \vdash F_3 \oplus F_4} \text{ax/ind}}{\bullet h_2 : \Delta_1 \vdash F_3 \oplus F_4} \oplus_{R_2}$$

- Case rule \oplus_{R_1}

$$\frac{\frac{h_2 : \mathbf{1}, \Delta_1 \vdash F_3}{\bullet h_2 : \mathbf{1}, \Delta_1 \vdash F_3 \oplus F_4} \oplus_{R_1}}{\bullet h_2 : \mathbf{1}, \Delta_1 \vdash F_3 \oplus F_4} \oplus_{R_1} \rightarrow \frac{\frac{h_2 : \Delta_1 \vdash F_3}{\bullet h_2 : \Delta_1 \vdash F_3 \oplus F_4} \text{ax/ind}}{\bullet h_2 : \Delta_1 \vdash F_3 \oplus F_4} \oplus_{R_1}$$

- Case rule $\mathbf{1}_L$

$$\frac{\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_3} \mathbf{1}_L}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_3} \mathbf{1}_L \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2 \vdash F_3} \text{ax}}{\bullet h_1 : \Delta_2 \vdash F_3} \text{H}$$

- Case rule \otimes_R

$$\frac{h_1 : \mathbf{1}, \Delta_5 \vdash F_3 \quad h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : (\mathbf{1}, \Delta_5), \Delta_2 \vdash F_3 \otimes F_4} \otimes_R \rightarrow \frac{\frac{h_1 : \Delta_5 \vdash F_3}{\bullet h_1 : \Delta_2, \Delta_5 \vdash F_3 \otimes F_4} \text{ ax/ind} \quad \frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2, \Delta_5 \vdash F_3 \otimes F_4} \text{ ax}}{\bullet h_1 : \Delta_2, \Delta_5 \vdash F_3 \otimes F_4} \otimes_R$$

$$\frac{h_1 : \Delta_2 \vdash F_3 \quad h_1 : \mathbf{1}, \Delta_5 \vdash F_4}{\bullet h_1 : \Delta_2, \mathbf{1}, \Delta_5 \vdash F_3 \otimes F_4} \otimes_R \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2, \Delta_5 \vdash F_3 \otimes F_4} \text{ ax} \quad \frac{h_1 : \Delta_5 \vdash F_4}{\bullet h_1 : \Delta_2, \Delta_5 \vdash F_3 \otimes F_4} \text{ ax/ind}}{\bullet h_1 : \Delta_2, \Delta_5 \vdash F_3 \otimes F_4} \otimes_R$$

- Case rule W

$$\frac{h_1 : \mathbf{1}, \Delta_4 \vdash F_3}{\bullet h_1 : (\mathbf{1}, \Delta_4), !F_2 \vdash F_3} W \rightarrow \frac{\frac{h_1 : \Delta_4 \vdash F_3}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} \text{ ax/ind}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} W$$

- Case rule C

$$\frac{h_1 : \mathbf{1}, \Delta_4, !F_2, !F_2 \vdash F_3}{\bullet h_1 : (\mathbf{1}, \Delta_4), !F_2 \vdash F_3} C \rightarrow \frac{\frac{h_1 : \Delta_4, !F_2, !F_2 \vdash F_3}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} \text{ ax/ind}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} C$$

- Case rule $!L$

$$\frac{h_1 : \mathbf{1}, \Delta_4, F_2 \vdash F_3}{\bullet h_1 : (\mathbf{1}, \Delta_4), !F_2 \vdash F_3} !L \rightarrow \frac{\frac{h_1 : \Delta_4, F_2 \vdash F_3}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} \text{ ax/ind}}{\bullet h_1 : \Delta_4, !F_2 \vdash F_3} !L$$

- Case rule $\&_{L2}$

$$\frac{h_1 : \mathbf{1}, \Delta_5, F_3 \vdash F_4}{\bullet h_1 : (\mathbf{1}, \Delta_5), F_2 \& F_3 \vdash F_4} \&_{L2} \rightarrow \frac{\frac{h_1 : \Delta_5, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_4} \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{h_1 : \mathbf{1}, \Delta_5, F_2 \vdash F_4}{\bullet h_1 : (\mathbf{1}, \Delta_5), F_2 \& F_3 \vdash F_4} \&_{L1} \rightarrow \frac{\frac{h_1 : \Delta_5, F_2 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_1 : \Delta_5, F_2 \& F_3 \vdash F_4} \&_{L1}$$

- Case rule \otimes_L

$$\frac{h_1 : \mathbf{1}, \Delta_5, F_2, F_3 \vdash F_4}{\bullet h_1 : (\mathbf{1}, \Delta_5), F_2 \otimes F_3 \vdash F_4} \otimes_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_1 : \Delta_5, F_2 \otimes F_3 \vdash F_4} \otimes_L$$

- Case rule \oplus_L

$$\frac{h_1 : \mathbf{1}, \Delta_5, F_2 \vdash F_4 \quad h_1 : \mathbf{1}, \Delta_5, F_3 \vdash F_4}{\bullet h_1 : (\mathbf{1}, \Delta_5), F_2 \oplus F_3 \vdash F_4} \oplus_L \rightarrow \frac{\frac{h_1 : \Delta_5, F_2 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_4} \text{ ax/ind} \quad \frac{h_1 : \Delta_5, F_3 \vdash F_4}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_1 : \Delta_5, F_2 \oplus F_3 \vdash F_4} \oplus_L$$

- Case rule \multimap_L

$$\frac{h_1 : \mathbf{1}, \Delta_6 \vdash F_3 \quad h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : (\mathbf{1}, \Delta_6), \Delta_2, F_3 \multimap F_4 \vdash F_5} \multimap_L \rightarrow \frac{\frac{h_1 : \Delta_6 \vdash F_3}{\bullet h_1 : \Delta_2, \Delta_6, F_3 \multimap F_4 \vdash F_5} \text{ ax/ind} \quad \frac{h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, \Delta_6, F_3 \multimap F_4 \vdash F_5} \text{ ax}}{\bullet h_1 : \Delta_2, \Delta_6, F_3 \multimap F_4 \vdash F_5} \multimap_L$$

$$\frac{h_1 : \Delta_2 \vdash F_3 \quad h_1 : \mathbf{1}, \Delta_6, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, (\mathbf{1}, \Delta_6), F_3 \multimap F_4 \vdash F_5} \multimap_L \rightarrow \frac{\frac{h_1 : \Delta_2 \vdash F_3}{\bullet h_1 : \Delta_2, \Delta_6, F_3 \multimap F_4 \vdash F_5} \text{ ax} \quad \frac{h_1 : \Delta_6, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, \Delta_6, F_3 \multimap F_4 \vdash F_5} \text{ ax/ind}}{\bullet h_1 : \Delta_2, \Delta_6, F_3 \multimap F_4 \vdash F_5} \multimap_L$$

- Case rule I

2.10 Status of \otimes_R : (Left Premise): Non invertible

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule \top
- Case rule $\&_R$
- Case rule $\neg\circ_R$
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule $\mathbf{1}_L$

$$\frac{h_4 : \Delta_1, \Delta_5 \vdash F_2 \otimes F_3}{\bullet h_4 : \mathbf{1}, \Delta_1, \Delta_5 \vdash F_2 \otimes F_3} \mathbf{1}_L \rightarrow \frac{}{h_4 : \Delta_5 \vdash F_2} \text{ax/ind} \mathbf{1}_L$$

$$\frac{h_4 : \Delta_1, \Delta_5 \vdash F_2 \otimes F_3}{\bullet h_4 : \mathbf{1}, \Delta_1, \Delta_5 \vdash F_2 \otimes F_3} \mathbf{1}_L \rightarrow \frac{}{h_4 : \Delta_1 \vdash F_2} \text{ax/ind} \text{H}$$

- Case rule \otimes_R

$$\frac{h_1 : \Delta_4, \Delta_5 \vdash F_2 \quad h_1 : \Delta_6, \Delta_7 \vdash F_3}{\bullet h_1 : (\Delta_4, \Delta_5), \Delta_6, \Delta_7 \vdash F_2 \otimes F_3} \otimes_R \rightarrow \frac{}{\bullet h_1 : \Delta_4, \Delta_6 \vdash F_2} \text{fail}$$

- Case rule W

$$\frac{h_4 : \Delta_1, \Delta_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_6), !F_5 \vdash F_2 \otimes F_3} W \rightarrow \frac{}{h_4 : \Delta_6 \vdash F_2} \text{ax/ind} W$$

$$\frac{h_4 : \Delta_1, \Delta_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_6), !F_5 \vdash F_2 \otimes F_3} W \rightarrow \frac{}{h_4 : \Delta_1 \vdash F_2} \text{ax/ind} \text{H}$$

- Case rule C

$$\frac{h_4 : \Delta_1, \Delta_6, !F_5, !F_5 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_6), !F_5 \vdash F_2 \otimes F_3} C \rightarrow \frac{}{h_4 : \Delta_6 \vdash F_2} \text{ax/ind} W$$

$$\frac{h_4 : \Delta_1, \Delta_6, !F_5, !F_5 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_6), !F_5 \vdash F_2 \otimes F_3} C \rightarrow \frac{}{h_4 : \Delta_1 \vdash F_2} \text{ax/ind} \text{H}$$

- Case rule $!L$

$$\frac{h_4 : \Delta_1, \Delta_6, F_5 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_6), !F_5 \vdash F_2 \otimes F_3} !L \rightarrow \frac{h_4 : \Delta_6 \vdash F_2}{\bullet h_4 : \Delta_6, !F_5 \vdash F_2} \text{ax/ind } W$$

$$\frac{h_4 : \Delta_1, \Delta_6, F_5 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_6), !F_5 \vdash F_2 \otimes F_3} !L \rightarrow \frac{h_4 : \Delta_1 \vdash F_2}{\bullet h_4 : \Delta_1 \vdash F_2} \text{ax/ind } H$$

- Case rule $\&_{L2}$

$$\frac{h_4 : \Delta_1, \Delta_7, F_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \& F_6 \vdash F_2 \otimes F_3} \&_{L2} \rightarrow \frac{h_4 : \Delta_7, F_6 \vdash F_2}{\bullet h_4 : \Delta_7, F_5 \& F_6 \vdash F_2} \text{ax/ind } \&_{L2}$$

$$\frac{h_4 : \Delta_1, \Delta_7, F_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \& F_6 \vdash F_2 \otimes F_3} \&_{L2} \rightarrow \frac{h_4 : \Delta_1 \vdash F_2}{\bullet h_4 : \Delta_1 \vdash F_2} \text{ax/ind } H$$

- Case rule $\&_{L1}$

$$\frac{h_4 : \Delta_1, \Delta_7, F_5 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \& F_6 \vdash F_2 \otimes F_3} \&_{L1} \rightarrow \frac{h_4 : \Delta_7, F_5 \vdash F_2}{\bullet h_4 : \Delta_7, F_5 \& F_6 \vdash F_2} \text{ax/ind } \&_{L1}$$

$$\frac{h_4 : \Delta_1, \Delta_7, F_5 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \& F_6 \vdash F_2 \otimes F_3} \&_{L1} \rightarrow \frac{h_4 : \Delta_1 \vdash F_2}{\bullet h_4 : \Delta_1 \vdash F_2} \text{ax/ind } H$$

- Case rule \otimes_L

$$\frac{h_4 : \Delta_1, \Delta_7, F_5, F_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \otimes F_6 \vdash F_2 \otimes F_3} \otimes_L \rightarrow \frac{h_4 : \Delta_7, F_5, F_6 \vdash F_2}{\bullet h_4 : \Delta_7, F_5 \otimes F_6 \vdash F_2} \text{ax/ind } \otimes_L$$

$$\frac{h_4 : \Delta_1, \Delta_7, F_5, F_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \otimes F_6 \vdash F_2 \otimes F_3} \otimes_L \rightarrow \frac{h_4 : \Delta_1 \vdash F_2}{\bullet h_4 : \Delta_1 \vdash F_2} \text{ax/ind } H$$

- Case rule \oplus_L

$$\frac{h_4 : \Delta_1, \Delta_7, F_5 \vdash F_2 \otimes F_3 \quad h_4 : \Delta_1, \Delta_7, F_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \oplus F_6 \vdash F_2 \otimes F_3} \oplus_L \rightarrow \frac{h_4 : \Delta_7, F_5 \vdash F_2}{\bullet h_4 : \Delta_7, F_5 \oplus F_6 \vdash F_2} \text{ax/ind } \oplus_L \quad \frac{h_4 : \Delta_7, F_6 \vdash F_2}{\bullet h_4 : \Delta_7, F_5 \oplus F_6 \vdash F_2} \text{ax/ind } \oplus_L$$

$$\frac{h_4 : \Delta_1, \Delta_7, F_5 \vdash F_2 \otimes F_3 \quad h_4 : \Delta_1, \Delta_7, F_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \oplus F_6 \vdash F_2 \otimes F_3} \oplus_L \rightarrow \frac{h_4 : \Delta_1 \vdash F_2}{\bullet h_4 : \Delta_1 \vdash F_2} \text{ax/ind } H$$

- Case rule $\neg\circ_L$

$$\frac{h_3 : \Delta_6, \Delta_7 \vdash F_4 \quad h_3 : \Delta_8, \Delta_9, F_5 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_6, \Delta_7), (\Delta_8, \Delta_9), F_4 \neg\circ F_5 \vdash F_1 \otimes F_2} \neg\circ_L \rightarrow \frac{}{\bullet h_3 : \Delta_6, \Delta_8, F_4 \neg\circ F_5 \vdash F_1} \text{fail}$$

$$\frac{h_3 : \Delta_6, \Delta_7 \vdash F_4 \quad h_3 : \Delta_8, \Delta_9, F_5 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_6, \Delta_7), (\Delta_8, \Delta_9), F_4 \neg\circ F_5 \vdash F_1 \otimes F_2} \neg\circ_L \rightarrow \frac{}{\bullet h_3 : \Delta_6, \Delta_8 \vdash F_1} \text{fail}$$

- Case rule I

2.11 Status of \otimes_R (Right Premise): : Non invertible

- Case rule $!R$
- Case rule 1_R
- Case rule \top
- Case rule $\&_R$
- Case rule $\neg\circ_R$
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule 1_L

$$\frac{h_4 : \Delta_1, \Delta_5 \vdash F_2 \otimes F_3}{\bullet h_4 : 1, \Delta_1, \Delta_5 \vdash F_2 \otimes F_3} 1_L \rightarrow \frac{}{\bullet h_4 : \Delta_1 \vdash F_3} \frac{\text{ax/ind}}{H}$$

$$\frac{h_4 : \Delta_1, \Delta_5 \vdash F_2 \otimes F_3}{\bullet h_4 : 1, \Delta_1, \Delta_5 \vdash F_2 \otimes F_3} 1_L \rightarrow \frac{}{\bullet h_4 : 1, \Delta_5 \vdash F_3} \frac{\text{ax/ind}}{1_L}$$

- Case rule \otimes_R

$$\frac{h_1 : \Delta_4, \Delta_5 \vdash F_2 \quad h_1 : \Delta_6, \Delta_7 \vdash F_3}{\bullet h_1 : (\Delta_4, \Delta_5), \Delta_6, \Delta_7 \vdash F_2 \otimes F_3} \otimes_R \rightarrow \frac{}{\bullet h_1 : \Delta_5, \Delta_7 \vdash F_3} \text{fail}$$

- Case rule W

$$\frac{h_4 : \Delta_1, \Delta_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_6), !F_5 \vdash F_2 \otimes F_3} W \rightarrow \frac{}{\bullet h_4 : \Delta_1 \vdash F_3} \frac{\text{ax/ind}}{H}$$

$$\frac{h_4 : \Delta_1, \Delta_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_6), !F_5 \vdash F_2 \otimes F_3} W \rightarrow \frac{}{\bullet h_4 : \Delta_6, !F_5 \vdash F_3} \frac{\text{ax/ind}}{W}$$

- Case rule C

$$\frac{h_4 : \Delta_1, \Delta_6, !F_5, !F_5 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_6), !F_5 \vdash F_2 \otimes F_3} C \rightarrow \frac{}{\bullet h_4 : \Delta_1 \vdash F_3} \frac{\text{ax/ind}}{H}$$

$$\frac{h_4 : \Delta_1, \Delta_6, !F_5, !F_5 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_6), !F_5 \vdash F_2 \otimes F_3} C \rightarrow \frac{}{\bullet h_4 : \Delta_6, !F_5 \vdash F_3} \frac{\text{ax/ind}}{W}$$

- Case rule $!L$

$$\frac{h_4 : \Delta_1, \Delta_6, F_5 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_6), !F_5 \vdash F_2 \otimes F_3} !L \rightarrow \frac{\overline{h_4 : \Delta_1 \vdash F_3}}{\bullet h_4 : \Delta_1 \vdash F_3} \text{ax/ind}_H$$

$$\frac{h_4 : \Delta_1, \Delta_6, F_5 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_6), !F_5 \vdash F_2 \otimes F_3} !L \rightarrow \frac{\overline{h_4 : \Delta_6 \vdash F_3}}{\bullet h_4 : \Delta_6, !F_5 \vdash F_3} \text{ax/ind}_W$$

- Case rule $\&_{L2}$

$$\frac{h_4 : \Delta_1, \Delta_7, F_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \& F_6 \vdash F_2 \otimes F_3} \&_{L2} \rightarrow \frac{\overline{h_4 : \Delta_1 \vdash F_3}}{\bullet h_4 : \Delta_1 \vdash F_3} \text{ax/ind}_H$$

$$\frac{h_4 : \Delta_1, \Delta_7, F_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \& F_6 \vdash F_2 \otimes F_3} \&_{L2} \rightarrow \frac{\overline{h_4 : \Delta_7, F_6 \vdash F_3}}{\bullet h_4 : \Delta_7, F_5 \& F_6 \vdash F_3} \text{ax/ind}_{\&_{L2}}$$

- Case rule $\&_{L1}$

$$\frac{h_4 : \Delta_1, \Delta_7, F_5 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \& F_6 \vdash F_2 \otimes F_3} \&_{L1} \rightarrow \frac{\overline{h_4 : \Delta_1 \vdash F_3}}{\bullet h_4 : \Delta_1 \vdash F_3} \text{ax/ind}_H$$

$$\frac{h_4 : \Delta_1, \Delta_7, F_5 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \& F_6 \vdash F_2 \otimes F_3} \&_{L1} \rightarrow \frac{\overline{h_4 : \Delta_7, F_5 \vdash F_3}}{\bullet h_4 : \Delta_7, F_5 \& F_6 \vdash F_3} \text{ax/ind}_{\&_{L1}}$$

- Case rule \otimes_L

$$\frac{h_4 : \Delta_1, \Delta_7, F_5, F_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \otimes F_6 \vdash F_2 \otimes F_3} \otimes_L \rightarrow \frac{\overline{h_4 : \Delta_1 \vdash F_3}}{\bullet h_4 : \Delta_1 \vdash F_3} \text{ax/ind}_H$$

$$\frac{h_4 : \Delta_1, \Delta_7, F_5, F_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \otimes F_6 \vdash F_2 \otimes F_3} \otimes_L \rightarrow \frac{\overline{h_4 : \Delta_7, F_5, F_6 \vdash F_3}}{\bullet h_4 : \Delta_7, F_5 \otimes F_6 \vdash F_3} \text{ax/ind}_{\otimes_L}$$

- Case rule \oplus_L

$$\frac{h_4 : \Delta_1, \Delta_7, F_5 \vdash F_2 \otimes F_3 \quad h_4 : \Delta_1, \Delta_7, F_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \oplus F_6 \vdash F_2 \otimes F_3} \oplus_L \rightarrow \frac{\overline{h_4 : \Delta_1 \vdash F_3}}{\bullet h_4 : \Delta_1 \vdash F_3} \text{ax/ind}_H$$

$$\frac{h_4 : \Delta_1, \Delta_7, F_5 \vdash F_2 \otimes F_3 \quad h_4 : \Delta_1, \Delta_7, F_6 \vdash F_2 \otimes F_3}{\bullet h_4 : (\Delta_1, \Delta_7), F_5 \oplus F_6 \vdash F_2 \otimes F_3} \oplus_L \rightarrow \frac{\overline{h_4 : \Delta_7, F_5 \vdash F_3}}{\bullet h_4 : \Delta_7, F_5 \oplus F_6 \vdash F_3} \text{ax/ind}_{\oplus_L} \quad \frac{\overline{h_4 : \Delta_7, F_6 \vdash F_3}}{\bullet h_4 : \Delta_7, F_5 \oplus F_6 \vdash F_3} \text{ax/ind}_{\oplus_L}$$

- Case rule $\neg\circ_L$

$$\frac{h_3 : \Delta_6, \Delta_7 \vdash F_4 \quad h_3 : \Delta_8, \Delta_9, F_5 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_6, \Delta_7), (\Delta_8, \Delta_9), F_4 \neg\circ F_5 \vdash F_1 \otimes F_2} \neg\circ_L \rightarrow \frac{}{\bullet h_3 : \Delta_7, \Delta_9 \vdash F_2} \text{fail}$$

$$\frac{h_3 : \Delta_6, \Delta_7 \vdash F_4 \quad h_3 : \Delta_8, \Delta_9, F_5 \vdash F_1 \otimes F_2}{\bullet h_3 : (\Delta_6, \Delta_7), (\Delta_8, \Delta_9), F_4 \neg\circ F_5 \vdash F_1 \otimes F_2} \neg\circ_L \rightarrow \frac{}{\bullet h_3 : \Delta_7, \Delta_9, F_4 \neg\circ F_5 \vdash F_2} \text{fail}$$

- Case rule I

2.12 Status of W : : Non invertible

- Case rule $!R$

$$\frac{h_3 : !\Upsilon 1, !F_2 \vdash F_4}{\bullet h_3 : !\Upsilon 1, !F_2 \vdash !F_4} !R \rightarrow \frac{\overline{h_3 : !\Upsilon 1 \vdash F_4} \text{ ax/ind}}{\bullet h_3 : !\Upsilon 1 \vdash !F_4} !R$$

- Case rule $\mathbf{1}_R$

- Case rule \top

$$\overline{\bullet h_3 : \Delta_1, !F_2 \vdash \top} \top \rightarrow \overline{\bullet h_3 : \Delta_1 \vdash \top} \top$$

- Case rule $\&_R$

$$\frac{h_3 : \Delta_1, !F_2 \vdash F_4 \quad h_3 : \Delta_1, !F_2 \vdash F_5}{\bullet h_3 : \Delta_1, !F_2 \vdash F_4 \& F_5} \&_R \rightarrow \frac{\overline{h_3 : \Delta_1 \vdash F_4} \text{ ax/ind} \quad \overline{h_3 : \Delta_1 \vdash F_5} \text{ ax/ind}}{\bullet h_3 : \Delta_1 \vdash F_4 \& F_5} \&_R$$

- Case rule $-o_R$

$$\frac{h_3 : \Delta_1, F_4, !F_2 \vdash F_5}{\bullet h_3 : \Delta_1, !F_2 \vdash F_4 -o F_5} -o_R \rightarrow \frac{\overline{h_3 : \Delta_1, F_4 \vdash F_5} \text{ ax/ind}}{\bullet h_3 : \Delta_1 \vdash F_4 -o F_5} -o_R$$

- Case rule \oplus_{R_2}

$$\frac{h_3 : \Delta_1, !F_2 \vdash F_5}{\bullet h_3 : \Delta_1, !F_2 \vdash F_4 \oplus F_5} \oplus_{R_2} \rightarrow \frac{\overline{h_3 : \Delta_1 \vdash F_5} \text{ ax/ind}}{\bullet h_3 : \Delta_1 \vdash F_4 \oplus F_5} \oplus_{R_2}$$

- Case rule \oplus_{R_1}

$$\frac{h_3 : \Delta_1, !F_2 \vdash F_4}{\bullet h_3 : \Delta_1, !F_2 \vdash F_4 \oplus F_5} \oplus_{R_1} \rightarrow \frac{\overline{h_3 : \Delta_1 \vdash F_4} \text{ ax/ind}}{\bullet h_3 : \Delta_1 \vdash F_4 \oplus F_5} \oplus_{R_1}$$

- Case rule $\mathbf{1}_L$

$$\frac{h_2 : \Delta_4, !F_1 \vdash F_3}{\bullet h_2 : \mathbf{1}, \Delta_4, !F_1 \vdash F_3} \mathbf{1}_L \rightarrow \frac{\overline{h_2 : \Delta_4 \vdash F_3} \text{ ax/ind}}{\bullet h_2 : \mathbf{1}, \Delta_4 \vdash F_3} \mathbf{1}_L$$

- Case rule \otimes_R

$$\frac{h_2 : \Delta_6, !F_1 \vdash F_4 \quad h_2 : \Delta_3 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), \Delta_3 \vdash F_4 \otimes F_5} \otimes_R \rightarrow \frac{\overline{h_2 : \Delta_6 \vdash F_4} \text{ ax/ind} \quad \overline{h_2 : \Delta_3 \vdash F_5} \text{ ax}}{\bullet h_2 : \Delta_3, \Delta_6 \vdash F_4 \otimes F_5} \otimes_R$$

$$\frac{h_2 : \Delta_3 \vdash F_4 \quad h_2 : \Delta_6, !F_1 \vdash F_5}{\bullet h_2 : \Delta_3, \Delta_6, !F_1 \vdash F_4 \otimes F_5} \otimes_R \rightarrow \frac{\overline{h_2 : \Delta_3 \vdash F_4} \text{ ax} \quad \overline{h_2 : \Delta_6 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_3, \Delta_6 \vdash F_4 \otimes F_5} \otimes_R$$

- Case rule W

$$\frac{h_2 : \Delta_5, !F_1 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_1), !F_3 \vdash F_4} W \rightarrow \frac{\overline{h_2 : \Delta_5 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, !F_3 \vdash F_4} W$$

$$\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} W \rightarrow \frac{\overline{h_1 : \Delta_2 \vdash F_4} \text{ ax}}{\bullet h_1 : \Delta_2 \vdash F_4} H$$

- Case rule C

$$\frac{h_2 : \Delta_5, !F_1, !F_3, !F_3 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_1), !F_3 \vdash F_4} C \rightarrow \frac{\overline{h_2 : \Delta_5, !F_3, !F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, !F_3 \vdash F_4} C$$

$$\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_4}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} C \rightarrow \frac{\overline{\bullet h_1 : \Delta_2 \vdash F_4} \text{ fail}}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} C$$

- Case rule $!L$

$$\frac{h_2 : \Delta_5, F_3, !F_1 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_1), !F_3 \vdash F_4} !L \rightarrow \frac{\overline{h_2 : \Delta_5, F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, !F_3 \vdash F_4} !L$$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} !L \rightarrow \frac{\overline{\bullet h_1 : \Delta_2 \vdash F_4} \text{ fail}}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} !L$$

- Case rule $\&_{L2}$

$$\frac{h_2 : \Delta_6, F_4, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \& F_4 \vdash F_5} \&_{L2} \rightarrow \frac{\overline{h_2 : \Delta_6, F_4 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, F_3 \& F_4 \vdash F_5} \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{h_2 : \Delta_6, F_3, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \& F_4 \vdash F_5} \&_{L1} \rightarrow \frac{\overline{h_2 : \Delta_6, F_3 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, F_3 \& F_4 \vdash F_5} \&_{L1}$$

- Case rule \otimes_L

$$\frac{h_2 : \Delta_6, F_3, F_4, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \otimes F_4 \vdash F_5} \otimes_L \rightarrow \frac{\overline{h_2 : \Delta_6, F_3, F_4 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, F_3 \otimes F_4 \vdash F_5} \otimes_L$$

- Case rule \oplus_L

$$\frac{h_2 : \Delta_6, F_3, !F_1 \vdash F_5 \quad h_2 : \Delta_6, F_4, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \oplus F_4 \vdash F_5} \oplus_L \rightarrow \frac{\overline{h_2 : \Delta_6, F_3 \vdash F_5} \text{ ax/ind} \quad \overline{h_2 : \Delta_6, F_4 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, F_3 \oplus F_4 \vdash F_5} \oplus_L$$

- Case rule \multimap_L

$$\frac{h_2 : \Delta_7, !F_1 \vdash F_4 \quad h_2 : \Delta_3, F_5 \vdash F_6}{\bullet h_2 : (\Delta_7, !F_1), \Delta_3, F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\overline{h_2 : \Delta_7 \vdash F_4} \text{ ax/ind} \quad \overline{h_2 : \Delta_3, F_5 \vdash F_6} \text{ ax}}{\bullet h_2 : \Delta_3, \Delta_7, F_4 \multimap F_5 \vdash F_6} \multimap_L$$

$$\frac{h_2 : \Delta_3 \vdash F_4 \quad h_2 : \Delta_7, F_5, !F_1 \vdash F_6}{\bullet h_2 : \Delta_3, (\Delta_7, !F_1), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\overline{h_2 : \Delta_3 \vdash F_4} \text{ ax} \quad \overline{h_2 : \Delta_7, F_5 \vdash F_6} \text{ ax/ind}}{\bullet h_2 : \Delta_3, \Delta_7, F_4 \multimap F_5 \vdash F_6} \multimap_L$$

- Case rule I

2.13 Status of C : : Non invertible

- Case rule $!R$

$$\frac{h_3 : !\top 1, !F_2 \vdash F_4}{\bullet h_3 : !\top 1, !F_2 \vdash !F_4} !R \rightarrow \frac{\overline{h_3 : !\top 1, !F_2, !F_2 \vdash F_4}}{\bullet h_3 : !\top 1, !F_2, !F_2 \vdash !F_4} !R \text{ ax/ind}$$

- Case rule 1_R

- Case rule \top

$$\frac{}{\bullet h_3 : \Delta_1, !F_2 \vdash \top} \top \rightarrow \frac{}{\bullet h_3 : \Delta_1, !F_2, !F_2 \vdash \top} \top$$

- Case rule $\&_R$

$$\frac{h_3 : \Delta_1, !F_2 \vdash F_4 \quad h_3 : \Delta_1, !F_2 \vdash F_5}{\bullet h_3 : \Delta_1, !F_2 \vdash F_4 \& F_5} \&_R \rightarrow \frac{\overline{h_3 : \Delta_1, !F_2, !F_2 \vdash F_4} \text{ ax/ind} \quad \overline{h_3 : \Delta_1, !F_2, !F_2 \vdash F_5} \text{ ax/ind}}{\bullet h_3 : \Delta_1, !F_2, !F_2 \vdash F_4 \& F_5} \&_R$$

- Case rule $\neg\circ_R$

$$\frac{h_3 : \Delta_1, F_4, !F_2 \vdash F_5}{\bullet h_3 : \Delta_1, !F_2 \vdash F_4 \neg\circ F_5} \neg\circ_R \rightarrow \frac{\overline{h_3 : \Delta_1, F_4, !F_2, !F_2 \vdash F_5} \text{ ax/ind}}{\bullet h_3 : \Delta_1, !F_2, !F_2 \vdash F_4 \neg\circ F_5} \neg\circ_R$$

- Case rule \oplus_{R_2}

$$\frac{h_3 : \Delta_1, !F_2 \vdash F_5}{\bullet h_3 : \Delta_1, !F_2 \vdash F_4 \oplus F_5} \oplus_{R_2} \rightarrow \frac{\overline{h_3 : \Delta_1, !F_2, !F_2 \vdash F_5} \text{ ax/ind}}{\bullet h_3 : \Delta_1, !F_2, !F_2 \vdash F_4 \oplus F_5} \oplus_{R_2}$$

- Case rule \oplus_{R_1}

$$\frac{h_3 : \Delta_1, !F_2 \vdash F_4}{\bullet h_3 : \Delta_1, !F_2 \vdash F_4 \oplus F_5} \oplus_{R_1} \rightarrow \frac{\overline{h_3 : \Delta_1, !F_2, !F_2 \vdash F_4} \text{ ax/ind}}{\bullet h_3 : \Delta_1, !F_2, !F_2 \vdash F_4 \oplus F_5} \oplus_{R_1}$$

- Case rule 1_L

$$\frac{h_2 : \Delta_4, !F_1 \vdash F_3}{\bullet h_2 : 1, \Delta_4, !F_1 \vdash F_3} 1_L \rightarrow \frac{\overline{h_2 : \Delta_4, !F_1, !F_1 \vdash F_3} \text{ ax/ind}}{\bullet h_2 : 1, \Delta_4, !F_1, !F_1 \vdash F_3} 1_L$$

- Case rule \otimes_R

$$\frac{h_2 : \Delta_6, !F_1 \vdash F_4 \quad h_2 : \Delta_3 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), \Delta_3 \vdash F_4 \otimes F_5} \otimes_R \rightarrow \frac{\overline{h_2 : \Delta_6, !F_1, !F_1 \vdash F_4} \text{ ax/ind} \quad \overline{h_2 : \Delta_3 \vdash F_5} \text{ ax}}{\bullet h_2 : \Delta_3, \Delta_6, !F_1, !F_1 \vdash F_4 \otimes F_5} \otimes_R$$

$$\frac{h_2 : \Delta_3 \vdash F_4 \quad h_2 : \Delta_6, !F_1 \vdash F_5}{\bullet h_2 : \Delta_3, \Delta_6, !F_1 \vdash F_4 \otimes F_5} \otimes_R \rightarrow \frac{\overline{h_2 : \Delta_3 \vdash F_4} \text{ ax} \quad \overline{h_2 : \Delta_6, !F_1, !F_1 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_3, \Delta_6, !F_1, !F_1 \vdash F_4 \otimes F_5} \otimes_R$$

- Case rule W

$$\frac{h_2 : \Delta_5, !F_1 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_1), !F_3 \vdash F_4} W \rightarrow \frac{\overline{h_2 : \Delta_5, !F_1, !F_1 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, !F_1, !F_1, !F_3 \vdash F_4} W$$

$$\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} W \rightarrow \overline{\bullet h_1 : \Delta_2, !F_3, !F_3 \vdash F_4} \text{ fail}$$

- Case rule C

$$\frac{h_2 : \Delta_5, !F_1, !F_3, !F_3 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_1), !F_3 \vdash F_4} C \rightarrow \frac{\overline{h_2 : \Delta_5, !F_1, !F_1, !F_3, !F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, !F_1, !F_1, !F_3 \vdash F_4} C$$

$$\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_4}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} C \rightarrow \frac{\overline{h_1 : \Delta_2, !F_3, !F_3, !F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_1 : \Delta_2, !F_3, !F_3 \vdash F_4} C$$

- Case rule $!L$

$$\frac{h_2 : \Delta_5, F_3, !F_1 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_1), !F_3 \vdash F_4} !L \rightarrow \frac{\overline{h_2 : \Delta_5, F_3, !F_1, !F_1 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, !F_1, !F_1, !F_3 \vdash F_4} !L$$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} !L \rightarrow \overline{\bullet h_1 : \Delta_2, !F_3, !F_3 \vdash F_4} \text{ fail}$$

- Case rule $\&_{L2}$

$$\frac{h_2 : \Delta_6, F_4, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \& F_4 \vdash F_5} \&_{L2} \rightarrow \frac{\overline{h_2 : \Delta_6, F_4, !F_1, !F_1 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, !F_1, !F_1, F_3 \& F_4 \vdash F_5} \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{h_2 : \Delta_6, F_3, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \& F_4 \vdash F_5} \&_{L1} \rightarrow \frac{\overline{h_2 : \Delta_6, F_3, !F_1, !F_1 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, !F_1, !F_1, F_3 \& F_4 \vdash F_5} \&_{L1}$$

- Case rule \otimes_L

$$\frac{h_2 : \Delta_6, F_3, F_4, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \otimes F_4 \vdash F_5} \otimes_L \rightarrow \frac{\overline{h_2 : \Delta_6, F_3, F_4, !F_1, !F_1 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, !F_1, !F_1, F_3 \otimes F_4 \vdash F_5} \otimes_L$$

- Case rule \oplus_L

$$\frac{h_2 : \Delta_6, F_3, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \oplus F_4 \vdash F_5} \oplus_L \rightarrow \frac{\overline{h_2 : \Delta_6, F_3, !F_1, !F_1 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, !F_1, !F_1, F_3 \oplus F_4 \vdash F_5} \oplus_L$$

- Case rule \multimap_L

$$\frac{h_2 : \Delta_7, !F_1 \vdash F_4 \quad h_2 : \Delta_3, F_5 \vdash F_6}{\bullet h_2 : (\Delta_7, !F_1), \Delta_3, F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\overline{h_2 : \Delta_7, !F_1, !F_1 \vdash F_4} \text{ ax/ind} \quad \overline{h_2 : \Delta_3, F_5 \vdash F_6} \text{ ax}}{\bullet h_2 : \Delta_3, \Delta_7, !F_1, !F_1, F_4 \multimap F_5 \vdash F_6} \multimap_L$$

$$\frac{h_2 : \Delta_3 \vdash F_4 \quad h_2 : \Delta_7, F_5, !F_1 \vdash F_6}{\bullet h_2 : \Delta_3, (\Delta_7, !F_1), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\overline{h_2 : \Delta_3 \vdash F_4} \text{ ax} \quad \overline{h_2 : \Delta_7, F_5, !F_1, !F_1 \vdash F_6} \text{ ax/ind}}{\bullet h_2 : \Delta_3, \Delta_7, !F_1, !F_1, F_4 \multimap F_5 \vdash F_6} \multimap_L$$

- Case rule I

2.14 Status of $!L$: : Non invertible

- Case rule $!R$

$$\frac{h_3 : !\top 1, !F_2 \vdash F_4}{\bullet h_3 : !\top 1, !F_2 \vdash !F_4} !R \rightarrow \frac{}{\bullet h_3 : F_2, !\top 1 \vdash !F_4} \text{fail}$$

- Case rule 1_R

- Case rule \top

$$\frac{}{\bullet h_3 : \Delta_1, !F_2 \vdash \top} \top \rightarrow \frac{}{\bullet h_3 : \Delta_1, F_2 \vdash \top} \top$$

- Case rule $\&_R$

$$\frac{h_3 : \Delta_1, !F_2 \vdash F_4 \quad h_3 : \Delta_1, !F_2 \vdash F_5}{\bullet h_3 : \Delta_1, !F_2 \vdash F_4 \& F_5} \&_R \rightarrow \frac{\frac{}{h_3 : \Delta_1, F_2 \vdash F_4} \text{ax/ind} \quad \frac{}{h_3 : \Delta_1, F_2 \vdash F_5} \text{ax/ind}}{\bullet h_3 : \Delta_1, F_2 \vdash F_4 \& F_5} \&_R$$

- Case rule $\neg\circ_R$

$$\frac{h_3 : \Delta_1, F_4, !F_2 \vdash F_5}{\bullet h_3 : \Delta_1, !F_2 \vdash F_4 \neg\circ F_5} \neg\circ_R \rightarrow \frac{\frac{}{h_3 : \Delta_1, F_2, F_4 \vdash F_5} \text{ax/ind}}{\bullet h_3 : \Delta_1, F_2 \vdash F_4 \neg\circ F_5} \neg\circ_R$$

- Case rule \oplus_{R_2}

$$\frac{h_3 : \Delta_1, !F_2 \vdash F_5}{\bullet h_3 : \Delta_1, !F_2 \vdash F_4 \oplus F_5} \oplus_{R_2} \rightarrow \frac{\frac{}{h_3 : \Delta_1, F_2 \vdash F_5} \text{ax/ind}}{\bullet h_3 : \Delta_1, F_2 \vdash F_4 \oplus F_5} \oplus_{R_2}$$

- Case rule \oplus_{R_1}

$$\frac{h_3 : \Delta_1, !F_2 \vdash F_4}{\bullet h_3 : \Delta_1, !F_2 \vdash F_4 \oplus F_5} \oplus_{R_1} \rightarrow \frac{\frac{}{h_3 : \Delta_1, F_2 \vdash F_4} \text{ax/ind}}{\bullet h_3 : \Delta_1, F_2 \vdash F_4 \oplus F_5} \oplus_{R_1}$$

- Case rule 1_L

$$\frac{h_2 : \Delta_4, !F_1 \vdash F_3}{\bullet h_2 : 1, \Delta_4, !F_1 \vdash F_3} 1_L \rightarrow \frac{\frac{}{h_2 : \Delta_4, F_1 \vdash F_3} \text{ax/ind}}{\bullet h_2 : 1, \Delta_4, F_1 \vdash F_3} 1_L$$

- Case rule \otimes_R

$$\frac{h_2 : \Delta_6, !F_1 \vdash F_4 \quad h_2 : \Delta_3 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), \Delta_3 \vdash F_4 \otimes F_5} \otimes_R \rightarrow \frac{\frac{}{h_2 : \Delta_6, F_1 \vdash F_4} \text{ax/ind} \quad \frac{}{h_2 : \Delta_3 \vdash F_5} \text{ax}}{\bullet h_2 : \Delta_3, \Delta_6, F_1 \vdash F_4 \otimes F_5} \otimes_R$$

$$\frac{h_2 : \Delta_3 \vdash F_4 \quad h_2 : \Delta_6, !F_1 \vdash F_5}{\bullet h_2 : \Delta_3, \Delta_6, !F_1 \vdash F_4 \otimes F_5} \otimes_R \rightarrow \frac{\frac{}{h_2 : \Delta_3 \vdash F_4} \text{ax} \quad \frac{}{h_2 : \Delta_6, F_1 \vdash F_5} \text{ax/ind}}{\bullet h_2 : \Delta_3, \Delta_6, F_1 \vdash F_4 \otimes F_5} \otimes_R$$

- Case rule W

$$\frac{h_2 : \Delta_5, !F_1 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_1), !F_3 \vdash F_4} W \rightarrow \frac{\overline{h_2 : \Delta_5, F_1 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, F_1, !F_3 \vdash F_4} W$$

$$\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} W \rightarrow \frac{\overline{\bullet h_1 : \Delta_2, F_3 \vdash F_4} \text{ fail}}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4}$$

- Case rule C

$$\frac{h_2 : \Delta_5, !F_1, !F_3, !F_3 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_1), !F_3 \vdash F_4} C \rightarrow \frac{\overline{h_2 : \Delta_5, F_1, !F_3, !F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, F_1, !F_3 \vdash F_4} C$$

$$\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_4}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} C \rightarrow \frac{\overline{\bullet h_1 : \Delta_2, F_3 \vdash F_4} \text{ fail}}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4}$$

- Case rule $!L$

$$\frac{h_2 : \Delta_5, F_3, !F_1 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_1), !F_3 \vdash F_4} !L \rightarrow \frac{\overline{h_2 : \Delta_5, F_1, F_3 \vdash F_4} \text{ ax/ind}}{\bullet h_2 : \Delta_5, F_1, !F_3 \vdash F_4} !L$$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_4}{\bullet h_1 : \Delta_2, !F_3 \vdash F_4} !L \rightarrow \frac{\overline{h_1 : \Delta_2, F_3 \vdash F_4} \text{ ax}}{\bullet h_1 : \Delta_2, F_3 \vdash F_4} H$$

- Case rule $\&_{L2}$

$$\frac{h_2 : \Delta_6, F_4, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \& F_4 \vdash F_5} \&_{L2} \rightarrow \frac{\overline{h_2 : \Delta_6, F_1, F_4 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, F_1, F_3 \& F_4 \vdash F_5} \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{h_2 : \Delta_6, F_3, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \& F_4 \vdash F_5} \&_{L1} \rightarrow \frac{\overline{h_2 : \Delta_6, F_1, F_3 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, F_1, F_3 \& F_4 \vdash F_5} \&_{L1}$$

- Case rule \otimes_L

$$\frac{h_2 : \Delta_6, F_3, F_4, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \otimes F_4 \vdash F_5} \otimes_L \rightarrow \frac{\overline{h_2 : \Delta_6, F_1, F_3, F_4 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, F_1, F_3 \otimes F_4 \vdash F_5} \otimes_L$$

- Case rule \oplus_L

$$\frac{h_2 : \Delta_6, F_3, !F_1 \vdash F_5 \quad h_2 : \Delta_6, F_4, !F_1 \vdash F_5}{\bullet h_2 : (\Delta_6, !F_1), F_3 \oplus F_4 \vdash F_5} \oplus_L \rightarrow \frac{\overline{h_2 : \Delta_6, F_1, F_3 \vdash F_5} \text{ ax/ind} \quad \overline{h_2 : \Delta_6, F_1, F_4 \vdash F_5} \text{ ax/ind}}{\bullet h_2 : \Delta_6, F_1, F_3 \oplus F_4 \vdash F_5} \oplus_L$$

- Case rule \multimap_L

$$\frac{h_2 : \Delta_7, !F_1 \vdash F_4 \quad h_2 : \Delta_3, F_5 \vdash F_6}{\bullet h_2 : (\Delta_7, !F_1), \Delta_3, F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\overline{h_2 : \Delta_7, F_1 \vdash F_4} \text{ ax/ind} \quad \overline{h_2 : \Delta_3, F_5 \vdash F_6} \text{ ax}}{\bullet h_2 : \Delta_3, \Delta_7, F_1, F_4 \multimap F_5 \vdash F_6} \multimap_L$$

$$\frac{h_2 : \Delta_3 \vdash F_4 \quad h_2 : \Delta_7, F_5, !F_1 \vdash F_6}{\bullet h_2 : \Delta_3, (\Delta_7, !F_1), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\overline{h_2 : \Delta_3 \vdash F_4} \text{ ax} \quad \overline{h_2 : \Delta_7, F_1, F_5 \vdash F_6} \text{ ax/ind}}{\bullet h_2 : \Delta_3, \Delta_7, F_1, F_4 \multimap F_5 \vdash F_6} \multimap_L$$

- Case rule I

2.15 Status of $\&_{L2}$: : Non invertible

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule \top

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \& F_3 \vdash \top} \top \rightarrow \frac{}{\bullet h_4 : \Delta_1, F_3 \vdash \top} \top$$

- Case rule $\&_R$

$$\frac{\frac{h_4 : \Delta_1, F_2 \& F_3 \vdash F_5 \quad h_4 : \Delta_1, F_2 \& F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \& F_3 \vdash F_5 \& F_6} \&_R}{\bullet h_4 : \Delta_1, F_3 \vdash F_5} \text{ax/ind} \rightarrow \frac{\frac{h_4 : \Delta_1, F_3 \vdash F_5 \quad h_4 : \Delta_1, F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \& F_6} \&_R}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \& F_6} \text{ax/ind}$$

- Case rule $\neg\circ_R$

$$\frac{\frac{h_4 : \Delta_1, F_5, F_2 \& F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \& F_3 \vdash F_5 \neg\circ F_6} \neg\circ_R}{\bullet h_4 : \Delta_1, F_3, F_5 \vdash F_6} \text{ax/ind} \rightarrow \frac{\frac{h_4 : \Delta_1, F_3, F_5 \vdash F_6}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \neg\circ F_6} \neg\circ_R}{\bullet h_4 : \Delta_1, F_3, F_5 \vdash F_6} \text{ax/ind}$$

- Case rule \oplus_{R2}

$$\frac{\frac{h_4 : \Delta_1, F_2 \& F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \& F_3 \vdash F_5 \oplus F_6} \oplus_{R2}}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \oplus F_6} \text{ax/ind} \rightarrow \frac{\frac{h_4 : \Delta_1, F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \oplus F_6} \oplus_{R2}}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \oplus F_6} \text{ax/ind}$$

- Case rule \oplus_{R1}

$$\frac{\frac{h_4 : \Delta_1, F_2 \& F_3 \vdash F_5}{\bullet h_4 : \Delta_1, F_2 \& F_3 \vdash F_5 \oplus F_6} \oplus_{R1}}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \oplus F_6} \text{ax/ind} \rightarrow \frac{\frac{h_4 : \Delta_1, F_3 \vdash F_5}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \oplus F_6} \oplus_{R1}}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \oplus F_6} \text{ax/ind}$$

- Case rule $\mathbf{1}_L$

$$\frac{\frac{h_3 : \Delta_5, F_1 \& F_2 \vdash F_4}{\bullet h_3 : \mathbf{1}, \Delta_5, F_1 \& F_2 \vdash F_4} \mathbf{1}_L}{\bullet h_3 : \mathbf{1}, \Delta_5, F_2 \vdash F_4} \text{ax/ind} \rightarrow \frac{\frac{h_3 : \Delta_5, F_2 \vdash F_4}{\bullet h_3 : \mathbf{1}, \Delta_5, F_2 \vdash F_4} \mathbf{1}_L}{\bullet h_3 : \mathbf{1}, \Delta_5, F_2 \vdash F_4} \text{ax/ind}$$

- Case rule \otimes_R

$$\frac{\frac{h_3 : \Delta_7, F_1 \& F_2 \vdash F_5 \quad h_3 : \Delta_4 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \& F_2), \Delta_4 \vdash F_5 \otimes F_6} \otimes_R}{\bullet h_3 : \Delta_4, \Delta_7, F_2 \vdash F_5 \otimes F_6} \text{ax/ind} \rightarrow \frac{\frac{h_3 : \Delta_7, F_2 \vdash F_5 \quad h_3 : \Delta_4 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_2 \vdash F_5 \otimes F_6} \otimes_R}{\bullet h_3 : \Delta_4, \Delta_7, F_2 \vdash F_5 \otimes F_6} \text{ax}$$

$$\frac{\frac{h_3 : \Delta_4 \vdash F_5 \quad h_3 : \Delta_7, F_1 \& F_2 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_1 \& F_2 \vdash F_5 \otimes F_6} \otimes_R}{\bullet h_3 : \Delta_4, \Delta_7, F_2 \vdash F_5 \otimes F_6} \text{ax/ind} \rightarrow \frac{\frac{h_3 : \Delta_4 \vdash F_5 \quad h_3 : \Delta_7, F_2 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_2 \vdash F_5 \otimes F_6} \otimes_R}{\bullet h_3 : \Delta_4, \Delta_7, F_2 \vdash F_5 \otimes F_6} \text{ax/ind}$$

- Case rule W

$$\frac{h_3 : \Delta_6, F_1 \& F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \& F_2), !F_4 \vdash F_5} W \rightarrow \frac{\overline{h_3 : \Delta_6, F_2 \vdash F_5}}{\bullet h_3 : \Delta_6, F_2, !F_4 \vdash F_5} \text{ax/ind } W$$

- Case rule C

$$\frac{h_3 : \Delta_6, !F_4, !F_4, F_1 \& F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \& F_2), !F_4 \vdash F_5} C \rightarrow \frac{\overline{h_3 : \Delta_6, F_2, !F_4, !F_4 \vdash F_5}}{\bullet h_3 : \Delta_6, F_2, !F_4 \vdash F_5} \text{ax/ind } C$$

- Case rule $!L$

$$\frac{h_3 : \Delta_6, F_4, F_1 \& F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \& F_2), !F_4 \vdash F_5} !L \rightarrow \frac{\overline{h_3 : \Delta_6, F_2, F_4 \vdash F_5}}{\bullet h_3 : \Delta_6, F_2, !F_4 \vdash F_5} \text{ax/ind } !L$$

- Case rule $\&_{L2}$

$$\frac{h_3 : \Delta_7, F_5, F_1 \& F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \& F_2), F_4 \& F_5 \vdash F_6} \&_{L2} \rightarrow \frac{\overline{h_3 : \Delta_7, F_2, F_5 \vdash F_6}}{\bullet h_3 : \Delta_7, F_2, F_4 \& F_5 \vdash F_6} \text{ax/ind } \&_{L2}$$

$$\frac{h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_5} \&_{L2} \rightarrow \frac{\overline{h_1 : \Delta_2, F_4 \vdash F_5}}{\bullet h_1 : \Delta_2, F_4 \vdash F_5} \text{ax } H$$

- Case rule $\&_{L1}$

$$\frac{h_3 : \Delta_7, F_4, F_1 \& F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \& F_2), F_4 \& F_5 \vdash F_6} \&_{L1} \rightarrow \frac{\overline{h_3 : \Delta_7, F_2, F_4 \vdash F_6}}{\bullet h_3 : \Delta_7, F_2, F_4 \& F_5 \vdash F_6} \text{ax/ind } \&_{L1}$$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_5} \&_{L1} \rightarrow \overline{\bullet h_1 : \Delta_2, F_4 \vdash F_5} \text{ fail}$$

- Case rule \otimes_L

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \& F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \& F_2), F_4 \otimes F_5 \vdash F_6} \otimes_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_2, F_4, F_5 \vdash F_6}}{\bullet h_3 : \Delta_7, F_2, F_4 \otimes F_5 \vdash F_6} \text{ax/ind } \otimes_L$$

- Case rule \oplus_L

$$\frac{h_3 : \Delta_7, F_4, F_1 \& F_2 \vdash F_6 \quad h_3 : \Delta_7, F_5, F_1 \& F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \& F_2), F_4 \oplus F_5 \vdash F_6} \oplus_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_2, F_4 \vdash F_6} \quad \overline{h_3 : \Delta_7, F_2, F_5 \vdash F_6}}{\bullet h_3 : \Delta_7, F_2, F_4 \oplus F_5 \vdash F_6} \text{ax/ind } \oplus_L$$

- Case rule $\neg\circ_L$

$$\frac{h_3 : \Delta_8, F_1 \& F_2 \vdash F_5 \quad h_3 : \Delta_4, F_6 \vdash F_7}{\bullet h_3 : (\Delta_8, F_1 \& F_2), \Delta_4, F_5 \neg\circ F_6 \vdash F_7} \neg\circ_L \rightarrow \frac{\overline{h_3 : \Delta_8, F_2 \vdash F_5} \quad \overline{h_3 : \Delta_4, F_6 \vdash F_7}}{\bullet h_3 : \Delta_4, \Delta_8, F_2, F_5 \neg\circ F_6 \vdash F_7} \text{ax/ind } \neg\circ_L$$

$$\frac{h_3 : \Delta_4 \vdash F_5 \quad h_3 : \Delta_8, F_6, F_1 \& F_2 \vdash F_7}{\bullet h_3 : \Delta_4, (\Delta_8, F_1 \& F_2), F_5 \neg\circ F_6 \vdash F_7} \neg\circ_L \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash F_5} \quad \overline{h_3 : \Delta_8, F_2, F_6 \vdash F_7}}{\bullet h_3 : \Delta_4, \Delta_8, F_2, F_5 \neg\circ F_6 \vdash F_7} \text{ax/ind } \neg\circ_L$$

- Case rule I

2.16 Status of $\&_{L1}$: Non invertible

- Case rule $!R$
- Case rule 1_R
- Case rule \top

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \& F_3 \vdash \top} \top \rightarrow \frac{}{\bullet h_4 : \Delta_1, F_2 \vdash \top} \top$$

- Case rule $\&_R$

$$\frac{\frac{h_4 : \Delta_1, F_2 \& F_3 \vdash F_5 \quad h_4 : \Delta_1, F_2 \& F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \& F_3 \vdash F_5 \& F_6} \&_R}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \& F_6} \text{ax/ind} \rightarrow \frac{\frac{h_4 : \Delta_1, F_2 \vdash F_5}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \& F_6} \text{ax/ind} \quad \frac{h_4 : \Delta_1, F_2 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \& F_6} \text{ax/ind}}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \& F_6} \&_R$$

- Case rule $\neg\circ_R$

$$\frac{\frac{h_4 : \Delta_1, F_5, F_2 \& F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \& F_3 \vdash F_5 \neg\circ F_6} \neg\circ_R}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \neg\circ F_6} \text{ax/ind} \rightarrow \frac{\frac{h_4 : \Delta_1, F_2, F_5 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \neg\circ F_6} \neg\circ_R}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \neg\circ F_6} \text{ax/ind}$$

- Case rule \oplus_{R2}

$$\frac{\frac{h_4 : \Delta_1, F_2 \& F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \& F_3 \vdash F_5 \oplus F_6} \oplus_{R2}}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \oplus F_6} \text{ax/ind} \rightarrow \frac{\frac{h_4 : \Delta_1, F_2 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \oplus F_6} \oplus_{R2}}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \oplus F_6} \text{ax/ind}$$

- Case rule \oplus_{R1}

$$\frac{\frac{h_4 : \Delta_1, F_2 \& F_3 \vdash F_5}{\bullet h_4 : \Delta_1, F_2 \& F_3 \vdash F_5 \oplus F_6} \oplus_{R1}}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \oplus F_6} \text{ax/ind} \rightarrow \frac{\frac{h_4 : \Delta_1, F_2 \vdash F_5}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \oplus F_6} \oplus_{R1}}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \oplus F_6} \text{ax/ind}$$

- Case rule 1_L

$$\frac{\frac{h_3 : \Delta_5, F_1 \& F_2 \vdash F_4}{\bullet h_3 : 1, \Delta_5, F_1 \& F_2 \vdash F_4} 1_L}{\bullet h_3 : 1, \Delta_5, F_1 \vdash F_4} \text{ax/ind} \rightarrow \frac{\frac{h_3 : \Delta_5, F_1 \vdash F_4}{\bullet h_3 : 1, \Delta_5, F_1 \vdash F_4} 1_L}{\bullet h_3 : 1, \Delta_5, F_1 \vdash F_4} \text{ax/ind}$$

- Case rule \otimes_R

$$\frac{\frac{h_3 : \Delta_7, F_1 \& F_2 \vdash F_5 \quad h_3 : \Delta_4 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \& F_2), \Delta_4 \vdash F_5 \otimes F_6} \otimes_R}{\bullet h_3 : \Delta_4, \Delta_7, F_1 \vdash F_5 \otimes F_6} \text{ax/ind} \rightarrow \frac{\frac{h_3 : \Delta_7, F_1 \vdash F_5}{\bullet h_3 : \Delta_4, \Delta_7, F_1 \vdash F_5 \otimes F_6} \text{ax/ind} \quad \frac{h_3 : \Delta_4 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_1 \vdash F_5 \otimes F_6} \text{ax}}{\bullet h_3 : \Delta_4, \Delta_7, F_1 \vdash F_5 \otimes F_6} \otimes_R$$

$$\frac{\frac{h_3 : \Delta_4 \vdash F_5 \quad h_3 : \Delta_7, F_1 \& F_2 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_1 \& F_2 \vdash F_5 \otimes F_6} \otimes_R}{\bullet h_3 : \Delta_4, \Delta_7, F_1 \vdash F_5 \otimes F_6} \text{ax} \rightarrow \frac{\frac{h_3 : \Delta_4 \vdash F_5}{\bullet h_3 : \Delta_4, \Delta_7, F_1 \vdash F_5 \otimes F_6} \text{ax} \quad \frac{h_3 : \Delta_7, F_1 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_1 \vdash F_5 \otimes F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4, \Delta_7, F_1 \vdash F_5 \otimes F_6} \otimes_R$$

- Case rule W

$$\frac{\frac{h_3 : \Delta_6, F_1 \& F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \& F_2), !F_4 \vdash F_5} W}{\bullet h_3 : \Delta_6, F_1, !F_4 \vdash F_5} \text{ax/ind} \rightarrow \frac{\frac{h_3 : \Delta_6, F_1 \vdash F_5}{\bullet h_3 : \Delta_6, F_1, !F_4 \vdash F_5} W}{\bullet h_3 : \Delta_6, F_1, !F_4 \vdash F_5} \text{ax/ind}$$

- Case rule C

$$\frac{h_3 : \Delta_6, !F_4, !F_4, F_1 \& F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \& F_2), !F_4 \vdash F_5} C \rightarrow \frac{\overline{h_3 : \Delta_6, F_1, !F_4, !F_4 \vdash F_5}}{\bullet h_3 : \Delta_6, F_1, !F_4 \vdash F_5} \text{ax/ind} C$$

- Case rule $!L$

$$\frac{h_3 : \Delta_6, F_4, F_1 \& F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \& F_2), !F_4 \vdash F_5} !L \rightarrow \frac{\overline{h_3 : \Delta_6, F_1, F_4 \vdash F_5}}{\bullet h_3 : \Delta_6, F_1, !F_4 \vdash F_5} \text{ax/ind} !L$$

- Case rule $\&L_2$

$$\frac{h_3 : \Delta_7, F_5, F_1 \& F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \& F_2), F_4 \& F_5 \vdash F_6} \&L_2 \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_5 \vdash F_6}}{\bullet h_3 : \Delta_7, F_1, F_4 \& F_5 \vdash F_6} \text{ax/ind} \&L_2$$

$$\frac{h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_5} \&L_2 \rightarrow \frac{\overline{\bullet h_1 : \Delta_2, F_3 \vdash F_5}}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_5} \text{fail}$$

- Case rule $\&L_1$

$$\frac{h_3 : \Delta_7, F_4, F_1 \& F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \& F_2), F_4 \& F_5 \vdash F_6} \&L_1 \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_4 \vdash F_6}}{\bullet h_3 : \Delta_7, F_1, F_4 \& F_5 \vdash F_6} \text{ax/ind} \&L_1$$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_5} \&L_1 \rightarrow \frac{\overline{h_1 : \Delta_2, F_3 \vdash F_5}}{\bullet h_1 : \Delta_2, F_3 \vdash F_5} \text{ax} H$$

- Case rule \otimes_L

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \& F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \& F_2), F_4 \otimes F_5 \vdash F_6} \otimes_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_4, F_5 \vdash F_6}}{\bullet h_3 : \Delta_7, F_1, F_4 \otimes F_5 \vdash F_6} \text{ax/ind} \otimes_L$$

- Case rule \oplus_L

$$\frac{h_3 : \Delta_7, F_4, F_1 \& F_2 \vdash F_6 \quad h_3 : \Delta_7, F_5, F_1 \& F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \& F_2), F_4 \oplus F_5 \vdash F_6} \oplus_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_4 \vdash F_6} \text{ax/ind} \quad \overline{h_3 : \Delta_7, F_1, F_5 \vdash F_6} \text{ax/ind}}{\bullet h_3 : \Delta_7, F_1, F_4 \oplus F_5 \vdash F_6} \oplus_L$$

- Case rule \multimap_L

$$\frac{h_3 : \Delta_8, F_1 \& F_2 \vdash F_5 \quad h_3 : \Delta_4, F_6 \vdash F_7}{\bullet h_3 : (\Delta_8, F_1 \& F_2), \Delta_4, F_5 \multimap F_6 \vdash F_7} \multimap_L \rightarrow \frac{\overline{h_3 : \Delta_8, F_1 \vdash F_5} \text{ax/ind} \quad \overline{h_3 : \Delta_4, F_6 \vdash F_7} \text{ax}}{\bullet h_3 : \Delta_4, \Delta_8, F_1, F_5 \multimap F_6 \vdash F_7} \multimap_L$$

$$\frac{h_3 : \Delta_4 \vdash F_5 \quad h_3 : \Delta_8, F_6, F_1 \& F_2 \vdash F_7}{\bullet h_3 : \Delta_4, (\Delta_8, F_1 \& F_2), F_5 \multimap F_6 \vdash F_7} \multimap_L \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash F_5} \text{ax} \quad \overline{h_3 : \Delta_8, F_1, F_6 \vdash F_7} \text{ax/ind}}{\bullet h_3 : \Delta_4, \Delta_8, F_1, F_5 \multimap F_6 \vdash F_7} \multimap_L$$

- Case rule I

2.17 Status of \otimes_L : : Invertible

- Case rule $!R$
- Case rule 1_R
- Case rule \top

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \otimes F_3 \vdash \top} \top \rightarrow \frac{}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash \top} \top$$

- Case rule $\&_R$

$$\frac{\frac{h_4 : \Delta_1, F_2 \otimes F_3 \vdash F_5 \quad h_4 : \Delta_1, F_2 \otimes F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \otimes F_3 \vdash F_5 \& F_6} \&_R}{\frac{h_4 : \Delta_1, F_2, F_3 \vdash F_5 \quad ax/ind \quad \frac{h_4 : \Delta_1, F_2, F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \& F_6} \&_R}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \& F_6} \&_R} \rightarrow \frac{h_4 : \Delta_1, F_2, F_3 \vdash F_5 \quad ax/ind \quad \frac{h_4 : \Delta_1, F_2, F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \& F_6} \&_R}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \& F_6} \&_R} \&_R$$

- Case rule $\neg\circ_R$

$$\frac{\frac{h_4 : \Delta_1, F_5, F_2 \otimes F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \otimes F_3 \vdash F_5 \neg\circ F_6} \neg\circ_R}{\frac{h_4 : \Delta_1, F_2, F_3 \vdash F_5 \quad ax/ind \quad \frac{h_4 : \Delta_1, F_2, F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \neg\circ F_6} \neg\circ_R}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \neg\circ F_6} \neg\circ_R} \rightarrow \frac{h_4 : \Delta_1, F_2, F_3 \vdash F_5 \quad ax/ind \quad \frac{h_4 : \Delta_1, F_2, F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \neg\circ F_6} \neg\circ_R}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \neg\circ F_6} \neg\circ_R} \neg\circ_R$$

- Case rule \oplus_{R_2}

$$\frac{\frac{h_4 : \Delta_1, F_2 \otimes F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \otimes F_3 \vdash F_5 \oplus F_6} \oplus_{R_2}}{\frac{h_4 : \Delta_1, F_2, F_3 \vdash F_5 \quad ax/ind \quad \frac{h_4 : \Delta_1, F_2, F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \oplus F_6} \oplus_{R_2}}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \oplus F_6} \oplus_{R_2}} \rightarrow \frac{h_4 : \Delta_1, F_2, F_3 \vdash F_5 \quad ax/ind \quad \frac{h_4 : \Delta_1, F_2, F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \oplus F_6} \oplus_{R_2}}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \oplus F_6} \oplus_{R_2}} \oplus_{R_2}$$

- Case rule \oplus_{R_1}

$$\frac{\frac{h_4 : \Delta_1, F_2 \otimes F_3 \vdash F_5}{\bullet h_4 : \Delta_1, F_2 \otimes F_3 \vdash F_5 \oplus F_6} \oplus_{R_1}}{\frac{h_4 : \Delta_1, F_2, F_3 \vdash F_5 \quad ax/ind \quad \frac{h_4 : \Delta_1, F_2, F_3 \vdash F_5}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \oplus F_6} \oplus_{R_1}}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \oplus F_6} \oplus_{R_1}} \rightarrow \frac{h_4 : \Delta_1, F_2, F_3 \vdash F_5 \quad ax/ind \quad \frac{h_4 : \Delta_1, F_2, F_3 \vdash F_5}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \oplus F_6} \oplus_{R_1}}{\bullet h_4 : \Delta_1, F_2, F_3 \vdash F_5 \oplus F_6} \oplus_{R_1}} \oplus_{R_1}$$

- Case rule 1_L

$$\frac{\frac{h_3 : \Delta_5, F_1 \otimes F_2 \vdash F_4}{\bullet h_3 : \mathbf{1}, \Delta_5, F_1 \otimes F_2 \vdash F_4} 1_L}{\frac{h_3 : \Delta_5, F_1, F_2 \vdash F_4 \quad ax/ind \quad \frac{h_3 : \Delta_5, F_1, F_2 \vdash F_4}{\bullet h_3 : \mathbf{1}, \Delta_5, F_1, F_2 \vdash F_4} 1_L}{\bullet h_3 : \mathbf{1}, \Delta_5, F_1, F_2 \vdash F_4} 1_L} \rightarrow \frac{h_3 : \Delta_5, F_1, F_2 \vdash F_4 \quad ax/ind \quad \frac{h_3 : \Delta_5, F_1, F_2 \vdash F_4}{\bullet h_3 : \mathbf{1}, \Delta_5, F_1, F_2 \vdash F_4} 1_L}{\bullet h_3 : \mathbf{1}, \Delta_5, F_1, F_2 \vdash F_4} 1_L} 1_L$$

- Case rule \otimes_R

$$\frac{\frac{h_3 : \Delta_7, F_1 \otimes F_2 \vdash F_5 \quad h_3 : \Delta_4 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \otimes F_2), \Delta_4 \vdash F_5 \otimes F_6} \otimes_R}{\frac{h_3 : \Delta_7, F_1, F_2 \vdash F_5 \quad ax/ind \quad \frac{h_3 : \Delta_4 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_1, F_2 \vdash F_5 \otimes F_6} \otimes_R}{\bullet h_3 : \Delta_4, \Delta_7, F_1, F_2 \vdash F_5 \otimes F_6} \otimes_R} \rightarrow \frac{h_3 : \Delta_7, F_1, F_2 \vdash F_5 \quad ax/ind \quad \frac{h_3 : \Delta_4 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_1, F_2 \vdash F_5 \otimes F_6} \otimes_R}{\bullet h_3 : \Delta_4, \Delta_7, F_1, F_2 \vdash F_5 \otimes F_6} \otimes_R} \otimes_R$$

$$\frac{\frac{h_3 : \Delta_4 \vdash F_5 \quad h_3 : \Delta_7, F_1 \otimes F_2 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_1 \otimes F_2 \vdash F_5 \otimes F_6} \otimes_R}{\frac{h_3 : \Delta_4 \vdash F_5 \quad ax \quad \frac{h_3 : \Delta_7, F_1, F_2 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_1, F_2 \vdash F_5 \otimes F_6} \otimes_R}{\bullet h_3 : \Delta_4, \Delta_7, F_1, F_2 \vdash F_5 \otimes F_6} \otimes_R} \rightarrow \frac{h_3 : \Delta_4 \vdash F_5 \quad ax \quad \frac{h_3 : \Delta_7, F_1, F_2 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_1, F_2 \vdash F_5 \otimes F_6} \otimes_R}{\bullet h_3 : \Delta_4, \Delta_7, F_1, F_2 \vdash F_5 \otimes F_6} \otimes_R} \otimes_R$$

- Case rule W

$$\frac{\frac{h_3 : \Delta_6, F_1 \otimes F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \otimes F_2), !F_4 \vdash F_5} W}{\frac{h_3 : \Delta_6, F_1, F_2 \vdash F_5 \quad ax/ind \quad \frac{h_3 : \Delta_6, F_1, F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1, F_2, !F_4 \vdash F_5} W}{\bullet h_3 : \Delta_6, F_1, F_2, !F_4 \vdash F_5} W} \rightarrow \frac{h_3 : \Delta_6, F_1, F_2 \vdash F_5 \quad ax/ind \quad \frac{h_3 : \Delta_6, F_1, F_2 \vdash F_5}{\bullet h_3 : \Delta_6, F_1, F_2, !F_4 \vdash F_5} W}{\bullet h_3 : \Delta_6, F_1, F_2, !F_4 \vdash F_5} W} W$$

- Case rule C

$$\frac{h_3 : \Delta_6, !F_4, !F_4, F_1 \otimes F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \otimes F_2), !F_4 \vdash F_5} C \rightarrow \frac{h_3 : \Delta_6, F_1, F_2, !F_4, !F_4 \vdash F_5}{\bullet h_3 : \Delta_6, F_1, F_2, !F_4 \vdash F_5} \text{ax/ind} C$$

- Case rule $!L$

$$\frac{h_3 : \Delta_6, F_4, F_1 \otimes F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \otimes F_2), !F_4 \vdash F_5} !L \rightarrow \frac{h_3 : \Delta_6, F_1, F_2, F_4 \vdash F_5}{\bullet h_3 : \Delta_6, F_1, F_2, !F_4 \vdash F_5} \text{ax/ind} !L$$

- Case rule $\&_{L2}$

$$\frac{h_3 : \Delta_7, F_5, F_1 \otimes F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \otimes F_2), F_4 \& F_5 \vdash F_6} \&_{L2} \rightarrow \frac{h_3 : \Delta_7, F_1, F_2, F_5 \vdash F_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \& F_5 \vdash F_6} \text{ax/ind} \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{h_3 : \Delta_7, F_4, F_1 \otimes F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \otimes F_2), F_4 \& F_5 \vdash F_6} \&_{L1} \rightarrow \frac{h_3 : \Delta_7, F_1, F_2, F_4 \vdash F_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \& F_5 \vdash F_6} \text{ax/ind} \&_{L1}$$

- Case rule \otimes_L

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \otimes F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \otimes F_2), F_4 \otimes F_5 \vdash F_6} \otimes_L \rightarrow \frac{h_3 : \Delta_7, F_1, F_2, F_4, F_5 \vdash F_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \otimes F_5 \vdash F_6} \text{ax/ind} \otimes_L$$

$$\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_5} \otimes_L \rightarrow \frac{h_1 : \Delta_2, F_3, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3, F_4 \vdash F_5} \text{ax} H$$

- Case rule \oplus_L

$$\frac{h_3 : \Delta_7, F_4, F_1 \otimes F_2 \vdash F_6 \quad h_3 : \Delta_7, F_5, F_1 \otimes F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \otimes F_2), F_4 \oplus F_5 \vdash F_6} \oplus_L \rightarrow \frac{h_3 : \Delta_7, F_1, F_2, F_4 \vdash F_6 \quad h_3 : \Delta_7, F_1, F_2, F_5 \vdash F_6}{\bullet h_3 : \Delta_7, F_1, F_2, F_4 \oplus F_5 \vdash F_6} \text{ax/ind} \oplus_L$$

- Case rule \multimap_L

$$\frac{h_3 : \Delta_8, F_1 \otimes F_2 \vdash F_5 \quad h_3 : \Delta_4, F_6 \vdash F_7}{\bullet h_3 : (\Delta_8, F_1 \otimes F_2), \Delta_4, F_5 \multimap F_6 \vdash F_7} \multimap_L \rightarrow \frac{h_3 : \Delta_8, F_1, F_2 \vdash F_5 \quad h_3 : \Delta_4, F_6 \vdash F_7}{\bullet h_3 : \Delta_4, \Delta_8, F_1, F_2, F_5 \multimap F_6 \vdash F_7} \text{ax/ind} \multimap_L$$

$$\frac{h_3 : \Delta_4 \vdash F_5 \quad h_3 : \Delta_8, F_6, F_1 \otimes F_2 \vdash F_7}{\bullet h_3 : \Delta_4, (\Delta_8, F_1 \otimes F_2), F_5 \multimap F_6 \vdash F_7} \multimap_L \rightarrow \frac{h_3 : \Delta_4 \vdash F_5 \quad h_3 : \Delta_8, F_1, F_2, F_6 \vdash F_7}{\bullet h_3 : \Delta_4, \Delta_8, F_1, F_2, F_5 \multimap F_6 \vdash F_7} \text{ax/ind} \multimap_L$$

- Case rule I

2.18 Status of \oplus_L : (Left Premise): Invertible

- Case rule $!R$
- Case rule 1_R
- Case rule \top

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \oplus F_3 \vdash \top} \top \rightarrow \frac{}{\bullet h_4 : \Delta_1, F_2 \vdash \top} \top$$

- Case rule $\&_R$

$$\frac{\frac{h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_5 \quad h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_5 \& F_6} \&_R}{\frac{h_4 : \Delta_1, F_2 \vdash F_5 \quad \text{ax/ind} \quad \frac{h_4 : \Delta_1, F_2 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \& F_6} \&_R}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \& F_6} \text{ax/ind} \&_R} \rightarrow$$

- Case rule $\neg\circ_R$

$$\frac{\frac{h_4 : \Delta_1, F_5, F_2 \oplus F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_5 \neg\circ F_6} \neg\circ_R}{\frac{h_4 : \Delta_1, F_2, F_5 \vdash F_6 \quad \text{ax/ind}}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \neg\circ F_6} \neg\circ_R} \rightarrow$$

- Case rule \oplus_{R_2}

$$\frac{\frac{h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_5 \oplus F_6} \oplus_{R_2}}{\frac{h_4 : \Delta_1, F_2 \vdash F_6 \quad \text{ax/ind}}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \oplus F_6} \oplus_{R_2}} \rightarrow$$

- Case rule \oplus_{R_1}

$$\frac{\frac{h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_5}{\bullet h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_5 \oplus F_6} \oplus_{R_1}}{\frac{h_4 : \Delta_1, F_2 \vdash F_5 \quad \text{ax/ind}}{\bullet h_4 : \Delta_1, F_2 \vdash F_5 \oplus F_6} \oplus_{R_1}} \rightarrow$$

- Case rule 1_L

$$\frac{\frac{h_3 : \Delta_5, F_1 \oplus F_2 \vdash F_4}{\bullet h_3 : 1, \Delta_5, F_1 \oplus F_2 \vdash F_4} 1_L}{\frac{h_3 : \Delta_5, F_1 \vdash F_4 \quad \text{ax/ind}}{\bullet h_3 : 1, \Delta_5, F_1 \vdash F_4} 1_L} \rightarrow$$

- Case rule \otimes_R

$$\frac{\frac{h_3 : \Delta_7, F_1 \oplus F_2 \vdash F_5 \quad h_3 : \Delta_4 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \oplus F_2), \Delta_4 \vdash F_5 \otimes F_6} \otimes_R}{\frac{h_3 : \Delta_7, F_1 \vdash F_5 \quad \text{ax/ind} \quad \frac{h_3 : \Delta_4 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_1 \vdash F_5 \otimes F_6} \text{ax}}{\bullet h_3 : \Delta_4, \Delta_7, F_1 \vdash F_5 \otimes F_6} \otimes_R} \rightarrow$$

$$\frac{\frac{h_3 : \Delta_4 \vdash F_5 \quad h_3 : \Delta_7, F_1 \oplus F_2 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_1 \oplus F_2 \vdash F_5 \otimes F_6} \otimes_R}{\frac{h_3 : \Delta_4 \vdash F_5 \quad \text{ax} \quad \frac{h_3 : \Delta_7, F_1 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_1 \vdash F_5 \otimes F_6} \text{ax/ind}}{\bullet h_3 : \Delta_4, \Delta_7, F_1 \vdash F_5 \otimes F_6} \otimes_R} \rightarrow$$

- Case rule W

$$\frac{\frac{h_3 : \Delta_6, F_1 \oplus F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \oplus F_2), !F_4 \vdash F_5} W}{\frac{h_3 : \Delta_6, F_1 \vdash F_5 \quad \text{ax/ind}}{\bullet h_3 : \Delta_6, F_1, !F_4 \vdash F_5} W} \rightarrow$$

- Case rule C

$$\frac{h_3 : \Delta_6, !F_4, !F_4, F_1 \oplus F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \oplus F_2), !F_4 \vdash F_5} C \rightarrow \frac{\overline{h_3 : \Delta_6, F_1, !F_4, !F_4 \vdash F_5}}{\bullet h_3 : \Delta_6, F_1, !F_4 \vdash F_5} \text{ax/ind} C$$

- Case rule $!L$

$$\frac{h_3 : \Delta_6, F_4, F_1 \oplus F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \oplus F_2), !F_4 \vdash F_5} !L \rightarrow \frac{\overline{h_3 : \Delta_6, F_1, F_4 \vdash F_5}}{\bullet h_3 : \Delta_6, F_1, !F_4 \vdash F_5} \text{ax/ind} !L$$

- Case rule $\&_{L2}$

$$\frac{h_3 : \Delta_7, F_5, F_1 \oplus F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \oplus F_2), F_4 \& F_5 \vdash F_6} \&_{L2} \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_5 \vdash F_6}}{\bullet h_3 : \Delta_7, F_1, F_4 \& F_5 \vdash F_6} \text{ax/ind} \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{h_3 : \Delta_7, F_4, F_1 \oplus F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \oplus F_2), F_4 \& F_5 \vdash F_6} \&_{L1} \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_4 \vdash F_6}}{\bullet h_3 : \Delta_7, F_1, F_4 \& F_5 \vdash F_6} \text{ax/ind} \&_{L1}$$

- Case rule \otimes_L

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \oplus F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \oplus F_2), F_4 \otimes F_5 \vdash F_6} \otimes_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_4, F_5 \vdash F_6}}{\bullet h_3 : \Delta_7, F_1, F_4 \otimes F_5 \vdash F_6} \text{ax/ind} \otimes_L$$

- Case rule \oplus_L

$$\frac{h_3 : \Delta_7, F_4, F_1 \oplus F_2 \vdash F_6 \quad h_3 : \Delta_7, F_5, F_1 \oplus F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \oplus F_2), F_4 \oplus F_5 \vdash F_6} \oplus_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_1, F_4 \vdash F_6} \quad \overline{h_3 : \Delta_7, F_1, F_5 \vdash F_6}}{\bullet h_3 : \Delta_7, F_1, F_4 \oplus F_5 \vdash F_6} \text{ax/ind} \oplus_L$$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_5 \quad h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_5} \oplus_L \rightarrow \frac{\overline{h_1 : \Delta_2, F_3 \vdash F_5}}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_5} \text{ax} H$$

- Case rule \multimap_L

$$\frac{h_3 : \Delta_8, F_1 \oplus F_2 \vdash F_5 \quad h_3 : \Delta_4, F_6 \vdash F_7}{\bullet h_3 : (\Delta_8, F_1 \oplus F_2), \Delta_4, F_5 \multimap F_6 \vdash F_7} \multimap_L \rightarrow \frac{\overline{h_3 : \Delta_8, F_1 \vdash F_5} \quad \overline{h_3 : \Delta_4, F_6 \vdash F_7}}{\bullet h_3 : \Delta_4, \Delta_8, F_1, F_5 \multimap F_6 \vdash F_7} \text{ax/ind} \multimap_L$$

$$\frac{h_3 : \Delta_4 \vdash F_5 \quad h_3 : \Delta_8, F_6, F_1 \oplus F_2 \vdash F_7}{\bullet h_3 : \Delta_4, (\Delta_8, F_1 \oplus F_2), F_5 \multimap F_6 \vdash F_7} \multimap_L \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash F_5} \quad \overline{h_3 : \Delta_8, F_1, F_6 \vdash F_7}}{\bullet h_3 : \Delta_4, \Delta_8, F_1, F_5 \multimap F_6 \vdash F_7} \text{ax/ind} \multimap_L$$

- Case rule I

2.19 Status of \oplus_L (Right Premise): : Invertible

- Case rule $!R$
- Case rule 1_R
- Case rule \top

$$\frac{}{\bullet h_4 : \Delta_1, F_2 \oplus F_3 \vdash \top} \top \rightarrow \frac{}{\bullet h_4 : \Delta_1, F_3 \vdash \top} \top$$

- Case rule $\&_R$

$$\frac{\frac{h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_5 \quad h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_5 \& F_6} \&_R}{\frac{h_4 : \Delta_1, F_3 \vdash F_5 \quad ax/ind \quad h_4 : \Delta_1, F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \& F_6} \&_R} \rightarrow \frac{}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \& F_6} \&_R$$

- Case rule $\neg\circ_R$

$$\frac{\frac{h_4 : \Delta_1, F_5, F_2 \oplus F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_5 \neg\circ F_6} \neg\circ_R}{\frac{h_4 : \Delta_1, F_3, F_5 \vdash F_6 \quad ax/ind}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \neg\circ F_6} \neg\circ_R} \rightarrow \frac{}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \neg\circ F_6} \neg\circ_R$$

- Case rule \oplus_{R_2}

$$\frac{\frac{h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_6}{\bullet h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_5 \oplus F_6} \oplus_{R_2}}{\frac{h_4 : \Delta_1, F_3 \vdash F_6 \quad ax/ind}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \oplus F_6} \oplus_{R_2}} \rightarrow \frac{}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \oplus F_6} \oplus_{R_2}$$

- Case rule \oplus_{R_1}

$$\frac{\frac{h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_5}{\bullet h_4 : \Delta_1, F_2 \oplus F_3 \vdash F_5 \oplus F_6} \oplus_{R_1}}{\frac{h_4 : \Delta_1, F_3 \vdash F_5 \quad ax/ind}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \oplus F_6} \oplus_{R_1}} \rightarrow \frac{}{\bullet h_4 : \Delta_1, F_3 \vdash F_5 \oplus F_6} \oplus_{R_1}$$

- Case rule 1_L

$$\frac{\frac{h_3 : \Delta_5, F_1 \oplus F_2 \vdash F_4}{\bullet h_3 : 1, \Delta_5, F_1 \oplus F_2 \vdash F_4} 1_L}{\frac{h_3 : \Delta_5, F_2 \vdash F_4 \quad ax/ind}{\bullet h_3 : 1, \Delta_5, F_2 \vdash F_4} 1_L} \rightarrow \frac{}{\bullet h_3 : 1, \Delta_5, F_2 \vdash F_4} 1_L$$

- Case rule \otimes_R

$$\frac{\frac{h_3 : \Delta_7, F_1 \oplus F_2 \vdash F_5 \quad h_3 : \Delta_4 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \oplus F_2), \Delta_4 \vdash F_5 \otimes F_6} \otimes_R}{\frac{h_3 : \Delta_7, F_2 \vdash F_5 \quad ax/ind \quad h_3 : \Delta_4 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_2 \vdash F_5 \otimes F_6} \otimes_R} \rightarrow \frac{}{\bullet h_3 : \Delta_4, \Delta_7, F_2 \vdash F_5 \otimes F_6} \otimes_R$$

$$\frac{\frac{h_3 : \Delta_4 \vdash F_5 \quad h_3 : \Delta_7, F_1 \oplus F_2 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_1 \oplus F_2 \vdash F_5 \otimes F_6} \otimes_R}{\frac{h_3 : \Delta_4 \vdash F_5 \quad ax \quad h_3 : \Delta_7, F_2 \vdash F_6}{\bullet h_3 : \Delta_4, \Delta_7, F_2 \vdash F_5 \otimes F_6} \otimes_R} \rightarrow \frac{}{\bullet h_3 : \Delta_4, \Delta_7, F_2 \vdash F_5 \otimes F_6} \otimes_R$$

- Case rule W

$$\frac{\frac{h_3 : \Delta_6, F_1 \oplus F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \oplus F_2), !F_4 \vdash F_5} W}{\frac{h_3 : \Delta_6, F_2 \vdash F_5 \quad ax/ind}{\bullet h_3 : \Delta_6, F_2, !F_4 \vdash F_5} W} \rightarrow \frac{}{\bullet h_3 : \Delta_6, F_2, !F_4 \vdash F_5} W$$

- Case rule C

$$\frac{h_3 : \Delta_6, !F_4, !F_4, F_1 \oplus F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \oplus F_2), !F_4 \vdash F_5} C \rightarrow \frac{\overline{h_3 : \Delta_6, F_2, !F_4, !F_4 \vdash F_5}}{\bullet h_3 : \Delta_6, F_2, !F_4 \vdash F_5} \text{ax/ind } C$$

- Case rule $!L$

$$\frac{h_3 : \Delta_6, F_4, F_1 \oplus F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, F_1 \oplus F_2), !F_4 \vdash F_5} !L \rightarrow \frac{\overline{h_3 : \Delta_6, F_2, F_4 \vdash F_5}}{\bullet h_3 : \Delta_6, F_2, !F_4 \vdash F_5} \text{ax/ind } !L$$

- Case rule $\&_{L2}$

$$\frac{h_3 : \Delta_7, F_5, F_1 \oplus F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \oplus F_2), F_4 \& F_5 \vdash F_6} \&_{L2} \rightarrow \frac{\overline{h_3 : \Delta_7, F_2, F_5 \vdash F_6}}{\bullet h_3 : \Delta_7, F_2, F_4 \& F_5 \vdash F_6} \text{ax/ind } \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{h_3 : \Delta_7, F_4, F_1 \oplus F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \oplus F_2), F_4 \& F_5 \vdash F_6} \&_{L1} \rightarrow \frac{\overline{h_3 : \Delta_7, F_2, F_4 \vdash F_6}}{\bullet h_3 : \Delta_7, F_2, F_4 \& F_5 \vdash F_6} \text{ax/ind } \&_{L1}$$

- Case rule \otimes_L

$$\frac{h_3 : \Delta_7, F_4, F_5, F_1 \oplus F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \oplus F_2), F_4 \otimes F_5 \vdash F_6} \otimes_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_2, F_4, F_5 \vdash F_6}}{\bullet h_3 : \Delta_7, F_2, F_4 \otimes F_5 \vdash F_6} \text{ax/ind } \otimes_L$$

- Case rule \oplus_L

$$\frac{h_3 : \Delta_7, F_4, F_1 \oplus F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, F_1 \oplus F_2), F_4 \oplus F_5 \vdash F_6} \oplus_L \rightarrow \frac{\overline{h_3 : \Delta_7, F_2, F_4 \vdash F_6}}{\bullet h_3 : \Delta_7, F_2, F_4 \oplus F_5 \vdash F_6} \text{ax/ind } \oplus_L$$

$$\frac{h_1 : \Delta_2, F_3 \vdash F_5 \quad h_1 : \Delta_2, F_4 \vdash F_5}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_5} \oplus_L \rightarrow \frac{\overline{h_1 : \Delta_2, F_4 \vdash F_5}}{\bullet h_1 : \Delta_2, F_4 \vdash F_5} \text{ax } H$$

- Case rule \multimap_L

$$\frac{h_3 : \Delta_8, F_1 \oplus F_2 \vdash F_5 \quad h_3 : \Delta_4, F_6 \vdash F_7}{\bullet h_3 : (\Delta_8, F_1 \oplus F_2), \Delta_4, F_5 \multimap F_6 \vdash F_7} \multimap_L \rightarrow \frac{\overline{h_3 : \Delta_8, F_2 \vdash F_5}}{\bullet h_3 : \Delta_4, \Delta_8, F_2, F_5 \multimap F_6 \vdash F_7} \text{ax/ind } \multimap_L$$

$$\frac{h_3 : \Delta_4 \vdash F_5 \quad h_3 : \Delta_8, F_6, F_1 \oplus F_2 \vdash F_7}{\bullet h_3 : \Delta_4, (\Delta_8, F_1 \oplus F_2), F_5 \multimap F_6 \vdash F_7} \multimap_L \rightarrow \frac{\overline{h_3 : \Delta_4 \vdash F_5}}{\bullet h_3 : \Delta_4, \Delta_8, F_2, F_5 \multimap F_6 \vdash F_7} \text{ax/ind } \multimap_L$$

- Case rule I

2.20 Status of $\neg\circ_L$: (Left Premise): Non invertible

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule \top

$$\frac{}{\bullet h_5 : \Delta_1, \Delta_2, F_3, \neg\circ F_4 \vdash \top} \top \rightarrow \frac{}{\bullet h_5 : \Delta_1 \vdash F_3} \text{fail}$$

- Case rule $\&_R$

$$\frac{h_5 : \Delta_1, \Delta_2, F_3, \neg\circ F_4 \vdash F_6 \quad h_5 : \Delta_1, \Delta_2, F_3, \neg\circ F_4 \vdash F_7}{\bullet h_5 : \Delta_1, \Delta_2, F_3, \neg\circ F_4 \vdash F_6 \& F_7} \&_R \rightarrow \frac{h_5 : \Delta_1 \vdash F_3}{\bullet h_5 : \Delta_1 \vdash F_3} \text{ax/ind}_H$$

- Case rule $\neg\circ_R$

$$\frac{h_5 : \Delta_1, \Delta_2, F_6, F_3, \neg\circ F_4 \vdash F_7}{\bullet h_5 : \Delta_1, \Delta_2, F_3, \neg\circ F_4 \vdash F_6 \neg\circ F_7} \neg\circ_R \rightarrow \frac{h_5 : \Delta_1 \vdash F_3}{\bullet h_5 : \Delta_1 \vdash F_3} \text{ax/ind}_H$$

- Case rule \oplus_{R_2}

$$\frac{h_5 : \Delta_1, \Delta_2, F_3, \neg\circ F_4 \vdash F_7}{\bullet h_5 : \Delta_1, \Delta_2, F_3, \neg\circ F_4 \vdash F_6 \oplus F_7} \oplus_{R_2} \rightarrow \frac{h_5 : \Delta_1 \vdash F_3}{\bullet h_5 : \Delta_1 \vdash F_3} \text{ax/ind}_H$$

- Case rule \oplus_{R_1}

$$\frac{h_5 : \Delta_1, \Delta_2, F_3, \neg\circ F_4 \vdash F_6}{\bullet h_5 : \Delta_1, \Delta_2, F_3, \neg\circ F_4 \vdash F_6 \oplus F_7} \oplus_{R_1} \rightarrow \frac{h_5 : \Delta_1 \vdash F_3}{\bullet h_5 : \Delta_1 \vdash F_3} \text{ax/ind}_H$$

- Case rule $\mathbf{1}_L$

$$\frac{h_4 : \Delta_1, \Delta_6, F_2, \neg\circ F_3 \vdash F_5}{\bullet h_4 : \mathbf{1}, \Delta_1, \Delta_6, F_2, \neg\circ F_3 \vdash F_5} \mathbf{1}_L \rightarrow \frac{h_4 : \Delta_6 \vdash F_2}{\bullet h_4 : \mathbf{1}, \Delta_6 \vdash F_2} \text{ax/ind}_{\mathbf{1}_L}$$

$$\frac{h_4 : \Delta_1, \Delta_6, F_2, \neg\circ F_3 \vdash F_5}{\bullet h_4 : \mathbf{1}, \Delta_1, \Delta_6, F_2, \neg\circ F_3 \vdash F_5} \mathbf{1}_L \rightarrow \frac{h_4 : \Delta_1 \vdash F_2}{\bullet h_4 : \Delta_1 \vdash F_2} \text{ax/ind}_H$$

- Case rule \otimes_R

$$\frac{h_3 : \Delta_6, \Delta_7, F_1, \neg\circ F_2 \vdash F_4 \quad h_3 : \Delta_8, \Delta_9 \vdash F_5}{\bullet h_3 : (\Delta_6, \Delta_7, F_1, \neg\circ F_2), \Delta_8, \Delta_9 \vdash F_4 \otimes F_5} \otimes_R \rightarrow \frac{}{\bullet h_3 : \Delta_6, \Delta_8 \vdash F_1} \text{fail}$$

$$\frac{h_3 : \Delta_6, \Delta_7 \vdash F_4 \quad h_3 : \Delta_8, \Delta_9, F_1, \neg\circ F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, \Delta_7), \Delta_8, \Delta_9, F_1, \neg\circ F_2 \vdash F_4 \otimes F_5} \otimes_R \rightarrow \frac{}{\bullet h_3 : \Delta_6, \Delta_8 \vdash F_1} \text{fail}$$

- Case rule W

$$\frac{h_4 : \Delta_1, \Delta_7, F_2 \multimap F_3 \vdash F_6}{\bullet h_4 : (\Delta_1, \Delta_7, F_2 \multimap F_3), !F_5 \vdash F_6} W \rightarrow \frac{h_4 : \Delta_7 \vdash F_2}{\bullet h_4 : \Delta_7, !F_5 \vdash F_2} \text{ax/ind } W$$

$$\frac{h_4 : \Delta_1, \Delta_7, F_2 \multimap F_3 \vdash F_6}{\bullet h_4 : (\Delta_1, \Delta_7, F_2 \multimap F_3), !F_5 \vdash F_6} W \rightarrow \frac{h_4 : \Delta_1 \vdash F_2}{\bullet h_4 : \Delta_1 \vdash F_2} \text{ax/ind } H$$

- Case rule C

$$\frac{h_4 : \Delta_1, \Delta_7, !F_5, !F_5, F_2 \multimap F_3 \vdash F_6}{\bullet h_4 : (\Delta_1, \Delta_7, F_2 \multimap F_3), !F_5 \vdash F_6} C \rightarrow \frac{h_4 : \Delta_7 \vdash F_2}{\bullet h_4 : \Delta_7, !F_5 \vdash F_2} \text{ax/ind } W$$

$$\frac{h_4 : \Delta_1, \Delta_7, !F_5, !F_5, F_2 \multimap F_3 \vdash F_6}{\bullet h_4 : (\Delta_1, \Delta_7, F_2 \multimap F_3), !F_5 \vdash F_6} C \rightarrow \frac{h_4 : \Delta_1 \vdash F_2}{\bullet h_4 : \Delta_1 \vdash F_2} \text{ax/ind } H$$

- Case rule $!L$

$$\frac{h_4 : \Delta_1, \Delta_7, F_5, F_2 \multimap F_3 \vdash F_6}{\bullet h_4 : (\Delta_1, \Delta_7, F_2 \multimap F_3), !F_5 \vdash F_6} !L \rightarrow \frac{h_4 : \Delta_7 \vdash F_2}{\bullet h_4 : \Delta_7, !F_5 \vdash F_2} \text{ax/ind } W$$

$$\frac{h_4 : \Delta_1, \Delta_7, F_5, F_2 \multimap F_3 \vdash F_6}{\bullet h_4 : (\Delta_1, \Delta_7, F_2 \multimap F_3), !F_5 \vdash F_6} !L \rightarrow \frac{h_4 : \Delta_1 \vdash F_2}{\bullet h_4 : \Delta_1 \vdash F_2} \text{ax/ind } H$$

- Case rule $\&_{L2}$

$$\frac{h_4 : \Delta_1, \Delta_8, F_6, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \& F_6 \vdash F_7} \&_{L2} \rightarrow \frac{h_4 : \Delta_8, F_6 \vdash F_2}{\bullet h_4 : \Delta_8, F_5 \& F_6 \vdash F_2} \text{ax/ind } \&_{L2}$$

$$\frac{h_4 : \Delta_1, \Delta_8, F_6, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \& F_6 \vdash F_7} \&_{L2} \rightarrow \frac{h_4 : \Delta_1 \vdash F_2}{\bullet h_4 : \Delta_1 \vdash F_2} \text{ax/ind } H$$

- Case rule $\&_{L1}$

$$\frac{h_4 : \Delta_1, \Delta_8, F_5, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \& F_6 \vdash F_7} \&_{L1} \rightarrow \frac{h_4 : \Delta_8, F_5 \vdash F_2}{\bullet h_4 : \Delta_8, F_5 \& F_6 \vdash F_2} \text{ax/ind } \&_{L1}$$

$$\frac{h_4 : \Delta_1, \Delta_8, F_5, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \& F_6 \vdash F_7} \&_{L1} \rightarrow \frac{h_4 : \Delta_1 \vdash F_2}{\bullet h_4 : \Delta_1 \vdash F_2} \text{ax/ind } H$$

- Case rule \otimes_L

$$\frac{h_4 : \Delta_1, \Delta_8, F_5, F_6, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \otimes F_6 \vdash F_7} \otimes_L \rightarrow \frac{h_4 : \Delta_8, F_5, F_6 \vdash F_2}{\bullet h_4 : \Delta_8, F_5 \otimes F_6 \vdash F_2} \text{ax/ind } \otimes_L$$

$$\frac{h_4 : \Delta_1, \Delta_8, F_5, F_6, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \otimes F_6 \vdash F_7} \otimes_L \rightarrow \frac{h_4 : \Delta_1 \vdash F_2}{\bullet h_4 : \Delta_1 \vdash F_2} \text{ax/ind } H$$

- Case rule \oplus_L

$$\frac{h_4 : \Delta_1, \Delta_8, F_5, F_2 \multimap F_3 \vdash F_7 \quad h_4 : \Delta_1, \Delta_8, F_6, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \oplus F_6 \vdash F_7} \oplus_L \rightarrow \frac{\frac{h_4 : \Delta_8, F_5 \vdash F_2}{\bullet h_4 : \Delta_8, F_5 \oplus F_6 \vdash F_2} \text{ax/ind} \quad \frac{h_4 : \Delta_8, F_6 \vdash F_2}{\bullet h_4 : \Delta_8, F_5 \oplus F_6 \vdash F_2} \text{ax/ind}}{\bullet h_4 : \Delta_8, F_5 \oplus F_6 \vdash F_2} \oplus_L$$

$$\frac{h_4 : \Delta_1, \Delta_8, F_5, F_2 \multimap F_3 \vdash F_7 \quad h_4 : \Delta_1, \Delta_8, F_6, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \oplus F_6 \vdash F_7} \oplus_L \rightarrow \frac{\frac{h_4 : \Delta_1 \vdash F_2}{\bullet h_4 : \Delta_1 \vdash F_2} \text{ax/ind}}{\bullet h_4 : \Delta_1 \vdash F_2} \text{H}$$

- Case rule \multimap_L

$$\frac{h_3 : \Delta_7, \Delta_8, F_1 \multimap F_2 \vdash F_4 \quad h_3 : \Delta_9, \Delta_{10}, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8, F_1 \multimap F_2), (\Delta_9, \Delta_{10}), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{}{\bullet h_3 : \Delta_7, \Delta_9, F_4 \multimap F_5 \vdash F_1} \text{fail}$$

$$\frac{h_3 : \Delta_7, \Delta_8 \vdash F_4 \quad h_3 : \Delta_9, \Delta_{10}, F_5, F_1 \multimap F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, F_1 \multimap F_2), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{}{\bullet h_3 : \Delta_7, \Delta_9, F_4 \multimap F_5 \vdash F_1} \text{fail}$$

$$\frac{h_3 : \Delta_7, \Delta_8, F_1 \multimap F_2 \vdash F_4 \quad h_3 : \Delta_9, \Delta_{10}, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8, F_1 \multimap F_2), (\Delta_9, \Delta_{10}), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{}{\bullet h_3 : \Delta_7, \Delta_9 \vdash F_1} \text{fail}$$

$$\frac{h_3 : \Delta_7, \Delta_8 \vdash F_4 \quad h_3 : \Delta_9, \Delta_{10}, F_5, F_1 \multimap F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, F_1 \multimap F_2), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{}{\bullet h_3 : \Delta_7, \Delta_9 \vdash F_1} \text{fail}$$

$$\frac{h_1 : \Delta_5, \Delta_6 \vdash F_2 \quad h_1 : \Delta_7, \Delta_8, F_3 \vdash F_4}{\bullet h_1 : (\Delta_5, \Delta_6), (\Delta_7, \Delta_8), F_2 \multimap F_3 \vdash F_4} \multimap_L \rightarrow \frac{}{\bullet h_1 : \Delta_5, \Delta_7 \vdash F_2} \text{fail}$$

- Case rule I

2.21 Status of \multimap_L (Right Premise): : Non invertible

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule \top

$$\frac{}{\bullet h_5 : \Delta_1, \Delta_2, F_3 \multimap F_4 \vdash \top} \top \rightarrow \frac{}{\bullet h_5 : \Delta_2, F_4 \vdash \top} \top$$

- Case rule $\&_R$

$$\frac{h_5 : \Delta_1, \Delta_2, F_3 \multimap F_4 \vdash F_6 \quad h_5 : \Delta_1, \Delta_2, F_3 \multimap F_4 \vdash F_7}{\bullet h_5 : \Delta_1, \Delta_2, F_3 \multimap F_4 \vdash F_6 \& F_7} \&_R \rightarrow \frac{\frac{h_5 : \Delta_2, F_4 \vdash F_6}{\bullet h_5 : \Delta_2, F_4 \vdash F_6 \& F_7} \text{ax/ind} \quad \frac{h_5 : \Delta_2, F_4 \vdash F_7}{\bullet h_5 : \Delta_2, F_4 \vdash F_6 \& F_7} \text{ax/ind}}{\bullet h_5 : \Delta_2, F_4 \vdash F_6 \& F_7} \&_R$$

- Case rule \multimap_R

$$\frac{h_5 : \Delta_1, \Delta_2, F_6, F_3 \multimap F_4 \vdash F_7}{\bullet h_5 : \Delta_1, \Delta_2, F_3 \multimap F_4 \vdash F_6 \multimap F_7} \multimap_R \rightarrow \frac{\frac{h_5 : \Delta_2, F_4, F_6 \vdash F_7}{\bullet h_5 : \Delta_2, F_4 \vdash F_6 \multimap F_7} \text{ax/ind}}{\bullet h_5 : \Delta_2, F_4 \vdash F_6 \multimap F_7} \multimap_R$$

- Case rule \oplus_{R2}

$$\frac{h_5 : \Delta_1, \Delta_2, F_3 \multimap F_4 \vdash F_7}{\bullet h_5 : \Delta_1, \Delta_2, F_3 \multimap F_4 \vdash F_6 \oplus F_7} \oplus_{R2} \rightarrow \frac{\overline{h_5 : \Delta_2, F_4 \vdash F_7} \text{ ax/ind}}{\bullet h_5 : \Delta_2, F_4 \vdash F_6 \oplus F_7} \oplus_{R2}$$

- Case rule \oplus_{R1}

$$\frac{h_5 : \Delta_1, \Delta_2, F_3 \multimap F_4 \vdash F_6}{\bullet h_5 : \Delta_1, \Delta_2, F_3 \multimap F_4 \vdash F_6 \oplus F_7} \oplus_{R1} \rightarrow \frac{\overline{h_5 : \Delta_2, F_4 \vdash F_6} \text{ ax/ind}}{\bullet h_5 : \Delta_2, F_4 \vdash F_6 \oplus F_7} \oplus_{R1}$$

- Case rule 1_L

$$\frac{h_4 : \Delta_1, \Delta_6, F_2 \multimap F_3 \vdash F_5}{\bullet h_4 : 1, \Delta_1, \Delta_6, F_2 \multimap F_3 \vdash F_5} 1_L \rightarrow \frac{\overline{h_4 : \Delta_1, F_3 \vdash F_5} \text{ ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash F_5} H$$

$$\frac{h_4 : \Delta_1, \Delta_6, F_2 \multimap F_3 \vdash F_5}{\bullet h_4 : 1, \Delta_1, \Delta_6, F_2 \multimap F_3 \vdash F_5} 1_L \rightarrow \frac{\overline{h_4 : \Delta_6, F_3 \vdash F_5} \text{ ax/ind}}{\bullet h_4 : 1, \Delta_6, F_3 \vdash F_5} 1_L$$

- Case rule \otimes_R

$$\frac{h_3 : \Delta_6, \Delta_7, F_1 \multimap F_2 \vdash F_4 \quad h_3 : \Delta_8, \Delta_9 \vdash F_5}{\bullet h_3 : (\Delta_6, \Delta_7, F_1 \multimap F_2), \Delta_8, \Delta_9 \vdash F_4 \otimes F_5} \otimes_R \rightarrow \frac{\overline{\bullet h_3 : \Delta_7, \Delta_9, F_2 \vdash F_4 \otimes F_5} \text{ fail}}{\bullet h_3 : \Delta_7, \Delta_9, F_2 \vdash F_4 \otimes F_5} \text{ fail}$$

$$\frac{h_3 : \Delta_6, \Delta_7 \vdash F_4 \quad h_3 : \Delta_8, \Delta_9, F_1 \multimap F_2 \vdash F_5}{\bullet h_3 : (\Delta_6, \Delta_7), \Delta_8, \Delta_9, F_1 \multimap F_2 \vdash F_4 \otimes F_5} \otimes_R \rightarrow \frac{\overline{\bullet h_3 : \Delta_7, \Delta_9, F_2 \vdash F_4 \otimes F_5} \text{ fail}}{\bullet h_3 : \Delta_7, \Delta_9, F_2 \vdash F_4 \otimes F_5} \text{ fail}$$

- Case rule W

$$\frac{h_4 : \Delta_1, \Delta_7, F_2 \multimap F_3 \vdash F_6}{\bullet h_4 : (\Delta_1, \Delta_7, F_2 \multimap F_3), !F_5 \vdash F_6} W \rightarrow \frac{\overline{h_4 : \Delta_1, F_3 \vdash F_6} \text{ ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash F_6} H$$

$$\frac{h_4 : \Delta_1, \Delta_7, F_2 \multimap F_3 \vdash F_6}{\bullet h_4 : (\Delta_1, \Delta_7, F_2 \multimap F_3), !F_5 \vdash F_6} W \rightarrow \frac{\overline{h_4 : \Delta_7, F_3 \vdash F_6} \text{ ax/ind}}{\bullet h_4 : \Delta_7, F_3, !F_5 \vdash F_6} W$$

- Case rule C

$$\frac{h_4 : \Delta_1, \Delta_7, !F_5, !F_5, F_2 \multimap F_3 \vdash F_6}{\bullet h_4 : (\Delta_1, \Delta_7, F_2 \multimap F_3), !F_5 \vdash F_6} C \rightarrow \frac{\overline{h_4 : \Delta_1, F_3 \vdash F_6} \text{ ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash F_6} H$$

$$\frac{h_4 : \Delta_1, \Delta_7, !F_5, !F_5, F_2 \multimap F_3 \vdash F_6}{\bullet h_4 : (\Delta_1, \Delta_7, F_2 \multimap F_3), !F_5 \vdash F_6} C \rightarrow \frac{\overline{h_4 : \Delta_7, F_3 \vdash F_6} \text{ ax/ind}}{\bullet h_4 : \Delta_7, F_3, !F_5 \vdash F_6} W$$

- Case rule $!L$

$$\frac{h_4 : \Delta_1, \Delta_7, F_5, F_2 \multimap F_3 \vdash F_6}{\bullet h_4 : (\Delta_1, \Delta_7, F_2 \multimap F_3), !F_5 \vdash F_6} !L \rightarrow \frac{\overline{h_4 : \Delta_1, F_3 \vdash F_6} \text{ ax/ind}}{\bullet h_4 : \Delta_1, F_3 \vdash F_6} H$$

$$\frac{h_4 : \Delta_1, \Delta_7, F_5, F_2 \multimap F_3 \vdash F_6}{\bullet h_4 : (\Delta_1, \Delta_7, F_2 \multimap F_3), !F_5 \vdash F_6} !L \rightarrow \frac{\overline{h_4 : \Delta_7, F_3 \vdash F_6} \text{ ax/ind}}{\bullet h_4 : \Delta_7, F_3, !F_5 \vdash F_6} W$$

- Case rule $\&_{L2}$

$$\frac{h_4 : \Delta_1, \Delta_8, F_6, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \& F_6 \vdash F_7} \&_{L2} \rightarrow \frac{\overline{h_4 : \Delta_1, F_3 \vdash F_7}}{\bullet h_4 : \Delta_1, F_3 \vdash F_7} \text{ax/ind} \text{ H}$$

$$\frac{h_4 : \Delta_1, \Delta_8, F_6, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \& F_6 \vdash F_7} \&_{L2} \rightarrow \frac{\overline{h_4 : \Delta_8, F_3, F_6 \vdash F_7}}{\bullet h_4 : \Delta_8, F_3, F_5 \& F_6 \vdash F_7} \text{ax/ind} \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{h_4 : \Delta_1, \Delta_8, F_5, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \& F_6 \vdash F_7} \&_{L1} \rightarrow \frac{\overline{h_4 : \Delta_1, F_3 \vdash F_7}}{\bullet h_4 : \Delta_1, F_3 \vdash F_7} \text{ax/ind} \text{ H}$$

$$\frac{h_4 : \Delta_1, \Delta_8, F_5, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \& F_6 \vdash F_7} \&_{L1} \rightarrow \frac{\overline{h_4 : \Delta_8, F_3, F_5 \vdash F_7}}{\bullet h_4 : \Delta_8, F_3, F_5 \& F_6 \vdash F_7} \text{ax/ind} \&_{L1}$$

- Case rule \otimes_L

$$\frac{h_4 : \Delta_1, \Delta_8, F_5, F_6, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \otimes F_6 \vdash F_7} \otimes_L \rightarrow \frac{\overline{h_4 : \Delta_1, F_3 \vdash F_7}}{\bullet h_4 : \Delta_1, F_3 \vdash F_7} \text{ax/ind} \text{ H}$$

$$\frac{h_4 : \Delta_1, \Delta_8, F_5, F_6, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \otimes F_6 \vdash F_7} \otimes_L \rightarrow \frac{\overline{h_4 : \Delta_8, F_3, F_5, F_6 \vdash F_7}}{\bullet h_4 : \Delta_8, F_3, F_5 \otimes F_6 \vdash F_7} \text{ax/ind} \otimes_L$$

- Case rule \oplus_L

$$\frac{h_4 : \Delta_1, \Delta_8, F_5, F_2 \multimap F_3 \vdash F_7 \quad h_4 : \Delta_1, \Delta_8, F_6, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \oplus F_6 \vdash F_7} \oplus_L \rightarrow \frac{\overline{h_4 : \Delta_1, F_3 \vdash F_7}}{\bullet h_4 : \Delta_1, F_3 \vdash F_7} \text{ax/ind} \text{ H}$$

$$\frac{h_4 : \Delta_1, \Delta_8, F_5, F_2 \multimap F_3 \vdash F_7 \quad h_4 : \Delta_1, \Delta_8, F_6, F_2 \multimap F_3 \vdash F_7}{\bullet h_4 : (\Delta_1, \Delta_8, F_2 \multimap F_3), F_5 \oplus F_6 \vdash F_7} \oplus_L \rightarrow \frac{\overline{h_4 : \Delta_8, F_3, F_5 \vdash F_7} \quad \overline{h_4 : \Delta_8, F_3, F_6 \vdash F_7}}{\bullet h_4 : \Delta_8, F_3, F_5 \oplus F_6 \vdash F_7} \text{ax/ind} \oplus_L$$

- Case rule \multimap_L

$$\frac{h_3 : \Delta_7, \Delta_8, F_1 \multimap F_2 \vdash F_4 \quad h_3 : \Delta_9, \Delta_{10}, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8, F_1 \multimap F_2), (\Delta_9, \Delta_{10}), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\overline{\bullet h_3 : \Delta_8, \Delta_{10}, F_2 \vdash F_6}}{\bullet h_3 : \Delta_8, \Delta_{10}, F_2 \vdash F_6} \text{fail}$$

$$\frac{h_3 : \Delta_7, \Delta_8 \vdash F_4 \quad h_3 : \Delta_9, \Delta_{10}, F_5, F_1 \multimap F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, F_1 \multimap F_2), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\overline{\bullet h_3 : \Delta_8, \Delta_{10}, F_2 \vdash F_6}}{\bullet h_3 : \Delta_8, \Delta_{10}, F_2 \vdash F_6} \text{fail}$$

$$\frac{h_3 : \Delta_7, \Delta_8, F_1 \multimap F_2 \vdash F_4 \quad h_3 : \Delta_9, \Delta_{10}, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8, F_1 \multimap F_2), (\Delta_9, \Delta_{10}), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\overline{\bullet h_3 : \Delta_8, \Delta_{10}, F_2, F_4 \multimap F_5 \vdash F_6}}{\bullet h_3 : \Delta_8, \Delta_{10}, F_2, F_4 \multimap F_5 \vdash F_6} \text{fail}$$

$$\frac{h_3 : \Delta_7, \Delta_8 \vdash F_4 \quad h_3 : \Delta_9, \Delta_{10}, F_5, F_1 \multimap F_2 \vdash F_6}{\bullet h_3 : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, F_1 \multimap F_2), F_4 \multimap F_5 \vdash F_6} \multimap_L \rightarrow \frac{\overline{\bullet h_3 : \Delta_8, \Delta_{10}, F_2, F_4 \multimap F_5 \vdash F_6}}{\bullet h_3 : \Delta_8, \Delta_{10}, F_2, F_4 \multimap F_5 \vdash F_6} \text{fail}$$

$$\frac{h_1 : \Delta_5, \Delta_6 \vdash F_2 \quad h_1 : \Delta_7, \Delta_8, F_3 \vdash F_4}{\bullet h_1 : (\Delta_5, \Delta_6), (\Delta_7, \Delta_8), F_2 \multimap F_3 \vdash F_4} \multimap_L \rightarrow \frac{\overline{\bullet h_1 : \Delta_6, \Delta_8, F_3 \vdash F_4}}{\bullet h_1 : \Delta_6, \Delta_8, F_3 \vdash F_4} \text{fail}$$

- Case rule I

2.22 Status of I : : Invertible

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule \top
- Case rule $\&_R$
- Case rule \multimap_R
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L_2}$
- Case rule $\&_{L_1}$
- Case rule \otimes_L
- Case rule \oplus_L
- Case rule \multimap_L
- Case rule I

$$\frac{\bullet h_1 : p(n_2) \vdash p(n_2)}{I} \rightarrow \text{trivial}$$

3 Identity-Expansion

$$\frac{}{- : \top \vdash \top} \top$$

$$\frac{\frac{}{- : F_0 \vdash F_0} \text{IH} \quad \frac{}{- : F_1 \vdash F_1} \text{IH}}{- : F_0 \vdash F_0 \oplus F_1} \oplus_{R_1} \quad \frac{}{- : F_1 \vdash F_1} \text{IH} \quad \frac{}{- : F_1 \vdash F_0 \oplus F_1} \oplus_{R_2}}{- : F_0 \oplus F_1 \vdash F_0 \oplus F_1} \oplus_L$$

$$\frac{\frac{}{- : F_0 \vdash F_0} \text{IH} \quad \frac{}{- : F_1 \vdash F_1} \text{IH}}{- : F_0, F_0 \multimap F_1 \vdash F_1} \multimap_L \quad \frac{}{- : F_0, F_0 \multimap F_1 \vdash F_1} \multimap_R}{- : F_0 \multimap F_1 \vdash F_0 \multimap F_1} \multimap_R$$

$$\frac{\frac{}{- : F_0 \vdash F_0} \text{IH} \quad \frac{}{- : F_1 \vdash F_1} \text{IH}}{- : F_0, F_1 \vdash F_0 \otimes F_1} \otimes_R \quad \frac{}{- : F_0, F_1 \vdash F_0 \otimes F_1} \otimes_L}{- : F_0 \otimes F_1 \vdash F_0 \otimes F_1} \otimes_L$$

$$\frac{\frac{}{- : F_0 \vdash F_0} \text{IH} \quad \frac{}{- : F_1 \vdash F_1} \text{IH}}{- : F_0 \& F_1 \vdash F_0} \&_{L1} \quad \frac{}{- : F_1 \vdash F_1} \text{IH} \quad \frac{}{- : F_0 \& F_1 \vdash F_1} \&_{L2}}{- : F_0 \& F_1 \vdash F_0 \& F_1} \&_R$$

$$\frac{}{- : * \vdash \mathbf{1}} \mathbf{1}_R$$

$$\frac{}{- : \mathbf{1} \vdash \mathbf{1}} \mathbf{1}_L$$

4 Weakening on bang: $\Gamma \vdash !F$ implies $\Gamma \vdash F$.

- Case(s) rule $!R$

$$\frac{h_1 : !\top_2 \vdash F_3}{\bullet h_1 : !\top_2 \vdash !F_3} !R \rightarrow \frac{\overline{h_1 : !\top_2 \vdash F_3}^{\text{ax}}}{\bullet h_1 : !\top_2 \vdash F_3} \text{H}$$

- Case(s) rule $\mathbf{1}_R$
- Case(s) rule \top
- Case(s) rule $\&_R$
- Case(s) rule $\neg\circ_R$
- Case(s) rule \oplus_{R_2}
- Case(s) rule \oplus_{R_1}
- Case(s) rule $\mathbf{1}_L$

$$\frac{h_2 : \Delta_3 \vdash !F_1}{\bullet h_2 : \mathbf{1}, \Delta_3 \vdash !F_1} \mathbf{1}_L \rightarrow \frac{\overline{h_2 : \Delta_3 \vdash !F_1}^{\text{ax}}}{h_2 : \Delta_3 \vdash F_1} \text{IH}}{\bullet h_2 : \mathbf{1}, \Delta_3 \vdash F_1} \mathbf{1}_L$$

- Case(s) rule \otimes_R
- Case(s) rule W

$$\frac{h_2 : \Delta_3 \vdash !F_1}{\bullet h_2 : \Delta_3, !F_4 \vdash !F_1} W \rightarrow \frac{\overline{h_2 : \Delta_3 \vdash !F_1}^{\text{ax}}}{h_2 : \Delta_3 \vdash F_1} \text{IH}}{\bullet h_2 : \Delta_3, !F_4 \vdash F_1} W$$

- Case(s) rule C

$$\frac{h_2 : \Delta_3, !F_4, !F_4 \vdash !F_1}{\bullet h_2 : \Delta_3, !F_4 \vdash !F_1} C \rightarrow \frac{\overline{h_2 : \Delta_3, !F_4, !F_4 \vdash !F_1}^{\text{ax}}}{h_2 : \Delta_3, !F_4, !F_4 \vdash F_1} \text{IH}}{\bullet h_2 : \Delta_3, !F_4 \vdash F_1} C$$

- Case(s) rule $!L$

$$\frac{h_2 : \Delta_3, F_4 \vdash !F_1}{\bullet h_2 : \Delta_3, !F_4 \vdash !F_1} !L \rightarrow \frac{\overline{h_2 : \Delta_3, F_4 \vdash !F_1}^{\text{ax}}}{h_2 : \Delta_3, F_4 \vdash F_1} \text{IH}}{\bullet h_2 : \Delta_3, !F_4 \vdash F_1} !L$$

- Case(s) rule $\&_{L2}$

$$\frac{h_2 : \Delta_3, F_5 \vdash !F_1}{\bullet h_2 : \Delta_3, F_4 \& F_5 \vdash !F_1} \&_{L2} \rightarrow \frac{\overline{h_2 : \Delta_3, F_5 \vdash !F_1}^{\text{ax}}}{h_2 : \Delta_3, F_5 \vdash F_1} \text{IH}}{\bullet h_2 : \Delta_3, F_4 \& F_5 \vdash F_1} \&_{L2}$$

- Case(s) rule $\&_{L1}$

$$\frac{h_2 : \Delta_3, F_4 \vdash !F_1}{\bullet h_2 : \Delta_3, F_4 \& F_5 \vdash !F_1} \&_{L1} \rightarrow \frac{\overline{h_2 : \Delta_3, F_4 \vdash !F_1}^{\text{ax}}}{h_2 : \Delta_3, F_4 \vdash F_1} \text{IH}}{\bullet h_2 : \Delta_3, F_4 \& F_5 \vdash F_1} \&_{L1}$$

- Case(s) rule \otimes_L

$$\frac{\frac{h_2 : \Delta_3, F_4, F_5 \vdash !F_1}{\bullet h_2 : \Delta_3, F_4 \otimes F_5 \vdash !F_1} \otimes_L}{\bullet h_2 : \Delta_3, F_4 \otimes F_5 \vdash F_1} \rightarrow \frac{\frac{\frac{h_2 : \Delta_3, F_4, F_5 \vdash !F_1}{\bullet h_2 : \Delta_3, F_4, F_5 \vdash F_1} \text{IH}}{\bullet h_2 : \Delta_3, F_4 \otimes F_5 \vdash F_1} \otimes_L}{\bullet h_2 : \Delta_3, F_4 \otimes F_5 \vdash F_1} \text{ax}$$

- Case(s) rule \oplus_L

$$\frac{\frac{h_2 : \Delta_3, F_4 \vdash !F_1}{\bullet h_2 : \Delta_3, F_4 \oplus F_5 \vdash !F_1} \oplus_L \quad \frac{h_2 : \Delta_3, F_5 \vdash !F_1}{\bullet h_2 : \Delta_3, F_4 \oplus F_5 \vdash !F_1} \oplus_L}{\bullet h_2 : \Delta_3, F_4 \oplus F_5 \vdash F_1} \rightarrow \frac{\frac{\frac{h_2 : \Delta_3, F_4 \vdash !F_1}{\bullet h_2 : \Delta_3, F_4 \vdash F_1} \text{IH}}{\bullet h_2 : \Delta_3, F_4 \oplus F_5 \vdash F_1} \oplus_L \quad \frac{\frac{h_2 : \Delta_3, F_5 \vdash !F_1}{\bullet h_2 : \Delta_3, F_5 \vdash F_1} \text{IH}}{\bullet h_2 : \Delta_3, F_4 \oplus F_5 \vdash F_1} \oplus_L}{\bullet h_2 : \Delta_3, F_4 \oplus F_5 \vdash F_1} \oplus_L \text{ax}}$$

- Case(s) rule $\neg\circ_L$

$$\frac{\frac{h_2 : \Delta_3 \vdash F_5}{\bullet h_2 : \Delta_3, \Delta_4, F_5 \neg\circ F_6 \vdash !F_1} \neg\circ_L \quad \frac{h_2 : \Delta_4, F_6 \vdash !F_1}{\bullet h_2 : \Delta_3, \Delta_4, F_5 \neg\circ F_6 \vdash !F_1} \neg\circ_L}{\bullet h_2 : \Delta_3, \Delta_4, F_5 \neg\circ F_6 \vdash F_1} \rightarrow \frac{\frac{\frac{h_2 : \Delta_3 \vdash F_5}{\bullet h_2 : \Delta_3 \vdash F_5} \text{ax}}{\bullet h_2 : \Delta_3, \Delta_4, F_5 \neg\circ F_6 \vdash F_1} \neg\circ_L \quad \frac{\frac{h_2 : \Delta_4, F_6 \vdash !F_1}{\bullet h_2 : \Delta_4, F_6 \vdash F_1} \text{IH}}{\bullet h_2 : \Delta_3, \Delta_4, F_5 \neg\circ F_6 \vdash F_1} \neg\circ_L}{\bullet h_2 : \Delta_3, \Delta_4, F_5 \neg\circ F_6 \vdash F_1} \neg\circ_L \text{ax}}$$

- Case(s) rule I

5 Cut-Elimination

5.1 Status of !R: OK

- Case rule !R

$$\begin{array}{c}
 \frac{h_1 : !\Upsilon 2 \vdash F_5}{\bullet h_1 : !\Upsilon 2 \vdash !F_5} !R \quad \frac{h_6 : !\Upsilon 4, !F_5 \vdash F_7}{\bullet h_6 : !\Upsilon 4, !F_5 \vdash !F_7} !R}{- : !\Upsilon 2, !\Upsilon 4 \vdash !F_7} \text{Cut} \\
 \rightarrow \\
 \frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_5}{- : !\Upsilon 2, !\Upsilon 4 \vdash F_7} \text{ax/W} \quad \frac{h_6 : !\Upsilon 4, !F_5 \vdash F_7}{- : !\Upsilon 2, !\Upsilon 4 \vdash !F_7} \text{ax/W}}{- : !\Upsilon 2, !\Upsilon 4 \vdash !F_7} \text{hCut} \\
 \text{!R}
 \end{array}$$

- Case rule $\mathbf{1}_R$

- Case rule \top

$$\begin{array}{c}
 \frac{h_1 : !\Upsilon 2 \vdash F_5}{\bullet h_1 : !\Upsilon 2 \vdash !F_5} !R \quad \frac{}{\bullet h_6 : \Delta_4, !F_5 \vdash \top} \top}{- : !\Upsilon 2, \Delta_4 \vdash \top} \text{Cut} \\
 \rightarrow \\
 - : !\Upsilon 2, \Delta_4 \vdash \top \quad \top
 \end{array}$$

- Case rule $\&_R$

$$\begin{array}{c}
 \frac{h_1 : !\Upsilon 2 \vdash F_5}{\bullet h_1 : !\Upsilon 2 \vdash !F_5} !R \quad \frac{h_6 : \Delta_4, !F_5 \vdash F_7 \quad h_6 : \Delta_4, !F_5 \vdash F_8}{\bullet h_6 : \Delta_4, !F_5 \vdash F_7 \& F_8} \&_R}{- : !\Upsilon 2, \Delta_4 \vdash F_7 \& F_8} \text{Cut} \\
 \rightarrow \\
 \frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_5}{- : !\Upsilon 2, \Delta_4 \vdash F_7} \text{ax/W} \quad \frac{h_6 : \Delta_4, !F_5 \vdash F_7}{- : !\Upsilon 2, \Delta_4 \vdash F_7} \text{ax/W} \quad \frac{\bullet h_1 : !\Upsilon 2 \vdash !F_5}{- : !\Upsilon 2, \Delta_4 \vdash F_8} \text{ax/W} \quad \frac{h_6 : \Delta_4, !F_5 \vdash F_8}{- : !\Upsilon 2, \Delta_4 \vdash F_8} \text{ax/W}}{- : !\Upsilon 2, \Delta_4 \vdash F_7 \& F_8} \text{hCut} \quad \&_R
 \end{array}$$

- Case rule $\neg\circ_R$

$$\begin{array}{c}
 \frac{h_1 : !\Upsilon 2 \vdash F_5}{\bullet h_1 : !\Upsilon 2 \vdash !F_5} !R \quad \frac{h_6 : \Delta_4, F_7, !F_5 \vdash F_8}{\bullet h_6 : \Delta_4, !F_5 \vdash F_7 \neg\circ F_8} \neg\circ_R}{- : !\Upsilon 2, \Delta_4 \vdash F_7 \neg\circ F_8} \text{Cut} \\
 \rightarrow \\
 \frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_5}{- : !\Upsilon 2, \Delta_4, F_7 \vdash F_8} \text{ax/W} \quad \frac{h_6 : \Delta_4, F_7, !F_5 \vdash F_8}{- : !\Upsilon 2, \Delta_4, F_7 \vdash F_8} \text{ax/W}}{- : !\Upsilon 2, \Delta_4 \vdash F_7 \neg\circ F_8} \text{hCut} \\
 \neg\circ_R
 \end{array}$$

- Case rule \oplus_{R_2}

$$\begin{array}{c}
 \frac{h_1 : !\Upsilon 2 \vdash F_5}{\bullet h_1 : !\Upsilon 2 \vdash !F_5} !R \quad \frac{h_6 : \Delta_4, !F_5 \vdash F_8}{\bullet h_6 : \Delta_4, !F_5 \vdash F_7 \oplus F_8} \oplus_{R_2}}{- : !\Upsilon 2, \Delta_4 \vdash F_7 \oplus F_8} \text{Cut} \\
 \rightarrow \\
 \frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_5}{- : !\Upsilon 2, \Delta_4 \vdash F_8} \text{ax/W} \quad \frac{h_6 : \Delta_4, !F_5 \vdash F_8}{- : !\Upsilon 2, \Delta_4 \vdash F_7 \oplus F_8} \text{ax/W}}{- : !\Upsilon 2, \Delta_4 \vdash F_7 \oplus F_8} \text{hCut} \\
 \oplus_{R_2}
 \end{array}$$

- Case rule \oplus_{R_1}

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_5}{\bullet h_1 : !\Upsilon 2 \vdash !F_5} !R \quad \frac{h_6 : \Delta_4, !F_5 \vdash F_7}{\bullet h_6 : \Delta_4, !F_5 \vdash F_7 \oplus F_8} \oplus_{R1} \\
\hline
- : !\Upsilon 2, \Delta_4 \vdash F_7 \oplus F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : !\Upsilon 2 \vdash F_5}{\bullet h_1 : !\Upsilon 2 \vdash !F_5} \text{ax/W} \quad \frac{h_6 : \Delta_4, !F_5 \vdash F_7}{h_6 : \Delta_4, !F_5 \vdash F_7 \oplus F_8} \text{ax/W}}{- : !\Upsilon 2, \Delta_4 \vdash F_7} \text{hCut} \\
\hline
- : !\Upsilon 2, \Delta_4 \vdash F_7 \oplus F_8 \quad \oplus_{R1}
\end{array}$$

- Case rule 1_L

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_7, !F_4 \vdash F_6}{\bullet h_5 : (1, \Delta_7), !F_4 \vdash F_6} 1_L \\
\hline
- : !\Upsilon 2, 1, \Delta_7 \vdash F_6 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} \text{ax/W} \quad \frac{h_5 : 1, \Delta_7, !F_4 \vdash F_6}{h_5 : 1, \Delta_7, !F_4 \vdash F_6} \text{ax/W}}{- : 1, !\Upsilon 2, \Delta_7 \vdash F_6} \text{hCut}
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_9, !F_4 \vdash F_7 \quad h_5 : \Delta_6 \vdash F_8}{\bullet h_5 : (\Delta_6, \Delta_9), !F_4 \vdash F_7 \otimes F_8} \otimes_R \\
\hline
- : !\Upsilon 2, \Delta_6, \Delta_9 \vdash F_7 \otimes F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} \text{ax/W} \quad \frac{h_5 : \Delta_9, !F_4 \vdash F_7}{h_5 : \Delta_9, !F_4 \vdash F_7} \text{ax/W}}{- : !\Upsilon 2, \Delta_9 \vdash F_7} \text{hCut} \quad \frac{- : \Delta_6 \vdash F_8}{- : \Delta_6 \vdash F_8} \otimes_R \\
\hline
- : !\Upsilon 2, \Delta_6, \Delta_9 \vdash F_7 \otimes F_8 \quad \otimes_R \\
\rightarrow \\
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_6 \vdash F_7 \quad h_5 : \Delta_9, !F_4 \vdash F_8}{\bullet h_5 : (\Delta_6, \Delta_9), !F_4 \vdash F_7 \otimes F_8} \otimes_R \\
\hline
- : !\Upsilon 2, \Delta_6, \Delta_9 \vdash F_7 \otimes F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_6 \vdash F_7}{- : \Delta_6 \vdash F_7} \text{ax/W} \quad \frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} \text{ax/W} \quad \frac{h_5 : \Delta_9, !F_4 \vdash F_8}{h_5 : \Delta_9, !F_4 \vdash F_8} \text{ax/W}}{- : !\Upsilon 2, \Delta_9 \vdash F_8} \text{hCut} \\
\hline
- : !\Upsilon 2, \Delta_6, \Delta_9 \vdash F_7 \otimes F_8 \quad \otimes_R
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_8, !F_4 \vdash F_7}{\bullet h_5 : (\Delta_8, !F_6), !F_4 \vdash F_7} W \\
\hline
- : !\Upsilon 2, \Delta_8, !F_6 \vdash F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} \text{ax/W} \quad \frac{h_5 : \Delta_8, !F_4, !F_6 \vdash F_7}{h_5 : \Delta_8, !F_4, !F_6 \vdash F_7} \text{ax/W}}{- : !\Upsilon 2, \Delta_8, !F_6 \vdash F_7} \text{hCut} \\
\rightarrow \\
\frac{h_1 : !\Upsilon 2 \vdash F_6}{\bullet h_1 : !\Upsilon 2 \vdash !F_6} !R \quad \frac{h_4 : \Delta_5 \vdash F_7}{\bullet h_4 : \Delta_5, !F_6 \vdash F_7} W \\
\hline
- : !\Upsilon 2, \Delta_5 \vdash F_7 \quad \text{Cut} \\
\rightarrow \\
- : !\Upsilon 2, \Delta_5 \vdash F_7 \quad \text{ax/W}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_8, !F_4, !F_6 \vdash F_7}{\bullet h_5 : (\Delta_8, !F_6), !F_4 \vdash F_7} C \\
\hline
- : !\Upsilon 2, \Delta_8, !F_6 \vdash F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} \text{ax/W} \quad \frac{h_5 : \Delta_8, !F_4, !F_6 \vdash F_7}{h_5 : \Delta_8, !F_4, !F_6 \vdash F_7} \text{ax/W}}{- : !\Upsilon 2, \Delta_8, !F_6 \vdash F_7} \text{hCut} \\
\hline
- : !\Upsilon 2, \Delta_8, !F_6 \vdash F_7 \quad C
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_6}{\bullet h_1 : !\Upsilon 2 \vdash !F_6} !R \quad \frac{h_4 : \Delta_5, !F_6, !F_6 \vdash F_7}{\bullet h_4 : \Delta_5, !F_6 \vdash F_7} C \\
\hline
- : !\Upsilon 2, \Delta_5 \vdash F_7 \\
\rightarrow \\
\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_6 \quad ax/W \quad \frac{h_4 : \Delta_5, !F_6, !F_6 \vdash F_7}{- : !\Upsilon 2, \Delta_5 \vdash F_7} mCut}{- : !\Upsilon 2, \Delta_5 \vdash F_7} ax/W
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_8, F_6, !F_4 \vdash F_7}{\bullet h_5 : (\Delta_8, !F_6), !F_4 \vdash F_7} !L \\
\hline
- : !\Upsilon 2, \Delta_8, !F_6 \vdash F_7 \\
\rightarrow \\
\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4 \quad ax/W \quad \frac{h_5 : \Delta_8, F_6, !F_4 \vdash F_7}{- : !\Upsilon 2, \Delta_8, F_6 \vdash F_7} hCut}{- : !\Upsilon 2, \Delta_8, !F_6 \vdash F_7} !L \\
\rightarrow \\
\frac{h_1 : !\Upsilon 2 \vdash F_6}{\bullet h_1 : !\Upsilon 2 \vdash !F_6} !R \quad \frac{h_4 : \Delta_5, F_6 \vdash F_7}{\bullet h_4 : \Delta_5, !F_6 \vdash F_7} !L \\
\hline
- : !\Upsilon 2, \Delta_5 \vdash F_7 \\
\rightarrow \\
\frac{- : !\Upsilon 2 \vdash F_6 \quad ax/W \quad \frac{- : \Delta_5, F_6 \vdash F_7}{- : !\Upsilon 2, \Delta_5 \vdash F_7} sCut}{- : !\Upsilon 2, \Delta_5 \vdash F_7} ax/W
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_9, F_7, !F_4 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \& F_7), !F_4 \vdash F_8} \&_{L2} \\
\hline
- : !\Upsilon 2, \Delta_9, F_6 \& F_7 \vdash F_8 \\
\rightarrow \\
\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4 \quad ax/W \quad \frac{h_5 : \Delta_9, F_7, !F_4 \vdash F_8}{- : !\Upsilon 2, \Delta_9, F_7 \vdash F_8} hCut}{- : !\Upsilon 2, \Delta_9, F_6 \& F_7 \vdash F_8} \&_{L2}
\end{array}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_9, F_6, !F_4 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \& F_7), !F_4 \vdash F_8} \&_{L1} \\
\hline
- : !\Upsilon 2, \Delta_9, F_6 \& F_7 \vdash F_8 \\
\rightarrow \\
\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4 \quad ax/W \quad \frac{h_5 : \Delta_9, F_6, !F_4 \vdash F_8}{- : !\Upsilon 2, \Delta_9, F_6 \vdash F_8} hCut}{- : !\Upsilon 2, \Delta_9, F_6 \& F_7 \vdash F_8} \&_{L1}
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_9, F_6, F_7, !F_4 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \otimes F_7), !F_4 \vdash F_8} \otimes_L \\
\hline
- : !\Upsilon 2, \Delta_9, F_6 \otimes F_7 \vdash F_8 \\
\rightarrow \\
\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4 \quad ax/W \quad \frac{h_5 : \Delta_9, F_6, F_7, !F_4 \vdash F_8}{- : !\Upsilon 2, \Delta_9, F_6, F_7 \vdash F_8} hCut}{- : !\Upsilon 2, \Delta_9, F_6 \otimes F_7 \vdash F_8} \otimes_L
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_9, F_6, !F_4 \vdash F_8 \quad h_5 : \Delta_9, F_7, !F_4 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \oplus F_7), !F_4 \vdash F_8} \oplus_L}{- : !\Upsilon 2, \Delta_9, F_6 \oplus F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : !\Upsilon 2 \vdash !F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_6, !F_4 \vdash F_8}{h_5 : \Delta_9, F_7, !F_4 \vdash F_8} \text{ax/W}}{- : !\Upsilon 2, \Delta_9, F_6 \vdash F_8} \text{hCut} \quad \frac{\frac{h_1 : !\Upsilon 2 \vdash !F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} \text{ax/W} \quad \frac{h_5 : \Delta_9, F_7, !F_4 \vdash F_8}{h_5 : \Delta_9, F_7, !F_4 \vdash F_8} \text{ax/W}}{- : !\Upsilon 2, \Delta_9, F_7 \vdash F_8} \text{hCut}}{- : !\Upsilon 2, \Delta_9, F_6 \oplus F_7 \vdash F_8} \oplus_L
\end{array}$$

- Case rule $\neg\circ_L$

$$\begin{array}{c}
\frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_{10}, !F_4 \vdash F_7 \quad h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : (\Delta_6, \Delta_{10}, F_7 \neg\circ F_8), !F_4 \vdash F_9} \neg\circ_L}{- : !\Upsilon 2, \Delta_6, \Delta_{10}, F_7 \neg\circ F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : !\Upsilon 2 \vdash !F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} \text{ax/W} \quad \frac{h_5 : \Delta_{10}, !F_4 \vdash F_7}{h_5 : \Delta_{10}, !F_4 \vdash F_7} \text{ax/W}}{- : !\Upsilon 2, \Delta_{10} \vdash F_7} \text{hCut} \quad \frac{- : \Delta_6, F_8 \vdash F_9}{- : \Delta_6, F_8 \vdash F_9} \text{ax/W}}{- : !\Upsilon 2, \Delta_{10}, \Delta_6, F_7 \neg\circ F_8 \vdash F_9} \neg\circ_L \\
\frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_5 : \Delta_6 \vdash F_7 \quad h_5 : \Delta_{10}, F_8, !F_4 \vdash F_9}{\bullet h_5 : (\Delta_6, \Delta_{10}, F_7 \neg\circ F_8), !F_4 \vdash F_9} \neg\circ_L}{- : !\Upsilon 2, \Delta_6, \Delta_{10}, F_7 \neg\circ F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_6 \vdash F_7}{- : \Delta_6 \vdash F_7} \text{ax/W} \quad \frac{\frac{h_1 : !\Upsilon 2 \vdash !F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} \text{ax/W} \quad \frac{h_5 : \Delta_{10}, F_8, !F_4 \vdash F_9}{h_5 : \Delta_{10}, F_8, !F_4 \vdash F_9} \text{ax/W}}{- : !\Upsilon 2, \Delta_{10}, F_8 \vdash F_9} \text{hCut}}{- : !\Upsilon 2, \Delta_{10}, \Delta_6, F_7 \neg\circ F_8 \vdash F_9} \neg\circ_L
\end{array}$$

- Case rule I

5.2 Status of 1_R : OK

- Case rule $!R$
- Case rule 1_R
- Case rule \top

$$\frac{\frac{\frac{h_1 : * \vdash \mathbf{1}}{\bullet h_1 : * \vdash \mathbf{1}} 1_R \quad \frac{h_3 : \Delta_2, \mathbf{1} \vdash \top}{\bullet h_3 : \Delta_2, \mathbf{1} \vdash \top} \top}{- : *, \Delta_2 \vdash \top} \text{Cut}}{- : \Delta_2 \vdash \top} \top$$

- Case rule $\&_R$

$$\begin{array}{c}
\frac{\frac{\frac{h_3 : \mathbf{1}, \Delta_2 \vdash F_4}{\bullet h_3 : \Delta_2, \mathbf{1} \vdash F_4} 1_R \quad \frac{h_3 : \mathbf{1}, \Delta_2 \vdash F_5}{\bullet h_3 : \Delta_2, \mathbf{1} \vdash F_4 \& F_5} \&_R}{- : *, \Delta_2 \vdash F_4 \& F_5} \text{Cut}}{- : *, \Delta_2 \vdash F_4 \& F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : * \vdash \mathbf{1}}{\bullet h_1 : * \vdash \mathbf{1}} \text{ax/W} \quad \frac{h_3 : \mathbf{1}, \Delta_2 \vdash F_4}{h_3 : \mathbf{1}, \Delta_2 \vdash F_4} \text{ax/W}}{- : \Delta_2 \vdash F_4} \text{hCut} \quad \frac{\frac{h_1 : * \vdash \mathbf{1}}{\bullet h_1 : * \vdash \mathbf{1}} \text{ax/W} \quad \frac{h_3 : \mathbf{1}, \Delta_2 \vdash F_5}{h_3 : \mathbf{1}, \Delta_2 \vdash F_5} \text{ax/W}}{- : \Delta_2 \vdash F_5} \text{hCut}}{- : \Delta_2 \vdash F_4 \& F_5} \&_R
\end{array}$$

- Case rule $\neg\circ_R$

$$\begin{array}{c}
\frac{}{\bullet h_1 : * \vdash \mathbf{1}} \mathbf{1}_R \quad \frac{h_3 : \mathbf{1}, \Delta_2, F_4 \vdash F_5}{\bullet h_3 : \Delta_2, \mathbf{1} \vdash F_4 \multimap F_5} \multimap_R \\
\hline
- : *, \Delta_2 \vdash F_4 \multimap F_5 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash \mathbf{1}} \mathbf{1}_R \quad \frac{h_3 : \mathbf{1}, \Delta_2, F_4 \vdash F_5}{\bullet h_3 : \Delta_2, \mathbf{1} \vdash F_4 \multimap F_5} \multimap_R}{- : \Delta_2, F_4 \vdash F_5} \text{ax/W}}{- : \Delta_2 \vdash F_4 \multimap F_5} \text{hCut} \quad \multimap_R
\end{array}$$

- Case rule \oplus_{R_2}

$$\begin{array}{c}
\frac{}{\bullet h_1 : * \vdash \mathbf{1}} \mathbf{1}_R \quad \frac{h_3 : \mathbf{1}, \Delta_2 \vdash F_5}{\bullet h_3 : \Delta_2, \mathbf{1} \vdash F_4 \oplus F_5} \oplus_{R_2} \\
\hline
- : *, \Delta_2 \vdash F_4 \oplus F_5 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash \mathbf{1}} \text{ax/W} \quad \frac{h_3 : \mathbf{1}, \Delta_2 \vdash F_5}{\bullet h_3 : \Delta_2, \mathbf{1} \vdash F_4 \oplus F_5} \oplus_{R_2}}{- : \Delta_2 \vdash F_5} \text{ax/W}}{- : \Delta_2 \vdash F_4 \oplus F_5} \text{hCut} \quad \oplus_{R_2}
\end{array}$$

- Case rule \oplus_{R_1}

$$\begin{array}{c}
\frac{}{\bullet h_1 : * \vdash \mathbf{1}} \mathbf{1}_R \quad \frac{h_3 : \mathbf{1}, \Delta_2 \vdash F_4}{\bullet h_3 : \Delta_2, \mathbf{1} \vdash F_4 \oplus F_5} \oplus_{R_1} \\
\hline
- : *, \Delta_2 \vdash F_4 \oplus F_5 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash \mathbf{1}} \text{ax/W} \quad \frac{h_3 : \mathbf{1}, \Delta_2 \vdash F_4}{\bullet h_3 : \Delta_2, \mathbf{1} \vdash F_4 \oplus F_5} \oplus_{R_1}}{- : \Delta_2 \vdash F_4} \text{ax/W}}{- : \Delta_2 \vdash F_4 \oplus F_5} \text{hCut} \quad \oplus_{R_1}
\end{array}$$

- Case rule $\mathbf{1}_L$

$$\begin{array}{c}
\frac{}{\bullet h_1 : * \vdash \mathbf{1}} \mathbf{1}_R \quad \frac{h_2 : \Delta_3 \vdash F_4}{\bullet h_2 : \Delta_3, \mathbf{1} \vdash F_4} \mathbf{1}_L \\
\hline
- : *, \Delta_3 \vdash F_4 \quad \text{Cut} \\
\rightarrow \\
- : \Delta_3 \vdash F_4 \quad \text{ax/W}
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{}{\bullet h_1 : * \vdash \mathbf{1}} \mathbf{1}_R \quad \frac{h_2 : \mathbf{1}, \Delta_6 \vdash F_4 \quad h_2 : \Delta_3 \vdash F_5}{\bullet h_2 : (\Delta_3, \Delta_6), \mathbf{1} \vdash F_4 \otimes F_5} \otimes_R \\
\hline
- : *, \Delta_3, \Delta_6 \vdash F_4 \otimes F_5 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash \mathbf{1}} \text{ax/W} \quad \frac{h_2 : \mathbf{1}, \Delta_6 \vdash F_4}{\bullet h_2 : (\Delta_3, \Delta_6), \mathbf{1} \vdash F_4 \otimes F_5} \otimes_R}{- : \Delta_6 \vdash F_4} \text{ax/W}}{- : \Delta_3, \Delta_6 \vdash F_4 \otimes F_5} \text{hCut} \quad \frac{- : \Delta_3 \vdash F_5}{\otimes_R} \text{ax/W} \\
\otimes_R \\
\frac{}{\bullet h_1 : * \vdash \mathbf{1}} \mathbf{1}_R \quad \frac{h_2 : \Delta_3 \vdash F_4 \quad h_2 : \mathbf{1}, \Delta_6 \vdash F_5}{\bullet h_2 : (\Delta_3, \Delta_6), \mathbf{1} \vdash F_4 \otimes F_5} \otimes_R \\
\hline
- : *, \Delta_3, \Delta_6 \vdash F_4 \otimes F_5 \quad \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_3 \vdash F_4}{\text{ax/W}} \quad \frac{\frac{}{\bullet h_1 : * \vdash \mathbf{1}} \mathbf{1}_R \quad \frac{h_2 : \mathbf{1}, \Delta_6 \vdash F_5}{\bullet h_2 : (\Delta_3, \Delta_6), \mathbf{1} \vdash F_4 \otimes F_5} \otimes_R}{- : \Delta_6 \vdash F_5} \text{ax/W}}{- : \Delta_3, \Delta_6 \vdash F_4 \otimes F_5} \text{hCut} \quad \otimes_R
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{}{\bullet h_1 : * \vdash \mathbf{1}} \mathbf{1}_R \quad \frac{h_2 : \mathbf{1}, \Delta_5 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_3), \mathbf{1} \vdash F_4} W \\
\hline
- : *, \Delta_5, !F_3 \vdash F_4 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash \mathbf{1}} \mathbf{1}_R \quad \frac{h_2 : \mathbf{1}, \Delta_5, !F_3 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_3), \mathbf{1} \vdash F_4} W}{- : \Delta_5, !F_3 \vdash F_4} \text{ax/W}}{- : \Delta_5, !F_3 \vdash F_4} \text{hCut}
\end{array}$$

- Case rule C

$$\frac{\frac{\frac{}{\bullet h_1 : * \vdash \mathbf{1}}{\mathbf{1}_R} \quad \frac{h_2 : \mathbf{1}, \Delta_5, !F_3, !F_3 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_3), \mathbf{1} \vdash F_4} C}{- : *, \Delta_5, !F_3 \vdash F_4} \text{Cut}}{\rightarrow} \quad \frac{\frac{\frac{}{\bullet h_1 : * \vdash \mathbf{1}}{\mathbf{1}_R} \quad \frac{h_2 : \mathbf{1}, \Delta_5, !F_3, !F_3 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_3), \mathbf{1} \vdash F_4} C}{- : \Delta_5, !F_3, !F_3 \vdash F_4} \text{ax/W}}{- : \Delta_5, !F_3 \vdash F_4} \text{hCut}}{- : \Delta_5, !F_3 \vdash F_4} C$$

- Case rule $!L$

$$\frac{\frac{\frac{}{\bullet h_1 : * \vdash \mathbf{1}}{\mathbf{1}_R} \quad \frac{h_2 : \mathbf{1}, \Delta_5, F_3 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_3), \mathbf{1} \vdash F_4} !L}{- : *, \Delta_5, !F_3 \vdash F_4} \text{Cut}}{\rightarrow} \quad \frac{\frac{\frac{}{\bullet h_1 : * \vdash \mathbf{1}}{\mathbf{1}_R} \quad \frac{h_2 : \mathbf{1}, \Delta_5, F_3 \vdash F_4}{\bullet h_2 : (\Delta_5, !F_3), \mathbf{1} \vdash F_4} !L}{- : \Delta_5, F_3 \vdash F_4} \text{ax/W}}{- : \Delta_5, !F_3 \vdash F_4} \text{hCut}}{- : \Delta_5, !F_3 \vdash F_4} !L$$

- Case rule $\&_{L2}$

$$\frac{\frac{\frac{}{\bullet h_1 : * \vdash \mathbf{1}}{\mathbf{1}_R} \quad \frac{h_2 : \mathbf{1}, \Delta_6, F_4 \vdash F_5}{\bullet h_2 : (\Delta_6, F_3 \& F_4), \mathbf{1} \vdash F_5} \&_{L2}}{- : *, \Delta_6, F_3 \& F_4 \vdash F_5} \text{Cut}}{\rightarrow} \quad \frac{\frac{\frac{}{\bullet h_1 : * \vdash \mathbf{1}}{\mathbf{1}_R} \quad \frac{h_2 : \mathbf{1}, \Delta_6, F_4 \vdash F_5}{\bullet h_2 : (\Delta_6, F_3 \& F_4), \mathbf{1} \vdash F_5} \&_{L2}}{- : \Delta_6, F_4 \vdash F_5} \text{ax/W}}{- : \Delta_6, F_3 \& F_4 \vdash F_5} \text{hCut}}{- : \Delta_6, F_3 \& F_4 \vdash F_5} \&_{L2}$$

- Case rule $\&_{L1}$

$$\frac{\frac{\frac{}{\bullet h_1 : * \vdash \mathbf{1}}{\mathbf{1}_R} \quad \frac{h_2 : \mathbf{1}, \Delta_6, F_3 \vdash F_5}{\bullet h_2 : (\Delta_6, F_3 \& F_4), \mathbf{1} \vdash F_5} \&_{L1}}{- : *, \Delta_6, F_3 \& F_4 \vdash F_5} \text{Cut}}{\rightarrow} \quad \frac{\frac{\frac{}{\bullet h_1 : * \vdash \mathbf{1}}{\mathbf{1}_R} \quad \frac{h_2 : \mathbf{1}, \Delta_6, F_3 \vdash F_5}{\bullet h_2 : (\Delta_6, F_3 \& F_4), \mathbf{1} \vdash F_5} \&_{L1}}{- : \Delta_6, F_3 \vdash F_5} \text{ax/W}}{- : \Delta_6, F_3 \& F_4 \vdash F_5} \text{hCut}}{- : \Delta_6, F_3 \& F_4 \vdash F_5} \&_{L1}$$

- Case rule \otimes_L

$$\frac{\frac{\frac{}{\bullet h_1 : * \vdash \mathbf{1}}{\mathbf{1}_R} \quad \frac{h_2 : \mathbf{1}, \Delta_6, F_3, F_4 \vdash F_5}{\bullet h_2 : (\Delta_6, F_3 \otimes F_4), \mathbf{1} \vdash F_5} \otimes_L}{- : *, \Delta_6, F_3 \otimes F_4 \vdash F_5} \text{Cut}}{\rightarrow} \quad \frac{\frac{\frac{}{\bullet h_1 : * \vdash \mathbf{1}}{\mathbf{1}_R} \quad \frac{h_2 : \mathbf{1}, \Delta_6, F_3, F_4 \vdash F_5}{\bullet h_2 : (\Delta_6, F_3 \otimes F_4), \mathbf{1} \vdash F_5} \otimes_L}{- : \Delta_6, F_3, F_4 \vdash F_5} \text{ax/W}}{- : \Delta_6, F_3 \otimes F_4 \vdash F_5} \text{hCut}}{- : \Delta_6, F_3 \otimes F_4 \vdash F_5} \otimes_L$$

- Case rule \oplus_L

$$\frac{\frac{\frac{}{\bullet h_1 : * \vdash \mathbf{1}}{\mathbf{1}_R} \quad \frac{h_2 : \mathbf{1}, \Delta_6, F_3 \vdash F_5 \quad h_2 : \mathbf{1}, \Delta_6, F_4 \vdash F_5}{\bullet h_2 : (\Delta_6, F_3 \oplus F_4), \mathbf{1} \vdash F_5} \oplus_L}{- : *, \Delta_6, F_3 \oplus F_4 \vdash F_5} \text{Cut}}{\rightarrow} \quad \frac{\frac{\frac{}{\bullet h_1 : * \vdash \mathbf{1}}{\mathbf{1}_R} \quad \frac{h_2 : \mathbf{1}, \Delta_6, F_3 \vdash F_5}{\bullet h_2 : (\Delta_6, F_3 \oplus F_4), \mathbf{1} \vdash F_5} \oplus_L}{- : \Delta_6, F_3 \vdash F_5} \text{ax/W}}{- : \Delta_6, F_3 \oplus F_4 \vdash F_5} \text{hCut}}{\frac{\frac{}{\bullet h_1 : * \vdash \mathbf{1}}{\mathbf{1}_R} \quad \frac{h_2 : \mathbf{1}, \Delta_6, F_4 \vdash F_5}{\bullet h_2 : (\Delta_6, F_3 \oplus F_4), \mathbf{1} \vdash F_5} \oplus_L}{- : \Delta_6, F_4 \vdash F_5} \text{ax/W}}{- : \Delta_6, F_3 \oplus F_4 \vdash F_5} \text{hCut}}{- : \Delta_6, F_3 \oplus F_4 \vdash F_5} \oplus_L$$

- Case rule $\neg\circ_L$

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_1 : * \vdash \mathbf{1}}{1_R} \quad \frac{h_2 : \mathbf{1}, \Delta_7 \vdash F_4 \quad h_2 : \Delta_3, F_5 \vdash F_6}{\bullet h_2 : (\Delta_3, \Delta_7, F_4 \neg\circ F_5), \mathbf{1} \vdash F_6}{} \neg\circ_L}{- : *, \Delta_3, \Delta_7, F_4 \neg\circ F_5 \vdash F_6} \text{Cut}}{\rightarrow} \\
\frac{\frac{\frac{}{\bullet h_1 : * \vdash \mathbf{1}}{1_R} \quad \frac{h_2 : \mathbf{1}, \Delta_7 \vdash F_4}{\text{ax/W}}}{- : \Delta_7 \vdash F_4} \quad \frac{h_2 : \Delta_3, F_5 \vdash F_6}{- : \Delta_3, F_5 \vdash F_6} \text{ax/W}}{- : \Delta_3, \Delta_7, F_4 \neg\circ F_5 \vdash F_6} \text{hCut} \neg\circ_L \\
\frac{\frac{\frac{}{\bullet h_1 : * \vdash \mathbf{1}}{1_R} \quad \frac{h_2 : \Delta_3 \vdash F_4 \quad h_2 : \mathbf{1}, \Delta_7, F_5 \vdash F_6}{\bullet h_2 : (\Delta_3, \Delta_7, F_4 \neg\circ F_5), \mathbf{1} \vdash F_6}{} \neg\circ_L}{- : *, \Delta_3, \Delta_7, F_4 \neg\circ F_5 \vdash F_6} \text{Cut}}{\rightarrow} \\
\frac{\frac{h_2 : \Delta_3 \vdash F_4}{- : \Delta_3 \vdash F_4} \text{ax/W} \quad \frac{\frac{}{\bullet h_1 : * \vdash \mathbf{1}}{1_R} \quad \frac{h_2 : \mathbf{1}, \Delta_7, F_5 \vdash F_6}{\text{ax/W}}}{- : \Delta_7, F_5 \vdash F_6} \text{hCut}}{- : \Delta_3, \Delta_7, F_4 \neg\circ F_5 \vdash F_6} \text{hCut} \neg\circ_L
\end{array}$$

- Case rule I

5.3 Status of \top : OK

- Case rule $!R$

- Case rule $\mathbf{1}_R$

- Case rule \top

$$\frac{\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top}{} \top \quad \frac{\frac{}{\bullet h_4 : \Delta_3, \top \vdash \top}{} \top}}{- : \Delta_2, \Delta_3 \vdash \top} \text{Cut}}{\rightarrow} \\
\frac{}{- : \Delta_2, \Delta_3 \vdash \top} \top$$

- Case rule $\&_R$

$$\frac{\frac{\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top}{} \top \quad \frac{h_4 : \top, \Delta_3 \vdash F_5 \quad h_4 : \top, \Delta_3 \vdash F_6}{\bullet h_4 : \Delta_3, \top \vdash F_5 \& F_6}{} \&_R}{- : \Delta_2, \Delta_3 \vdash F_5 \& F_6} \text{Cut}}{\rightarrow} \\
\frac{\frac{\frac{}{\bullet h_1 : * \vdash \top}{} \top \quad \frac{h_4 : \Delta_2, \Delta_3, \top \vdash F_5}{\text{ax/W}}}{- : \Delta_2, \Delta_3 \vdash F_5} \text{hCut} \quad \frac{\frac{}{\bullet h_1 : * \vdash \top}{} \top \quad \frac{h_4 : \Delta_2, \Delta_3, \top \vdash F_6}{\text{ax/W}}}{- : \Delta_2, \Delta_3 \vdash F_6} \text{hCut}}{- : \Delta_2, \Delta_3 \vdash F_5 \& F_6} \&_R$$

- Case rule $\neg\circ_R$

$$\frac{\frac{\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top}{} \top \quad \frac{h_4 : \top, \Delta_3, F_5 \vdash F_6}{\bullet h_4 : \Delta_3, \top \vdash F_5 \neg\circ F_6}{} \neg\circ_R}{- : \Delta_2, \Delta_3 \vdash F_5 \neg\circ F_6} \text{Cut}}{\rightarrow} \\
\frac{\frac{\frac{}{\bullet h_1 : * \vdash \top}{} \top \quad \frac{h_4 : \Delta_2, \Delta_3, F_5, \top \vdash F_6}{\text{ax/W}}}{- : \Delta_2, \Delta_3, F_5 \vdash F_6} \text{hCut}}{- : \Delta_2, \Delta_3 \vdash F_5 \neg\circ F_6} \neg\circ_R$$

- Case rule \oplus_{R_2}

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_4 : \top, \Delta_3 \vdash F_6}{\bullet h_4 : \Delta_3, \top \vdash F_5 \oplus F_6} \oplus_{R_2}}{- : \Delta_2, \Delta_3 \vdash F_5 \oplus F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_4 : \Delta_2, \Delta_3, \top \vdash F_6}{\bullet h_4 : \Delta_2, \Delta_3, \top \vdash F_6} \text{ax/W}}{- : \Delta_2, \Delta_3 \vdash F_6} \text{hCut} \\
\oplus_{R_2} \\
- : \Delta_2, \Delta_3 \vdash F_5 \oplus F_6
\end{array}$$

- Case rule \oplus_{R_1}

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_4 : \top, \Delta_3 \vdash F_5}{\bullet h_4 : \Delta_3, \top \vdash F_5 \oplus F_6} \oplus_{R_1}}{- : \Delta_2, \Delta_3 \vdash F_5 \oplus F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_4 : \Delta_2, \Delta_3, \top \vdash F_5}{\bullet h_4 : \Delta_2, \Delta_3, \top \vdash F_5} \text{ax/W}}{- : \Delta_2, \Delta_3 \vdash F_5} \text{hCut} \\
\oplus_{R_1} \\
- : \Delta_2, \Delta_3 \vdash F_5 \oplus F_6
\end{array}$$

- Case rule 1_L

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_5 \vdash F_4}{\bullet h_3 : (\mathbf{1}, \Delta_5), \top \vdash F_4} \mathbf{1}_L}{- : \Delta_2, \mathbf{1}, \Delta_5 \vdash F_4} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : \mathbf{1}, \Delta_2, \Delta_5, \top \vdash F_4}{\bullet h_3 : \mathbf{1}, \Delta_2, \Delta_5, \top \vdash F_4} \text{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_5 \vdash F_4} \text{hCut}
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_7 \vdash F_5 \quad h_3 : \Delta_4 \vdash F_6}{\bullet h_3 : (\Delta_4, \Delta_7), \top \vdash F_5 \otimes F_6} \otimes_R}{- : \Delta_2, \Delta_4, \Delta_7 \vdash F_5 \otimes F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : \Delta_7, \top \vdash F_5}{\bullet h_3 : \Delta_7, \top \vdash F_5} \text{ax/W}}{- : \Delta_7 \vdash F_5} \text{hCut} \quad \frac{- : \Delta_2, \Delta_4 \vdash F_6}{- : \Delta_2, \Delta_4, \Delta_7 \vdash F_5 \otimes F_6} \text{ax/W} \\
\otimes_R \\
- : \Delta_2, \Delta_4, \Delta_7 \vdash F_5 \otimes F_6 \\
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \Delta_4 \vdash F_5 \quad h_3 : \top, \Delta_7 \vdash F_6}{\bullet h_3 : (\Delta_4, \Delta_7), \top \vdash F_5 \otimes F_6} \otimes_R}{- : \Delta_2, \Delta_4, \Delta_7 \vdash F_5 \otimes F_6} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_4 \vdash F_5}{- : \Delta_2, \Delta_4, \Delta_7 \vdash F_5 \otimes F_6} \text{ax/W} \quad \frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : \Delta_2, \Delta_7, \top \vdash F_6}{\bullet h_3 : \Delta_2, \Delta_7, \top \vdash F_6} \text{ax/W}}{- : \Delta_2, \Delta_7 \vdash F_6} \text{hCut} \\
\otimes_R \\
- : \Delta_2, \Delta_4, \Delta_7 \vdash F_5 \otimes F_6
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_6 \vdash F_5}{\bullet h_3 : (\Delta_6, !F_4), \top \vdash F_5} W}{- : \Delta_2, \Delta_6, !F_4 \vdash F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : \Delta_2, \Delta_6, \top, !F_4 \vdash F_5}{\bullet h_3 : \Delta_2, \Delta_6, \top, !F_4 \vdash F_5} \text{ax/W}}{- : \Delta_2, \Delta_6, !F_4 \vdash F_5} \text{hCut}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_6, !F_4, !F_4 \vdash F_5}{\bullet h_3 : (\Delta_6, !F_4), \top \vdash F_5} C}{- : \Delta_2, \Delta_6, !F_4 \vdash F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : \Delta_2, \Delta_6, \top, !F_4, !F_4 \vdash F_5}{- : \Delta_2, \Delta_6, !F_4, !F_4 \vdash F_5} C}{- : \Delta_2, \Delta_6, !F_4 \vdash F_5} \text{hCut} \quad \text{ax/W}
\end{array}$$

- Case rule !L

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_6, F_4 \vdash F_5}{\bullet h_3 : (\Delta_6, !F_4), \top \vdash F_5} !L}{- : \Delta_2, \Delta_6, !F_4 \vdash F_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : \Delta_2, \Delta_6, F_4, \top \vdash F_5}{- : \Delta_2, \Delta_6, F_4 \vdash F_5} !L}{- : \Delta_2, \Delta_6, !F_4 \vdash F_5} \text{hCut} \quad \text{ax/W}
\end{array}$$

- Case rule &L2

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_7, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, F_4 \& F_5), \top \vdash F_6} \&L2}{- : \Delta_2, \Delta_7, F_4 \& F_5 \vdash F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : \Delta_2, \Delta_7, F_5, \top \vdash F_6}{- : \Delta_2, \Delta_7, F_5 \vdash F_6} \&L2}{- : \Delta_2, \Delta_7, F_4 \& F_5 \vdash F_6} \text{hCut} \quad \text{ax/W}
\end{array}$$

- Case rule &L1

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_7, F_4 \vdash F_6}{\bullet h_3 : (\Delta_7, F_4 \& F_5), \top \vdash F_6} \&L1}{- : \Delta_2, \Delta_7, F_4 \& F_5 \vdash F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : \Delta_2, \Delta_7, F_4, \top \vdash F_6}{- : \Delta_2, \Delta_7, F_4 \vdash F_6} \&L1}{- : \Delta_2, \Delta_7, F_4 \& F_5 \vdash F_6} \text{hCut} \quad \text{ax/W}
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_7, F_4, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, F_4 \otimes F_5), \top \vdash F_6} \otimes_L}{- : \Delta_2, \Delta_7, F_4 \otimes F_5 \vdash F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : \Delta_2, \Delta_7, F_4, F_5, \top \vdash F_6}{- : \Delta_2, \Delta_7, F_4, F_5 \vdash F_6} \otimes_L}{- : \Delta_2, \Delta_7, F_4 \otimes F_5 \vdash F_6} \text{hCut} \quad \text{ax/W}
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_7, F_4 \vdash F_6 \quad h_3 : \top, \Delta_7, F_5 \vdash F_6}{\bullet h_3 : (\Delta_7, F_4 \oplus F_5), \top \vdash F_6} \oplus_L}{- : \Delta_2, \Delta_7, F_4 \oplus F_5 \vdash F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : \Delta_2, \Delta_7, F_4, \top \vdash F_6}{- : \Delta_2, \Delta_7, F_4 \vdash F_6} \oplus_L \quad \frac{}{\bullet h_1 : * \vdash \top} \top \quad \frac{h_3 : \Delta_2, \Delta_7, F_5, \top \vdash F_6}{- : \Delta_2, \Delta_7, F_5 \vdash F_6} \oplus_L}{- : \Delta_2, \Delta_7, F_4 \oplus F_5 \vdash F_6} \text{hCut} \quad \text{ax/W}
\end{array}$$

- Case rule -\circ_L

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \top, \Delta_8 \vdash F_5 \quad h_3 : \Delta_4, F_6 \vdash F_7}{\bullet h_3 : (\Delta_4, \Delta_8, F_5 \multimap F_6), \top \vdash F_7} \multimap_L}{- : \Delta_2, \Delta_4, \Delta_8, F_5 \multimap F_6 \vdash F_7} \text{Cut}}{\rightarrow} \\
\frac{\frac{\frac{\bullet h_1 : * \vdash \top}{- : \Delta_8 \vdash F_5} \top \quad \frac{h_3 : \Delta_8, \top \vdash F_5}{\text{hCut}} \text{ax/W}}{- : \Delta_2, \Delta_4, \Delta_8, F_5 \multimap F_6 \vdash F_7} \multimap_L}{- : \Delta_2, \Delta_4, \Delta_8, F_5 \multimap F_6 \vdash F_7} \text{ax/W} \\
\frac{\frac{\frac{\frac{}{\bullet h_1 : \Delta_2 \vdash \top} \top \quad \frac{h_3 : \Delta_4 \vdash F_5 \quad h_3 : \top, \Delta_8, F_6 \vdash F_7}{\bullet h_3 : (\Delta_4, \Delta_8, F_5 \multimap F_6), \top \vdash F_7} \multimap_L}{- : \Delta_2, \Delta_4, \Delta_8, F_5 \multimap F_6 \vdash F_7} \text{Cut}}{\rightarrow} \\
\frac{\frac{- : \Delta_4 \vdash F_5}{\text{ax/W}} \quad \frac{\frac{\bullet h_1 : * \vdash \top}{- : \Delta_2, \Delta_8, F_6, \top \vdash F_7} \top \quad \frac{h_3 : \Delta_2, \Delta_8, F_6, \top \vdash F_7}{\text{hCut}} \text{ax/W}}{- : \Delta_2, \Delta_4, \Delta_8, F_5 \multimap F_6 \vdash F_7} \multimap_L}{- : \Delta_2, \Delta_4, \Delta_8, F_5 \multimap F_6 \vdash F_7} \multimap_L
\end{array}$$

- Case rule I

5.4 Status of $\&_R$: OK

- Case rule $!R$
- Case rule 1_R
- Case rule \top

$$\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \& F_7} \&_R \quad \frac{}{\bullet h_8 : \Delta_5, F_6 \& F_7 \vdash \top} \top}{- : \Delta_2, \Delta_5 \vdash \top} \text{Cut}}{\rightarrow} \\
- : \Delta_2, \Delta_5 \vdash \top \quad \top$$

- Case rule $\&_R$

$$\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \& F_7} \&_R \quad \frac{h_8 : \Delta_5, F_6 \& F_7 \vdash F_9 \quad h_8 : \Delta_5, F_6 \& F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \& F_7 \vdash F_9 \& F_{10}} \&_R}{- : \Delta_2, \Delta_5 \vdash F_9 \& F_{10}} \text{Cut}}{\rightarrow} \\
\frac{\frac{\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_6 \& F_7}{- : \Delta_2, \Delta_5 \vdash F_9} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \& F_7 \vdash F_9}{\text{hCut}} \text{ax/W}}{\bullet h_1 : \Delta_2 \vdash F_6 \& F_7 \quad h_8 : \Delta_5, F_6 \& F_7 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9 \& F_{10}} \&_R}{- : \Delta_2, \Delta_5 \vdash F_9 \& F_{10}} \text{hCut}$$

- Case rule \multimap_R

$$\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \& F_7} \&_R \quad \frac{h_8 : \Delta_5, F_9, F_6 \& F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \& F_7 \vdash F_9 \multimap F_{10}} \multimap_R}{- : \Delta_2, \Delta_5 \vdash F_9 \multimap F_{10}} \text{Cut}}{\rightarrow} \\
\frac{\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_6 \& F_7}{- : \Delta_2, \Delta_5, F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_9, F_6 \& F_7 \vdash F_{10}}{\text{hCut}} \text{ax/W}}{- : \Delta_2, \Delta_5, F_9 \vdash F_{10}} \multimap_R}{- : \Delta_2, \Delta_5 \vdash F_9 \multimap F_{10}} \multimap_R$$

- Case rule \oplus_{R_2}

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \& F_7} \&_R \quad \frac{h_8 : \Delta_5, F_6 \& F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \& F_7 \vdash F_9 \oplus F_{10}} \oplus_{R_2} \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_6 \& F_7}{- : \Delta_2, \Delta_5 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \& F_7 \vdash F_{10}}{h_8 : \Delta_5, F_6 \& F_7 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10}} \text{hCut} \oplus_{R_2} \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10}
\end{array}$$

- Case rule \oplus_{R_1}

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \& F_7} \&_R \quad \frac{h_8 : \Delta_5, F_6 \& F_7 \vdash F_9}{\bullet h_8 : \Delta_5, F_6 \& F_7 \vdash F_9 \oplus F_{10}} \oplus_{R_1} \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_6 \& F_7}{- : \Delta_2, \Delta_5 \vdash F_9} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \& F_7 \vdash F_9}{h_8 : \Delta_5, F_6 \& F_7 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10}} \text{hCut} \oplus_{R_1} \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10}
\end{array}$$

- Case rule 1_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_9, F_5 \& F_6 \vdash F_8}{\bullet h_7 : (\mathbf{1}, \Delta_9), F_5 \& F_6 \vdash F_8} \mathbf{1}_L \\
\hline
- : \Delta_2, \mathbf{1}, \Delta_9 \vdash F_8 \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \mathbf{1}, \Delta_2, \Delta_9 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \mathbf{1}, \Delta_9, F_5 \& F_6 \vdash F_8}{h_7 : \mathbf{1}, \Delta_9, F_5 \& F_6 \vdash F_8} \text{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_9 \vdash F_8} \text{hCut} \\
\hline
- : \mathbf{1}, \Delta_2, \Delta_9 \vdash F_8
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{11}, F_5 \& F_6 \vdash F_9 \quad h_7 : \Delta_8 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_5 \& F_6 \vdash F_9 \otimes F_{10}} \otimes_R \\
\hline
- : \Delta_2, \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \Delta_{11}, \Delta_2 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_5 \& F_6 \vdash F_9}{h_7 : \Delta_{11}, F_5 \& F_6 \vdash F_9} \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8 \vdash F_9 \otimes F_{10}} \text{hCut} \otimes_R \\
\hline
- : \Delta_{11}, \Delta_2, \Delta_8 \vdash F_9 \otimes F_{10}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{11}, F_5 \& F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_5 \& F_6 \vdash F_9 \otimes F_{10}} \otimes_R \\
\hline
- : \Delta_2, \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10} \\
\rightarrow \\
\frac{\frac{- : \Delta_8 \vdash F_9}{- : \Delta_{11}, \Delta_2 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \Delta_{11}, \Delta_2 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8 \vdash F_9 \otimes F_{10}} \text{hCut} \otimes_R \\
\hline
- : \Delta_{11}, \Delta_2, \Delta_8 \vdash F_9 \otimes F_{10}
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{10}, F_5 \& F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \& F_6 \vdash F_9} W \\
\hline
- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9 \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, !F_8, F_5 \& F_6 \vdash F_9}{h_7 : \Delta_{10}, !F_8, F_5 \& F_6 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{10}, !F_8, !F_8, F_5 \& F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \& F_6 \vdash F_9} C \\
\hline
- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9 \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \Delta_{10}, \Delta_2, !F_8, !F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, !F_8, !F_8, F_5 \& F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} C
\end{array}$$

- Case rule !L

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \& F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \& F_6 \vdash F_9} !L \\
\hline
- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9 \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \Delta_{10}, \Delta_2, F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \& F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} !L
\end{array}$$

- Case rule &L2

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \& F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_5 \& F_6 \vdash F_{10}} \&_{L2} \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \& F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \& F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \&_{L2} \\
\frac{h_1 : \Delta_2 \vdash F_7 \quad h_1 : \Delta_2 \vdash F_8}{\bullet h_1 : \Delta_2 \vdash F_7 \& F_8} \&_R \quad \frac{h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \& F_8 \vdash F_9} \&_{L2} \\
\hline
- : \Delta_2, \Delta_6 \vdash F_9 \\
\rightarrow \\
\frac{\frac{- : \Delta_2 \vdash F_8}{- : \Delta_2, \Delta_6 \vdash F_9} \text{ax/W} \quad \frac{- : \Delta_6, F_8 \vdash F_9}{- : \Delta_2, \Delta_6 \vdash F_9} \text{sCut}}{- : \Delta_2, \Delta_6 \vdash F_9}
\end{array}$$

- Case rule &L1

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \& F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_5 \& F_6 \vdash F_{10}} \&_{L1} \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \& F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \& F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \&_{L1} \\
\frac{h_1 : \Delta_2 \vdash F_7 \quad h_1 : \Delta_2 \vdash F_8}{\bullet h_1 : \Delta_2 \vdash F_7 \& F_8} \&_R \quad \frac{h_5 : \Delta_6, F_7 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \& F_8 \vdash F_9} \&_{L1} \\
\hline
- : \Delta_2, \Delta_6 \vdash F_9 \\
\rightarrow \\
\frac{\frac{- : \Delta_2 \vdash F_7}{- : \Delta_2, \Delta_6 \vdash F_9} \text{ax/W} \quad \frac{- : \Delta_6, F_7 \vdash F_9}{- : \Delta_2, \Delta_6 \vdash F_9} \text{sCut}}{- : \Delta_2, \Delta_6 \vdash F_9}
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \& F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \otimes F_9), F_5 \& F_6 \vdash F_{10}} \otimes_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \otimes F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{- : \Delta_{11}, \Delta_2, F_8, F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \& F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \otimes F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, F_8 \otimes F_9 \vdash F_{10}} \otimes_L
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \& F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \& F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \oplus F_9), F_5 \& F_6 \vdash F_{10}} \oplus_L}{\vdash : \Delta_2, \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{\vdash : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \& F_6 \vdash F_{10}}{\vdash : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{ax/W}}{\vdash : \Delta_{11}, \Delta_2, F_8 \oplus F_9 \vdash F_{10}} \text{hCut} \quad \frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{\vdash : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \& F_6 \vdash F_{10}}{\vdash : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{ax/W}}{\vdash : \Delta_{11}, \Delta_2, F_8 \oplus F_9 \vdash F_{10}} \text{hCut} \oplus_L
\end{array}$$

- Case rule $\neg\circ_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_{12}, F_5 \& F_6 \vdash F_9 \quad h_7 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \neg\circ F_{10}), F_5 \& F_6 \vdash F_{11}} \neg\circ_L}{\vdash : \Delta_2, \Delta_8, \Delta_{12}, F_9 \neg\circ F_{10} \vdash F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{\vdash : \Delta_{12}, \Delta_2 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, F_5 \& F_6 \vdash F_9}{\vdash : \Delta_{12}, \Delta_2, \Delta_8, F_9 \neg\circ F_{10} \vdash F_{11}} \text{hCut} \quad \frac{\vdash : \Delta_8, F_{10} \vdash F_{11}}{\vdash : \Delta_8, F_{10} \vdash F_{11}} \text{ax/W}}{\vdash : \Delta_{12}, \Delta_2, \Delta_8, F_9 \neg\circ F_{10} \vdash F_{11}} \neg\circ_L \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \quad h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6} \&_R \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{12}, F_{10}, F_5 \& F_6 \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \neg\circ F_{10}), F_5 \& F_6 \vdash F_{11}} \neg\circ_L}{\vdash : \Delta_2, \Delta_8, \Delta_{12}, F_9 \neg\circ F_{10} \vdash F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\vdash : \Delta_8 \vdash F_9}{\vdash : \Delta_{12}, \Delta_2, \Delta_8, F_9 \neg\circ F_{10} \vdash F_{11}} \text{ax/W} \quad \frac{\bullet h_1 : \Delta_2 \vdash F_5 \& F_6}{\vdash : \Delta_{12}, \Delta_2, F_{10} \vdash F_{11}} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, F_{10}, F_5 \& F_6 \vdash F_{11}}{\vdash : \Delta_{12}, \Delta_2, F_{10} \vdash F_{11}} \text{hCut}}{\vdash : \Delta_{12}, \Delta_2, \Delta_8, F_9 \neg\circ F_{10} \vdash F_{11}} \neg\circ_L
\end{array}$$

- Case rule I

5.5 Status of $\neg\circ_R$: OK

- Case rule $!R$

- Case rule 1_R

- Case rule \top

$$\frac{\frac{h_1 : \Delta_2, F_6 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \neg\circ F_7} \neg\circ_R \quad \frac{\vdash}{\bullet h_8 : \Delta_5, F_6 \neg\circ F_7 \vdash \top} \top}{\vdash : \Delta_2, \Delta_5 \vdash \top} \text{Cut} \\
\rightarrow \\
\frac{\vdash}{\vdash : \Delta_2, \Delta_5 \vdash \top} \top$$

- Case rule $\&_R$

$$\frac{\frac{h_1 : \Delta_2, F_6 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \neg\circ F_7} \neg\circ_R \quad \frac{h_8 : \Delta_5, F_6 \neg\circ F_7 \vdash F_9 \quad h_8 : \Delta_5, F_6 \neg\circ F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \neg\circ F_7 \vdash F_9 \& F_{10}} \&_R}{\vdash : \Delta_2, \Delta_5 \vdash F_9 \& F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_6 \neg\circ F_7}{\vdash : \Delta_2, \Delta_5 \vdash F_9} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \neg\circ F_7 \vdash F_9}{\vdash : \Delta_2, \Delta_5 \vdash F_9 \& F_{10}} \text{ax/W}}{\vdash : \Delta_2, \Delta_5 \vdash F_9 \& F_{10}} \text{hCut} \quad \frac{\bullet h_1 : \Delta_2 \vdash F_6 \neg\circ F_7}{\vdash : \Delta_2, \Delta_5 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \neg\circ F_7 \vdash F_{10}}{\vdash : \Delta_2, \Delta_5 \vdash F_{10}} \text{ax/W}}{\vdash : \Delta_2, \Delta_5 \vdash F_9 \& F_{10}} \text{hCut} \&_R$$

- Case rule $\neg\circ_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_6 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \multimap F_7} \multimap_R \quad \frac{h_8 : \Delta_5, F_9, F_6 \multimap F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \multimap F_7 \vdash F_9 \multimap F_{10}} \multimap_R}{- : \Delta_2, \Delta_5 \vdash F_9 \multimap F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_6 \multimap F_7}{- : \Delta_2, \Delta_5, F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_9, F_6 \multimap F_7 \vdash F_{10}}{- : \Delta_2, \Delta_5 \vdash F_9 \multimap F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9 \multimap F_{10}} \text{hCut} \multimap_R
\end{array}$$

- Case rule \oplus_{R_2}

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_6 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \multimap F_7} \multimap_R \quad \frac{h_8 : \Delta_5, F_6 \multimap F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \multimap F_7 \vdash F_9 \oplus F_{10}} \oplus_{R_2}}{- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_6 \multimap F_7}{- : \Delta_2, \Delta_5 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \multimap F_7 \vdash F_{10}}{- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10}} \text{hCut} \oplus_{R_2}
\end{array}$$

- Case rule \oplus_{R_1}

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_6 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \multimap F_7} \multimap_R \quad \frac{h_8 : \Delta_5, F_6 \multimap F_7 \vdash F_9}{\bullet h_8 : \Delta_5, F_6 \multimap F_7 \vdash F_9 \oplus F_{10}} \oplus_{R_1}}{- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_6 \multimap F_7}{- : \Delta_2, \Delta_5 \vdash F_9} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \multimap F_7 \vdash F_9}{- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10}} \text{hCut} \oplus_{R_1}
\end{array}$$

- Case rule $\mathbf{1}_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_9, F_5 \multimap F_6 \vdash F_8}{\bullet h_7 : (\mathbf{1}, \Delta_9), F_5 \multimap F_6 \vdash F_8} \mathbf{1}_L}}{- : \Delta_2, \mathbf{1}, \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \mathbf{1}, \Delta_2, \Delta_9 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \mathbf{1}, \Delta_9, F_5 \multimap F_6 \vdash F_8}{- : \mathbf{1}, \Delta_2, \Delta_9 \vdash F_8} \text{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_9 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_{11}, F_5 \multimap F_6 \vdash F_9 \quad h_7 : \Delta_8 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_5 \multimap F_6 \vdash F_9 \otimes F_{10}} \otimes_R}}{- : \Delta_2, \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \Delta_{11}, \Delta_2 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_5 \multimap F_6 \vdash F_9}{- : \Delta_{11}, \Delta_2, \Delta_8 \vdash F_9 \otimes F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8 \vdash F_9 \otimes F_{10}} \text{hCut} \quad \frac{- : \Delta_8 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, \Delta_8 \vdash F_9 \otimes F_{10}} \otimes_R} \\
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{11}, F_5 \multimap F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_5 \multimap F_6 \vdash F_9 \otimes F_{10}} \otimes_R}}{- : \Delta_2, \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_8 \vdash F_9 \quad \frac{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \Delta_{11}, \Delta_2 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_5 \multimap F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8 \vdash F_9 \otimes F_{10}} \text{hCut} \otimes_R
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_{10}, F_5 \multimap F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \multimap F_6 \vdash F_9} W \\
\hline
- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9 \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, !F_8, F_5 \multimap F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W} \\
\hline
- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9 \quad \text{hCut}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_{10}, !F_8, !F_8, F_5 \multimap F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \multimap F_6 \vdash F_9} C \\
\hline
- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9 \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \Delta_{10}, \Delta_2, !F_8, !F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, !F_8, !F_8, F_5 \multimap F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W} \\
\hline
- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9 \quad \text{hCut} \\
- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9 \quad C
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \multimap F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \multimap F_6 \vdash F_9} !L \\
\hline
- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9 \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \Delta_{10}, \Delta_2, F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \multimap F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, F_8 \vdash F_9} \text{ax/W} \\
\hline
- : \Delta_{10}, \Delta_2, F_8 \vdash F_9 \quad \text{hCut} \\
- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9 \quad !L
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \multimap F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_5 \multimap F_6 \vdash F_{10}} \&_{L2} \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \& F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \multimap F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \text{ax/W} \\
\hline
- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10} \quad \&_{L2} \\
- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10} \quad \text{hCut}
\end{array}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \multimap F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_5 \multimap F_6 \vdash F_{10}} \&_{L1} \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \& F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \multimap F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \text{ax/W} \\
\hline
- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10} \quad \&_{L1} \\
- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10} \quad \text{hCut}
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \multimap F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \otimes F_9), F_5 \multimap F_6 \vdash F_{10}} \otimes_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \otimes F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \Delta_{11}, \Delta_2, F_8, F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \multimap F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \otimes F_9 \vdash F_{10}} \text{ax/W} \\
\hline
- : \Delta_{11}, \Delta_2, F_8, F_9 \vdash F_{10} \quad \otimes_L \\
- : \Delta_{11}, \Delta_2, F_8 \otimes F_9 \vdash F_{10} \quad \text{hCut}
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \multimap F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \multimap F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \oplus F_9), F_5 \multimap F_6 \vdash F_{10}} \oplus_L}{- : \Delta_2, \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \multimap F_6 \vdash F_{10}}{h\text{Cut}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \multimap F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \oplus F_9 \vdash F_{10}} \oplus_L}{- : \Delta_{11}, \Delta_2, F_8 \oplus F_9 \vdash F_{10}} \text{hCut}
\end{array}$$

- Case rule \multimap_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_{12}, F_5 \multimap F_6 \vdash F_9 \quad h_7 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \multimap F_{10}), F_5 \multimap F_6 \vdash F_{11}} \multimap_L}{- : \Delta_2, \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \Delta_{12}, \Delta_2 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, F_5 \multimap F_6 \vdash F_9}{h\text{Cut}} \text{ax/W} \quad \frac{h_7 : \Delta_8, F_{10} \vdash F_{11}}{- : \Delta_8, F_{10} \vdash F_{11}} \text{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_8, F_9 \multimap F_{10} \vdash F_{11}} \multimap_L}{- : \Delta_2, \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2, F_5 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \multimap F_6} \multimap_R \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{12}, F_{10}, F_5 \multimap F_6 \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \multimap F_{10}), F_5 \multimap F_6 \vdash F_{11}} \multimap_L}{- : \Delta_2, \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_8 \vdash F_9}{- : \Delta_{12}, \Delta_2, \Delta_8, F_9 \multimap F_{10} \vdash F_{11}} \text{ax/W} \quad \frac{\frac{h_1 : \Delta_2 \vdash F_5 \multimap F_6}{- : \Delta_{12}, \Delta_2, F_{10} \vdash F_{11}} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, F_{10}, F_5 \multimap F_6 \vdash F_{11}}{h\text{Cut}} \text{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_8, F_9 \multimap F_{10} \vdash F_{11}} \multimap_L}{- : \Delta_2, \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2, F_8 \vdash F_9}{\bullet h_1 : \Delta_2 \vdash F_8 \multimap F_9} \multimap_R \quad \frac{h_5 : \Delta_6 \vdash F_8 \quad h_5 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_5 : (\Delta_6, \Delta_7), F_8 \multimap F_9 \vdash F_{10}} \multimap_L}{- : \Delta_2, \Delta_6, \Delta_7 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_6 \vdash F_8}{- : \Delta_2, \Delta_6, \Delta_7 \vdash F_{10}} \text{ax/W} \quad \frac{- : \Delta_2, F_8 \vdash F_9}{- : \Delta_2, \Delta_7, F_8 \vdash F_{10}} \text{ax/W} \quad \frac{- : \Delta_7, F_9 \vdash F_{10}}{s\text{Cut}} \text{ax/W}}{- : \Delta_2, \Delta_6, \Delta_7 \vdash F_{10}} s\text{Cut}
\end{array}$$

- Case rule I

5.6 Status of \oplus_{R2} : OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule \top

$$\frac{\frac{h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \oplus_{R2} \quad \frac{}{\bullet h_8 : \Delta_5, F_6 \oplus F_7 \vdash \top} \top}{- : \Delta_2, \Delta_5 \vdash \top} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_5 \vdash \top} \top$$

- Case rule $\&_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \oplus_{R_2} \quad \frac{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9 \quad h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9 \& F_{10}} \&_R}{- : \Delta_2, \Delta_5 \vdash F_9 \& F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_6 \oplus F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9}{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9} \text{hCut} \quad \frac{\frac{h_1 : \Delta_2 \vdash F_6 \oplus F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_{10}}{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_{10}} \text{hCut}}{- : \Delta_2, \Delta_5 \vdash F_9 \& F_{10}} \&_R
\end{array}$$

- Case rule $\neg o_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \oplus_{R_2} \quad \frac{h_8 : \Delta_5, F_9, F_6 \oplus F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9 \neg o F_{10}} \neg o_R}{- : \Delta_2, \Delta_5 \vdash F_9 \neg o F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_6 \oplus F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_9, F_6 \oplus F_7 \vdash F_{10}}{h_8 : \Delta_5, F_9, F_6 \oplus F_7 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_5, F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_2, \Delta_5 \vdash F_9 \neg o F_{10}} \neg o_R
\end{array}$$

- Case rule \oplus_{R_2}

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \oplus_{R_2} \quad \frac{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9 \oplus F_{10}} \oplus_{R_2}}{- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_6 \oplus F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_{10}}{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_{10}} \text{hCut}}{- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10}} \oplus_{R_2}
\end{array}$$

- Case rule \oplus_{R_1}

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \oplus_{R_2} \quad \frac{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9}{\bullet h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9 \oplus F_{10}} \oplus_{R_1}}{- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_6 \oplus F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9}{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9} \text{hCut}}{- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10}} \oplus_{R_1}
\end{array}$$

- Case rule $\mathbf{1}_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R_2} \quad \frac{h_7 : \Delta_9, F_5 \oplus F_6 \vdash F_8}{\bullet h_7 : (\mathbf{1}, \Delta_9), F_5 \oplus F_6 \vdash F_8} \mathbf{1}_L}{- : \Delta_2, \mathbf{1}, \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \text{ax/W} \quad \frac{h_7 : \mathbf{1}, \Delta_9, F_5 \oplus F_6 \vdash F_8}{h_7 : \mathbf{1}, \Delta_9, F_5 \oplus F_6 \vdash F_8} \text{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_9 \vdash F_8} \text{hCut}}{- : \mathbf{1}, \Delta_2, \Delta_9 \vdash F_8}
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R_2} \quad \frac{h_7 : \Delta_{11}, F_5 \oplus F_6 \vdash F_9 \quad h_7 : \Delta_8 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_5 \oplus F_6 \vdash F_9 \otimes F_{10}} \otimes_R}{- : \Delta_2, \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_5 \oplus F_6 \vdash F_9}{h_7 : \Delta_{11}, F_5 \oplus F_6 \vdash F_9} \text{ax/W}}{- : \Delta_{11}, \Delta_2 \vdash F_9} \text{hCut} \quad \frac{h_7 : \Delta_8 \vdash F_{10}}{- : \Delta_8 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8 \vdash F_9 \otimes F_{10}} \otimes_R
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R2} \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{11}, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_5 \oplus F_6 \vdash F_9 \otimes F_{10}} \otimes_R}{- : \Delta_2, \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10}} \text{Cut}}{\rightarrow} \\
\frac{- : \Delta_8 \vdash F_9 \quad \frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{11}, \Delta_2 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_5 \oplus F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8 \vdash F_9 \otimes F_{10}} \otimes_R}{- : \Delta_{11}, \Delta_2, \Delta_8 \vdash F_9 \otimes F_{10}} \text{hCut}}
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R2} \quad \frac{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \oplus F_6 \vdash F_9} W}{- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9} \text{Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, !F_8, F_5 \oplus F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{hCut}}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R2} \quad \frac{h_7 : \Delta_{10}, !F_8, F_5 \oplus F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \oplus F_6 \vdash F_9} C}{- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9} \text{Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{10}, \Delta_2, !F_8, !F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, !F_8, F_5 \oplus F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{hCut}}
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R2} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \oplus F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \oplus F_6 \vdash F_9} !L}{- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9} \text{Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{10}, \Delta_2, F_8 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \oplus F_6 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{hCut}}
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R2} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_5 \oplus F_6 \vdash F_{10}} \&_{L2}}{- : \Delta_2, \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \text{Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \oplus F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \text{hCut}}
\end{array}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R2} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_5 \oplus F_6 \vdash F_{10}} \&_{L1}}{- : \Delta_2, \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \text{Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \oplus F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \text{hCut}}
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R2} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \otimes F_9), F_5 \oplus F_6 \vdash F_{10}} \otimes_L}{- : \Delta_2, \Delta_{11}, F_8 \otimes F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{11}, \Delta_2, F_8, F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \oplus F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \otimes F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \otimes F_9 \vdash F_{10}} \text{hCut} \otimes_L}{- : \Delta_{11}, \Delta_2, F_8 \otimes F_9 \vdash F_{10}} \text{hCut}
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R2} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \oplus F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \oplus F_9), F_5 \oplus F_6 \vdash F_{10}} \oplus_L}{- : \Delta_2, \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \oplus F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \oplus F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \oplus F_9 \vdash F_{10}} \text{hCut} \oplus_L \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \oplus F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \oplus F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, F_8 \oplus F_9 \vdash F_{10}} \oplus_L \\
\oplus_L \\
\frac{h_1 : \Delta_2 \vdash F_8}{\bullet h_1 : \Delta_2 \vdash F_7 \oplus F_8} \oplus_{R2} \quad \frac{h_5 : \Delta_6, F_7 \vdash F_9 \quad h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \oplus F_8 \vdash F_9} \oplus_L}{- : \Delta_2, \Delta_6 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_2 \vdash F_8 \quad \text{ax/W} \quad - : \Delta_6, F_8 \vdash F_9 \quad \text{ax/W}}{- : \Delta_2, \Delta_6 \vdash F_9} \text{sCut}
\end{array}$$

- Case rule $\neg\circ_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R2} \quad \frac{h_7 : \Delta_{12}, F_5 \oplus F_6 \vdash F_9 \quad h_7 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \neg\circ F_{10}), F_5 \oplus F_6 \vdash F_{11}} \neg\circ_L}{- : \Delta_2, \Delta_8, \Delta_{12}, F_9 \neg\circ F_{10} \vdash F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{12}, \Delta_2 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, F_5 \oplus F_6 \vdash F_9}{- : \Delta_{12}, \Delta_2, \Delta_8, F_9 \neg\circ F_{10} \vdash F_{11}} \text{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_8, F_9 \neg\circ F_{10} \vdash F_{11}} \text{hCut} \neg\circ_L \quad \frac{h_7 : \Delta_8, F_{10} \vdash F_{11}}{- : \Delta_8, F_{10} \vdash F_{11}} \text{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_8, F_9 \neg\circ F_{10} \vdash F_{11}} \neg\circ_L \\
\neg\circ_L \\
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R2} \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{12}, F_{10}, F_5 \oplus F_6 \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \neg\circ F_{10}), F_5 \oplus F_6 \vdash F_{11}} \neg\circ_L}{- : \Delta_2, \Delta_8, \Delta_{12}, F_9 \neg\circ F_{10} \vdash F_{11}} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_8 \vdash F_9 \quad \text{ax/W} \quad \frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{12}, \Delta_2, \Delta_8, F_9 \neg\circ F_{10} \vdash F_{11}} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, F_{10}, F_5 \oplus F_6 \vdash F_{11}}{- : \Delta_{12}, \Delta_2, F_{10} \vdash F_{11}} \text{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_8, F_9 \neg\circ F_{10} \vdash F_{11}} \text{hCut} \neg\circ_L}{- : \Delta_{12}, \Delta_2, \Delta_8, F_9 \neg\circ F_{10} \vdash F_{11}} \neg\circ_L
\end{array}$$

- Case rule I

5.7 Status of \oplus_{R1} : OK

- Case rule $!R$
- Case rule 1_R
- Case rule \top

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \oplus_{R1} \quad \frac{}{\bullet h_8 : \Delta_5, F_6 \oplus F_7 \vdash \top} \top}{- : \Delta_2, \Delta_5 \vdash \top} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_5 \vdash \top} \top
\end{array}$$

- Case rule $\&_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \oplus_{R1} \quad \frac{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9 \quad h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9 \& F_{10}} \&_R \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \& F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \oplus F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9}{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9} \text{hCut} \quad \frac{\frac{h_1 : \Delta_2 \vdash F_6 \oplus F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_{10}}{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_{10}} \text{hCut} \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \& F_{10} \quad \&_R
\end{array}$$

- Case rule $\neg\circ_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \oplus_{R1} \quad \frac{h_8 : \Delta_5, F_9, F_6 \oplus F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9 \neg\circ F_{10}} \neg\circ_R \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \neg\circ F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \oplus F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_9, F_6 \oplus F_7 \vdash F_{10}}{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_5, F_9 \vdash F_{10}} \text{hCut} \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \neg\circ F_{10} \quad \neg\circ_R
\end{array}$$

- Case rule \oplus_{R2}

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \oplus_{R1} \quad \frac{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_{10}}{\bullet h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9 \oplus F_{10}} \oplus_{R2} \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \oplus F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_{10}}{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_{10}} \text{hCut} \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10} \quad \oplus_{R2}
\end{array}$$

- Case rule \oplus_{R1}

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \oplus_{R1} \quad \frac{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9}{\bullet h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9 \oplus F_{10}} \oplus_{R1} \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \oplus F_7}{\bullet h_1 : \Delta_2 \vdash F_6 \oplus F_7} \text{ax/W} \quad \frac{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9}{h_8 : \Delta_5, F_6 \oplus F_7 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_5 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_2, \Delta_5 \vdash F_9 \oplus F_{10} \quad \oplus_{R1}
\end{array}$$

- Case rule $\mathbf{1}_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_9, F_5 \oplus F_6 \vdash F_8}{\bullet h_7 : (\mathbf{1}, \Delta_9), F_5 \oplus F_6 \vdash F_8} \mathbf{1}_L \\
\hline
- : \Delta_2, \mathbf{1}, \Delta_9 \vdash F_8 \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \text{ax/W} \quad \frac{h_7 : \mathbf{1}, \Delta_9, F_5 \oplus F_6 \vdash F_8}{h_7 : \mathbf{1}, \Delta_9, F_5 \oplus F_6 \vdash F_8} \text{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_9 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{11}, F_5 \oplus F_6 \vdash F_9 \quad h_7 : \Delta_8 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_5 \oplus F_6 \vdash F_9 \otimes F_{10}} \otimes_R \\
\hline
- : \Delta_2, \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10} \quad \text{Cut} \\
\hline
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_5 \oplus F_6 \vdash F_9}{h_7 : \Delta_{11}, F_5 \oplus F_6 \vdash F_9} \text{ax/W}}{- : \Delta_{11}, \Delta_2 \vdash F_9} \text{hCut} \quad \frac{- : \Delta_8 \vdash F_{10}}{- : \Delta_8 \vdash F_{10}} \text{ax/W} \\
\hline
- : \Delta_{11}, \Delta_2, \Delta_8 \vdash F_9 \otimes F_{10} \quad \otimes_R
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{11}, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_5 \oplus F_6 \vdash F_9 \otimes F_{10}} \otimes_R}{- : \Delta_2, \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10}} \text{Cut}}{\rightarrow} \\
\frac{- : \Delta_8 \vdash F_9 \quad \text{ax/W} \quad \frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6 \quad \text{ax/W} \quad h_7 : \Delta_{11}, F_5 \oplus F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, \Delta_8 \vdash F_9 \otimes F_{10}} \otimes_R
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{10}, F_5 \oplus F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \oplus F_6 \vdash F_9} W}{- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9} \text{Cut}}{\rightarrow} \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6 \quad \text{ax/W} \quad h_7 : \Delta_{10}, !F_8, F_5 \oplus F_6 \vdash F_9 \quad \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} \text{hCut}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{10}, !F_8, F_5 \oplus F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \oplus F_6 \vdash F_9} C}{- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9} \text{Cut}}{\rightarrow} \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6 \quad \text{ax/W} \quad h_7 : \Delta_{10}, !F_8, !F_8, F_5 \oplus F_6 \vdash F_9 \quad \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_8, !F_8 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} C
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{10}, F_8, F_5 \oplus F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_5 \oplus F_6 \vdash F_9} !L}{- : \Delta_2, \Delta_{10}, !F_8 \vdash F_9} \text{Cut}}{\rightarrow} \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6 \quad \text{ax/W} \quad h_7 : \Delta_{10}, F_8, F_5 \oplus F_6 \vdash F_9 \quad \text{ax/W}}{- : \Delta_{10}, \Delta_2, F_8 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_2, !F_8 \vdash F_9} !L
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{11}, F_9, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_5 \oplus F_6 \vdash F_{10}} \&_{L2}}{- : \Delta_2, \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \text{Cut}}{\rightarrow} \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6 \quad \text{ax/W} \quad h_7 : \Delta_{11}, F_9, F_5 \oplus F_6 \vdash F_{10} \quad \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \&_{L2}
\end{array}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_5 \oplus F_6 \vdash F_{10}} \&_{L1}}{- : \Delta_2, \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \text{Cut}}{\rightarrow} \\
\frac{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6 \quad \text{ax/W} \quad h_7 : \Delta_{11}, F_8, F_5 \oplus F_6 \vdash F_{10} \quad \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9 \vdash F_{10}} \&_{L1}
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \otimes F_9), F_5 \oplus F_6 \vdash F_{10}} \otimes_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \otimes F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{11}, \Delta_2, F_8, F_9 \vdash F_{10}} \text{ ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, F_5 \oplus F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \otimes F_9 \vdash F_{10}} \text{ ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \otimes F_9 \vdash F_{10}} \text{ hCut} \otimes_L \\
\text{Cut}
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \oplus F_6 \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, F_5 \oplus F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \oplus F_9), F_5 \oplus F_6 \vdash F_{10}} \oplus_L \\
\hline
- : \Delta_2, \Delta_{11}, F_8 \oplus F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{11}, \Delta_2, F_8 \vdash F_{10}} \text{ ax/W} \quad \frac{h_7 : \Delta_{11}, F_8, F_5 \oplus F_6 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \oplus F_9 \vdash F_{10}} \text{ ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \oplus F_9 \vdash F_{10}} \text{ hCut} \oplus_L \\
\text{Cut}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_7}{\bullet h_1 : \Delta_2 \vdash F_7 \oplus F_8} \oplus_{R1} \quad \frac{h_5 : \Delta_6, F_7 \vdash F_9 \quad h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \oplus F_8 \vdash F_9} \oplus_L \\
\hline
- : \Delta_2, \Delta_6 \vdash F_9 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_7}{- : \Delta_2 \vdash F_7} \text{ ax/W} \quad \frac{h_5 : \Delta_6, F_7 \vdash F_9}{- : \Delta_6, F_7 \vdash F_9} \text{ ax/W}}{- : \Delta_2, \Delta_6 \vdash F_9} \text{ sCut} \\
\text{Cut}
\end{array}$$

- Case rule \multimap_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_{12}, F_5 \oplus F_6 \vdash F_9 \quad h_7 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \multimap F_{10}), F_5 \oplus F_6 \vdash F_{11}} \multimap_L \\
\hline
- : \Delta_2, \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_{12}, \Delta_2 \vdash F_9} \text{ ax/W} \quad \frac{h_7 : \Delta_{12}, F_5 \oplus F_6 \vdash F_9}{- : \Delta_{12}, \Delta_2, \Delta_8, F_9 \multimap F_{10} \vdash F_{11}} \text{ ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_8, F_9 \multimap F_{10} \vdash F_{11}} \text{ hCut} \multimap_L \\
\text{Cut}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5 \oplus F_6} \oplus_{R1} \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{12}, F_{10}, F_5 \oplus F_6 \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \multimap F_{10}), F_5 \oplus F_6 \vdash F_{11}} \multimap_L \\
\hline
- : \Delta_2, \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5 \oplus F_6}{- : \Delta_8 \vdash F_9} \text{ ax/W} \quad \frac{h_7 : \Delta_{12}, F_{10}, F_5 \oplus F_6 \vdash F_{11}}{- : \Delta_{12}, \Delta_2, F_{10} \vdash F_{11}} \text{ ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_8, F_9 \multimap F_{10} \vdash F_{11}} \text{ hCut} \multimap_L \\
\text{Cut}
\end{array}$$

- Case rule I

5.8 Status of 1_L : OK

- Case rule $!R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash 5}{\bullet h_1 : 1, \Delta_2 \vdash 5} 1_L \quad \frac{h_6 : !\Upsilon 4, 5 \vdash F_7}{\bullet h_6 : !\Upsilon 4, 5 \vdash !F_7} !R \\
\hline
- : (1, \Delta_2), !\Upsilon 4 \vdash !F_7 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash 5}{- : 1, !\Upsilon 4, \Delta_2 \vdash !F_7} \text{ ax/W} \quad \frac{h_6 : 1, 5, !\Upsilon 4 \vdash !F_7}{- : 1, !\Upsilon 4, \Delta_2 \vdash !F_7} \text{ ax/W}}{- : 1, !\Upsilon 4, \Delta_2 \vdash !F_7} \text{ hCut} \\
\text{Cut}
\end{array}$$

- Case rule 1_R

- Case rule \top

$$\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_5} \mathbf{1}_L}{- : (\mathbf{1}, \Delta_2), \Delta_4 \vdash \top} \top}{\rightarrow} \text{Cut}}{- : \mathbf{1}, \Delta_2, \Delta_4 \vdash \top} \top$$

- Case rule $\&_R$

$$\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_5} \mathbf{1}_L}{- : (\mathbf{1}, \Delta_2), \Delta_4 \vdash F_7 \& F_8} \text{Cut}}{\rightarrow} \frac{\frac{h_6 : \Delta_4, F_5 \vdash F_7}{\bullet h_6 : \Delta_4, F_5 \vdash F_7 \& F_8} \text{Cut}}{h_6 : \Delta_4, F_5 \vdash F_8} \&_R}{\rightarrow} \frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_5} \text{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_4 \vdash F_7 \& F_8} \text{hCut}}{- : \mathbf{1}, \Delta_2, \Delta_4 \vdash F_7 \& F_8} \text{ax/W}$$

- Case rule $\neg\circ_R$

$$\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_5} \mathbf{1}_L}{- : (\mathbf{1}, \Delta_2), \Delta_4 \vdash F_7 \neg\circ F_8} \text{Cut}}{\rightarrow} \frac{\frac{h_6 : \Delta_4, F_5, F_7 \vdash F_8}{\bullet h_6 : \Delta_4, F_5 \vdash F_7 \neg\circ F_8} \neg\circ_R}{- : (\mathbf{1}, \Delta_2), \Delta_4 \vdash F_7 \neg\circ F_8} \text{Cut}}{\rightarrow} \frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_5} \text{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_4 \vdash F_7 \neg\circ F_8} \text{hCut}}{- : \mathbf{1}, \Delta_2, \Delta_4 \vdash F_7 \neg\circ F_8} \text{ax/W}$$

- Case rule \oplus_{R_2}

$$\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_5} \mathbf{1}_L}{- : (\mathbf{1}, \Delta_2), \Delta_4 \vdash F_7 \oplus F_8} \text{Cut}}{\rightarrow} \frac{\frac{h_6 : \Delta_4, F_5 \vdash F_8}{\bullet h_6 : \Delta_4, F_5 \vdash F_7 \oplus F_8} \oplus_{R_2}}{- : (\mathbf{1}, \Delta_2), \Delta_4 \vdash F_7 \oplus F_8} \text{Cut}}{\rightarrow} \frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_5} \text{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_4 \vdash F_7 \oplus F_8} \text{hCut}}{- : \mathbf{1}, \Delta_2, \Delta_4 \vdash F_7 \oplus F_8} \text{ax/W}$$

- Case rule \oplus_{R_1}

$$\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_5} \mathbf{1}_L}{- : (\mathbf{1}, \Delta_2), \Delta_4 \vdash F_7 \oplus F_8} \text{Cut}}{\rightarrow} \frac{\frac{h_6 : \Delta_4, F_5 \vdash F_7}{\bullet h_6 : \Delta_4, F_5 \vdash F_7 \oplus F_8} \oplus_{R_1}}{- : (\mathbf{1}, \Delta_2), \Delta_4 \vdash F_7 \oplus F_8} \text{Cut}}{\rightarrow} \frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_5} \text{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_4 \vdash F_7 \oplus F_8} \text{hCut}}{- : \mathbf{1}, \Delta_2, \Delta_4 \vdash F_7 \oplus F_8} \text{ax/W}$$

- Case rule $\mathbf{1}_L$

$$\frac{\frac{\frac{h_1 : \Delta_2 \vdash \mathbf{1}}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash \mathbf{1}} \mathbf{1}_L}{- : (\mathbf{1}, \Delta_2), \Delta_5 \vdash F_6} \text{Cut}}{\rightarrow} \frac{\frac{h_4 : \Delta_5 \vdash F_6}{\bullet h_4 : \Delta_5, \mathbf{1} \vdash F_6} \mathbf{1}_L}{- : (\mathbf{1}, \Delta_2), \Delta_5 \vdash F_6} \text{Cut}}{\rightarrow} \frac{\frac{h_1 : \Delta_2 \vdash \mathbf{1}}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash \mathbf{1}} \text{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_5 \vdash F_6} \text{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_5 \vdash F_6} \text{ax/W}$$

$$\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_4} \mathbf{1}_L}{- : (\mathbf{1}, \Delta_2), \mathbf{1}, \Delta_7 \vdash F_6} \text{Cut}}{\rightarrow} \frac{\frac{h_5 : \Delta_7, F_4 \vdash F_6}{\bullet h_5 : (\mathbf{1}, \Delta_7), F_4 \vdash F_6} \mathbf{1}_L}{- : (\mathbf{1}, \Delta_2), \mathbf{1}, \Delta_7 \vdash F_6} \text{Cut}}{\rightarrow} \frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_4} \text{ax/W}}{- : \mathbf{1}, \mathbf{1}, \Delta_2, \Delta_7 \vdash F_6} \text{hCut}}{- : \mathbf{1}, \mathbf{1}, \Delta_2, \Delta_7 \vdash F_6} \text{ax/W}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_4} \mathbf{1}_L \quad \frac{h_5 : \Delta_9, F_4, F_7 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \& F_7), F_4 \vdash F_8}}{- : (\mathbf{1}, \Delta_2), \Delta_9, F_6 \& F_7 \vdash F_8} \text{Cut}}{\rightarrow} \&_{L2} \\
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \mathbf{1}, \Delta_2, \Delta_9, F_6 \& F_7 \vdash F_8} \text{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_9, F_6 \& F_7 \vdash F_8} \text{hCut}}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \& F_7}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_6 \& F_7} \mathbf{1}_L \quad \frac{h_4 : \Delta_5, F_7 \vdash F_8}{\bullet h_4 : \Delta_5, F_6 \& F_7 \vdash F_8}}{- : (\mathbf{1}, \Delta_2), \Delta_5 \vdash F_8} \text{Cut}}{\rightarrow} \&_{L2} \\
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \& F_7}{\bullet h_1 : \mathbf{1}, \Delta_2, \Delta_5 \vdash F_8} \text{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_5 \vdash F_8} \text{hCut}}{\rightarrow}
\end{array}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_4} \mathbf{1}_L \quad \frac{h_5 : \Delta_9, F_4, F_6 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \& F_7), F_4 \vdash F_8}}{- : (\mathbf{1}, \Delta_2), \Delta_9, F_6 \& F_7 \vdash F_8} \text{Cut}}{\rightarrow} \&_{L1} \\
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \mathbf{1}, \Delta_2, \Delta_9, F_6 \& F_7 \vdash F_8} \text{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_9, F_6 \& F_7 \vdash F_8} \text{hCut}}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \& F_7}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_6 \& F_7} \mathbf{1}_L \quad \frac{h_4 : \Delta_5, F_6 \vdash F_8}{\bullet h_4 : \Delta_5, F_6 \& F_7 \vdash F_8}}{- : (\mathbf{1}, \Delta_2), \Delta_5 \vdash F_8} \text{Cut}}{\rightarrow} \&_{L1} \\
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \& F_7}{\bullet h_1 : \mathbf{1}, \Delta_2, \Delta_5 \vdash F_8} \text{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_5 \vdash F_8} \text{hCut}}{\rightarrow}
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_4} \mathbf{1}_L \quad \frac{h_5 : \Delta_9, F_4, F_6, F_7 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \otimes F_7), F_4 \vdash F_8}}{- : (\mathbf{1}, \Delta_2), \Delta_9, F_6 \otimes F_7 \vdash F_8} \text{Cut}}{\rightarrow} \otimes_L \\
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \mathbf{1}, \Delta_2, \Delta_9, F_6 \otimes F_7 \vdash F_8} \text{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_9, F_6 \otimes F_7 \vdash F_8} \text{hCut}}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \otimes F_7}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_6 \otimes F_7} \mathbf{1}_L \quad \frac{h_4 : \Delta_5, F_6, F_7 \vdash F_8}{\bullet h_4 : \Delta_5, F_6 \otimes F_7 \vdash F_8}}{- : (\mathbf{1}, \Delta_2), \Delta_5 \vdash F_8} \text{Cut}}{\rightarrow} \otimes_L \\
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \otimes F_7}{\bullet h_1 : \mathbf{1}, \Delta_2, \Delta_5 \vdash F_8} \text{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_5 \vdash F_8} \text{hCut}}{\rightarrow}
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_4} \mathbf{1}_L \quad \frac{h_5 : \Delta_9, F_4, F_6 \vdash F_8 \quad h_5 : \Delta_9, F_4, F_7 \vdash F_8}{\bullet h_5 : (\Delta_9, F_6 \oplus F_7), F_4 \vdash F_8}}{- : (\mathbf{1}, \Delta_2), \Delta_9, F_6 \oplus F_7 \vdash F_8} \text{Cut}}{\rightarrow} \oplus_L \\
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \mathbf{1}, \Delta_2, \Delta_9, F_6 \oplus F_7 \vdash F_8} \text{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_9, F_6 \oplus F_7 \vdash F_8} \text{hCut}}{\rightarrow}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \oplus F_7}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_6 \oplus F_7} \mathbf{1}_L \quad \frac{h_4 : \Delta_5, F_6 \vdash F_8 \quad h_4 : \Delta_5, F_7 \vdash F_8}{\bullet h_4 : \Delta_5, F_6 \oplus F_7 \vdash F_8} \oplus_L}{- : (\mathbf{1}, \Delta_2), \Delta_5 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \oplus F_7}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_6 \oplus F_7} \text{ax/W} \quad \frac{h_4 : \Delta_5, F_6 \oplus F_7 \vdash F_8}{\bullet h_4 : \mathbf{1}, \Delta_5, F_6 \oplus F_7 \vdash F_8} \text{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_5 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule $\neg\circ_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_4} \mathbf{1}_L \quad \frac{h_5 : \Delta_{10}, F_4 \vdash F_7 \quad h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : (\Delta_6, \Delta_{10}, F_7 \neg\circ F_8), F_4 \vdash F_9} \neg\circ_L}{- : (\mathbf{1}, \Delta_2), \Delta_6, \Delta_{10}, F_7 \neg\circ F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_4} \text{ax/W} \quad \frac{h_5 : \Delta_{10}, \Delta_6, F_4, F_7 \neg\circ F_8 \vdash F_9}{\bullet h_5 : \mathbf{1}, \Delta_{10}, \Delta_6, F_7 \neg\circ F_8 \vdash F_9} \text{ax/W}}{- : \mathbf{1}, \Delta_{10}, \Delta_2, \Delta_6, F_7 \neg\circ F_8 \vdash F_9} \text{hCut} \\
\\
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_4} \mathbf{1}_L \quad \frac{h_5 : \Delta_6 \vdash F_7 \quad h_5 : \Delta_{10}, F_4, F_8 \vdash F_9}{\bullet h_5 : (\Delta_6, \Delta_{10}, F_7 \neg\circ F_8), F_4 \vdash F_9} \neg\circ_L}{- : (\mathbf{1}, \Delta_2), \Delta_6, \Delta_{10}, F_7 \neg\circ F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_4}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_4} \text{ax/W} \quad \frac{h_5 : \Delta_6, F_7 \neg\circ F_8 \vdash F_9}{\bullet h_5 : \mathbf{1}, \Delta_{10}, \Delta_6, F_7 \neg\circ F_8 \vdash F_9} \text{ax/W}}{- : \mathbf{1}, \Delta_{10}, \Delta_2, \Delta_6, F_7 \neg\circ F_8 \vdash F_9} \text{hCut} \\
\\
\frac{\frac{h_1 : \Delta_2 \vdash F_7 \neg\circ F_8}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_7 \neg\circ F_8} \mathbf{1}_L \quad \frac{h_4 : \Delta_5 \vdash F_7 \quad h_4 : \Delta_6, F_8 \vdash F_9}{\bullet h_4 : (\Delta_5, \Delta_6), F_7 \neg\circ F_8 \vdash F_9} \neg\circ_L}{- : (\mathbf{1}, \Delta_2), \Delta_5, \Delta_6 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_7 \neg\circ F_8}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash F_7 \neg\circ F_8} \text{ax/W} \quad \frac{h_4 : \Delta_5, \Delta_6, F_7 \neg\circ F_8 \vdash F_9}{\bullet h_4 : \mathbf{1}, \Delta_5, \Delta_6, F_7 \neg\circ F_8 \vdash F_9} \text{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_5, \Delta_6 \vdash F_9} \text{hCut}
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash p(n_5)}{\bullet h_1 : \mathbf{1}, \Delta_2 \vdash p(n_5)} \mathbf{1}_L \quad \frac{h_4 : *, p(n_5) \vdash p(n_5)}{\bullet h_4 : *, p(n_5) \vdash p(n_5)} I}{- : (\mathbf{1}, \Delta_2), * \vdash p(n_5)} \text{Cut} \\
\rightarrow \\
\frac{}{- : \mathbf{1}, \Delta_2 \vdash p(n_5)} \text{ax/W}
\end{array}$$

5.9 Status of \otimes_R : OK

- Case rule $!R$

- Case rule $\mathbf{1}_R$

- Case rule \top

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_7 \quad h_1 : \Delta_3 \vdash F_8}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_7 \otimes F_8} \otimes_R \quad \frac{h_9 : \Delta_6, F_7 \otimes F_8 \vdash \top}{\bullet h_9 : \Delta_6, F_7 \otimes F_8 \vdash \top} \top}{- : (\Delta_2, \Delta_3), \Delta_6 \vdash \top} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_3, \Delta_6 \vdash \top} \top
\end{array}$$

- Case rule $\&_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_7 \quad h_1 : \Delta_3 \vdash F_8}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_7 \otimes F_8} \otimes_R \quad \frac{h_9 : \Delta_6, F_7 \otimes F_8 \vdash F_{10} \quad h_9 : \Delta_6, F_7 \otimes F_8 \vdash F_{11}}{\bullet h_9 : \Delta_6, F_7 \otimes F_8 \vdash F_{10} \& F_{11}} \&_R}{- : (\Delta_2, \Delta_3), \Delta_6 \vdash F_{10} \& F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_7 \quad h_1 : \Delta_3 \vdash F_8}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_7 \otimes F_8} \text{ax/W} \quad \frac{h_9 : \Delta_6, F_7 \otimes F_8 \vdash F_{10}}{\text{hCut}} \text{ax/W}}{- : \Delta_2, \Delta_3, \Delta_6 \vdash F_{10}} \quad \frac{\frac{h_9 : \Delta_6, F_7 \otimes F_8 \vdash F_{11}}{\text{hCut}} \text{ax/W}}{- : \Delta_2, \Delta_3, \Delta_6 \vdash F_{11}} \text{ax/W}}{- : \Delta_2, \Delta_3, \Delta_6 \vdash F_{10} \& F_{11}} \&_R
\end{array}$$

- Case rule $\neg\circ_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_7 \quad h_1 : \Delta_3 \vdash F_8}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_7 \otimes F_8} \otimes_R \quad \frac{h_9 : \Delta_6, F_{10}, F_7 \otimes F_8 \vdash F_{11}}{\bullet h_9 : \Delta_6, F_7 \otimes F_8 \vdash F_{10} \neg\circ F_{11}} \neg\circ_R}{- : (\Delta_2, \Delta_3), \Delta_6 \vdash F_{10} \neg\circ F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_7 \quad h_1 : \Delta_3 \vdash F_8}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_7 \otimes F_8} \text{ax/W} \quad \frac{h_9 : \Delta_6, F_{10}, F_7 \otimes F_8 \vdash F_{11}}{\text{hCut}} \text{ax/W}}{- : \Delta_2, \Delta_3, \Delta_6, F_{10} \vdash F_{11}} \text{hCut}}{- : \Delta_2, \Delta_3, \Delta_6 \vdash F_{10} \neg\circ F_{11}} \neg\circ_R
\end{array}$$

- Case rule \oplus_{R_2}

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_7 \quad h_1 : \Delta_3 \vdash F_8}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_7 \otimes F_8} \otimes_R \quad \frac{h_9 : \Delta_6, F_7 \otimes F_8 \vdash F_{11}}{\bullet h_9 : \Delta_6, F_7 \otimes F_8 \vdash F_{10} \oplus F_{11}} \oplus_{R_2}}{- : (\Delta_2, \Delta_3), \Delta_6 \vdash F_{10} \oplus F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_7 \quad h_1 : \Delta_3 \vdash F_8}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_7 \otimes F_8} \text{ax/W} \quad \frac{h_9 : \Delta_6, F_7 \otimes F_8 \vdash F_{11}}{\text{hCut}} \text{ax/W}}{- : \Delta_2, \Delta_3, \Delta_6 \vdash F_{11}} \text{hCut}}{- : \Delta_2, \Delta_3, \Delta_6 \vdash F_{10} \oplus F_{11}} \oplus_{R_2}
\end{array}$$

- Case rule \oplus_{R_1}

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_7 \quad h_1 : \Delta_3 \vdash F_8}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_7 \otimes F_8} \otimes_R \quad \frac{h_9 : \Delta_6, F_7 \otimes F_8 \vdash F_{10}}{\bullet h_9 : \Delta_6, F_7 \otimes F_8 \vdash F_{10} \oplus F_{11}} \oplus_{R_1}}{- : (\Delta_2, \Delta_3), \Delta_6 \vdash F_{10} \oplus F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_7 \quad h_1 : \Delta_3 \vdash F_8}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_7 \otimes F_8} \text{ax/W} \quad \frac{h_9 : \Delta_6, F_7 \otimes F_8 \vdash F_{10}}{\text{hCut}} \text{ax/W}}{- : \Delta_2, \Delta_3, \Delta_6 \vdash F_{10}} \text{hCut}}{- : \Delta_2, \Delta_3, \Delta_6 \vdash F_{10} \oplus F_{11}} \oplus_{R_1}
\end{array}$$

- Case rule $\mathbf{1}_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{10}, F_6 \otimes F_7 \vdash F_9}{\bullet h_8 : (\mathbf{1}, \Delta_{10}), F_6 \otimes F_7 \vdash F_9} \mathbf{1}_L}{- : (\Delta_2, \Delta_3), \mathbf{1}, \Delta_{10} \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7} \text{ax/W} \quad \frac{h_8 : \mathbf{1}, \Delta_{10}, F_6 \otimes F_7 \vdash F_9}{\text{hCut}} \text{ax/W}}{- : \mathbf{1}, \Delta_{10}, \Delta_2, \Delta_3 \vdash F_9} \text{hCut}
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{12}, F_6 \otimes F_7 \vdash F_{10} \quad h_8 : \Delta_9 \vdash F_{11}}{\bullet h_8 : (\Delta_9, \Delta_{12}), F_6 \otimes F_7 \vdash F_{10} \otimes F_{11}} \otimes_R}{- : (\Delta_2, \Delta_3), \Delta_9, \Delta_{12} \vdash F_{10} \otimes F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7} \text{ax/W} \quad \frac{h_8 : \Delta_{12}, F_6 \otimes F_7 \vdash F_{10}}{\text{hCut}} \text{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_3 \vdash F_{10}} \text{hCut} \quad \frac{- : \Delta_9 \vdash F_{11}}{\otimes_R} \text{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_3, \Delta_9 \vdash F_{10} \otimes F_{11}} \otimes_R
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_9 \vdash F_{10} \quad h_8 : \Delta_{12}, F_6 \otimes F_7 \vdash F_{11}}{\bullet h_8 : (\Delta_9, \Delta_{12}), F_6 \otimes F_7 \vdash F_{10} \otimes F_{11}} \otimes_R}{- : (\Delta_2, \Delta_3), \Delta_9, \Delta_{12} \vdash F_{10} \otimes F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_9 \vdash F_{10} \quad \bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7}{- : \Delta_{12}, \Delta_2, \Delta_3, \Delta_9 \vdash F_{10} \otimes F_{11}} \text{ax/W} \quad \frac{h_8 : \Delta_{12}, F_6 \otimes F_7 \vdash F_{11}}{- : \Delta_{12}, \Delta_2, \Delta_3 \vdash F_{11}} \text{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_3, \Delta_9 \vdash F_{10} \otimes F_{11}} \text{hCut} \otimes_R
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{11}, F_6 \otimes F_7 \vdash F_{10}}{\bullet h_8 : (\Delta_{11}, !F_9), F_6 \otimes F_7 \vdash F_{10}} W}{- : (\Delta_2, \Delta_3), \Delta_{11}, !F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7 \quad \text{ax/W} \quad h_8 : \Delta_{11}, !F_9, F_6 \otimes F_7 \vdash F_{10} \quad \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_3, !F_9 \vdash F_{10}} \text{hCut}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{11}, !F_9, !F_9, F_6 \otimes F_7 \vdash F_{10}}{\bullet h_8 : (\Delta_{11}, !F_9), F_6 \otimes F_7 \vdash F_{10}} C}{- : (\Delta_2, \Delta_3), \Delta_{11}, !F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7 \quad \text{ax/W} \quad h_8 : \Delta_{11}, !F_9, !F_9, F_6 \otimes F_7 \vdash F_{10} \quad \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_3, !F_9, !F_9 \vdash F_{10}} \text{hCut} \\
\text{---} \\
- : \Delta_{11}, \Delta_2, \Delta_3, !F_9 \vdash F_{10} \quad C
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{11}, F_9, F_6 \otimes F_7 \vdash F_{10}}{\bullet h_8 : (\Delta_{11}, !F_9), F_6 \otimes F_7 \vdash F_{10}} !L}{- : (\Delta_2, \Delta_3), \Delta_{11}, !F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7 \quad \text{ax/W} \quad h_8 : \Delta_{11}, F_9, F_6 \otimes F_7 \vdash F_{10} \quad \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_3, F_9 \vdash F_{10}} \text{hCut} \\
\text{---} \\
- : \Delta_{11}, \Delta_2, \Delta_3, !F_9 \vdash F_{10} \quad !L
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{12}, F_{10}, F_6 \otimes F_7 \vdash F_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \& F_{10}), F_6 \otimes F_7 \vdash F_{11}} \&_{L2}}{- : (\Delta_2, \Delta_3), \Delta_{12}, F_9 \& F_{10} \vdash F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7 \quad \text{ax/W} \quad h_8 : \Delta_{12}, F_{10}, F_6 \otimes F_7 \vdash F_{11} \quad \text{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_3, F_{10} \vdash F_{11}} \text{hCut} \\
\text{---} \\
- : \Delta_{12}, \Delta_2, \Delta_3, F_9 \& F_{10} \vdash F_{11} \quad \&_{L2}
\end{array}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \otimes F_7 \vdash F_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \& F_{10}), F_6 \otimes F_7 \vdash F_{11}} \&_{L1}}{- : (\Delta_2, \Delta_3), \Delta_{12}, F_9 \& F_{10} \vdash F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7 \quad \text{ax/W} \quad h_8 : \Delta_{12}, F_9, F_6 \otimes F_7 \vdash F_{11} \quad \text{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_3, F_9 \vdash F_{11}} \text{hCut} \\
\text{---} \\
- : \Delta_{12}, \Delta_2, \Delta_3, F_9 \& F_{10} \vdash F_{11} \quad \&_{L1}
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{12}, F_9, F_{10}, F_6 \otimes F_7 \vdash F_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \otimes F_{10}), F_6 \otimes F_7 \vdash F_{11}} \otimes_L}{- : (\Delta_2, \Delta_3), \Delta_{12}, F_9 \otimes F_{10} \vdash F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7}{- : \Delta_{12}, \Delta_2, \Delta_3, F_{10}, F_9 \vdash F_{11}} \text{ax/W} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_9, F_6 \otimes F_7 \vdash F_{11}}{- : \Delta_{12}, \Delta_2, \Delta_3, F_9 \otimes F_{10} \vdash F_{11}} \text{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_3, F_9 \otimes F_{10} \vdash F_{11}} \text{hCut} \otimes_L \\
\frac{h_1 : \Delta_2 \vdash F_8 \quad h_1 : \Delta_3 \vdash F_9}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_8 \otimes F_9} \otimes_R \quad \frac{h_6 : \Delta_7, F_8, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \otimes F_9 \vdash F_{10}} \otimes_L}{- : (\Delta_2, \Delta_3), \Delta_7 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_2 \vdash F_8}{- : \Delta_2, \Delta_3, \Delta_7 \vdash F_{10}} \text{ax/W} \quad \frac{- : \Delta_3 \vdash F_9}{- : \Delta_3, \Delta_7, F_8 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_3, \Delta_7 \vdash F_{10}} \text{sCut} \quad \frac{- : \Delta_7, F_8, F_9 \vdash F_{10}}{- : \Delta_2, \Delta_3, \Delta_7 \vdash F_{10}} \text{sCut}}{- : \Delta_2, \Delta_3, \Delta_7 \vdash F_{10}} \text{sCut}
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \otimes F_7 \vdash F_{11} \quad h_8 : \Delta_{12}, F_{10}, F_6 \otimes F_7 \vdash F_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \oplus F_{10}), F_6 \otimes F_7 \vdash F_{11}} \oplus_L}{- : (\Delta_2, \Delta_3), \Delta_{12}, F_9 \oplus F_{10} \vdash F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7}{- : \Delta_{12}, \Delta_2, \Delta_3, F_9 \vdash F_{11}} \text{ax/W} \quad \frac{h_8 : \Delta_{12}, F_9, F_6 \otimes F_7 \vdash F_{11}}{- : \Delta_{12}, \Delta_2, \Delta_3, F_{10} \vdash F_{11}} \text{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_3, F_9 \oplus F_{10} \vdash F_{11}} \text{hCut} \quad \frac{\frac{h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7}{- : \Delta_{12}, \Delta_2, \Delta_3, F_{10} \vdash F_{11}} \text{ax/W} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_6 \otimes F_7 \vdash F_{11}}{- : \Delta_{12}, \Delta_2, \Delta_3, F_9 \oplus F_{10} \vdash F_{11}} \text{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_3, F_9 \oplus F_{10} \vdash F_{11}} \text{hCut}}{- : \Delta_{12}, \Delta_2, \Delta_3, F_9 \oplus F_{10} \vdash F_{11}} \oplus_L
\end{array}$$

- Case rule \multimap_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_{13}, F_6 \otimes F_7 \vdash F_{10} \quad h_8 : \Delta_9, F_{11} \vdash F_{12}}{\bullet h_8 : (\Delta_9, \Delta_{13}, F_{10} \multimap F_{11}), F_6 \otimes F_7 \vdash F_{12}} \multimap_L}{- : (\Delta_2, \Delta_3), \Delta_9, \Delta_{13}, F_{10} \multimap F_{11} \vdash F_{12}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7}{- : \Delta_{13}, \Delta_2, \Delta_3 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_{13}, F_6 \otimes F_7 \vdash F_{10}}{- : \Delta_{13}, \Delta_2, \Delta_3, \Delta_9, F_{10} \multimap F_{11} \vdash F_{12}} \text{ax/W}}{- : \Delta_{13}, \Delta_2, \Delta_3, \Delta_9, F_{10} \multimap F_{11} \vdash F_{12}} \text{hCut} \quad \frac{- : \Delta_9, F_{11} \vdash F_{12}}{- : \Delta_{13}, \Delta_2, \Delta_3, \Delta_9, F_{10} \multimap F_{11} \vdash F_{12}} \text{ax/W}}{- : \Delta_{13}, \Delta_2, \Delta_3, \Delta_9, F_{10} \multimap F_{11} \vdash F_{12}} \multimap_L \\
\frac{h_1 : \Delta_2 \vdash F_6 \quad h_1 : \Delta_3 \vdash F_7}{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7} \otimes_R \quad \frac{h_8 : \Delta_9 \vdash F_{10} \quad h_8 : \Delta_{13}, F_{11}, F_6 \otimes F_7 \vdash F_{12}}{\bullet h_8 : (\Delta_9, \Delta_{13}, F_{10} \multimap F_{11}), F_6 \otimes F_7 \vdash F_{12}} \multimap_L}{- : (\Delta_2, \Delta_3), \Delta_9, \Delta_{13}, F_{10} \multimap F_{11} \vdash F_{12}} \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_9 \vdash F_{10}}{- : \Delta_{13}, \Delta_2, \Delta_3, \Delta_9, F_{10} \multimap F_{11} \vdash F_{12}} \text{ax/W} \quad \frac{\bullet h_1 : \Delta_2, \Delta_3 \vdash F_6 \otimes F_7}{- : \Delta_{13}, \Delta_2, \Delta_3, F_{11} \vdash F_{12}} \text{ax/W} \quad \frac{h_8 : \Delta_{13}, F_{11}, F_6 \otimes F_7 \vdash F_{12}}{- : \Delta_{13}, \Delta_2, \Delta_3, F_{11} \vdash F_{12}} \text{hCut}}{- : \Delta_{13}, \Delta_2, \Delta_3, \Delta_9, F_{10} \multimap F_{11} \vdash F_{12}} \text{hCut}}{- : \Delta_{13}, \Delta_2, \Delta_3, \Delta_9, F_{10} \multimap F_{11} \vdash F_{12}} \multimap_L
\end{array}$$

- Case rule I

5.10 Status of W : OK

- Case rule $!R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash 6 \quad W \quad h_7 : !\Upsilon 5, 6 \vdash F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash 6} \quad \frac{h_7 : !\Upsilon 5, 6 \vdash F_8}{\bullet h_7 : !\Upsilon 5, 6 \vdash !F_8} \quad !R}{- : (\Delta_2, !F_3), !\Upsilon 5 \vdash !F_8} \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_2 \vdash 6}{- : !\Upsilon 5, \Delta_2, !F_3 \vdash !F_8} \text{ax/W} \quad \frac{\bullet h_7 : 6, !\Upsilon 5, !F_3 \vdash !F_8}{- : !\Upsilon 5, \Delta_2, !F_3 \vdash !F_8} \text{hCut}}{- : !\Upsilon 5, \Delta_2, !F_3 \vdash !F_8} \text{ax/W}
\end{array}$$

- Case rule 1_R

- Case rule \top

$$\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} W \quad \frac{}{\bullet h_7 : \Delta_5, F_6 \vdash \top} \top}{- : (\Delta_2, !F_3), \Delta_5 \vdash \top} \text{Cut} \quad \rightarrow \quad \frac{}{- : \Delta_2, \Delta_5, !F_3 \vdash \top} \top$$

- Case rule $\&_R$

$$\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} W \quad \frac{h_7 : \Delta_5, F_6 \vdash F_8 \quad h_7 : \Delta_5, F_6 \vdash F_9}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \& F_9} \&_R}{- : (\Delta_2, !F_3), \Delta_5 \vdash F_8 \& F_9} \text{Cut} \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} \text{ax/W} \quad \frac{h_7 : \Delta_5, F_6 \vdash F_8 \quad h_7 : \Delta_5, F_6 \vdash F_9}{\bullet h_7 : \Delta_5, F_6, !F_3 \vdash F_8 \& F_9} \text{ax/W}}{- : \Delta_2, \Delta_5, !F_3 \vdash F_8 \& F_9} \text{hCut}$$

- Case rule $\neg\circ_R$

$$\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} W \quad \frac{h_7 : \Delta_5, F_6, F_8 \vdash F_9}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \neg\circ F_9} \neg\circ_R}{- : (\Delta_2, !F_3), \Delta_5 \vdash F_8 \neg\circ F_9} \text{Cut} \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} \text{ax/W} \quad \frac{h_7 : \Delta_5, F_6, F_8 \vdash F_9}{\bullet h_7 : \Delta_5, F_6, !F_3 \vdash F_8 \neg\circ F_9} \text{ax/W}}{- : \Delta_2, \Delta_5, !F_3 \vdash F_8 \neg\circ F_9} \text{hCut}$$

- Case rule \oplus_{R_2}

$$\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} W \quad \frac{h_7 : \Delta_5, F_6 \vdash F_9}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \oplus F_9} \oplus_{R_2}}{- : (\Delta_2, !F_3), \Delta_5 \vdash F_8 \oplus F_9} \text{Cut} \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} \text{ax/W} \quad \frac{h_7 : \Delta_5, F_6 \vdash F_9}{\bullet h_7 : \Delta_5, F_6, !F_3 \vdash F_8 \oplus F_9} \text{ax/W}}{- : \Delta_2, \Delta_5, !F_3 \vdash F_8 \oplus F_9} \text{hCut}$$

- Case rule \oplus_{R_1}

$$\frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} W \quad \frac{h_7 : \Delta_5, F_6 \vdash F_8}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \oplus F_9} \oplus_{R_1}}{- : (\Delta_2, !F_3), \Delta_5 \vdash F_8 \oplus F_9} \text{Cut} \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_2 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} \text{ax/W} \quad \frac{h_7 : \Delta_5, F_6 \vdash F_8}{\bullet h_7 : \Delta_5, F_6, !F_3 \vdash F_8 \oplus F_9} \text{ax/W}}{- : \Delta_2, \Delta_5, !F_3 \vdash F_8 \oplus F_9} \text{hCut}$$

- Case rule $\mathbf{1}_L$

$$\frac{\frac{h_1 : \Delta_2 \vdash \mathbf{1}}{\bullet h_1 : \Delta_2, !F_3 \vdash \mathbf{1}} W \quad \frac{h_5 : \Delta_6 \vdash F_7}{\bullet h_5 : \Delta_6, \mathbf{1} \vdash F_7} \mathbf{1}_L}{- : (\Delta_2, !F_3), \Delta_6 \vdash F_7} \text{Cut} \quad \rightarrow \quad \frac{}{- : \Delta_2, \Delta_6, !F_3 \vdash F_7} \text{ax/W}$$

$$\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} W \quad \frac{h_6 : \Delta_8, F_5 \vdash F_7}{\bullet h_6 : (\mathbf{1}, \Delta_8), F_5 \vdash F_7} \mathbf{1}_L}{- : (\Delta_2, !F_3), \mathbf{1}, \Delta_8 \vdash F_7} \text{Cut} \quad \rightarrow \quad \frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \text{ax/W} \quad \frac{h_6 : \Delta_8, F_5 \vdash F_7}{\bullet h_6 : \mathbf{1}, \Delta_8, F_5, !F_3 \vdash F_7} \text{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_8, !F_3 \vdash F_7} \text{hCut}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} W \quad \frac{h_6 : \Delta_{10}, F_5 \vdash F_8 \quad h_6 : \Delta_7 \vdash F_9}{\bullet h_6 : (\Delta_7, \Delta_{10}), F_5 \vdash F_8 \otimes F_9} \otimes_R}{- : (\Delta_2, !F_3), \Delta_7, \Delta_{10} \vdash F_8 \otimes F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{- : \Delta_2, \Delta_7, !F_3 \vdash F_8 \otimes F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, \Delta_7, F_5, !F_3 \vdash F_8 \otimes F_9}{- : \Delta_{10}, \Delta_2, \Delta_7, !F_3 \vdash F_8 \otimes F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, \Delta_7, !F_3 \vdash F_8 \otimes F_9} \text{hCut} \\
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} W \quad \frac{h_6 : \Delta_7 \vdash F_8 \quad h_6 : \Delta_{10}, F_5 \vdash F_9}{\bullet h_6 : (\Delta_7, \Delta_{10}), F_5 \vdash F_8 \otimes F_9} \otimes_R}{- : (\Delta_2, !F_3), \Delta_7, \Delta_{10} \vdash F_8 \otimes F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{- : \Delta_2, \Delta_7, !F_3 \vdash F_8 \otimes F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, \Delta_7, F_5, !F_3 \vdash F_8 \otimes F_9}{- : \Delta_{10}, \Delta_2, \Delta_7, !F_3 \vdash F_8 \otimes F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, \Delta_7, !F_3 \vdash F_8 \otimes F_9} \text{hCut}
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} W \quad \frac{h_6 : \Delta_9, F_5 \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_5 \vdash F_8} W}{- : (\Delta_2, !F_3), \Delta_9, !F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_9, F_5, !F_3, !F_7 \vdash F_8}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \text{ax/W}}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \text{hCut} \\
\frac{h_1 : \Delta_2 \vdash !F_7}{\bullet h_1 : \Delta_2, !F_3 \vdash !F_7} W \quad \frac{h_5 : \Delta_6 \vdash F_8}{\bullet h_5 : \Delta_6, !F_7 \vdash F_8} W}{- : (\Delta_2, !F_3), \Delta_6 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_2, \Delta_6, !F_3 \vdash F_8}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8} \text{ax/W}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} W \quad \frac{h_6 : \Delta_9, F_5, !F_7, !F_7 \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_5 \vdash F_8} C}{- : (\Delta_2, !F_3), \Delta_9, !F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_9, F_5, !F_3, !F_7 \vdash F_8}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \text{ax/W}}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \text{hCut} \\
\frac{h_1 : \Delta_2 \vdash !F_7}{\bullet h_1 : \Delta_2, !F_3 \vdash !F_7} W \quad \frac{h_5 : \Delta_6, !F_7, !F_7 \vdash F_8}{\bullet h_5 : \Delta_6, !F_7 \vdash F_8} C}{- : (\Delta_2, !F_3), \Delta_6 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash !F_7}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_6, !F_3, !F_7 \vdash F_8}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8} \text{ax/W}}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} W \quad \frac{h_6 : \Delta_9, F_5, F_7 \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_5 \vdash F_8} !L}{- : (\Delta_2, !F_3), \Delta_9, !F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_9, F_5, !F_3, !F_7 \vdash F_8}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \text{ax/W}}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \text{hCut} \\
\frac{h_1 : \Delta_2 \vdash !F_7}{\bullet h_1 : \Delta_2, !F_3 \vdash !F_7} W \quad \frac{h_5 : \Delta_6, F_7 \vdash F_8}{\bullet h_5 : \Delta_6, !F_7 \vdash F_8} !L}{- : (\Delta_2, !F_3), \Delta_6 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash !F_7}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_6, !F_3, !F_7 \vdash F_8}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8} \text{ax/W}}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} W \quad \frac{h_6 : \Delta_{10}, F_5, F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \& F_8), F_5 \vdash F_9}}{- : (\Delta_2, !F_3), \Delta_{10}, F_7 \& F_8 \vdash F_9}}{\rightarrow} \&_{L2} \text{Cut} \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, F_5, !F_3, F_7 \& F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, \Delta_2, !F_3, F_7 \& F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_3, F_7 \& F_8 \vdash F_9}}{\text{hCut}} \\
\frac{\frac{h_1 : \Delta_2 \vdash F_7 \& F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \& F_8} W \quad \frac{h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \& F_8 \vdash F_9}}{- : (\Delta_2, !F_3), \Delta_6 \vdash F_9}}{\rightarrow} \&_{L2} \text{Cut} \\
\frac{\frac{h_1 : \Delta_2 \vdash F_7 \& F_8}{\bullet h_1 : \Delta_2 \vdash F_7 \& F_8} \text{ax/W} \quad \frac{h_5 : \Delta_6, !F_3, F_7 \& F_8 \vdash F_9}{\bullet h_5 : \Delta_6, !F_3, F_7 \& F_8 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_6, !F_3 \vdash F_9}}{\text{hCut}}
\end{array}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} W \quad \frac{h_6 : \Delta_{10}, F_5, F_7 \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \& F_8), F_5 \vdash F_9}}{- : (\Delta_2, !F_3), \Delta_{10}, F_7 \& F_8 \vdash F_9}}{\rightarrow} \&_{L1} \text{Cut} \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, F_5, !F_3, F_7 \& F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, \Delta_2, !F_3, F_7 \& F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_3, F_7 \& F_8 \vdash F_9}}{\text{hCut}} \\
\frac{\frac{h_1 : \Delta_2 \vdash F_7 \& F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \& F_8} W \quad \frac{h_5 : \Delta_6, F_7 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \& F_8 \vdash F_9}}{- : (\Delta_2, !F_3), \Delta_6 \vdash F_9}}{\rightarrow} \&_{L1} \text{Cut} \\
\frac{\frac{h_1 : \Delta_2 \vdash F_7 \& F_8}{\bullet h_1 : \Delta_2 \vdash F_7 \& F_8} \text{ax/W} \quad \frac{h_5 : \Delta_6, !F_3, F_7 \& F_8 \vdash F_9}{\bullet h_5 : \Delta_6, !F_3, F_7 \& F_8 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_6, !F_3 \vdash F_9}}{\text{hCut}}
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} W \quad \frac{h_6 : \Delta_{10}, F_5, F_7, F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \otimes F_8), F_5 \vdash F_9}}{- : (\Delta_2, !F_3), \Delta_{10}, F_7 \otimes F_8 \vdash F_9}}{\rightarrow} \otimes_L \text{Cut} \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, F_5, !F_3, F_7 \otimes F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, \Delta_2, !F_3, F_7 \otimes F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_3, F_7 \otimes F_8 \vdash F_9}}{\text{hCut}} \\
\frac{\frac{h_1 : \Delta_2 \vdash F_7 \otimes F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \otimes F_8} W \quad \frac{h_5 : \Delta_6, F_7, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \otimes F_8 \vdash F_9}}{- : (\Delta_2, !F_3), \Delta_6 \vdash F_9}}{\rightarrow} \otimes_L \text{Cut} \\
\frac{\frac{h_1 : \Delta_2 \vdash F_7 \otimes F_8}{\bullet h_1 : \Delta_2 \vdash F_7 \otimes F_8} \text{ax/W} \quad \frac{h_5 : \Delta_6, !F_3, F_7 \otimes F_8 \vdash F_9}{\bullet h_5 : \Delta_6, !F_3, F_7 \otimes F_8 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_6, !F_3 \vdash F_9}}{\text{hCut}}
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} W \quad \frac{h_6 : \Delta_{10}, F_5, F_7 \vdash F_9 \quad h_6 : \Delta_{10}, F_5, F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \oplus F_8), F_5 \vdash F_9}}{- : (\Delta_2, !F_3), \Delta_{10}, F_7 \oplus F_8 \vdash F_9}}{\rightarrow} \oplus_L \text{Cut} \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2 \vdash F_5} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, F_5, !F_3, F_7 \oplus F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, \Delta_2, !F_3, F_7 \oplus F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_3, F_7 \oplus F_8 \vdash F_9}}{\text{hCut}}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_7 \oplus F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \oplus F_8} \quad W \quad \frac{h_5 : \Delta_6, F_7 \vdash F_9 \quad h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \oplus F_8 \vdash F_9} \oplus L \\
\hline
- : (\Delta_2, !F_3), \Delta_6 \vdash F_9 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_7 \oplus F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \oplus F_8} \quad \text{ax/W} \quad \frac{h_5 : \Delta_6, !F_3, F_7 \oplus F_8 \vdash F_9}{\bullet h_5 : \Delta_6, !F_3, F_7 \oplus F_8 \vdash F_9} \quad \text{ax/W}}{- : \Delta_2, \Delta_6, !F_3 \vdash F_9} \text{hCut}
\end{array}$$

- Case rule \multimap_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \quad W \quad \frac{h_6 : \Delta_{11}, F_5 \vdash F_8 \quad h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : (\Delta_7, \Delta_{11}, F_8 \multimap F_9), F_5 \vdash F_{10}} \multimap_L \\
\hline
- : (\Delta_2, !F_3), \Delta_7, \Delta_{11}, F_8 \multimap F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \quad \text{ax/W} \quad \frac{h_6 : \Delta_{11}, \Delta_7, F_5, !F_3, F_8 \multimap F_9 \vdash F_{10}}{\bullet h_6 : \Delta_{11}, \Delta_7, F_5, !F_3, F_8 \multimap F_9 \vdash F_{10}} \quad \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_7, !F_3, F_8 \multimap F_9 \vdash F_{10}} \text{hCut} \\
\\
\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \quad W \quad \frac{h_6 : \Delta_7 \vdash F_8 \quad h_6 : \Delta_{11}, F_5, F_9 \vdash F_{10}}{\bullet h_6 : (\Delta_7, \Delta_{11}, F_8 \multimap F_9), F_5 \vdash F_{10}} \multimap_L \\
\hline
- : (\Delta_2, !F_3), \Delta_7, \Delta_{11}, F_8 \multimap F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \quad \text{ax/W} \quad \frac{h_6 : \Delta_{11}, \Delta_7, F_5, !F_3, F_8 \multimap F_9 \vdash F_{10}}{\bullet h_6 : \Delta_{11}, \Delta_7, F_5, !F_3, F_8 \multimap F_9 \vdash F_{10}} \quad \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_7, !F_3, F_8 \multimap F_9 \vdash F_{10}} \text{hCut} \\
\\
\frac{h_1 : \Delta_2 \vdash F_8 \multimap F_9}{\bullet h_1 : \Delta_2, !F_3 \vdash F_8 \multimap F_9} \quad W \quad \frac{h_5 : \Delta_6 \vdash F_8 \quad h_5 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_5 : (\Delta_6, \Delta_7), F_8 \multimap F_9 \vdash F_{10}} \multimap_L \\
\hline
- : (\Delta_2, !F_3), \Delta_6, \Delta_7 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2 \vdash F_8 \multimap F_9}{\bullet h_1 : \Delta_2, !F_3 \vdash F_8 \multimap F_9} \quad \text{ax/W} \quad \frac{h_5 : \Delta_6, \Delta_7, !F_3, F_8 \multimap F_9 \vdash F_{10}}{\bullet h_5 : \Delta_6, \Delta_7, !F_3, F_8 \multimap F_9 \vdash F_{10}} \quad \text{ax/W}}{- : \Delta_2, \Delta_6, \Delta_7, !F_3 \vdash F_{10}} \text{hCut}
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash p(n_6)}{\bullet h_1 : \Delta_2, !F_3 \vdash p(n_6)} \quad W \quad \frac{}{\bullet h_5 : *, p(n_6) \vdash p(n_6)} I \\
\hline
- : (\Delta_2, !F_3), * \vdash p(n_6) \\
\rightarrow \\
\frac{}{- : \Delta_2, !F_3 \vdash p(n_6)} \text{ax/W}
\end{array}$$

5.11 Status of C : OK

- Case rule $!R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash 6}{\bullet h_1 : \Delta_2, !F_3 \vdash 6} \quad C \quad \frac{h_7 : !\Upsilon 5, 6 \vdash F_8}{\bullet h_7 : !\Upsilon 5, 6 \vdash !F_8} !R \\
\hline
- : (\Delta_2, !F_3), !\Upsilon 5 \vdash !F_8 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash 6}{\bullet h_1 : \Delta_2, !F_3 \vdash 6} \quad \text{ax/W} \quad \frac{h_7 : 6, !\Upsilon 5 \vdash !F_8}{\bullet h_7 : 6, !\Upsilon 5 \vdash !F_8} \quad \text{ax/W}}{- : !\Upsilon 5, \Delta_2, !F_3, !F_3 \vdash !F_8} \text{hCut} \\
\frac{}{- : !\Upsilon 5, \Delta_2, !F_3 \vdash !F_8} C
\end{array}$$

- Case rule 1_R

- Case rule \top

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} \quad C \quad \frac{}{\bullet h_7 : \Delta_5, F_6 \vdash \top} \top \\
\hline
- : (\Delta_2, !F_3), \Delta_5 \vdash \top \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_5, !F_3 \vdash \top} \top
\end{array}$$

- Case rule $\&_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} C \quad \frac{h_7 : \Delta_5, F_6 \vdash F_8 \quad h_7 : \Delta_5, F_6 \vdash F_9}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \& F_9} \&_R \\
\hline
- : (\Delta_2, !F_3), \Delta_5 \vdash F_8 \& F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} \text{ax/W} \quad \frac{h_7 : \Delta_5, F_6 \vdash F_8 \& F_9}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \& F_9} \text{ax/W}}{- : \Delta_2, \Delta_5, !F_3, !F_3 \vdash F_8 \& F_9} \text{hCut} \\
\hline
- : \Delta_2, \Delta_5, !F_3 \vdash F_8 \& F_9 \quad C
\end{array}$$

- Case rule $\neg\circ_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} C \quad \frac{h_7 : \Delta_5, F_6, F_8 \vdash F_9}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \neg\circ F_9} \neg\circ_R \\
\hline
- : (\Delta_2, !F_3), \Delta_5 \vdash F_8 \neg\circ F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} \text{ax/W} \quad \frac{h_7 : \Delta_5, F_6, F_8 \vdash F_9}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \neg\circ F_9} \text{ax/W}}{- : \Delta_2, \Delta_5, F_8, !F_3 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_2, \Delta_5, !F_3 \vdash F_8 \neg\circ F_9 \quad \neg\circ_R
\end{array}$$

- Case rule \oplus_{R_2}

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} C \quad \frac{h_7 : \Delta_5, F_6 \vdash F_9}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \oplus F_9} \oplus_{R_2} \\
\hline
- : (\Delta_2, !F_3), \Delta_5 \vdash F_8 \oplus F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} \text{ax/W} \quad \frac{h_7 : \Delta_5, F_6 \vdash F_9}{\bullet h_7 : \Delta_5, F_6 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_5, !F_3 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_2, \Delta_5, !F_3 \vdash F_8 \oplus F_9 \quad \oplus_{R_2}
\end{array}$$

- Case rule \oplus_{R_1}

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} C \quad \frac{h_7 : \Delta_5, F_6 \vdash F_8}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \oplus F_9} \oplus_{R_1} \\
\hline
- : (\Delta_2, !F_3), \Delta_5 \vdash F_8 \oplus F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} \text{ax/W} \quad \frac{h_7 : \Delta_5, F_6 \vdash F_8}{\bullet h_7 : \Delta_5, F_6 \vdash F_8} \text{ax/W}}{- : \Delta_2, \Delta_5, !F_3 \vdash F_8} \text{hCut} \\
\hline
- : \Delta_2, \Delta_5, !F_3 \vdash F_8 \oplus F_9 \quad \oplus_{R_1}
\end{array}$$

- Case rule $\mathbf{1}_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash \mathbf{1}}{\bullet h_1 : \Delta_2, !F_3 \vdash \mathbf{1}} C \quad \frac{h_5 : \Delta_6 \vdash F_7}{\bullet h_5 : \Delta_6, \mathbf{1} \vdash F_7} \mathbf{1}_L \\
\hline
- : (\Delta_2, !F_3), \Delta_6 \vdash F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash \mathbf{1}}{\bullet h_1 : \Delta_2, !F_3 \vdash \mathbf{1}} \text{ax/W} \quad \frac{h_5 : \Delta_6 \vdash F_7}{\bullet h_5 : \Delta_6, \mathbf{1} \vdash F_7} \mathbf{1}_L}}{- : \Delta_2, \Delta_6, !F_3 \vdash F_7} \text{hCut} \\
\hline
- : \Delta_2, \Delta_6, !F_3 \vdash F_7
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} C \quad \frac{h_6 : \Delta_8, F_5 \vdash F_7}{\bullet h_6 : (\mathbf{1}, \Delta_8), F_5 \vdash F_7} \mathbf{1}_L \\
\hline
- : (\Delta_2, !F_3), \mathbf{1}, \Delta_8 \vdash F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \text{ax/W} \quad \frac{h_6 : \Delta_8, F_5 \vdash F_7}{\bullet h_6 : (\mathbf{1}, \Delta_8), F_5 \vdash F_7} \text{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_8, !F_3 \vdash F_7} \text{hCut} \\
\hline
- : \mathbf{1}, \Delta_2, \Delta_8, !F_3 \vdash F_7
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} C \quad \frac{h_6 : \Delta_{10}, F_5 \vdash F_8 \quad h_6 : \Delta_7 \vdash F_9}{\bullet h_6 : (\Delta_7, \Delta_{10}), F_5 \vdash F_8 \otimes F_9} \otimes_R \\
\hline
- : (\Delta_2, !F_3), \Delta_7, \Delta_{10} \vdash F_8 \otimes F_9 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \text{ ax/W} \quad \frac{h_6 : \Delta_{10}, \Delta_7, F_5 \vdash F_8 \otimes F_9}{\bullet h_6 : \Delta_{10}, \Delta_7, F_5 \vdash F_8 \otimes F_9} \text{ ax/W}}{- : \Delta_{10}, \Delta_2, \Delta_7, !F_3, !F_3 \vdash F_8 \otimes F_9} \text{ hCut}}{- : \Delta_{10}, \Delta_2, \Delta_7, !F_3 \vdash F_8 \otimes F_9} C \\
\hline
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} C \quad \frac{h_6 : \Delta_7 \vdash F_8 \quad h_6 : \Delta_{10}, F_5 \vdash F_9}{\bullet h_6 : (\Delta_7, \Delta_{10}), F_5 \vdash F_8 \otimes F_9} \otimes_R \\
\hline
- : (\Delta_2, !F_3), \Delta_7, \Delta_{10} \vdash F_8 \otimes F_9 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \text{ ax/W} \quad \frac{h_6 : \Delta_{10}, \Delta_7, F_5 \vdash F_8 \otimes F_9}{\bullet h_6 : \Delta_{10}, \Delta_7, F_5 \vdash F_8 \otimes F_9} \text{ ax/W}}{- : \Delta_{10}, \Delta_2, \Delta_7, !F_3, !F_3 \vdash F_8 \otimes F_9} \text{ hCut}}{- : \Delta_{10}, \Delta_2, \Delta_7, !F_3 \vdash F_8 \otimes F_9} C
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} C \quad \frac{h_6 : \Delta_9, F_5 \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_5 \vdash F_8} W \\
\hline
- : (\Delta_2, !F_3), \Delta_9, !F_7 \vdash F_8 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \text{ ax/W} \quad \frac{h_6 : \Delta_9, F_5, !F_7 \vdash F_8}{\bullet h_6 : \Delta_9, F_5, !F_7 \vdash F_8} \text{ ax/W}}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \text{ hCut}}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \\
\hline
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash !F_7}{\bullet h_1 : \Delta_2, !F_3 \vdash !F_7} C \quad \frac{h_5 : \Delta_6 \vdash F_8}{\bullet h_5 : \Delta_6, !F_7 \vdash F_8} W \\
\hline
- : (\Delta_2, !F_3), \Delta_6 \vdash F_8 \\
\rightarrow \\
- : \Delta_2, \Delta_6, !F_3 \vdash F_8 \quad \text{ ax/W}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} C \quad \frac{h_6 : \Delta_9, F_5, !F_7, !F_7 \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_5 \vdash F_8} C \\
\hline
- : (\Delta_2, !F_3), \Delta_9, !F_7 \vdash F_8 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \text{ ax/W} \quad \frac{h_6 : \Delta_9, F_5, !F_7 \vdash F_8}{\bullet h_6 : \Delta_9, F_5, !F_7 \vdash F_8} \text{ ax/W}}{- : \Delta_2, \Delta_9, !F_3, !F_3, !F_7 \vdash F_8} \text{ hCut}}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} C \\
\hline
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash !F_7}{\bullet h_1 : \Delta_2, !F_3 \vdash !F_7} C \quad \frac{h_5 : \Delta_6, !F_7, !F_7 \vdash F_8}{\bullet h_5 : \Delta_6, !F_7 \vdash F_8} C \\
\hline
- : (\Delta_2, !F_3), \Delta_6 \vdash F_8 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash !F_7}{\bullet h_1 : \Delta_2, !F_3 \vdash !F_7} \text{ ax/W} \quad \frac{h_5 : \Delta_6, !F_7, !F_7 \vdash F_8}{\bullet h_5 : \Delta_6, !F_7 \vdash F_8} \text{ ax/W}}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8} \text{ mCut}}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8}
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} C \quad \frac{h_6 : \Delta_9, F_5, F_7 \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_5 \vdash F_8} !L \\
\hline
- : (\Delta_2, !F_3), \Delta_9, !F_7 \vdash F_8 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \text{ ax/W} \quad \frac{h_6 : \Delta_9, F_5, !F_7 \vdash F_8}{\bullet h_6 : \Delta_9, F_5, !F_7 \vdash F_8} \text{ ax/W}}{- : \Delta_2, \Delta_9, !F_3, !F_3, !F_7 \vdash F_8} \text{ hCut}}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} C
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash !F_7}{\bullet h_1 : \Delta_2, !F_3 \vdash !F_7} C \quad \frac{h_5 : \Delta_6, F_7 \vdash F_8}{\bullet h_5 : \Delta_6, !F_7 \vdash F_8} !L \\
\hline
- : (\Delta_2, !F_3), \Delta_6 \vdash F_8 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash !F_7}{- : \Delta_2, \Delta_6, !F_3, !F_3 \vdash F_8} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_6, !F_7 \vdash F_8}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8} \text{ax/W}}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8} \text{hCut} \\
\hline
- : \Delta_2, \Delta_6, !F_3 \vdash F_8 \quad C
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} C \quad \frac{h_6 : \Delta_{10}, F_5, F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \& F_8), F_5 \vdash F_9} \&_{L2} \\
\hline
- : (\Delta_2, !F_3), \Delta_{10}, F_7 \& F_8 \vdash F_9 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{- : \Delta_{10}, \Delta_2, !F_3, !F_3, F_7 \& F_8 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_5, F_7 \& F_8 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_3, F_7 \& F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_3, F_7 \& F_8 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_2, !F_3, F_7 \& F_8 \vdash F_9 \quad C \\
\hline
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_7 \& F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \& F_8} C \quad \frac{h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \& F_8 \vdash F_9} \&_{L2} \\
\hline
- : (\Delta_2, !F_3), \Delta_6 \vdash F_9 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_7 \& F_8}{- : \Delta_2, \Delta_6, !F_3, !F_3 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_6, F_7 \& F_8 \vdash F_9}{- : \Delta_2, \Delta_6, !F_3 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_6, !F_3 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_2, \Delta_6, !F_3 \vdash F_9 \quad C
\end{array}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} C \quad \frac{h_6 : \Delta_{10}, F_5, F_7 \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \& F_8), F_5 \vdash F_9} \&_{L1} \\
\hline
- : (\Delta_2, !F_3), \Delta_{10}, F_7 \& F_8 \vdash F_9 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{- : \Delta_{10}, \Delta_2, !F_3, !F_3, F_7 \& F_8 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_5, F_7 \& F_8 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_3, F_7 \& F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_3, F_7 \& F_8 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_2, !F_3, F_7 \& F_8 \vdash F_9 \quad C \\
\hline
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_7 \& F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \& F_8} C \quad \frac{h_5 : \Delta_6, F_7 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \& F_8 \vdash F_9} \&_{L1} \\
\hline
- : (\Delta_2, !F_3), \Delta_6 \vdash F_9 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_7 \& F_8}{- : \Delta_2, \Delta_6, !F_3, !F_3 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_6, F_7 \& F_8 \vdash F_9}{- : \Delta_2, \Delta_6, !F_3 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_6, !F_3 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_2, \Delta_6, !F_3 \vdash F_9 \quad C
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} C \quad \frac{h_6 : \Delta_{10}, F_5, F_7, F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \otimes F_8), F_5 \vdash F_9} \otimes_L \\
\hline
- : (\Delta_2, !F_3), \Delta_{10}, F_7 \otimes F_8 \vdash F_9 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{- : \Delta_{10}, \Delta_2, !F_3, !F_3, F_7 \otimes F_8 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_5, F_7 \otimes F_8 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_3, F_7 \otimes F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_3, F_7 \otimes F_8 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_2, !F_3, F_7 \otimes F_8 \vdash F_9 \quad C \\
\hline
\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_7 \otimes F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \otimes F_8} C \quad \frac{h_5 : \Delta_6, F_7, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \otimes F_8 \vdash F_9} \otimes_L \\
\hline
- : (\Delta_2, !F_3), \Delta_6 \vdash F_9 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_7 \otimes F_8}{- : \Delta_2, \Delta_6, !F_3, !F_3 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_6, F_7 \otimes F_8 \vdash F_9}{- : \Delta_2, \Delta_6, !F_3 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_6, !F_3 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_2, \Delta_6, !F_3 \vdash F_9 \quad C
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} C \quad \frac{h_6 : \Delta_{10}, F_5, F_7 \vdash F_9 \quad h_6 : \Delta_{10}, F_5, F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \oplus F_8), F_5 \vdash F_9} \oplus_L}{- : (\Delta_2, !F_3), \Delta_{10}, F_7 \oplus F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{- : \Delta_{10}, \Delta_2, !F_3, !F_3, F_7 \oplus F_8 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{10}, F_5, F_7 \oplus F_8 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_3, F_7 \oplus F_8 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_2, !F_3, F_7 \oplus F_8 \vdash F_9} C \\
\\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_7 \oplus F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \oplus F_8} C \quad \frac{h_5 : \Delta_6, F_7 \vdash F_9 \quad h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \oplus F_8 \vdash F_9} \oplus_L}{- : (\Delta_2, !F_3), \Delta_6 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_7 \oplus F_8}{- : \Delta_2, \Delta_6, !F_3, !F_3 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_6, F_7 \oplus F_8 \vdash F_9}{- : \Delta_2, \Delta_6, !F_3 \vdash F_9} \text{hCut}}{- : \Delta_2, \Delta_6, !F_3 \vdash F_9} C
\end{array}$$

- Case rule \multimap_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} C \quad \frac{h_6 : \Delta_{11}, F_5 \vdash F_8 \quad h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : (\Delta_7, \Delta_{11}, F_8 \multimap F_9), F_5 \vdash F_{10}} \multimap_L}{- : (\Delta_2, !F_3), \Delta_7, \Delta_{11}, F_8 \multimap F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{- : \Delta_{11}, \Delta_2, \Delta_7, !F_3, !F_3, F_8 \multimap F_9 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{11}, \Delta_7, F_5, F_8 \multimap F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, \Delta_7, !F_3, F_8 \multimap F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, \Delta_7, !F_3, F_8 \multimap F_9 \vdash F_{10}} C \\
\\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} C \quad \frac{h_6 : \Delta_7 \vdash F_8 \quad h_6 : \Delta_{11}, F_5, F_9 \vdash F_{10}}{\bullet h_6 : (\Delta_7, \Delta_{11}, F_8 \multimap F_9), F_5 \vdash F_{10}} \multimap_L}{- : (\Delta_2, !F_3), \Delta_7, \Delta_{11}, F_8 \multimap F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_5}{- : \Delta_{11}, \Delta_2, \Delta_7, !F_3, !F_3, F_8 \multimap F_9 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_{11}, \Delta_7, F_5, F_8 \multimap F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, \Delta_7, !F_3, F_8 \multimap F_9 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, \Delta_7, !F_3, F_8 \multimap F_9 \vdash F_{10}} C \\
\\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_8 \multimap F_9}{\bullet h_1 : \Delta_2, !F_3 \vdash F_8 \multimap F_9} C \quad \frac{h_5 : \Delta_6 \vdash F_8 \quad h_5 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_5 : (\Delta_6, \Delta_7), F_8 \multimap F_9 \vdash F_{10}} \multimap_L}{- : (\Delta_2, !F_3), \Delta_6, \Delta_7 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash F_8 \multimap F_9}{- : \Delta_2, \Delta_6, \Delta_7, !F_3, !F_3 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_5 : \Delta_6, \Delta_7, F_8 \multimap F_9 \vdash F_{10}}{- : \Delta_2, \Delta_6, \Delta_7, !F_3 \vdash F_{10}} \text{hCut}}{- : \Delta_2, \Delta_6, \Delta_7, !F_3 \vdash F_{10}} C
\end{array}$$

- Case rule I

$$\frac{\frac{h_1 : \Delta_2, !F_3, !F_3 \vdash p(n_6)}{\bullet h_1 : \Delta_2, !F_3 \vdash p(n_6)} C \quad \frac{}{\bullet h_5 : *, p(n_6) \vdash p(n_6)} I}{- : (\Delta_2, !F_3), * \vdash p(n_6)} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_2, !F_3 \vdash p(n_6)} \text{ax/W}$$

5.12 Status of $!L$: OK

- Case rule $!R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash 6}{\bullet h_1 : \Delta_2, !F_3 \vdash 6} !L \quad \frac{h_7 : !\Upsilon 5, 6 \vdash F_8}{\bullet h_7 : !\Upsilon 5, 6 \vdash !F_8} !R}{- : (\Delta_2, !F_3), !\Upsilon 5 \vdash !F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash 6}{\bullet h_1 : \Delta_2, !F_3 \vdash 6} \text{ax/W} \quad \frac{h_7 : 6, !\Upsilon 5 \vdash !F_8}{\bullet h_7 : 6, !\Upsilon 5 \vdash !F_8} \text{ax/W}}{- : !\Upsilon 5, \Delta_2, F_3 \vdash !F_8} \text{hCut}}{- : !\Upsilon 5, \Delta_2, !F_3 \vdash !F_8} !L
\end{array}$$

- Case rule $\mathbf{1}_R$

- Case rule \top

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} !L \quad \frac{}{\bullet h_7 : \Delta_5, F_6 \vdash \top} \top}{- : (\Delta_2, !F_3), \Delta_5 \vdash \top} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_5, !F_3 \vdash \top} \top
\end{array}$$

- Case rule $\&_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} !L \quad \frac{h_7 : \Delta_5, F_6 \vdash F_8 \quad h_7 : \Delta_5, F_6 \vdash F_9}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \& F_9} \&_R}{- : (\Delta_2, !F_3), \Delta_5 \vdash F_8 \& F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} \text{ax/W} \quad \frac{h_7 : \Delta_5, F_6 \vdash F_8 \& F_9}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \& F_9} \text{ax/W}}{- : \Delta_2, \Delta_5, F_3 \vdash F_8 \& F_9} \text{hCut}}{- : \Delta_2, \Delta_5, !F_3 \vdash F_8 \& F_9} !L
\end{array}$$

- Case rule $\neg\circ_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} !L \quad \frac{h_7 : \Delta_5, F_6, F_8 \vdash F_9}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \neg\circ F_9} \neg\circ_R}{- : (\Delta_2, !F_3), \Delta_5 \vdash F_8 \neg\circ F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} \text{ax/W} \quad \frac{h_7 : \Delta_5, F_6, F_8 \vdash F_9}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \neg\circ F_9} \text{ax/W}}{- : \Delta_2, \Delta_5, F_8, !F_3 \vdash F_9} \text{hCut}}{- : \Delta_2, \Delta_5, !F_3 \vdash F_8 \neg\circ F_9} \neg\circ_R
\end{array}$$

- Case rule \oplus_{R_2}

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} !L \quad \frac{h_7 : \Delta_5, F_6 \vdash F_9}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \oplus F_9} \oplus_{R_2}}{- : (\Delta_2, !F_3), \Delta_5 \vdash F_8 \oplus F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} \text{ax/W} \quad \frac{h_7 : \Delta_5, F_6 \vdash F_9}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \oplus F_9} \text{ax/W}}{- : \Delta_2, \Delta_5, !F_3 \vdash F_9} \text{hCut}}{- : \Delta_2, \Delta_5, !F_3 \vdash F_8 \oplus F_9} \oplus_{R_2}
\end{array}$$

- Case rule \oplus_{R_1}

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} !L \quad \frac{h_7 : \Delta_5, F_6 \vdash F_8}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \oplus F_9} \oplus_{R_1}}{- : (\Delta_2, !F_3), \Delta_5 \vdash F_8 \oplus F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, !F_3 \vdash F_6} \text{ax/W} \quad \frac{h_7 : \Delta_5, F_6 \vdash F_8}{\bullet h_7 : \Delta_5, F_6 \vdash F_8 \oplus F_9} \text{ax/W}}{- : \Delta_2, \Delta_5, !F_3 \vdash F_8} \text{hCut}}{- : \Delta_2, \Delta_5, !F_3 \vdash F_8 \oplus F_9} \oplus_{R_1}
\end{array}$$

- Case rule $\mathbf{1}_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash \mathbf{1}}{\bullet h_1 : \Delta_2, !F_3 \vdash \mathbf{1}} !L \quad \frac{h_5 : \Delta_6 \vdash F_7}{\bullet h_5 : \Delta_6, \mathbf{1} \vdash F_7} \mathbf{1}_L}{- : (\Delta_2, !F_3), \Delta_6 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_6, !F_3 \vdash F_7} \text{ax/W} \\
\\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} !L \quad \frac{h_6 : \Delta_8, F_5 \vdash F_7}{\bullet h_6 : (\mathbf{1}, \Delta_8), F_5 \vdash F_7} \mathbf{1}_L}{- : (\Delta_2, !F_3), \mathbf{1}, \Delta_8 \vdash F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \text{ax/W} \quad \frac{h_6 : \mathbf{1}, \Delta_8, F_5 \vdash F_7}{h_6 : \mathbf{1}, \Delta_8, F_5 \vdash F_7} \text{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_8, !F_3 \vdash F_7} \text{hCut}
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} !L \quad \frac{h_6 : \Delta_{10}, F_5 \vdash F_8 \quad h_6 : \Delta_7 \vdash F_9}{\bullet h_6 : (\Delta_7, \Delta_{10}), F_5 \vdash F_8 \otimes F_9} \otimes_R}{- : (\Delta_2, !F_3), \Delta_7, \Delta_{10} \vdash F_8 \otimes F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, \Delta_7, F_5 \vdash F_8 \otimes F_9}{\bullet h_6 : \Delta_{10}, \Delta_7, F_5 \vdash F_8 \otimes F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, \Delta_7, F_3 \vdash F_8 \otimes F_9} \text{hCut} \\
\rightarrow \\
\frac{}{- : \Delta_{10}, \Delta_2, \Delta_7, !F_3 \vdash F_8 \otimes F_9} !L \\
\\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} !L \quad \frac{h_6 : \Delta_7 \vdash F_8 \quad h_6 : \Delta_{10}, F_5 \vdash F_9}{\bullet h_6 : (\Delta_7, \Delta_{10}), F_5 \vdash F_8 \otimes F_9} \otimes_R}{- : (\Delta_2, !F_3), \Delta_7, \Delta_{10} \vdash F_8 \otimes F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, \Delta_7, F_5 \vdash F_8 \otimes F_9}{\bullet h_6 : \Delta_{10}, \Delta_7, F_5 \vdash F_8 \otimes F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, \Delta_7, F_3 \vdash F_8 \otimes F_9} \text{hCut} \\
\rightarrow \\
\frac{}{- : \Delta_{10}, \Delta_2, \Delta_7, !F_3 \vdash F_8 \otimes F_9} !L
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} !L \quad \frac{h_6 : \Delta_9, F_5 \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_5 \vdash F_8} W}{- : (\Delta_2, !F_3), \Delta_9, !F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_5, !F_7 \vdash F_8}{h_6 : \Delta_9, F_5, !F_7 \vdash F_8} \text{ax/W}}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} \text{hCut} \\
\\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash !F_7}{\bullet h_1 : \Delta_2, !F_3 \vdash !F_7} !L \quad \frac{h_5 : \Delta_6 \vdash F_8}{\bullet h_5 : \Delta_6, !F_7 \vdash F_8} W}{- : (\Delta_2, !F_3), \Delta_6 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8} \text{ax/W}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} !L \quad \frac{h_6 : \Delta_9, F_5, !F_7, !F_7 \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_5 \vdash F_8} C}{- : (\Delta_2, !F_3), \Delta_9, !F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \text{ax/W} \quad \frac{h_6 : \Delta_9, F_5, !F_7, !F_7 \vdash F_8}{h_6 : \Delta_9, F_5, !F_7, !F_7 \vdash F_8} \text{ax/W}}{- : \Delta_2, \Delta_9, !F_3, !F_7, !F_7 \vdash F_8} \text{hCut} \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} C
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash !F_7}{\bullet h_1 : \Delta_2, !F_3 \vdash !F_7} !L \quad \frac{h_5 : \Delta_6, !F_7, !F_7 \vdash F_8}{\bullet h_5 : \Delta_6, !F_7 \vdash F_8} C \\
\hline
- : (\Delta_2, !F_3), \Delta_6 \vdash F_8 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash !F_7}{\bullet h_1 : \Delta_2, !F_3 \vdash !F_7} \text{ ax/W} \quad \frac{h_5 : \Delta_6, !F_7, !F_7 \vdash F_8}{\bullet h_5 : \Delta_6, !F_7 \vdash F_8} \text{ ax/W}}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8} \text{ mCut}
\end{array}$$

- Case rule !L

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} !L \quad \frac{h_6 : \Delta_9, F_5, F_7 \vdash F_8}{\bullet h_6 : (\Delta_9, !F_7), F_5 \vdash F_8} !L \\
\hline
- : (\Delta_2, !F_3), \Delta_9, !F_7 \vdash F_8 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \text{ ax/W} \quad \frac{h_6 : \Delta_9, F_5, !F_7 \vdash F_8}{\bullet h_6 : \Delta_9, F_5, !F_7 \vdash F_8} \text{ ax/W}}{- : \Delta_2, \Delta_9, F_3, !F_7 \vdash F_8} \text{ hCut} \\
\frac{- : \Delta_2, \Delta_9, F_3, !F_7 \vdash F_8}{- : \Delta_2, \Delta_9, !F_3, !F_7 \vdash F_8} !L \\
\frac{h_1 : \Delta_2, F_3 \vdash !F_7}{\bullet h_1 : \Delta_2, !F_3 \vdash !F_7} !L \quad \frac{h_5 : \Delta_6, F_7 \vdash F_8}{\bullet h_5 : \Delta_6, !F_7 \vdash F_8} !L \\
\hline
- : (\Delta_2, !F_3), \Delta_6 \vdash F_8 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash !F_7}{\bullet h_1 : \Delta_2, !F_3 \vdash !F_7} \text{ ax/W} \quad \frac{h_5 : \Delta_6, !F_7 \vdash F_8}{\bullet h_5 : \Delta_6, !F_7 \vdash F_8} \text{ ax/W}}{- : \Delta_2, \Delta_6, F_3 \vdash F_8} \text{ hCut} \\
\frac{- : \Delta_2, \Delta_6, F_3 \vdash F_8}{- : \Delta_2, \Delta_6, !F_3 \vdash F_8} !L
\end{array}$$

- Case rule &L2

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} !L \quad \frac{h_6 : \Delta_{10}, F_5, F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \& F_8), F_5 \vdash F_9} \&L2 \\
\hline
- : (\Delta_2, !F_3), \Delta_{10}, F_7 \& F_8 \vdash F_9 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \text{ ax/W} \quad \frac{h_6 : \Delta_{10}, F_5, F_7 \& F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_5, F_7 \& F_8 \vdash F_9} \text{ ax/W}}{- : \Delta_{10}, \Delta_2, F_3, F_7 \& F_8 \vdash F_9} \text{ hCut} \\
\frac{- : \Delta_{10}, \Delta_2, F_3, F_7 \& F_8 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_3, F_7 \& F_8 \vdash F_9} !L \\
\frac{h_1 : \Delta_2, F_3 \vdash F_7 \& F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \& F_8} !L \quad \frac{h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \& F_8 \vdash F_9} \&L2 \\
\hline
- : (\Delta_2, !F_3), \Delta_6 \vdash F_9 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_7 \& F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \& F_8} \text{ ax/W} \quad \frac{h_5 : \Delta_6, F_7 \& F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \& F_8 \vdash F_9} \text{ ax/W}}{- : \Delta_2, \Delta_6, F_3 \vdash F_9} \text{ hCut} \\
\frac{- : \Delta_2, \Delta_6, F_3 \vdash F_9}{- : \Delta_2, \Delta_6, !F_3 \vdash F_9} !L
\end{array}$$

- Case rule &L1

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} !L \quad \frac{h_6 : \Delta_{10}, F_5, F_7 \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \& F_8), F_5 \vdash F_9} \&L1 \\
\hline
- : (\Delta_2, !F_3), \Delta_{10}, F_7 \& F_8 \vdash F_9 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} \text{ ax/W} \quad \frac{h_6 : \Delta_{10}, F_5, F_7 \& F_8 \vdash F_9}{\bullet h_6 : \Delta_{10}, F_5, F_7 \& F_8 \vdash F_9} \text{ ax/W}}{- : \Delta_{10}, \Delta_2, F_3, F_7 \& F_8 \vdash F_9} \text{ hCut} \\
\frac{- : \Delta_{10}, \Delta_2, F_3, F_7 \& F_8 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_3, F_7 \& F_8 \vdash F_9} !L \\
\frac{h_1 : \Delta_2, F_3 \vdash F_7 \& F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \& F_8} !L \quad \frac{h_5 : \Delta_6, F_7 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \& F_8 \vdash F_9} \&L1 \\
\hline
- : (\Delta_2, !F_3), \Delta_6 \vdash F_9 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_7 \& F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \& F_8} \text{ ax/W} \quad \frac{h_5 : \Delta_6, F_7 \& F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \& F_8 \vdash F_9} \text{ ax/W}}{- : \Delta_2, \Delta_6, F_3 \vdash F_9} \text{ hCut} \\
\frac{- : \Delta_2, \Delta_6, F_3 \vdash F_9}{- : \Delta_2, \Delta_6, !F_3 \vdash F_9} !L
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} !L \quad \frac{h_6 : \Delta_{10}, F_5, F_7, F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \otimes F_8), F_5 \vdash F_9} \otimes_L}{- : (\Delta_2, !F_3), \Delta_{10}, F_7 \otimes F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{- : \Delta_{10}, \Delta_2, F_3, F_7 \otimes F_8 \vdash F_9} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, F_5, F_7 \otimes F_8 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_3, F_7 \otimes F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, F_3, F_7 \otimes F_8 \vdash F_9} \text{hCut} \quad !L \\
- : \Delta_{10}, \Delta_2, !F_3, F_7 \otimes F_8 \vdash F_9 \\
\frac{h_1 : \Delta_2, F_3 \vdash F_7 \otimes F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \otimes F_8} !L \quad \frac{h_5 : \Delta_6, F_7, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \otimes F_8 \vdash F_9} \otimes_L}{- : (\Delta_2, !F_3), \Delta_6 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_7 \otimes F_8}{- : \Delta_2, \Delta_6, F_3 \vdash F_9} \text{ax/W} \quad \frac{h_5 : \Delta_6, F_7 \otimes F_8 \vdash F_9}{- : \Delta_2, \Delta_6, !F_3 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_6, F_3 \vdash F_9} \text{hCut} \quad !L \\
- : \Delta_2, \Delta_6, !F_3 \vdash F_9
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} !L \quad \frac{h_6 : \Delta_{10}, F_5, F_7 \vdash F_9 \quad h_6 : \Delta_{10}, F_5, F_8 \vdash F_9}{\bullet h_6 : (\Delta_{10}, F_7 \oplus F_8), F_5 \vdash F_9} \oplus_L}{- : (\Delta_2, !F_3), \Delta_{10}, F_7 \oplus F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{- : \Delta_{10}, \Delta_2, F_3, F_7 \oplus F_8 \vdash F_9} \text{ax/W} \quad \frac{h_6 : \Delta_{10}, F_5, F_7 \oplus F_8 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_3, F_7 \oplus F_8 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, F_3, F_7 \oplus F_8 \vdash F_9} \text{hCut} \quad !L \\
- : \Delta_{10}, \Delta_2, !F_3, F_7 \oplus F_8 \vdash F_9 \\
\frac{h_1 : \Delta_2, F_3 \vdash F_7 \oplus F_8}{\bullet h_1 : \Delta_2, !F_3 \vdash F_7 \oplus F_8} !L \quad \frac{h_5 : \Delta_6, F_7 \vdash F_9 \quad h_5 : \Delta_6, F_8 \vdash F_9}{\bullet h_5 : \Delta_6, F_7 \oplus F_8 \vdash F_9} \oplus_L}{- : (\Delta_2, !F_3), \Delta_6 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_7 \oplus F_8}{- : \Delta_2, \Delta_6, F_3 \vdash F_9} \text{ax/W} \quad \frac{h_5 : \Delta_6, F_7 \oplus F_8 \vdash F_9}{- : \Delta_2, \Delta_6, !F_3 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_6, F_3 \vdash F_9} \text{hCut} \quad !L \\
- : \Delta_2, \Delta_6, !F_3 \vdash F_9
\end{array}$$

- Case rule \multimap_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} !L \quad \frac{h_6 : \Delta_{11}, F_5 \vdash F_8 \quad h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : (\Delta_7, \Delta_{11}, F_8 \multimap F_9), F_5 \vdash F_{10}} \multimap_L}{- : (\Delta_2, !F_3), \Delta_7, \Delta_{11}, F_8 \multimap F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{- : \Delta_{11}, \Delta_2, \Delta_7, F_3, F_8 \multimap F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_6 : \Delta_{11}, \Delta_7, F_5, F_8 \multimap F_9 \vdash F_{10}}{\bullet h_6 : \Delta_{11}, \Delta_7, F_5, F_8 \multimap F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_7, F_3, F_8 \multimap F_9 \vdash F_{10}} \text{hCut} \quad !L \\
- : \Delta_{11}, \Delta_2, \Delta_7, !F_3, F_8 \multimap F_9 \vdash F_{10} \\
\frac{h_1 : \Delta_2, F_3 \vdash F_5}{\bullet h_1 : \Delta_2, !F_3 \vdash F_5} !L \quad \frac{h_6 : \Delta_7 \vdash F_8 \quad h_6 : \Delta_{11}, F_5, F_9 \vdash F_{10}}{\bullet h_6 : (\Delta_7, \Delta_{11}, F_8 \multimap F_9), F_5 \vdash F_{10}} \multimap_L}{- : (\Delta_2, !F_3), \Delta_7, \Delta_{11}, F_8 \multimap F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_5}{- : \Delta_{11}, \Delta_2, \Delta_7, F_3, F_8 \multimap F_9 \vdash F_{10}} \text{ax/W} \quad \frac{h_6 : \Delta_{11}, \Delta_7, F_5, F_8 \multimap F_9 \vdash F_{10}}{\bullet h_6 : \Delta_{11}, \Delta_7, F_5, F_8 \multimap F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_7, F_3, F_8 \multimap F_9 \vdash F_{10}} \text{hCut} \quad !L \\
- : \Delta_{11}, \Delta_2, \Delta_7, !F_3, F_8 \multimap F_9 \vdash F_{10} \\
\frac{h_1 : \Delta_2, F_3 \vdash F_8 \multimap F_9}{\bullet h_1 : \Delta_2, !F_3 \vdash F_8 \multimap F_9} !L \quad \frac{h_5 : \Delta_6 \vdash F_8 \quad h_5 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_5 : (\Delta_6, \Delta_7), F_8 \multimap F_9 \vdash F_{10}} \multimap_L}{- : (\Delta_2, !F_3), \Delta_6, \Delta_7 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_8 \multimap F_9}{- : \Delta_2, \Delta_6, \Delta_7, F_3 \vdash F_{10}} \text{ax/W} \quad \frac{h_5 : \Delta_6, \Delta_7, F_8 \multimap F_9 \vdash F_{10}}{\bullet h_5 : \Delta_6, \Delta_7, F_8 \multimap F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_6, \Delta_7, F_3 \vdash F_{10}} \text{hCut} \quad !L \\
- : \Delta_2, \Delta_6, \Delta_7, !F_3 \vdash F_{10}
\end{array}$$

- Case rule I

$$\frac{\frac{h_1 : \Delta_2, F_3 \vdash p(n_6)}{\bullet h_1 : \Delta_2, !F_3 \vdash p(n_6)} \text{!}L \quad \frac{}{\bullet h_5 : *, p(n_6) \vdash p(n_6)} I}{\frac{}{- : (\Delta_2, !F_3), * \vdash p(n_6)} \text{Cut}}{\rightarrow} \frac{}{- : \Delta_2, !F_3 \vdash p(n_6)} \text{ax/W}$$

5.13 Status of $\&_{L2}$: OK

- Case rule $!R$

$$\frac{\frac{h_1 : \Delta_2, F_4 \vdash 7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash 7} \&_{L2} \quad \frac{h_8 : !\Upsilon 6, 7 \vdash F_9}{\bullet h_8 : !\Upsilon 6, 7 \vdash !F_9} \text{!}R}{\frac{}{- : (\Delta_2, F_3 \& F_4), !\Upsilon 6 \vdash !F_9} \text{Cut}}{\rightarrow} \frac{\frac{h_1 : \Delta_2, F_4 \vdash 7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash 7} \text{ax/W} \quad \frac{h_8 : !\Upsilon 6, 7 \vdash F_9}{\bullet h_8 : 7, !\Upsilon 6 \vdash !F_9} \text{ax/W}}{\frac{}{- : !\Upsilon 6, \Delta_2, F_4 \vdash !F_9} \text{hCut}}{\rightarrow} \frac{}{- : !\Upsilon 6, \Delta_2, F_3 \& F_4 \vdash !F_9} \&_{L2}$$

- Case rule 1_R

- Case rule \top

$$\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7} \&_{L2} \quad \frac{}{\bullet h_8 : \Delta_6, F_7 \vdash \top} \top}{\frac{}{- : (\Delta_2, F_3 \& F_4), \Delta_6 \vdash \top} \text{Cut}}{\rightarrow} \frac{}{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash \top} \top$$

- Case rule $\&_R$

$$\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7} \&_{L2} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9 \quad h_8 : \Delta_6, F_7 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \& F_{10}} \&_R}{\frac{}{- : (\Delta_2, F_3 \& F_4), \Delta_6 \vdash F_9 \& F_{10}} \text{Cut}}{\rightarrow} \frac{\frac{h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9 \& F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \& F_{10}} \text{ax/W}}{\frac{}{- : \Delta_2, \Delta_6, F_4 \vdash F_9 \& F_{10}} \text{hCut}}{\rightarrow} \frac{}{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash F_9 \& F_{10}} \&_{L2}$$

- Case rule \multimap_R

$$\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7} \&_{L2} \quad \frac{h_8 : \Delta_6, F_7, F_9 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \multimap F_{10}} \multimap_R}{\frac{}{- : (\Delta_2, F_3 \& F_4), \Delta_6 \vdash F_9 \multimap F_{10}} \text{Cut}}{\rightarrow} \frac{\frac{h_1 : \Delta_2, F_3 \& F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7, F_9 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7, F_9 \vdash F_{10}} \text{ax/W}}{\frac{}{- : \Delta_2, \Delta_6, F_9, F_3 \& F_4 \vdash F_{10}} \text{hCut}}{\rightarrow} \frac{}{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash F_9 \multimap F_{10}} \multimap_R$$

- Case rule \oplus_{R2}

$$\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7} \&_{L2} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \oplus F_{10}} \oplus_{R2}}{\frac{}{- : (\Delta_2, F_3 \& F_4), \Delta_6 \vdash F_9 \oplus F_{10}} \text{Cut}}{\rightarrow} \frac{\frac{h_1 : \Delta_2, F_3 \& F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_{10}} \text{ax/W}}{\frac{}{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash F_{10}} \text{hCut}}{\rightarrow} \frac{}{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash F_9 \oplus F_{10}} \oplus_{R2}$$

- Case rule \oplus_{R_1}

$$\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7} \&_{L2} \frac{h_8 : \Delta_6, F_7 \vdash F_9}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \oplus F_{10}} \oplus_{R_1}}{- : (\Delta_2, F_3 \& F_4), \Delta_6 \vdash F_9 \oplus F_{10}} \text{Cut}$$

$$\xrightarrow{\text{ax/W}} \frac{\frac{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7}{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash F_9} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9}{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash F_9 \oplus F_{10}} \text{hCut}}{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash F_9 \oplus F_{10}} \oplus_{R_1}$$

- Case rule 1_L

$$\frac{\frac{h_1 : \Delta_2, F_4 \vdash \mathbf{1}}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash \mathbf{1}} \&_{L2} \frac{h_6 : \Delta_7 \vdash F_8}{\bullet h_6 : \Delta_7, \mathbf{1} \vdash F_8} 1_L}}{- : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_8} \text{Cut}$$

$$\xrightarrow{\text{ax/W}} \frac{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_8}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_8} \text{ax/W}$$

$$\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L2} \frac{h_7 : \Delta_9, F_6 \vdash F_8}{\bullet h_7 : (\mathbf{1}, \Delta_9), F_6 \vdash F_8} 1_L}}{- : (\Delta_2, F_3 \& F_4), \mathbf{1}, \Delta_9 \vdash F_8} \text{Cut}$$

$$\xrightarrow{\text{ax/W}} \frac{\frac{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6}{- : \mathbf{1}, \Delta_2, \Delta_9, F_3 \& F_4 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \mathbf{1}, \Delta_9, F_6 \vdash F_8}{- : \mathbf{1}, \Delta_2, \Delta_9, F_3 \& F_4 \vdash F_8} \text{hCut}}{- : \mathbf{1}, \Delta_2, \Delta_9, F_3 \& F_4 \vdash F_8} \text{hCut}$$

- Case rule \otimes_R

$$\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L2} \frac{h_7 : \Delta_{11}, F_6 \vdash F_9 \quad h_7 : \Delta_8 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_6 \vdash F_9 \otimes F_{10}} \otimes_R}}{- : (\Delta_2, F_3 \& F_4), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10}} \text{Cut}$$

$$\xrightarrow{\text{ax/W}} \frac{\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, \Delta_8, F_4 \vdash F_9 \otimes F_{10}} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_{11}, \Delta_8, F_6 \vdash F_9 \otimes F_{10}}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \& F_4 \vdash F_9 \otimes F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \& F_4 \vdash F_9 \otimes F_{10}} \&_{L2}}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \& F_4 \vdash F_9 \otimes F_{10}} \text{hCut}$$

$$\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L2} \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{11}, F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_6 \vdash F_9 \otimes F_{10}} \otimes_R}}{- : (\Delta_2, F_3 \& F_4), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10}} \text{Cut}$$

$$\xrightarrow{\text{ax/W}} \frac{\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, \Delta_8, F_4 \vdash F_9 \otimes F_{10}} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_{11}, \Delta_8, F_6 \vdash F_9 \otimes F_{10}}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \& F_4 \vdash F_9 \otimes F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \& F_4 \vdash F_9 \otimes F_{10}} \&_{L2}}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \& F_4 \vdash F_9 \otimes F_{10}} \text{hCut}$$

- Case rule W

$$\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L2} \frac{h_7 : \Delta_{10}, F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_6 \vdash F_9} W}}{- : (\Delta_2, F_3 \& F_4), \Delta_{10}, !F_8 \vdash F_9} \text{Cut}$$

$$\xrightarrow{\text{ax/W}} \frac{\frac{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \& F_4 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_6, !F_8 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \& F_4 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \& F_4 \vdash F_9} \text{hCut}$$

$$\frac{\frac{h_1 : \Delta_2, F_4 \vdash !F_8}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash !F_8} \&_{L2} \frac{h_6 : \Delta_7 \vdash F_9}{\bullet h_6 : \Delta_7, !F_8 \vdash F_9} W}}{- : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_9} \text{Cut}$$

$$\xrightarrow{\text{ax/W}} \frac{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_9}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_9} \text{ax/W}$$

- Case rule C

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L2} \quad \frac{h_7 : \Delta_{10}, F_6, !F_8, !F_8 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_6 \vdash F_9} C \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_{10}, !F_8 \vdash F_9 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_6, !F_8, !F_8 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_6 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_8, !F_8, F_3 \& F_4 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_2, !F_8, F_3 \& F_4 \vdash F_9 \quad C
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_4 \vdash !F_8}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash !F_8} \&_{L2} \quad \frac{h_6 : \Delta_7, !F_8, !F_8 \vdash F_9}{\bullet h_6 : \Delta_7, !F_8 \vdash F_9} C \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_9 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash !F_8}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash !F_8} \text{ax/W} \quad \frac{h_6 : \Delta_7, !F_8, !F_8 \vdash F_9}{\bullet h_6 : \Delta_7, !F_8 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_9} \text{mCut}
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L2} \quad \frac{h_7 : \Delta_{10}, F_6, F_8 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_6 \vdash F_9} !L \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_{10}, !F_8 \vdash F_9 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_6, F_8 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_6 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, F_8, F_3 \& F_4 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_2, !F_8, F_3 \& F_4 \vdash F_9 \quad !L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_4 \vdash !F_8}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash !F_8} \&_{L2} \quad \frac{h_6 : \Delta_7, F_8 \vdash F_9}{\bullet h_6 : \Delta_7, !F_8 \vdash F_9} !L \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_9 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash !F_8}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash !F_8} \text{ax/W} \quad \frac{h_6 : \Delta_7, F_8 \vdash F_9}{\bullet h_6 : \Delta_7, !F_8 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_7, F_4 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_9 \quad \&_{L2}
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L2} \quad \frac{h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_6 \vdash F_{10}} \&_{L2} \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_{11}, F_8 \& F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_6 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_4, F_8 \& F_9 \vdash F_{10}} \text{hCut} \\
\hline
- : \Delta_{11}, \Delta_2, F_3 \& F_4, F_8 \& F_9 \vdash F_{10} \quad \&_{L2}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_4 \vdash F_8 \& F_9}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_8 \& F_9} \&_{L2} \quad \frac{h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}} \&_{L2} \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_8 \& F_9}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_8 \& F_9} \text{ax/W} \quad \frac{h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_7, F_4 \vdash F_{10}} \text{hCut} \\
\hline
- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10} \quad \&_{L2}
\end{array}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L2} \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_6 \vdash F_{10}} \&_{L1} \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_{11}, F_8 \& F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_6 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_4, F_8 \& F_9 \vdash F_{10}} \text{hCut} \\
\hline
- : \Delta_{11}, \Delta_2, F_3 \& F_4, F_8 \& F_9 \vdash F_{10} \quad \&_{L2}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_4 \vdash F_8 \& F_9}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_8 \& F_9} \&_{L2} \quad \frac{h_6 : \Delta_7, F_8 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}} \&_{L1} \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_8 \& F_9}{- : \Delta_2, \Delta_7, F_4 \vdash F_{10}} \text{ ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10}} \text{ ax/W}}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10}} \text{ hCut} \&_{L2}
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L2} \quad \frac{h_7 : \Delta_{11}, F_6, F_8, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \otimes F_9), F_6 \vdash F_{10}} \otimes_L \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_{11}, F_8 \otimes F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_4, F_8 \otimes F_9 \vdash F_{10}} \text{ ax/W} \quad \frac{\bullet h_7 : \Delta_{11}, F_6, F_8 \otimes F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_3 \& F_4, F_8 \otimes F_9 \vdash F_{10}} \text{ ax/W}}{- : \Delta_{11}, \Delta_2, F_3 \& F_4, F_8 \otimes F_9 \vdash F_{10}} \text{ hCut} \&_{L2} \\
\frac{h_1 : \Delta_2, F_4 \vdash F_8 \otimes F_9}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_8 \otimes F_9} \&_{L2} \quad \frac{h_6 : \Delta_7, F_8, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \otimes F_9 \vdash F_{10}} \otimes_L \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_8 \otimes F_9}{- : \Delta_2, \Delta_7, F_4 \vdash F_{10}} \text{ ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \otimes F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10}} \text{ ax/W}}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10}} \text{ hCut} \&_{L2}
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L2} \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \vdash F_{10} \quad h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \oplus F_9), F_6 \vdash F_{10}} \oplus_L \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_4, F_8 \oplus F_9 \vdash F_{10}} \text{ ax/W} \quad \frac{\bullet h_7 : \Delta_{11}, F_6, F_8 \oplus F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_3 \& F_4, F_8 \oplus F_9 \vdash F_{10}} \text{ ax/W}}{- : \Delta_{11}, \Delta_2, F_3 \& F_4, F_8 \oplus F_9 \vdash F_{10}} \text{ hCut} \&_{L2} \\
\frac{h_1 : \Delta_2, F_4 \vdash F_8 \oplus F_9}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_8 \oplus F_9} \&_{L2} \quad \frac{h_6 : \Delta_7, F_8 \vdash F_{10} \quad h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \oplus F_9 \vdash F_{10}} \oplus_L \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_8 \oplus F_9}{- : \Delta_2, \Delta_7, F_4 \vdash F_{10}} \text{ ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \oplus F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10}} \text{ ax/W}}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10}} \text{ hCut} \&_{L2}
\end{array}$$

- Case rule \multimap_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L2} \quad \frac{h_7 : \Delta_{12}, F_6 \vdash F_9 \quad h_7 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \multimap F_{10}), F_6 \vdash F_{11}} \multimap_L \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_6}{- : \Delta_{12}, \Delta_2, \Delta_8, F_4, F_9 \multimap F_{10} \vdash F_{11}} \text{ ax/W} \quad \frac{\bullet h_7 : \Delta_{12}, \Delta_8, F_6, F_9 \multimap F_{10} \vdash F_{11}}{- : \Delta_{12}, \Delta_2, \Delta_8, F_3 \& F_4, F_9 \multimap F_{10} \vdash F_{11}} \text{ ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_8, F_3 \& F_4, F_9 \multimap F_{10} \vdash F_{11}} \text{ hCut} \&_{L2} \\
\frac{h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L2} \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{12}, F_6, F_{10} \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \multimap F_{10}), F_6 \vdash F_{11}} \multimap_L \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_6}{- : \Delta_{12}, \Delta_2, \Delta_8, F_4, F_9 \multimap F_{10} \vdash F_{11}} \text{ ax/W} \quad \frac{\bullet h_7 : \Delta_{12}, \Delta_8, F_6, F_9 \multimap F_{10} \vdash F_{11}}{- : \Delta_{12}, \Delta_2, \Delta_8, F_3 \& F_4, F_9 \multimap F_{10} \vdash F_{11}} \text{ ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_8, F_3 \& F_4, F_9 \multimap F_{10} \vdash F_{11}} \text{ hCut} \&_{L2}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_4 \vdash F_9 \multimap F_{10}}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_9 \multimap F_{10}} \&_{L2} \quad \frac{h_6 : \Delta_7 \vdash F_9 \quad h_6 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_6 : (\Delta_7, \Delta_8), F_9 \multimap F_{10} \vdash F_{11}} \multimap_L \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_7, \Delta_8 \vdash F_{11} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_4 \vdash F_9 \multimap F_{10}}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_9 \multimap F_{10}} \text{ ax/W} \quad \frac{h_6 : \Delta_7, \Delta_8, F_9 \multimap F_{10} \vdash F_{11}}{\bullet h_6 : \Delta_7, \Delta_8, F_9 \multimap F_{10} \vdash F_{11}} \text{ ax/W}}{- : \Delta_2, \Delta_7, \Delta_8, F_4 \vdash F_{11}} \text{ hCut} \\
\frac{- : \Delta_2, \Delta_7, \Delta_8, F_3 \& F_4 \vdash F_{11}}{- : \Delta_2, \Delta_7, \Delta_8, F_3 \& F_4 \vdash F_{11}} \&_{L2}
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_4 \vdash p(n_7)}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash p(n_7)} \&_{L2} \quad \frac{}{\bullet h_6 : *, p(n_7) \vdash p(n_7)} I \\
\hline
- : (\Delta_2, F_3 \& F_4), * \vdash p(n_7) \\
\rightarrow \\
\frac{}{- : \Delta_2, F_3 \& F_4 \vdash p(n_7)} \text{ ax/W}
\end{array}$$

5.14 Status of $\&_{L1}$: OK

- Case rule $!R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash 7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash 7} \&_{L1} \quad \frac{h_8 : !\Upsilon 6, 7 \vdash F_9}{\bullet h_8 : !\Upsilon 6, 7 \vdash !F_9} !R \\
\hline
- : (\Delta_2, F_3 \& F_4), !\Upsilon 6 \vdash F_9 \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash 7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash 7} \text{ ax/W} \quad \frac{h_8 : 7, !\Upsilon 6 \vdash !F_9}{\bullet h_8 : 7, !\Upsilon 6 \vdash !F_9} \text{ ax/W}}{- : !\Upsilon 6, \Delta_2, F_3 \vdash !F_9} \text{ hCut} \\
\frac{- : !\Upsilon 6, \Delta_2, F_3 \& F_4 \vdash !F_9}{- : !\Upsilon 6, \Delta_2, F_3 \& F_4 \vdash !F_9} \&_{L1}
\end{array}$$

- Case rule 1_R

- Case rule \top

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7} \&_{L1} \quad \frac{}{\bullet h_8 : \Delta_6, F_7 \vdash \top} \top \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_6 \vdash \top \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash \top} \top
\end{array}$$

- Case rule $\&_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7} \&_{L1} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9 \quad h_8 : \Delta_6, F_7 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \& F_{10}} \&_R \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_6 \vdash F_9 \& F_{10} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7} \text{ ax/W} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9 \& F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \& F_{10}} \text{ ax/W}}{- : \Delta_2, \Delta_6, F_3 \vdash F_9 \& F_{10}} \text{ hCut} \\
\frac{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash F_9 \& F_{10}}{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash F_9 \& F_{10}} \&_{L1}
\end{array}$$

- Case rule \multimap_R

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7} \&_{L1} \quad \frac{h_8 : \Delta_6, F_7, F_9 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \multimap F_{10}} \multimap_R \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_6 \vdash F_9 \multimap F_{10} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \& F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7} \text{ ax/W} \quad \frac{h_8 : \Delta_6, F_7, F_9 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7, F_9 \vdash F_{10}} \text{ ax/W}}{- : \Delta_2, \Delta_6, F_9, F_3 \& F_4 \vdash F_{10}} \text{ hCut} \\
\frac{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash F_9 \multimap F_{10}}{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash F_9 \multimap F_{10}} \multimap_R
\end{array}$$

- Case rule \oplus_{R_2}

$$\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7} \&_{L1} \frac{h_8 : \Delta_6, F_7 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \oplus F_{10}} \oplus_{R2}}{- : (\Delta_2, F_3 \& F_4), \Delta_6 \vdash F_9 \oplus F_{10}} \text{Cut} \rightarrow \frac{\frac{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7}{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_{10}}{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash F_9 \oplus F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash F_9 \oplus F_{10}} \text{hCut} \oplus_{R2}$$

- Case rule \oplus_{R_1}

$$\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7} \&_{L1} \frac{h_8 : \Delta_6, F_7 \vdash F_9}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \oplus F_{10}} \oplus_{R1}}{- : (\Delta_2, F_3 \& F_4), \Delta_6 \vdash F_9 \oplus F_{10}} \text{Cut} \rightarrow \frac{\frac{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_7}{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash F_9} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9}{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash F_9 \oplus F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_6, F_3 \& F_4 \vdash F_9 \oplus F_{10}} \text{hCut} \oplus_{R1}$$

- Case rule $\mathbf{1}_L$

$$\frac{\frac{h_1 : \Delta_2, F_3 \vdash \mathbf{1}}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash \mathbf{1}} \&_{L1} \frac{h_6 : \Delta_7 \vdash F_8}{\bullet h_6 : \Delta_7, \mathbf{1} \vdash F_8} \mathbf{1}_L}}{- : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_8} \text{Cut} \rightarrow \frac{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_8}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_8} \text{ax/W}$$

$$\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L1} \frac{h_7 : \Delta_9, F_6 \vdash F_8}{\bullet h_7 : (\mathbf{1}, \Delta_9), F_6 \vdash F_8} \mathbf{1}_L}}{- : (\Delta_2, F_3 \& F_4), \mathbf{1}, \Delta_9 \vdash F_8} \text{Cut} \rightarrow \frac{\frac{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6}{- : \mathbf{1}, \Delta_2, \Delta_9, F_3 \& F_4 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \mathbf{1}, \Delta_9, F_6 \vdash F_8}{- : \mathbf{1}, \Delta_2, \Delta_9, F_3 \& F_4 \vdash F_8} \text{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_9, F_3 \& F_4 \vdash F_8} \text{hCut}$$

- Case rule \otimes_R

$$\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L1} \frac{h_7 : \Delta_{11}, F_6 \vdash F_9 \quad h_7 : \Delta_8 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_6 \vdash F_9 \otimes F_{10}} \otimes_R}}{- : (\Delta_2, F_3 \& F_4), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10}} \text{Cut} \rightarrow \frac{\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \vdash F_9 \otimes F_{10}} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_{11}, \Delta_8, F_6 \vdash F_9 \otimes F_{10}}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \& F_4 \vdash F_9 \otimes F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \& F_4 \vdash F_9 \otimes F_{10}} \text{hCut} \quad \&_{L1}}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \& F_4 \vdash F_9 \otimes F_{10}} \text{hCut}$$

$$\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L1} \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{11}, F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_6 \vdash F_9 \otimes F_{10}} \otimes_R}}{- : (\Delta_2, F_3 \& F_4), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10}} \text{Cut} \rightarrow \frac{\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \vdash F_9 \otimes F_{10}} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_{11}, \Delta_8, F_6 \vdash F_9 \otimes F_{10}}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \& F_4 \vdash F_9 \otimes F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \& F_4 \vdash F_9 \otimes F_{10}} \text{hCut} \quad \&_{L1}}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \& F_4 \vdash F_9 \otimes F_{10}} \text{hCut}$$

- Case rule W

$$\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L1} \frac{h_7 : \Delta_{10}, F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_6 \vdash F_9} W}}{- : (\Delta_2, F_3 \& F_4), \Delta_{10}, !F_8 \vdash F_9} \text{Cut} \rightarrow \frac{\frac{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \& F_4 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_6, !F_8 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \& F_4 \vdash F_9} \text{ax/W}}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \& F_4 \vdash F_9} \text{hCut}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash !F_8}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash !F_8} \&_{L1} \quad \frac{h_6 : \Delta_7 \vdash F_9}{\bullet h_6 : \Delta_7, !F_8 \vdash F_9} W \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_9 \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_9} \text{ax/W} \\
\text{Cut}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L1} \quad \frac{h_7 : \Delta_{10}, F_6, !F_8, !F_8 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_6 \vdash F_9} C \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_{10}, !F_8 \vdash F_9 \\
\rightarrow \\
\frac{}{- : \Delta_{10}, \Delta_2, !F_8, !F_8, F_3 \& F_4 \vdash F_9} \text{ax/W} \\
\frac{}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \& F_4 \vdash F_9} C \\
\text{hCut} \\
\frac{h_1 : \Delta_2, F_3 \vdash !F_8}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash !F_8} \&_{L1} \quad \frac{h_6 : \Delta_7, !F_8, !F_8 \vdash F_9}{\bullet h_6 : \Delta_7, !F_8 \vdash F_9} C \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_9 \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_9} \text{ax/W} \\
\text{mCut}
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L1} \quad \frac{h_7 : \Delta_{10}, F_6, F_8 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_6 \vdash F_9} !L \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_{10}, !F_8 \vdash F_9 \\
\rightarrow \\
\frac{}{- : \Delta_{10}, \Delta_2, F_8, F_3 \& F_4 \vdash F_9} \text{ax/W} \\
\frac{}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \& F_4 \vdash F_9} !L \\
\text{hCut} \\
\frac{h_1 : \Delta_2, F_3 \vdash !F_8}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash !F_8} \&_{L1} \quad \frac{h_6 : \Delta_7, F_8 \vdash F_9}{\bullet h_6 : \Delta_7, !F_8 \vdash F_9} !L \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_9 \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_7, F_3 \vdash F_9} \text{ax/W} \\
\frac{}{- : \Delta_2, \Delta_7, !F_8 \vdash F_9} \text{hCut} \\
\frac{}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_9} \&_{L1}
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L1} \quad \frac{h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_6 \vdash F_{10}} \&_{L2} \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_{11}, F_8 \& F_9 \vdash F_{10} \\
\rightarrow \\
\frac{}{- : \Delta_{11}, \Delta_2, F_9, F_3 \& F_4 \vdash F_{10}} \text{ax/W} \\
\frac{}{- : \Delta_{11}, \Delta_2, F_3 \& F_4, F_8 \& F_9 \vdash F_{10}} \&_{L2} \\
\text{hCut} \\
\frac{h_1 : \Delta_2, F_3 \vdash F_8 \& F_9}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_8 \& F_9} \&_{L1} \quad \frac{h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}} \&_{L2} \\
\hline
- : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_{10} \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_7, F_3 \vdash F_{10}} \text{ax/W} \\
\frac{}{- : \Delta_2, \Delta_7, F_8 \& F_9 \vdash F_{10}} \text{hCut} \\
\frac{}{- : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10}} \&_{L1}
\end{array}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L1} \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_6 \vdash F_{10}} \&_{L1}}{\vdash : (\Delta_2, F_3 \& F_4), \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\vdash : \Delta_{11}, \Delta_2, F_3, F_8 \& F_9 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_{11}, F_6, F_8 \& F_9 \vdash F_{10}}{\vdash : \Delta_{11}, \Delta_2, F_3 \& F_4, F_8 \& F_9 \vdash F_{10}} \text{hCut}}{\vdash : \Delta_{11}, \Delta_2, F_3 \& F_4, F_8 \& F_9 \vdash F_{10}} \&_{L1}}{\vdash : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_8 \& F_9}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_8 \& F_9} \&_{L1} \quad \frac{h_6 : \Delta_7, F_8 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}} \&_{L1}}{\vdash : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_8 \& F_9}{\vdash : \Delta_2, \Delta_7, F_3 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}}{\vdash : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10}} \text{hCut}}{\vdash : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10}} \&_{L1}}{\vdash : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10}} \&_{L1}
\end{array}$$

• Case rule \otimes_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L1} \quad \frac{h_7 : \Delta_{11}, F_6, F_8, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \otimes F_9), F_6 \vdash F_{10}} \otimes_L}{\vdash : (\Delta_2, F_3 \& F_4), \Delta_{11}, F_8 \otimes F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\vdash : \Delta_{11}, \Delta_2, F_3, F_8 \otimes F_9 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_{11}, F_6, F_8 \otimes F_9 \vdash F_{10}}{\vdash : \Delta_{11}, \Delta_2, F_3 \& F_4, F_8 \otimes F_9 \vdash F_{10}} \text{hCut}}{\vdash : \Delta_{11}, \Delta_2, F_3 \& F_4, F_8 \otimes F_9 \vdash F_{10}} \&_{L1}}{\vdash : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_8 \otimes F_9}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_8 \otimes F_9} \&_{L1} \quad \frac{h_6 : \Delta_7, F_8, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \otimes F_9 \vdash F_{10}} \otimes_L}{\vdash : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_8 \otimes F_9}{\vdash : \Delta_2, \Delta_7, F_3 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \otimes F_9 \vdash F_{10}}{\vdash : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10}} \text{hCut}}{\vdash : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10}} \&_{L1}}{\vdash : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10}} \&_{L1}
\end{array}$$

• Case rule \oplus_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L1} \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \vdash F_{10} \quad h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \oplus F_9), F_6 \vdash F_{10}} \oplus_L}{\vdash : (\Delta_2, F_3 \& F_4), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\vdash : \Delta_{11}, \Delta_2, F_3, F_8 \oplus F_9 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_{11}, F_6, F_8 \oplus F_9 \vdash F_{10}}{\vdash : \Delta_{11}, \Delta_2, F_3 \& F_4, F_8 \oplus F_9 \vdash F_{10}} \text{hCut}}{\vdash : \Delta_{11}, \Delta_2, F_3 \& F_4, F_8 \oplus F_9 \vdash F_{10}} \&_{L1}}{\vdash : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_8 \oplus F_9}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_8 \oplus F_9} \&_{L1} \quad \frac{h_6 : \Delta_7, F_8 \vdash F_{10} \quad h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \oplus F_9 \vdash F_{10}} \oplus_L}{\vdash : (\Delta_2, F_3 \& F_4), \Delta_7 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_8 \oplus F_9}{\vdash : \Delta_2, \Delta_7, F_3 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \oplus F_9 \vdash F_{10}}{\vdash : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10}} \text{hCut}}{\vdash : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10}} \&_{L1}}{\vdash : \Delta_2, \Delta_7, F_3 \& F_4 \vdash F_{10}} \&_{L1}
\end{array}$$

• Case rule \multimap_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L1} \quad \frac{h_7 : \Delta_{12}, F_6 \vdash F_9 \quad h_7 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \multimap F_{10}), F_6 \vdash F_{11}} \multimap_L}{\vdash : (\Delta_2, F_3 \& F_4), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\vdash : \Delta_{12}, \Delta_2, \Delta_8, F_3, F_9 \multimap F_{10} \vdash F_{11}} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_{12}, \Delta_8, F_6, F_9 \multimap F_{10} \vdash F_{11}}{\vdash : \Delta_{12}, \Delta_2, \Delta_8, F_3 \& F_4, F_9 \multimap F_{10} \vdash F_{11}} \text{hCut}}{\vdash : \Delta_{12}, \Delta_2, \Delta_8, F_3 \& F_4, F_9 \multimap F_{10} \vdash F_{11}} \&_{L1}}{\vdash : \Delta_{12}, \Delta_2, \Delta_8, F_3 \& F_4, F_9 \multimap F_{10} \vdash F_{11}} \&_{L1}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_6} \&_{L1} \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{12}, F_6, F_{10} \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \multimap F_{10}), F_6 \vdash F_{11}} \multimap_L}{- : (\Delta_2, F_3 \& F_4), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11}} \text{Cut}}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6}{- : \Delta_{12}, \Delta_2, \Delta_8, F_3, F_9 \multimap F_{10} \vdash F_{11}} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_{12}, \Delta_8, F_6, F_9 \multimap F_{10} \vdash F_{11}}{- : \Delta_{12}, \Delta_2, \Delta_8, F_3 \& F_4, F_9 \multimap F_{10} \vdash F_{11}} \text{hCut}}{- : \Delta_{12}, \Delta_2, \Delta_8, F_3 \& F_4, F_9 \multimap F_{10} \vdash F_{11}} \&_{L1}}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_9 \multimap F_{10}}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash F_9 \multimap F_{10}} \&_{L1} \quad \frac{h_6 : \Delta_7 \vdash F_9 \quad h_6 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_6 : (\Delta_7, \Delta_8), F_9 \multimap F_{10} \vdash F_{11}} \multimap_L}{- : (\Delta_2, F_3 \& F_4), \Delta_7, \Delta_8 \vdash F_{11}} \text{Cut}}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_9 \multimap F_{10}}{- : \Delta_2, \Delta_7, \Delta_8, F_3 \vdash F_{11}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, \Delta_8, F_9 \multimap F_{10} \vdash F_{11}}{- : \Delta_2, \Delta_7, \Delta_8, F_3 \& F_4 \vdash F_{11}} \text{hCut}}{- : \Delta_2, \Delta_7, \Delta_8, F_3 \& F_4 \vdash F_{11}} \&_{L1}}{\rightarrow}
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash p(n_7)}{\bullet h_1 : \Delta_2, F_3 \& F_4 \vdash p(n_7)} \&_{L1} \quad \frac{}{\bullet h_6 : *, p(n_7) \vdash p(n_7)} I}{- : (\Delta_2, F_3 \& F_4), * \vdash p(n_7)} \text{Cut}}{\rightarrow} \\
\frac{}{- : \Delta_2, F_3 \& F_4 \vdash p(n_7)} \text{ax/W}
\end{array}$$

5.15 Status of \otimes_L : OK

- Case rule $!R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash 7}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash 7} \otimes_L \quad \frac{h_8 : !\Upsilon 6, 7 \vdash F_9}{\bullet h_8 : !\Upsilon 6, 7 \vdash !F_9} !R}{- : (\Delta_2, F_3 \otimes F_4), !\Upsilon 6 \vdash !F_9} \text{Cut}}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash 7}{- : !\Upsilon 6, \Delta_2, F_3, F_4 \vdash !F_9} \text{ax/W} \quad \frac{\bullet h_8 : 7, !\Upsilon 6 \vdash !F_9}{- : !\Upsilon 6, \Delta_2, F_3 \otimes F_4 \vdash !F_9} \text{hCut}}{- : !\Upsilon 6, \Delta_2, F_3 \otimes F_4 \vdash !F_9} \otimes_L
\end{array}$$

- Case rule 1_R

- Case rule \top

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_7} \otimes_L \quad \frac{}{\bullet h_8 : \Delta_6, F_7 \vdash \top} \top}{- : (\Delta_2, F_3 \otimes F_4), \Delta_6 \vdash \top} \text{Cut}}{\rightarrow} \\
\frac{}{- : \Delta_2, \Delta_6, F_3 \otimes F_4 \vdash \top} \top
\end{array}$$

- Case rule $\&_R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_7} \otimes_L \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9 \quad h_8 : \Delta_6, F_7 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \& F_{10}} \&_R}{- : (\Delta_2, F_3 \otimes F_4), \Delta_6 \vdash F_9 \& F_{10}} \text{Cut}}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_7}{- : \Delta_2, \Delta_6, F_3, F_4 \vdash F_9 \& F_{10}} \text{ax/W} \quad \frac{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \& F_{10}}{- : \Delta_2, \Delta_6, F_3 \otimes F_4 \vdash F_9 \& F_{10}} \text{hCut}}{- : \Delta_2, \Delta_6, F_3 \otimes F_4 \vdash F_9 \& F_{10}} \otimes_L
\end{array}$$

- Case rule \multimap_R

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_7} \otimes_L \quad \frac{h_8 : \Delta_6, F_7, F_9 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \multimap F_{10}} \multimap_R \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_6 \vdash F_9 \multimap F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_7}{- : \Delta_2, \Delta_6, F_3 \otimes F_4 \vdash F_{10}} \text{ ax/W} \quad \frac{h_8 : \Delta_6, F_7, F_9 \vdash F_{10}}{h_8 : \Delta_6, F_7 \vdash F_9 \multimap F_{10}} \text{ ax/W}}{- : \Delta_2, \Delta_6, F_3 \otimes F_4 \vdash F_9 \multimap F_{10}} \text{ hCut} \\
\multimap_R \\
- : \Delta_2, \Delta_6, F_3 \otimes F_4 \vdash F_9 \multimap F_{10}
\end{array}$$

- Case rule \oplus_{R_2}

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_7} \otimes_L \quad \frac{h_8 : \Delta_6, F_7 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \oplus F_{10}} \oplus_{R_2} \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_6 \vdash F_9 \oplus F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_7}{- : \Delta_2, \Delta_6, F_3 \otimes F_4 \vdash F_{10}} \text{ ax/W} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_{10}}{h_8 : \Delta_6, F_7 \vdash F_9 \oplus F_{10}} \text{ ax/W}}{- : \Delta_2, \Delta_6, F_3 \otimes F_4 \vdash F_9 \oplus F_{10}} \text{ hCut} \\
\oplus_{R_2} \\
- : \Delta_2, \Delta_6, F_3 \otimes F_4 \vdash F_9 \oplus F_{10}
\end{array}$$

- Case rule \oplus_{R_1}

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_7} \otimes_L \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \oplus F_{10}} \oplus_{R_1} \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_6 \vdash F_9 \oplus F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_7}{- : \Delta_2, \Delta_6, F_3 \otimes F_4 \vdash F_9} \text{ ax/W} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9}{h_8 : \Delta_6, F_7 \vdash F_9 \oplus F_{10}} \text{ ax/W}}{- : \Delta_2, \Delta_6, F_3 \otimes F_4 \vdash F_9 \oplus F_{10}} \text{ hCut} \\
\oplus_{R_1} \\
- : \Delta_2, \Delta_6, F_3 \otimes F_4 \vdash F_9 \oplus F_{10}
\end{array}$$

- Case rule $\mathbf{1}_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3, F_4 \vdash \mathbf{1}}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash \mathbf{1}} \otimes_L \quad \frac{h_6 : \Delta_7 \vdash F_8}{\bullet h_6 : \Delta_7, \mathbf{1} \vdash F_8} \mathbf{1}_L \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_7 \vdash F_8 \\
\rightarrow \\
- : \Delta_2, \Delta_7, F_3 \otimes F_4 \vdash F_8 \quad \text{ax/W} \\
\mathbf{1}_L \\
\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6} \otimes_L \quad \frac{h_7 : \Delta_9, F_6 \vdash F_8}{\bullet h_7 : (\mathbf{1}, \Delta_9), F_6 \vdash F_8} \mathbf{1}_L \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \mathbf{1}, \Delta_9 \vdash F_8 \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6}{- : \mathbf{1}, \Delta_2, \Delta_9, F_3 \otimes F_4 \vdash F_8} \text{ ax/W} \quad \frac{h_7 : \mathbf{1}, \Delta_9, F_6 \vdash F_8}{h_7 : \mathbf{1}, \Delta_9, F_6 \vdash F_8} \text{ ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_9, F_3 \otimes F_4 \vdash F_8} \text{ hCut}
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6} \otimes_L \quad \frac{h_7 : \Delta_{11}, F_6 \vdash F_9 \quad h_7 : \Delta_8 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_6 \vdash F_9 \otimes F_{10}} \otimes_R \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3, F_4 \vdash F_9 \otimes F_{10}} \text{ ax/W} \quad \frac{h_7 : \Delta_{11}, \Delta_8, F_6 \vdash F_9 \otimes F_{10}}{h_7 : \Delta_{11}, \Delta_8, F_6 \vdash F_9 \otimes F_{10}} \text{ ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \otimes F_4 \vdash F_9 \otimes F_{10}} \text{ hCut} \\
\otimes_L \\
- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \otimes F_4 \vdash F_9 \otimes F_{10} \\
\otimes_R \\
\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6} \otimes_L \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{11}, F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_6 \vdash F_9 \otimes F_{10}} \otimes_R \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3, F_4 \vdash F_9 \otimes F_{10}} \text{ ax/W} \quad \frac{h_7 : \Delta_{11}, \Delta_8, F_6 \vdash F_9 \otimes F_{10}}{h_7 : \Delta_{11}, \Delta_8, F_6 \vdash F_9 \otimes F_{10}} \text{ ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \otimes F_4 \vdash F_9 \otimes F_{10}} \text{ hCut} \\
\otimes_L \\
- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \otimes F_4 \vdash F_9 \otimes F_{10}
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6} \otimes L \quad \frac{h_7 : \Delta_{10}, F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_6 \vdash F_9} W \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_{10}, !F_8 \vdash F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \otimes F_4 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_6, !F_8 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \otimes F_4 \vdash F_9} \text{hCut} \\
\hline
\frac{h_1 : \Delta_2, F_3, F_4 \vdash !F_8}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash !F_8} \otimes L \quad \frac{h_6 : \Delta_7 \vdash F_9}{\bullet h_6 : \Delta_7, !F_8 \vdash F_9} W \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_7 \vdash F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_2, \Delta_7, F_3 \otimes F_4 \vdash F_9} \text{ax/W}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6} \otimes L \quad \frac{h_7 : \Delta_{10}, F_6, !F_8, !F_8 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_6 \vdash F_9} C \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_{10}, !F_8 \vdash F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6}{- : \Delta_{10}, \Delta_2, !F_8, !F_8, F_3 \otimes F_4 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_6, !F_8, !F_8 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \otimes F_4 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_2, !F_8, !F_8, F_3 \otimes F_4 \vdash F_9 \quad C \\
\hline
\frac{h_1 : \Delta_2, F_3, F_4 \vdash !F_8}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash !F_8} \otimes L \quad \frac{h_6 : \Delta_7, !F_8, !F_8 \vdash F_9}{\bullet h_6 : \Delta_7, !F_8 \vdash F_9} C \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_7 \vdash F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash !F_8}{- : \Delta_2, \Delta_7, F_3 \otimes F_4 \vdash F_9} \text{ax/W} \quad \frac{h_6 : \Delta_7, !F_8, !F_8 \vdash F_9}{- : \Delta_2, \Delta_7, F_3 \otimes F_4 \vdash F_9} \text{mCut}
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6} \otimes L \quad \frac{h_7 : \Delta_{10}, F_6, F_8 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_6 \vdash F_9} !L \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_{10}, !F_8 \vdash F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6}{- : \Delta_{10}, \Delta_2, F_8, F_3 \otimes F_4 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_6, F_8 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \otimes F_4 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_{10}, \Delta_2, F_8, F_3 \otimes F_4 \vdash F_9 \quad !L \\
\hline
\frac{h_1 : \Delta_2, F_3, F_4 \vdash !F_8}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash !F_8} \otimes L \quad \frac{h_6 : \Delta_7, F_8 \vdash F_9}{\bullet h_6 : \Delta_7, !F_8 \vdash F_9} !L \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_7 \vdash F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{h_1 : \Delta_2, F_3, F_4 \vdash !F_8}{- : \Delta_2, \Delta_7, F_3, F_4 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, !F_8 \vdash F_9}{- : \Delta_2, \Delta_7, F_3 \otimes F_4 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_2, \Delta_7, F_3, F_4 \vdash F_9 \quad \otimes L \\
\hline
- : \Delta_2, \Delta_7, F_3 \otimes F_4 \vdash F_9
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6} \otimes L \quad \frac{h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_6 \vdash F_{10}} \&_{L2} \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_{11}, F_8 \& F_9 \vdash F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_9, F_3 \otimes F_4 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9, F_3 \otimes F_4 \vdash F_{10}} \text{hCut} \\
\hline
- : \Delta_{11}, \Delta_2, F_9, F_3 \otimes F_4 \vdash F_{10} \quad \&_{L2} \\
\hline
- : \Delta_{11}, \Delta_2, F_8 \& F_9, F_3 \otimes F_4 \vdash F_{10}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_8 \& F_9}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_8 \& F_9} \otimes_L \quad \frac{h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}} \&_{L2} \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_7 \vdash F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_8 \& F_9}{- : \Delta_2, \Delta_7, F_3, F_4 \vdash F_{10}} \text{ ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_3 \otimes F_4 \vdash F_{10}} \text{ hCut}}{- : \Delta_2, \Delta_7, F_3 \otimes F_4 \vdash F_{10}} \otimes_L
\end{array}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6} \otimes_L \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_6 \vdash F_{10}} \&_{L1} \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_{11}, F_8 \& F_9 \vdash F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_8, F_3 \otimes F_4 \vdash F_{10}} \text{ ax/W} \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9, F_3 \otimes F_4 \vdash F_{10}} \text{ hCut}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9, F_3 \otimes F_4 \vdash F_{10}} \&_{L1} \\
\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_8 \& F_9}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_8 \& F_9} \otimes_L \quad \frac{h_6 : \Delta_7, F_8 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}} \&_{L1} \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_7 \vdash F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_8 \& F_9}{- : \Delta_2, \Delta_7, F_3, F_4 \vdash F_{10}} \text{ ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_3 \otimes F_4 \vdash F_{10}} \text{ hCut}}{- : \Delta_2, \Delta_7, F_3 \otimes F_4 \vdash F_{10}} \otimes_L
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6} \otimes_L \quad \frac{h_7 : \Delta_{11}, F_6, F_8, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \otimes F_9), F_6 \vdash F_{10}} \otimes_L \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_{11}, F_8 \otimes F_9 \vdash F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_3, F_4, F_8 \otimes F_9 \vdash F_{10}} \text{ ax/W} \quad \frac{\bullet h_7 : \Delta_{11}, F_6, F_8 \otimes F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_3 \otimes F_4, F_8 \otimes F_9 \vdash F_{10}} \text{ hCut}}{- : \Delta_{11}, \Delta_2, F_3 \otimes F_4, F_8 \otimes F_9 \vdash F_{10}} \otimes_L \\
\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_8 \otimes F_9}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_8 \otimes F_9} \otimes_L \quad \frac{h_6 : \Delta_7, F_8, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \otimes F_9 \vdash F_{10}} \otimes_L \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_7 \vdash F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_8 \otimes F_9}{- : \Delta_2, \Delta_7, F_3, F_4 \vdash F_{10}} \text{ ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \otimes F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_3 \otimes F_4 \vdash F_{10}} \text{ hCut}}{- : \Delta_2, \Delta_7, F_3 \otimes F_4 \vdash F_{10}} \otimes_L
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6} \otimes_L \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \vdash F_{10} \quad h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \oplus F_9), F_6 \vdash F_{10}} \oplus_L \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_3, F_4, F_8 \oplus F_9 \vdash F_{10}} \text{ ax/W} \quad \frac{\bullet h_7 : \Delta_{11}, F_6, F_8 \oplus F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_3 \otimes F_4, F_8 \oplus F_9 \vdash F_{10}} \text{ hCut}}{- : \Delta_{11}, \Delta_2, F_3 \otimes F_4, F_8 \oplus F_9 \vdash F_{10}} \otimes_L \\
\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_8 \oplus F_9}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_8 \oplus F_9} \otimes_L \quad \frac{h_6 : \Delta_7, F_8 \vdash F_{10} \quad h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \oplus F_9 \vdash F_{10}} \oplus_L \\
\hline
- : (\Delta_2, F_3 \otimes F_4), \Delta_7 \vdash F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_8 \oplus F_9}{- : \Delta_2, \Delta_7, F_3, F_4 \vdash F_{10}} \text{ ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \oplus F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_3 \otimes F_4 \vdash F_{10}} \text{ hCut}}{- : \Delta_2, \Delta_7, F_3 \otimes F_4 \vdash F_{10}} \otimes_L
\end{array}$$

- Case rule $\neg\circ_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6} \otimes_L \quad \frac{\frac{h_7 : \Delta_{12}, F_6 \vdash F_9 \quad h_7 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \neg\circ F_{10}), F_6 \vdash F_{11}} \neg\circ_L}{\neg : (\Delta_2, F_3 \otimes F_4), \Delta_8, \Delta_{12}, F_9 \neg\circ F_{10} \vdash F_{11}} \text{Cut}}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\neg : \Delta_{12}, \Delta_2, \Delta_8, F_3, F_4, F_9 \neg\circ F_{10} \vdash F_{11}} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, \Delta_8, F_6, F_9 \neg\circ F_{10} \vdash F_{11}}{\neg : \Delta_{12}, \Delta_2, \Delta_8, F_3 \otimes F_4, F_9 \neg\circ F_{10} \vdash F_{11}} \text{hCut}}{\otimes_L} \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_6} \otimes_L \quad \frac{\frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{12}, F_6, F_{10} \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \neg\circ F_{10}), F_6 \vdash F_{11}} \neg\circ_L}{\neg : (\Delta_2, F_3 \otimes F_4), \Delta_8, \Delta_{12}, F_9 \neg\circ F_{10} \vdash F_{11}} \text{Cut}}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_6}{\neg : \Delta_{12}, \Delta_2, \Delta_8, F_3, F_4, F_9 \neg\circ F_{10} \vdash F_{11}} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, \Delta_8, F_6, F_9 \neg\circ F_{10} \vdash F_{11}}{\neg : \Delta_{12}, \Delta_2, \Delta_8, F_3 \otimes F_4, F_9 \neg\circ F_{10} \vdash F_{11}} \text{hCut}}{\otimes_L} \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_9 \neg\circ F_{10}}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_9 \neg\circ F_{10}} \otimes_L \quad \frac{\frac{h_6 : \Delta_7 \vdash F_9 \quad h_6 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_6 : (\Delta_7, \Delta_8), F_9 \neg\circ F_{10} \vdash F_{11}} \neg\circ_L}{\neg : (\Delta_2, F_3 \otimes F_4), \Delta_7, \Delta_8 \vdash F_{11}} \text{Cut}}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_9 \neg\circ F_{10}}{\neg : \Delta_2, \Delta_7, \Delta_8, F_3, F_4 \vdash F_{11}} \text{ax/W} \quad \frac{h_6 : \Delta_7, \Delta_8, F_9 \neg\circ F_{10} \vdash F_{11}}{\neg : \Delta_2, \Delta_7, \Delta_8, F_3 \otimes F_4 \vdash F_{11}} \text{hCut}}{\otimes_L} \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_9 \neg\circ F_{10}}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash F_9 \neg\circ F_{10}} \otimes_L \quad \frac{\frac{h_6 : \Delta_7 \vdash F_9 \quad h_6 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_6 : (\Delta_7, \Delta_8), F_9 \neg\circ F_{10} \vdash F_{11}} \neg\circ_L}{\neg : (\Delta_2, F_3 \otimes F_4), \Delta_7, \Delta_8 \vdash F_{11}} \text{Cut}}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash F_9 \neg\circ F_{10}}{\neg : \Delta_2, \Delta_7, \Delta_8, F_3, F_4 \vdash F_{11}} \text{ax/W} \quad \frac{h_6 : \Delta_7, \Delta_8, F_9 \neg\circ F_{10} \vdash F_{11}}{\neg : \Delta_2, \Delta_7, \Delta_8, F_3 \otimes F_4 \vdash F_{11}} \text{hCut}}{\otimes_L}
\end{array}$$

- Case rule I

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3, F_4 \vdash p(n_7)}{\bullet h_1 : \Delta_2, F_3 \otimes F_4 \vdash p(n_7)} \otimes_L \quad \frac{I}{\bullet h_6 : *, p(n_7) \vdash p(n_7)} \text{Cut}}{\neg : (\Delta_2, F_3 \otimes F_4), * \vdash p(n_7)} \\
\rightarrow \\
\frac{\neg : \Delta_2, F_3 \otimes F_4 \vdash p(n_7)}{\text{ax/W}}
\end{array}$$

5.16 Status of \oplus_L : OK

- Case rule $!R$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash 7 \quad h_1 : \Delta_2, F_4 \vdash 7}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash 7} \oplus_L \quad \frac{\frac{h_8 : !\Upsilon 6, 7 \vdash F_9}{\bullet h_8 : !\Upsilon 6, 7 \vdash !F_9} !R}{\neg : (\Delta_2, F_3 \oplus F_4), !\Upsilon 6 \vdash !F_9} \text{Cut}}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash 7}{\neg : !\Upsilon 6, \Delta_2, F_3 \vdash !F_9} \text{ax/W} \quad \frac{h_8 : 7, !\Upsilon 6 \vdash !F_9}{\neg : !\Upsilon 6, \Delta_2, F_3 \oplus F_4 \vdash !F_9} \text{hCut} \quad \frac{h_1 : \Delta_2, F_4 \vdash 7}{\neg : !\Upsilon 6, \Delta_2, F_4 \vdash !F_9} \text{ax/W} \quad \frac{h_8 : 7, !\Upsilon 6 \vdash !F_9}{\neg : !\Upsilon 6, \Delta_2, F_4 \vdash !F_9} \text{hCut}}{\oplus_L}
\end{array}$$

- Case rule 1_R

- Case rule \top

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_7 \quad h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_7} \oplus_L \quad \frac{\top}{\bullet h_8 : \Delta_6, F_7 \vdash \top} \text{Cut}}{\neg : (\Delta_2, F_3 \oplus F_4), \Delta_6 \vdash \top} \\
\rightarrow \\
\frac{\neg : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash \top}{\top}
\end{array}$$

- Case rule $\&_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_7 \quad h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_7} \oplus_L \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9 \quad h_8 : \Delta_6, F_7 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \& F_{10}} \&_R \\
\hline
- : (\Delta_2, F_3 \oplus F_4), \Delta_6 \vdash F_9 \& F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_7}{- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_9} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9}{- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_9} \text{hCut} \quad \frac{\frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_7}{- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_9} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_{10}}{- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_{10}} \text{hCut} \\
\hline
- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_9 \& F_{10} \quad \&_R
\end{array}$$

- Case rule $\neg\circ_R$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_7 \quad h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_7} \oplus_L \quad \frac{h_8 : \Delta_6, F_7, F_9 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \neg\circ F_{10}} \neg\circ_R \\
\hline
- : (\Delta_2, F_3 \oplus F_4), \Delta_6 \vdash F_9 \neg\circ F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_7}{- : \Delta_2, \Delta_6, F_9, F_3 \oplus F_4 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7, F_9 \vdash F_{10}}{- : \Delta_2, \Delta_6, F_9, F_3 \oplus F_4 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_6, F_9, F_3 \oplus F_4 \vdash F_{10}} \text{hCut} \\
\hline
- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_9 \neg\circ F_{10} \quad \neg\circ_R
\end{array}$$

- Case rule \oplus_{R_2}

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_7 \quad h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_7} \oplus_L \quad \frac{h_8 : \Delta_6, F_7 \vdash F_{10}}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \oplus F_{10}} \oplus_{R_2} \\
\hline
- : (\Delta_2, F_3 \oplus F_4), \Delta_6 \vdash F_9 \oplus F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_7}{- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_{10}} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_{10}}{- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_{10}} \text{hCut} \\
\hline
- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_9 \oplus F_{10} \quad \oplus_{R_2}
\end{array}$$

- Case rule \oplus_{R_1}

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_7 \quad h_1 : \Delta_2, F_4 \vdash F_7}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_7} \oplus_L \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9}{\bullet h_8 : \Delta_6, F_7 \vdash F_9 \oplus F_{10}} \oplus_{R_1} \\
\hline
- : (\Delta_2, F_3 \oplus F_4), \Delta_6 \vdash F_9 \oplus F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_7}{- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_9} \text{ax/W} \quad \frac{h_8 : \Delta_6, F_7 \vdash F_9}{- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_9} \text{hCut} \\
\hline
- : \Delta_2, \Delta_6, F_3 \oplus F_4 \vdash F_9 \oplus F_{10} \quad \oplus_{R_1}
\end{array}$$

- Case rule $\mathbf{1}_L$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash \mathbf{1} \quad h_1 : \Delta_2, F_4 \vdash \mathbf{1}}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash \mathbf{1}} \oplus_L \quad \frac{h_6 : \Delta_7 \vdash F_8}{\bullet h_6 : \Delta_7, \mathbf{1} \vdash F_8} \mathbf{1}_L \\
\hline
- : (\Delta_2, F_3 \oplus F_4), \Delta_7 \vdash F_8 \quad \text{Cut} \\
\rightarrow \\
- : \Delta_2, \Delta_7, F_3 \oplus F_4 \vdash F_8 \quad \text{ax/W} \\
\hline
\frac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6} \oplus_L \quad \frac{h_7 : \Delta_9, F_6 \vdash F_8}{\bullet h_7 : (\mathbf{1}, \Delta_9), F_6 \vdash F_8} \mathbf{1}_L \\
\hline
- : (\Delta_2, F_3 \oplus F_4), \mathbf{1}, \Delta_9 \vdash F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6}{- : \mathbf{1}, \Delta_2, \Delta_9, F_3 \oplus F_4 \vdash F_8} \text{ax/W} \quad \frac{h_7 : \mathbf{1}, \Delta_9, F_6 \vdash F_8}{- : \mathbf{1}, \Delta_2, \Delta_9, F_3 \oplus F_4 \vdash F_8} \text{ax/W}}{- : \mathbf{1}, \Delta_2, \Delta_9, F_3 \oplus F_4 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6} \oplus L \quad \frac{h_7 : \Delta_{11}, F_6 \vdash F_9 \quad h_7 : \Delta_8 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_6 \vdash F_9 \otimes F_{10}} \otimes R}{- : (\Delta_2, F_3 \oplus F_4), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10}} \text{Cut}} \\
\rightarrow \\
\frac{\frac{\frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_3 \oplus F_4 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_6 \vdash F_9}{- : \Delta_8 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \oplus F_4 \vdash F_9 \otimes F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \oplus F_4 \vdash F_9 \otimes F_{10}} \otimes R} \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6} \oplus L \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{11}, F_6 \vdash F_{10}}{\bullet h_7 : (\Delta_8, \Delta_{11}), F_6 \vdash F_9 \otimes F_{10}} \otimes R}{- : (\Delta_2, F_3 \oplus F_4), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10}} \text{Cut}} \\
\rightarrow \\
\frac{\frac{- : \Delta_8 \vdash F_9}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \oplus F_4 \vdash F_9 \otimes F_{10}} \text{ax/W} \quad \frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_3 \oplus F_4 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_6 \vdash F_{10}}{- : \Delta_8 \vdash F_{10}} \text{hCut}}{- : \Delta_{11}, \Delta_2, \Delta_8, F_3 \oplus F_4 \vdash F_9 \otimes F_{10}} \otimes R}
\end{array}$$

• Case rule W

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6} \oplus L \quad \frac{h_7 : \Delta_{10}, F_6 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_6 \vdash F_9} W}{- : (\Delta_2, F_3 \oplus F_4), \Delta_{10}, !F_8 \vdash F_9} \text{Cut}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \oplus F_4 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_6, !F_8 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \oplus F_4 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \oplus F_4 \vdash F_9} \\
\frac{h_1 : \Delta_2, F_3 \vdash !F_8 \quad h_1 : \Delta_2, F_4 \vdash !F_8}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash !F_8} \oplus L \quad \frac{h_6 : \Delta_7 \vdash F_9}{\bullet h_6 : \Delta_7, !F_8 \vdash F_9} W}{- : (\Delta_2, F_3 \oplus F_4), \Delta_7 \vdash F_9} \text{Cut}} \\
\rightarrow \\
\frac{- : \Delta_2, \Delta_7, F_3 \oplus F_4 \vdash F_9}{- : \Delta_2, \Delta_7, F_3 \oplus F_4 \vdash F_9} \text{ax/W}
\end{array}$$

• Case rule C

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6} \oplus L \quad \frac{h_7 : \Delta_{10}, F_6, !F_8, !F_8 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_6 \vdash F_9} C}{- : (\Delta_2, F_3 \oplus F_4), \Delta_{10}, !F_8 \vdash F_9} \text{Cut}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6}{- : \Delta_{10}, \Delta_2, !F_8, !F_8, F_3 \oplus F_4 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_6, !F_8, !F_8 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \oplus F_4 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \oplus F_4 \vdash F_9} C} \\
\frac{h_1 : \Delta_2, F_3 \vdash !F_8 \quad h_1 : \Delta_2, F_4 \vdash !F_8}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash !F_8} \oplus L \quad \frac{h_6 : \Delta_7, !F_8, !F_8 \vdash F_9}{\bullet h_6 : \Delta_7, !F_8 \vdash F_9} C}{- : (\Delta_2, F_3 \oplus F_4), \Delta_7 \vdash F_9} \text{Cut}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash !F_8}{- : \Delta_2, \Delta_7, F_3 \oplus F_4 \vdash F_9} \text{ax/W} \quad \frac{h_6 : \Delta_7, !F_8, !F_8 \vdash F_9}{- : \Delta_2, \Delta_7, F_3 \oplus F_4 \vdash F_9} \text{mCut}}{- : \Delta_2, \Delta_7, F_3 \oplus F_4 \vdash F_9}
\end{array}$$

• Case rule $!L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6} \oplus L \quad \frac{h_7 : \Delta_{10}, F_6, F_8 \vdash F_9}{\bullet h_7 : (\Delta_{10}, !F_8), F_6 \vdash F_9} !L}{- : (\Delta_2, F_3 \oplus F_4), \Delta_{10}, !F_8 \vdash F_9} \text{Cut}} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6}{- : \Delta_{10}, \Delta_2, F_8, F_3 \oplus F_4 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{10}, F_6, F_8 \vdash F_9}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \oplus F_4 \vdash F_9} \text{hCut}}{- : \Delta_{10}, \Delta_2, !F_8, F_3 \oplus F_4 \vdash F_9} !L}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash !F_8 \quad h_1 : \Delta_2, F_4 \vdash !F_8}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash !F_8} \oplus L \quad \frac{h_6 : \Delta_7, F_8 \vdash F_9}{\bullet h_6 : \Delta_7, !F_8 \vdash F_9} !L}{- : (\Delta_2, F_3 \oplus F_4), \Delta_7 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash !F_8}{- : \Delta_2, \Delta_7, F_3 \vdash F_9} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, !F_8 \vdash F_9}{- : \Delta_2, \Delta_7, F_4 \vdash F_9} \text{ax/W}}{- : \Delta_2, \Delta_7, F_3 \oplus F_4 \vdash F_9} \text{hCut} \oplus L
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6} \oplus L \quad \frac{h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_6 \vdash F_{10}} \&_{L2}}{- : (\Delta_2, F_3 \oplus F_4), \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_9, F_3 \oplus F_4 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9, F_3 \oplus F_4 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9, F_3 \oplus F_4 \vdash F_{10}} \text{hCut} \&_{L2} \\
\frac{h_1 : \Delta_2, F_3 \vdash F_8 \& F_9 \quad h_1 : \Delta_2, F_4 \vdash F_8 \& F_9}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_8 \& F_9} \oplus L \quad \frac{h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}} \&_{L2}}{- : (\Delta_2, F_3 \oplus F_4), \Delta_7 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_8 \& F_9}{- : \Delta_2, \Delta_7, F_3 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_4 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_7, F_3 \oplus F_4 \vdash F_{10}} \text{hCut} \oplus L
\end{array}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6} \oplus L \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \& F_9), F_6 \vdash F_{10}} \&_{L1}}{- : (\Delta_2, F_3 \oplus F_4), \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_8, F_3 \oplus F_4 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9, F_3 \oplus F_4 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \& F_9, F_3 \oplus F_4 \vdash F_{10}} \text{hCut} \&_{L1} \\
\frac{h_1 : \Delta_2, F_3 \vdash F_8 \& F_9 \quad h_1 : \Delta_2, F_4 \vdash F_8 \& F_9}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_8 \& F_9} \oplus L \quad \frac{h_6 : \Delta_7, F_8 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}} \&_{L1}}{- : (\Delta_2, F_3 \oplus F_4), \Delta_7 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_8 \& F_9}{- : \Delta_2, \Delta_7, F_3 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \& F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_4 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_7, F_3 \oplus F_4 \vdash F_{10}} \text{hCut} \oplus L
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6} \oplus L \quad \frac{h_7 : \Delta_{11}, F_6, F_8, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \otimes F_9), F_6 \vdash F_{10}} \otimes L}}{- : (\Delta_2, F_3 \oplus F_4), \Delta_{11}, F_8 \otimes F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_8, F_9, F_3 \oplus F_4 \vdash F_{10}} \text{ax/W} \quad \frac{h_7 : \Delta_{11}, F_6, F_8, F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_8 \otimes F_9, F_3 \oplus F_4 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_8 \otimes F_9, F_3 \oplus F_4 \vdash F_{10}} \text{hCut} \otimes L \\
\frac{h_1 : \Delta_2, F_3 \vdash F_8 \otimes F_9 \quad h_1 : \Delta_2, F_4 \vdash F_8 \otimes F_9}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_8 \otimes F_9} \oplus L \quad \frac{h_6 : \Delta_7, F_8, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \otimes F_9 \vdash F_{10}} \otimes L}}{- : (\Delta_2, F_3 \oplus F_4), \Delta_7 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_8 \otimes F_9}{- : \Delta_2, \Delta_7, F_3 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \otimes F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_4 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_7, F_3 \oplus F_4 \vdash F_{10}} \text{hCut} \otimes L
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6} \oplus_L \quad \frac{h_7 : \Delta_{11}, F_6, F_8 \vdash F_{10} \quad h_7 : \Delta_{11}, F_6, F_9 \vdash F_{10}}{\bullet h_7 : (\Delta_{11}, F_8 \oplus F_9), F_6 \vdash F_{10}} \oplus_L}{- : (\Delta_2, F_3 \oplus F_4), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}} \text{Cut}}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6}{- : \Delta_{11}, \Delta_2, F_3, F_8 \oplus F_9 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_7 : \Delta_{11}, F_6, F_8 \oplus F_9 \vdash F_{10}}{- : \Delta_{11}, \Delta_2, F_4, F_8 \oplus F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, F_3 \oplus F_4, F_8 \oplus F_9 \vdash F_{10}} \text{hCut}}{\oplus_L} \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_8 \oplus F_9 \quad h_1 : \Delta_2, F_4 \vdash F_8 \oplus F_9}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_8 \oplus F_9} \oplus_L \quad \frac{h_6 : \Delta_7, F_8 \vdash F_{10} \quad h_6 : \Delta_7, F_9 \vdash F_{10}}{\bullet h_6 : \Delta_7, F_8 \oplus F_9 \vdash F_{10}} \oplus_L}{- : (\Delta_2, F_3 \oplus F_4), \Delta_7 \vdash F_{10}} \text{Cut}}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_8 \oplus F_9}{- : \Delta_2, \Delta_7, F_3 \vdash F_{10}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, F_8 \oplus F_9 \vdash F_{10}}{- : \Delta_2, \Delta_7, F_4 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_7, F_3 \oplus F_4 \vdash F_{10}} \text{hCut}}{\oplus_L} \text{hCut}
\end{array}$$

- Case rule $\neg\circ_L$

$$\begin{array}{c}
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6} \oplus_L \quad \frac{h_7 : \Delta_{12}, F_6 \vdash F_9 \quad h_7 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \neg\circ F_{10}), F_6 \vdash F_{11}} \neg\circ_L}{- : (\Delta_2, F_3 \oplus F_4), \Delta_8, \Delta_{12}, F_9 \neg\circ F_{10} \vdash F_{11}} \text{Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6}{- : \Delta_{12}, \Delta_2, F_3 \oplus F_4 \vdash F_9} \text{ax/W} \quad \frac{h_7 : \Delta_{12}, F_6 \vdash F_9}{- : \Delta_8, F_{10} \vdash F_{11}} \text{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_8, F_9 \neg\circ F_{10}, F_3 \oplus F_4 \vdash F_{11}} \text{hCut}}{\neg\circ_L} \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_6 \quad h_1 : \Delta_2, F_4 \vdash F_6}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6} \oplus_L \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{12}, F_6, F_{10} \vdash F_{11}}{\bullet h_7 : (\Delta_8, \Delta_{12}, F_9 \neg\circ F_{10}), F_6 \vdash F_{11}} \neg\circ_L}{- : (\Delta_2, F_3 \oplus F_4), \Delta_8, \Delta_{12}, F_9 \neg\circ F_{10} \vdash F_{11}} \text{Cut}}{\rightarrow} \\
\frac{\frac{- : \Delta_8 \vdash F_9}{- : \Delta_{12}, \Delta_2, \Delta_8, F_9 \neg\circ F_{10}, F_3 \oplus F_4 \vdash F_{11}} \text{ax/W} \quad \frac{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_6}{- : \Delta_{12}, \Delta_2, F_{10}, F_3 \oplus F_4 \vdash F_{11}} \text{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_8, F_9 \neg\circ F_{10}, F_3 \oplus F_4 \vdash F_{11}} \text{hCut}}{\neg\circ_L} \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_9 \neg\circ F_{10} \quad h_1 : \Delta_2, F_4 \vdash F_9 \neg\circ F_{10}}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash F_9 \neg\circ F_{10}} \oplus_L \quad \frac{h_6 : \Delta_7 \vdash F_9 \quad h_6 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_6 : (\Delta_7, \Delta_8), F_9 \neg\circ F_{10} \vdash F_{11}} \neg\circ_L}{- : (\Delta_2, F_3 \oplus F_4), \Delta_7, \Delta_8 \vdash F_{11}} \text{Cut}}{\rightarrow} \\
\frac{\frac{h_1 : \Delta_2, F_3 \vdash F_9 \neg\circ F_{10}}{- : \Delta_2, \Delta_7, \Delta_8, F_3 \vdash F_{11}} \text{ax/W} \quad \frac{\bullet h_6 : \Delta_7, \Delta_8, F_9 \neg\circ F_{10} \vdash F_{11}}{- : \Delta_2, \Delta_7, \Delta_8, F_4 \vdash F_{11}} \text{ax/W}}{- : \Delta_2, \Delta_7, \Delta_8, F_3 \oplus F_4 \vdash F_{11}} \text{hCut}}{\oplus_L} \text{hCut}
\end{array}$$

- Case rule I

$$\frac{\frac{h_1 : \Delta_2, F_3 \vdash p(n_7) \quad h_1 : \Delta_2, F_4 \vdash p(n_7)}{\bullet h_1 : \Delta_2, F_3 \oplus F_4 \vdash p(n_7)} \oplus_L \quad \frac{\bullet h_6 : *, p(n_7) \vdash p(n_7)}{- : (\Delta_2, F_3 \oplus F_4), * \vdash p(n_7)} I}{- : \Delta_2, F_3 \oplus F_4 \vdash p(n_7)} \text{Cut}$$

5.17 Status of $\neg\circ_L$: OK

- Case rule $!R$

- Case rule 1_L

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash \mathbf{1} \quad \text{-oL} \quad \frac{h_7 : \Delta_8 \vdash F_9}{\bullet h_7 : \Delta_8, \mathbf{1} \vdash F_9} \mathbf{1}_L}{\frac{\bullet h_1 : \Delta_2, \Delta_3, F_4 \text{-o} F_5 \vdash \mathbf{1}}{\text{-} : (\Delta_2, \Delta_3, F_4 \text{-o} F_5), \Delta_8 \vdash F_9} \text{Cut}}{\rightarrow} \text{ax/W} \\
\frac{\text{-} : \Delta_2, \Delta_3, \Delta_8, F_4 \text{-o} F_5 \vdash F_9}{}
\\
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash F_7 \quad \text{-oL} \quad \frac{h_8 : \Delta_{10}, F_7 \vdash F_9}{\bullet h_8 : (\mathbf{1}, \Delta_{10}), F_7 \vdash F_9} \mathbf{1}_L}{\frac{\bullet h_1 : \Delta_2, \Delta_3, F_4 \text{-o} F_5 \vdash F_7}{\text{-} : (\Delta_2, \Delta_3, F_4 \text{-o} F_5), \mathbf{1}, \Delta_{10} \vdash F_9} \text{Cut}}{\rightarrow} \text{ax/W} \\
\frac{\bullet h_1 : \Delta_2, \Delta_3, F_4 \text{-o} F_5 \vdash F_7 \quad \text{ax/W} \quad h_8 : \mathbf{1}, \Delta_{10}, F_7 \vdash F_9 \quad \text{ax/W}}{\text{-} : \mathbf{1}, \Delta_{10}, \Delta_2, \Delta_3, F_4 \text{-o} F_5 \vdash F_9} \text{hCut}
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash F_7 \quad \text{-oL} \quad \frac{h_8 : \Delta_{12}, F_7 \vdash F_{10} \quad h_8 : \Delta_9 \vdash F_{11}}{\bullet h_8 : (\Delta_9, \Delta_{12}), F_7 \vdash F_{10} \otimes F_{11}} \otimes_R}{\frac{\bullet h_1 : \Delta_2, \Delta_3, F_4 \text{-o} F_5 \vdash F_7}{\text{-} : (\Delta_2, \Delta_3, F_4 \text{-o} F_5), \Delta_9, \Delta_{12} \vdash F_{10} \otimes F_{11}} \text{Cut}}{\rightarrow} \text{ax/W} \\
\frac{\bullet h_1 : \Delta_2, \Delta_3, F_4 \text{-o} F_5 \vdash F_7 \quad \text{ax/W} \quad h_8 : \Delta_{12}, F_7 \vdash F_{10} \quad \text{ax/W}}{\text{-} : \Delta_{12}, \Delta_2, \Delta_3, F_4 \text{-o} F_5 \vdash F_{10}} \text{hCut} \\
\frac{\text{-} : \Delta_{12}, \Delta_2, \Delta_3, \Delta_9, F_4 \text{-o} F_5 \vdash F_{10} \otimes F_{11}}{\text{-} : \Delta_9 \vdash F_{11}} \otimes_R
\\
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash F_7 \quad \text{-oL} \quad \frac{h_8 : \Delta_9 \vdash F_{10} \quad h_8 : \Delta_{12}, F_7 \vdash F_{11}}{\bullet h_8 : (\Delta_9, \Delta_{12}), F_7 \vdash F_{10} \otimes F_{11}} \otimes_R}{\frac{\bullet h_1 : \Delta_2, \Delta_3, F_4 \text{-o} F_5 \vdash F_7}{\text{-} : (\Delta_2, \Delta_3, F_4 \text{-o} F_5), \Delta_9, \Delta_{12} \vdash F_{10} \otimes F_{11}} \text{Cut}}{\rightarrow} \text{ax/W} \\
\frac{\text{-} : \Delta_9 \vdash F_{10} \quad \text{ax/W} \quad \bullet h_1 : \Delta_2, \Delta_3, F_4 \text{-o} F_5 \vdash F_7 \quad \text{ax/W} \quad h_8 : \Delta_{12}, F_7 \vdash F_{11}}{\text{-} : \Delta_{12}, \Delta_2, \Delta_3, F_4 \text{-o} F_5 \vdash F_{11}} \text{hCut} \\
\frac{\text{-} : \Delta_{12}, \Delta_2, \Delta_3, \Delta_9, F_4 \text{-o} F_5 \vdash F_{10} \otimes F_{11}}{\text{-} : \Delta_{12}, \Delta_2, \Delta_3, \Delta_9, F_4 \text{-o} F_5 \vdash F_{10} \otimes F_{11}} \otimes_R
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash F_7 \quad \text{-oL} \quad \frac{h_8 : \Delta_{11}, F_7 \vdash F_{10}}{\bullet h_8 : (\Delta_{11}, !F_9), F_7 \vdash F_{10}} W}{\frac{\bullet h_1 : \Delta_2, \Delta_3, F_4 \text{-o} F_5 \vdash F_7}{\text{-} : (\Delta_2, \Delta_3, F_4 \text{-o} F_5), \Delta_{11}, !F_9 \vdash F_{10}} \text{Cut}}{\rightarrow} \text{ax/W} \\
\frac{\bullet h_1 : \Delta_2, \Delta_3, F_4 \text{-o} F_5 \vdash F_7 \quad \text{ax/W} \quad h_8 : \Delta_{11}, F_7, !F_9 \vdash F_{10} \quad \text{ax/W}}{\text{-} : \Delta_{11}, \Delta_2, \Delta_3, !F_9, F_4 \text{-o} F_5 \vdash F_{10}} \text{hCut}
\\
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash !F_9 \quad \text{-oL} \quad \frac{h_7 : \Delta_8 \vdash F_{10}}{\bullet h_7 : \Delta_8, !F_9 \vdash F_{10}} W}{\frac{\bullet h_1 : \Delta_2, \Delta_3, F_4 \text{-o} F_5 \vdash !F_9}{\text{-} : (\Delta_2, \Delta_3, F_4 \text{-o} F_5), \Delta_8 \vdash F_{10}} \text{Cut}}{\rightarrow} \text{ax/W} \\
\frac{\text{-} : \Delta_2, \Delta_3, \Delta_8, F_4 \text{-o} F_5 \vdash F_{10}}{}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash F_7 \quad \text{-oL} \quad \frac{h_8 : \Delta_{11}, F_7, !F_9, !F_9 \vdash F_{10}}{\bullet h_8 : (\Delta_{11}, !F_9), F_7 \vdash F_{10}} C}{\frac{\bullet h_1 : \Delta_2, \Delta_3, F_4 \text{-o} F_5 \vdash F_7}{\text{-} : (\Delta_2, \Delta_3, F_4 \text{-o} F_5), \Delta_{11}, !F_9 \vdash F_{10}} \text{Cut}}{\rightarrow} \text{ax/W} \\
\frac{\bullet h_1 : \Delta_2, \Delta_3, F_4 \text{-o} F_5 \vdash F_7 \quad \text{ax/W} \quad h_8 : \Delta_{11}, F_7, !F_9, !F_9 \vdash F_{10} \quad \text{ax/W}}{\text{-} : \Delta_{11}, \Delta_2, \Delta_3, !F_9, !F_9, F_4 \text{-o} F_5 \vdash F_{10}} \text{hCut} \\
\frac{\text{-} : \Delta_{11}, \Delta_2, \Delta_3, !F_9, F_4 \text{-o} F_5 \vdash F_{10}}{\text{-} : \Delta_{11}, \Delta_2, \Delta_3, !F_9, F_4 \text{-o} F_5 \vdash F_{10}} C
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash !F_9 \quad \text{-o}_L \quad \frac{h_7 : \Delta_8, !F_9, !F_9 \vdash F_{10}}{\bullet h_7 : \Delta_8, !F_9 \vdash F_{10}} \text{Cut}}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \text{-o} F_5 \vdash !F_9} \text{Cut} \\
\frac{}{- : (\Delta_2, \Delta_3, F_4 \text{-o} F_5), \Delta_8 \vdash F_{10}}{\rightarrow} \\
\frac{\bullet h_1 : \Delta_2, \Delta_3, F_4 \text{-o} F_5 \vdash !F_9 \quad \text{ax/W} \quad \frac{h_7 : \Delta_8, !F_9, !F_9 \vdash F_{10}}{\bullet h_7 : \Delta_8, !F_9 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_3, \Delta_8, F_4 \text{-o} F_5 \vdash F_{10}} \text{mCut}
\end{array}$$

- Case rule !L

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash F_7 \quad \text{-o}_L \quad \frac{h_8 : \Delta_{11}, F_7, F_9 \vdash F_{10}}{\bullet h_8 : (\Delta_{11}, !F_9), F_7 \vdash F_{10}} \text{!L}}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \text{-o} F_5 \vdash F_7} \text{Cut} \\
\frac{}{- : (\Delta_2, \Delta_3, F_4 \text{-o} F_5), \Delta_{11}, !F_9 \vdash F_{10}}{\rightarrow} \\
\frac{\bullet h_1 : \Delta_2, \Delta_3, F_4 \text{-o} F_5 \vdash F_7 \quad \text{ax/W} \quad \frac{h_8 : \Delta_{11}, F_7, F_9 \vdash F_{10}}{\bullet h_8 : (\Delta_{11}, !F_9), F_7 \vdash F_{10}} \text{ax/W}}{- : \Delta_{11}, \Delta_2, \Delta_3, F_9, F_4 \text{-o} F_5 \vdash F_{10}} \text{hCut} \\
\frac{}{- : \Delta_{11}, \Delta_2, \Delta_3, !F_9, F_4 \text{-o} F_5 \vdash F_{10}}{\text{!L}} \\
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash !F_9 \quad \text{-o}_L \quad \frac{h_7 : \Delta_8, F_9 \vdash F_{10}}{\bullet h_7 : \Delta_8, !F_9 \vdash F_{10}} \text{!L}}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \text{-o} F_5 \vdash !F_9} \text{Cut} \\
\frac{}{- : (\Delta_2, \Delta_3, F_4 \text{-o} F_5), \Delta_8 \vdash F_{10}}{\rightarrow} \\
\frac{- : \Delta_2 \vdash F_4 \quad \text{ax/W} \quad \frac{h_1 : \Delta_3, F_5 \vdash !F_9 \quad \bullet h_7 : \Delta_8, !F_9 \vdash F_{10}}{- : \Delta_3, \Delta_8, F_5 \vdash F_{10}} \text{ax/W}}{- : \Delta_2, \Delta_3, \Delta_8, F_4 \text{-o} F_5 \vdash F_{10}} \text{hCut} \\
\frac{}{- : \Delta_2, \Delta_3, \Delta_8, F_4 \text{-o} F_5 \vdash F_{10}}{\text{-o}_L}
\end{array}$$

- Case rule &L2

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash F_7 \quad \text{-o}_L \quad \frac{h_8 : \Delta_{12}, F_7, F_{10} \vdash F_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \& F_{10}), F_7 \vdash F_{11}} \text{&L2}}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \text{-o} F_5 \vdash F_7} \text{Cut} \\
\frac{}{- : (\Delta_2, \Delta_3, F_4 \text{-o} F_5), \Delta_{12}, F_9 \& F_{10} \vdash F_{11}}{\rightarrow} \\
\frac{\bullet h_1 : \Delta_2, \Delta_3, F_4 \text{-o} F_5 \vdash F_7 \quad \text{ax/W} \quad \frac{h_8 : \Delta_{12}, F_{10}, F_7 \vdash F_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \& F_{10}), F_7 \vdash F_{11}} \text{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_3, F_{10}, F_4 \text{-o} F_5 \vdash F_{11}} \text{hCut} \\
\frac{}{- : \Delta_{12}, \Delta_2, \Delta_3, F_9 \& F_{10}, F_4 \text{-o} F_5 \vdash F_{11}}{\text{&L2}} \\
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash F_9 \& F_{10} \quad \text{-o}_L \quad \frac{h_7 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_7 : \Delta_8, F_9 \& F_{10} \vdash F_{11}} \text{&L2}}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \text{-o} F_5 \vdash F_9 \& F_{10}} \text{Cut} \\
\frac{}{- : (\Delta_2, \Delta_3, F_4 \text{-o} F_5), \Delta_8 \vdash F_{11}}{\rightarrow} \\
\frac{- : \Delta_2 \vdash F_4 \quad \text{ax/W} \quad \frac{h_1 : \Delta_3, F_5 \vdash F_9 \& F_{10} \quad \bullet h_7 : \Delta_8, F_9 \& F_{10} \vdash F_{11}}{- : \Delta_3, \Delta_8, F_5 \vdash F_{11}} \text{ax/W}}{- : \Delta_2, \Delta_3, \Delta_8, F_4 \text{-o} F_5 \vdash F_{11}} \text{hCut} \\
\frac{}{- : \Delta_2, \Delta_3, \Delta_8, F_4 \text{-o} F_5 \vdash F_{11}}{\text{-o}_L}
\end{array}$$

- Case rule &L1

$$\begin{array}{c}
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash F_7 \quad \text{-o}_L \quad \frac{h_8 : \Delta_{12}, F_7, F_9 \vdash F_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \& F_{10}), F_7 \vdash F_{11}} \text{&L1}}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \text{-o} F_5 \vdash F_7} \text{Cut} \\
\frac{}{- : (\Delta_2, \Delta_3, F_4 \text{-o} F_5), \Delta_{12}, F_9 \& F_{10} \vdash F_{11}}{\rightarrow} \\
\frac{\bullet h_1 : \Delta_2, \Delta_3, F_4 \text{-o} F_5 \vdash F_7 \quad \text{ax/W} \quad \frac{h_8 : \Delta_{12}, F_7, F_9 \vdash F_{11}}{\bullet h_8 : (\Delta_{12}, F_9 \& F_{10}), F_7 \vdash F_{11}} \text{ax/W}}{- : \Delta_{12}, \Delta_2, \Delta_3, F_9, F_4 \text{-o} F_5 \vdash F_{11}} \text{hCut} \\
\frac{}{- : \Delta_{12}, \Delta_2, \Delta_3, F_9 \& F_{10}, F_4 \text{-o} F_5 \vdash F_{11}}{\text{&L1}} \\
\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash F_9 \& F_{10} \quad \text{-o}_L \quad \frac{h_7 : \Delta_8, F_9 \vdash F_{11}}{\bullet h_7 : \Delta_8, F_9 \& F_{10} \vdash F_{11}} \text{&L1}}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \text{-o} F_5 \vdash F_9 \& F_{10}} \text{Cut} \\
\frac{}{- : (\Delta_2, \Delta_3, F_4 \text{-o} F_5), \Delta_8 \vdash F_{11}}{\rightarrow} \\
\frac{- : \Delta_2 \vdash F_4 \quad \text{ax/W} \quad \frac{h_1 : \Delta_3, F_5 \vdash F_9 \& F_{10} \quad \bullet h_7 : \Delta_8, F_9 \& F_{10} \vdash F_{11}}{- : \Delta_3, \Delta_8, F_5 \vdash F_{11}} \text{ax/W}}{- : \Delta_2, \Delta_3, \Delta_8, F_4 \text{-o} F_5 \vdash F_{11}} \text{hCut} \\
\frac{}{- : \Delta_2, \Delta_3, \Delta_8, F_4 \text{-o} F_5 \vdash F_{11}}{\text{-o}_L}
\end{array}$$

- Case rule I

$$\frac{\frac{\frac{h_1 : \Delta_2 \vdash F_4 \quad h_1 : \Delta_3, F_5 \vdash p(n_8)}{\bullet h_1 : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash p(n_8)} \multimap_L \quad \frac{}{\bullet h_7 : *, p(n_8) \vdash p(n_8)} I}{\frac{}{- : (\Delta_2, \Delta_3, F_4 \multimap F_5), * \vdash p(n_8)} \text{Cut}} \rightarrow}{\frac{}{- : \Delta_2, \Delta_3, F_4 \multimap F_5 \vdash p(n_8)} \text{ax/W}}$$

5.18 Status of I : OK

- Case rule $!R$

- Case rule 1_R

- Case rule \top

$$\frac{\frac{}{\bullet h_1 : p(n_4) \vdash p(n_4)} I \quad \frac{}{\bullet h_5 : \Delta_3, p(n_4) \vdash \top} \top}{\frac{}{- : p(n_4), \Delta_3 \vdash \top} \text{Cut}} \rightarrow}{\frac{}{- : \Delta_3, p(n_4) \vdash \top} \top}$$

- Case rule $\&_R$

$$\frac{\frac{}{\bullet h_1 : p(n_4) \vdash p(n_4)} I \quad \frac{\frac{h_5 : \Delta_3, p(n_4) \vdash F_6 \quad h_5 : \Delta_3, p(n_4) \vdash F_7}{\bullet h_5 : \Delta_3, p(n_4) \vdash F_6 \& F_7} \&_R}{\frac{}{- : p(n_4), \Delta_3 \vdash F_6 \& F_7} \text{Cut}} \rightarrow}{\frac{}{- : \Delta_3, p(n_4) \vdash F_6 \& F_7} \text{ax/W}}$$

- Case rule \multimap_R

$$\frac{\frac{}{\bullet h_1 : p(n_4) \vdash p(n_4)} I \quad \frac{\frac{h_5 : \Delta_3, F_6, p(n_4) \vdash F_7}{\bullet h_5 : \Delta_3, p(n_4) \vdash F_6 \multimap F_7} \multimap_R}{\frac{}{- : p(n_4), \Delta_3 \vdash F_6 \multimap F_7} \text{Cut}} \rightarrow}{\frac{}{- : \Delta_3, p(n_4) \vdash F_6 \multimap F_7} \text{ax/W}}$$

- Case rule \oplus_{R_2}

$$\frac{\frac{}{\bullet h_1 : p(n_4) \vdash p(n_4)} I \quad \frac{\frac{h_5 : \Delta_3, p(n_4) \vdash F_7}{\bullet h_5 : \Delta_3, p(n_4) \vdash F_6 \oplus F_7} \oplus_{R_2}}{\frac{}{- : p(n_4), \Delta_3 \vdash F_6 \oplus F_7} \text{Cut}} \rightarrow}{\frac{}{- : \Delta_3, p(n_4) \vdash F_6 \oplus F_7} \text{ax/W}}$$

- Case rule \oplus_{R_1}

$$\frac{\frac{}{\bullet h_1 : p(n_4) \vdash p(n_4)} I \quad \frac{\frac{h_5 : \Delta_3, p(n_4) \vdash F_6}{\bullet h_5 : \Delta_3, p(n_4) \vdash F_6 \oplus F_7} \oplus_{R_1}}{\frac{}{- : p(n_4), \Delta_3 \vdash F_6 \oplus F_7} \text{Cut}} \rightarrow}{\frac{}{- : \Delta_3, p(n_4) \vdash F_6 \oplus F_7} \text{ax/W}}$$

- Case rule $\mathbf{1}_L$

$$\frac{\frac{\frac{}{\bullet \mathbf{h}_1 : p(\mathbf{n}_3) \vdash p(\mathbf{n}_3)} I \quad \frac{\mathbf{h}_4 : \Delta_6, p(\mathbf{n}_3) \vdash F_5}{\bullet \mathbf{h}_4 : (\mathbf{1}, \Delta_6), p(\mathbf{n}_3) \vdash F_5} \mathbf{1}_L}{- : p(\mathbf{n}_3), \mathbf{1}, \Delta_6 \vdash F_5} \text{Cut}}{\rightarrow} \quad \frac{}{\bullet \mathbf{h}_4 : (\mathbf{1}, \Delta_6), p(\mathbf{n}_3) \vdash F_5} \text{ax/W}}{- : \mathbf{1}, \Delta_6, p(\mathbf{n}_3) \vdash F_5}$$

- Case rule \otimes_R

$$\frac{\frac{\frac{}{\bullet \mathbf{h}_1 : p(\mathbf{n}_3) \vdash p(\mathbf{n}_3)} I \quad \frac{\mathbf{h}_4 : \Delta_8, p(\mathbf{n}_3) \vdash F_6 \quad \mathbf{h}_4 : \Delta_5 \vdash F_7}{\bullet \mathbf{h}_4 : (\Delta_5, \Delta_8), p(\mathbf{n}_3) \vdash F_6 \otimes F_7} \otimes_R}{- : p(\mathbf{n}_3), \Delta_5, \Delta_8 \vdash F_6 \otimes F_7} \text{Cut}}{\rightarrow} \quad \frac{}{\bullet \mathbf{h}_4 : (\Delta_5, \Delta_8), p(\mathbf{n}_3) \vdash F_6 \otimes F_7} \text{ax/W}}{- : \Delta_5, \Delta_8, p(\mathbf{n}_3) \vdash F_6 \otimes F_7}$$

$$\frac{\frac{\frac{}{\bullet \mathbf{h}_1 : p(\mathbf{n}_3) \vdash p(\mathbf{n}_3)} I \quad \frac{\mathbf{h}_4 : \Delta_5 \vdash F_6 \quad \mathbf{h}_4 : \Delta_8, p(\mathbf{n}_3) \vdash F_7}{\bullet \mathbf{h}_4 : (\Delta_5, \Delta_8), p(\mathbf{n}_3) \vdash F_6 \otimes F_7} \otimes_R}{- : p(\mathbf{n}_3), \Delta_5, \Delta_8 \vdash F_6 \otimes F_7} \text{Cut}}{\rightarrow} \quad \frac{}{\bullet \mathbf{h}_4 : (\Delta_5, \Delta_8), p(\mathbf{n}_3) \vdash F_6 \otimes F_7} \text{ax/W}}{- : \Delta_5, \Delta_8, p(\mathbf{n}_3) \vdash F_6 \otimes F_7}$$

- Case rule W

$$\frac{\frac{\frac{}{\bullet \mathbf{h}_1 : p(\mathbf{n}_3) \vdash p(\mathbf{n}_3)} I \quad \frac{\mathbf{h}_4 : \Delta_7, p(\mathbf{n}_3) \vdash F_6}{\bullet \mathbf{h}_4 : (\Delta_7, !F_5), p(\mathbf{n}_3) \vdash F_6} W}{- : p(\mathbf{n}_3), \Delta_7, !F_5 \vdash F_6} \text{Cut}}{\rightarrow} \quad \frac{}{\bullet \mathbf{h}_4 : (\Delta_7, !F_5), p(\mathbf{n}_3) \vdash F_6} \text{ax/W}}{- : \Delta_7, !F_5, p(\mathbf{n}_3) \vdash F_6}$$

- Case rule C

$$\frac{\frac{\frac{}{\bullet \mathbf{h}_1 : p(\mathbf{n}_3) \vdash p(\mathbf{n}_3)} I \quad \frac{\mathbf{h}_4 : \Delta_7, !F_5, !F_5, p(\mathbf{n}_3) \vdash F_6}{\bullet \mathbf{h}_4 : (\Delta_7, !F_5), p(\mathbf{n}_3) \vdash F_6} C}{- : p(\mathbf{n}_3), \Delta_7, !F_5 \vdash F_6} \text{Cut}}{\rightarrow} \quad \frac{}{\bullet \mathbf{h}_4 : (\Delta_7, !F_5), p(\mathbf{n}_3) \vdash F_6} \text{ax/W}}{- : \Delta_7, !F_5, p(\mathbf{n}_3) \vdash F_6}$$

- Case rule $!L$

$$\frac{\frac{\frac{}{\bullet \mathbf{h}_1 : p(\mathbf{n}_3) \vdash p(\mathbf{n}_3)} I \quad \frac{\mathbf{h}_4 : \Delta_7, F_5, p(\mathbf{n}_3) \vdash F_6}{\bullet \mathbf{h}_4 : (\Delta_7, !F_5), p(\mathbf{n}_3) \vdash F_6} !L}{- : p(\mathbf{n}_3), \Delta_7, !F_5 \vdash F_6} \text{Cut}}{\rightarrow} \quad \frac{}{\bullet \mathbf{h}_4 : (\Delta_7, !F_5), p(\mathbf{n}_3) \vdash F_6} \text{ax/W}}{- : \Delta_7, !F_5, p(\mathbf{n}_3) \vdash F_6}$$

- Case rule $\&_{L2}$

$$\frac{\frac{\frac{}{\bullet \mathbf{h}_1 : p(\mathbf{n}_3) \vdash p(\mathbf{n}_3)} I \quad \frac{\mathbf{h}_4 : \Delta_8, F_6, p(\mathbf{n}_3) \vdash F_7}{\bullet \mathbf{h}_4 : (\Delta_8, F_5 \& F_6), p(\mathbf{n}_3) \vdash F_7} \&_{L2}}{- : p(\mathbf{n}_3), \Delta_8, F_5 \& F_6 \vdash F_7} \text{Cut}}{\rightarrow} \quad \frac{}{\bullet \mathbf{h}_4 : (\Delta_8, F_5 \& F_6), p(\mathbf{n}_3) \vdash F_7} \text{ax/W}}{- : \Delta_8, p(\mathbf{n}_3), F_5 \& F_6 \vdash F_7}$$

- Case rule $\&_{L1}$

$$\frac{\frac{\frac{}{\bullet h_1 : p(\mathbf{n}_3) \vdash p(\mathbf{n}_3)}{I} \quad \frac{h_4 : \Delta_8, F_5, p(\mathbf{n}_3) \vdash F_7}{\bullet h_4 : (\Delta_8, F_5 \& F_6), p(\mathbf{n}_3) \vdash F_7}}{\text{Cut}}}{\rightarrow} \&_{L1} \quad \frac{}{- : p(\mathbf{n}_3), \Delta_8, F_5 \& F_6 \vdash F_7}}{\text{Cut}}}{\rightarrow} \text{ax/w} \quad \frac{}{- : \Delta_8, p(\mathbf{n}_3), F_5 \& F_6 \vdash F_7}$$

- Case rule \otimes_L

$$\frac{\frac{\frac{}{\bullet h_1 : p(\mathbf{n}_3) \vdash p(\mathbf{n}_3)}{I} \quad \frac{h_4 : \Delta_8, F_5, F_6, p(\mathbf{n}_3) \vdash F_7}{\bullet h_4 : (\Delta_8, F_5 \otimes F_6), p(\mathbf{n}_3) \vdash F_7}}{\text{Cut}}}{\rightarrow} \otimes_L \quad \frac{}{- : p(\mathbf{n}_3), \Delta_8, F_5 \otimes F_6 \vdash F_7}}{\text{Cut}}}{\rightarrow} \text{ax/w} \quad \frac{}{- : \Delta_8, p(\mathbf{n}_3), F_5 \otimes F_6 \vdash F_7}$$

- Case rule \oplus_L

$$\frac{\frac{\frac{}{\bullet h_1 : p(\mathbf{n}_3) \vdash p(\mathbf{n}_3)}{I} \quad \frac{h_4 : \Delta_8, F_5, p(\mathbf{n}_3) \vdash F_7 \quad h_4 : \Delta_8, F_6, p(\mathbf{n}_3) \vdash F_7}{\bullet h_4 : (\Delta_8, F_5 \oplus F_6), p(\mathbf{n}_3) \vdash F_7}}{\text{Cut}}}{\rightarrow} \oplus_L \quad \frac{}{- : p(\mathbf{n}_3), \Delta_8, F_5 \oplus F_6 \vdash F_7}}{\text{Cut}}}{\rightarrow} \text{ax/w} \quad \frac{}{- : \Delta_8, p(\mathbf{n}_3), F_5 \oplus F_6 \vdash F_7}$$

- Case rule \multimap_L

$$\frac{\frac{\frac{}{\bullet h_1 : p(\mathbf{n}_3) \vdash p(\mathbf{n}_3)}{I} \quad \frac{h_4 : \Delta_9, p(\mathbf{n}_3) \vdash F_6 \quad h_4 : \Delta_5, F_7 \vdash F_8}{\bullet h_4 : (\Delta_5, \Delta_9, F_6 \multimap F_7), p(\mathbf{n}_3) \vdash F_8}}{\text{Cut}}}{\rightarrow} \multimap_L \quad \frac{}{- : p(\mathbf{n}_3), \Delta_5, \Delta_9, F_6 \multimap F_7 \vdash F_8}}{\text{Cut}}}{\rightarrow} \text{ax/w} \quad \frac{}{- : \Delta_5, \Delta_9, p(\mathbf{n}_3), F_6 \multimap F_7 \vdash F_8}$$

$$\frac{\frac{\frac{}{\bullet h_1 : p(\mathbf{n}_3) \vdash p(\mathbf{n}_3)}{I} \quad \frac{h_4 : \Delta_5 \vdash F_6 \quad h_4 : \Delta_9, F_7, p(\mathbf{n}_3) \vdash F_8}{\bullet h_4 : (\Delta_5, \Delta_9, F_6 \multimap F_7), p(\mathbf{n}_3) \vdash F_8}}{\text{Cut}}}{\rightarrow} \multimap_L \quad \frac{}{- : p(\mathbf{n}_3), \Delta_5, \Delta_9, F_6 \multimap F_7 \vdash F_8}}{\text{Cut}}}{\rightarrow} \text{ax/w} \quad \frac{}{- : \Delta_5, \Delta_9, p(\mathbf{n}_3), F_6 \multimap F_7 \vdash F_8}$$

- Case rule I

$$\frac{\frac{\frac{}{\bullet h_1 : p(\mathbf{n}_4) \vdash p(\mathbf{n}_4)}{I} \quad \frac{}{\bullet h_3 : *, p(\mathbf{n}_4) \vdash p(\mathbf{n}_4)}{I}}{\text{Cut}}}{\rightarrow} \quad \frac{}{- : p(\mathbf{n}_4), * \vdash p(\mathbf{n}_4)}}{I} \quad \frac{}{- : p(\mathbf{n}_4) \vdash p(\mathbf{n}_4)}$$

6 Cut-Elimination

6.1 Status of $!R$: OK

- Case rule $!R$

$$\frac{\frac{h_1 : !\Upsilon 2 \vdash F_5}{\bullet h_1 : !\Upsilon 2 \vdash !F_5} !R \quad \frac{h_7 : !\Upsilon 4, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{\bullet h_7 : \text{contract}(sn_6, !F_5), !\Upsilon 4 \vdash !F_8} !R}{- : !\Upsilon 2, !\Upsilon 4 \vdash !F_8} \text{Cut}$$

$$\xrightarrow{\text{ax}} \frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_5}{- : !\Upsilon 2, !\Upsilon 4 \vdash F_8} \text{ax} \quad \frac{h_7 : !\Upsilon 4, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{- : !\Upsilon 2, !\Upsilon 4 \vdash !F_8} \text{ax}}{- : !\Upsilon 2, !\Upsilon 4 \vdash !F_8} \text{hCut} !R$$

- Case rule 1_R

- Case rule \top

$$\frac{\frac{h_1 : !\Upsilon 2 \vdash F_5}{\bullet h_1 : !\Upsilon 2 \vdash !F_5} !R \quad \frac{}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_4 \vdash \top} \top}{- : !\Upsilon 2, \Delta_4 \vdash \top} \text{Cut}$$

$$\xrightarrow{\top} \frac{}{- : !\Upsilon 2, \Delta_4 \vdash \top} \top$$

- Case rule $\&_R$

$$\frac{\frac{h_1 : !\Upsilon 2 \vdash F_5}{\bullet h_1 : !\Upsilon 2 \vdash !F_5} !R \quad \frac{h_7 : \Delta_4, !F_5, \text{contract}(n_6, !F_5) \vdash F_8 \quad h_7 : \Delta_4, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_4 \vdash F_8 \& F_9} \text{Cut}}{- : !\Upsilon 2, \Delta_4 \vdash F_8 \& F_9} \&_R$$

$$\xrightarrow{\text{ax}} \frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_5}{- : !\Upsilon 2, \Delta_4 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_4, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{- : !\Upsilon 2, \Delta_4 \vdash F_8} \text{ax} \quad \frac{\bullet h_1 : !\Upsilon 2 \vdash !F_5}{- : !\Upsilon 2, \Delta_4 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_4, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 2, \Delta_4 \vdash F_9} \text{ax}}{- : !\Upsilon 2, \Delta_4 \vdash F_8 \& F_9} \text{hCut} \&_R$$

- Case rule \multimap_R

$$\frac{\frac{h_1 : !\Upsilon 2 \vdash F_5}{\bullet h_1 : !\Upsilon 2 \vdash !F_5} !R \quad \frac{h_7 : \Delta_4, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_4 \vdash F_8 \multimap F_9} \multimap_R}{- : !\Upsilon 2, \Delta_4 \vdash F_8 \multimap F_9} \text{Cut}$$

$$\xrightarrow{\text{ax}} \frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_5}{- : !\Upsilon 2, \Delta_4 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_4, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 2, \Delta_4 \vdash F_8 \multimap F_9} \text{ax}}{- : !\Upsilon 2, \Delta_4 \vdash F_8 \multimap F_9} \text{hCut} \multimap_R$$

- Case rule \oplus_{R_2}

$$\frac{\frac{h_1 : !\Upsilon 2 \vdash F_5}{\bullet h_1 : !\Upsilon 2 \vdash !F_5} !R \quad \frac{h_7 : \Delta_4, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_4 \vdash F_8 \oplus F_9} \oplus_{R_2}}{- : !\Upsilon 2, \Delta_4 \vdash F_8 \oplus F_9} \text{Cut}$$

$$\xrightarrow{\text{ax}} \frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_5}{- : !\Upsilon 2, \Delta_4 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_4, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 2, \Delta_4 \vdash F_8 \oplus F_9} \text{ax}}{- : !\Upsilon 2, \Delta_4 \vdash F_8 \oplus F_9} \text{hCut} \oplus_{R_2}$$

- Case rule \oplus_{R_1}

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_5}{\bullet h_1 : !\Upsilon 2 \vdash !F_5} !R \quad \frac{h_7 : \Delta_4, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_4 \vdash F_8 \oplus F_9} \oplus_{R_1} \\
\hline
- : !\Upsilon 2, \Delta_4 \vdash F_8 \oplus F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_5}{- : !\Upsilon 2 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_4, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{- : !\Upsilon 2, \Delta_4 \vdash F_8} \text{ax}}{- : !\Upsilon 2, \Delta_4 \vdash F_8 \oplus F_9} \oplus_{R_1} \quad \text{hCut}
\end{array}$$

- Case rule 1_L

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_6 : \Delta_8, !F_4, \text{contract}(n_5, !F_4) \vdash F_7}{\bullet h_6 : \text{contract}(sn_5, !F_4), 1, \Delta_8 \vdash F_7} 1_L \\
\hline
- : !\Upsilon 2, 1, \Delta_8 \vdash F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2 \vdash !F_4} \text{ax} \quad \frac{h_6 : \Delta_8, !F_4, \text{contract}(n_5, !F_4) \vdash F_7}{- : !\Upsilon 2, \Delta_8 \vdash F_7} \text{ax}}{- : 1, !\Upsilon 2, \Delta_8 \vdash F_7} 1_L \quad \text{hCut}
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_6 : \Delta_{10}, !F_4, \text{contract}(n_5, !F_4) \vdash F_8 \quad h_6 : \Delta_7 \vdash F_9}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_7, \Delta_{10} \vdash F_8 \otimes F_9} \otimes_R \\
\hline
- : !\Upsilon 2, \Delta_7, \Delta_{10} \vdash F_8 \otimes F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2 \vdash !F_4} \text{ax} \quad \frac{h_6 : \Delta_{10}, !F_4, \text{contract}(n_5, !F_4) \vdash F_8}{- : !\Upsilon 2, \Delta_{10} \vdash F_8} \text{ax}}{- : !\Upsilon 2, \Delta_{10}, \Delta_7 \vdash F_8 \otimes F_9} \otimes_R \quad \frac{- : \Delta_7 \vdash F_9}{- : \Delta_7 \vdash F_9} \text{ax} \\
\frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_6 : \Delta_9, \text{contract}(n_5, !F_4) \vdash F_7 \quad h_6 : \Delta_{10}, !F_4 \vdash F_8}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_9, \Delta_{10} \vdash F_7 \otimes F_8} \otimes_R}{- : !\Upsilon 2, \Delta_9, \Delta_{10} \vdash F_7 \otimes F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2 \vdash !F_4} \text{ax} \quad \frac{h_6 : \Delta_9, \text{contract}(n_5, !F_4) \vdash F_7}{- : !\Upsilon 2, \Delta_9 \vdash F_7} \text{ax} \quad \frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2 \vdash !F_4} \text{ax} \quad \frac{h_6 : \Delta_{10}, !F_4 \vdash F_8}{- : !\Upsilon 2, \Delta_{10} \vdash F_8} \text{ax}}{- : !\Upsilon 2, \Delta_{10}, \Delta_9 \vdash F_7 \otimes F_8} \otimes_R \quad \text{hCut} \\
\frac{- : !\Upsilon 2, \Delta_{10}, \Delta_9 \vdash F_7 \otimes F_8}{- : !\Upsilon 2, \Delta_{10}, \Delta_9 \vdash F_7 \otimes F_8} C \\
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_6 : \Delta_9, !F_4 \vdash F_7 \quad h_6 : \Delta_{10}, \text{contract}(n_5, !F_4) \vdash F_8}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_9, \Delta_{10} \vdash F_7 \otimes F_8} \otimes_R \\
\hline
- : !\Upsilon 2, \Delta_9, \Delta_{10} \vdash F_7 \otimes F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2 \vdash !F_4} \text{ax} \quad \frac{h_6 : \Delta_9, !F_4 \vdash F_7}{- : !\Upsilon 2, \Delta_9 \vdash F_7} \text{ax} \quad \frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2 \vdash !F_4} \text{ax} \quad \frac{h_6 : \Delta_{10}, \text{contract}(n_5, !F_4) \vdash F_8}{- : !\Upsilon 2, \Delta_{10} \vdash F_8} \text{ax}}{- : !\Upsilon 2, \Delta_{10}, \Delta_9 \vdash F_7 \otimes F_8} \otimes_R \quad \text{hCut} \\
\frac{- : !\Upsilon 2, \Delta_{10}, \Delta_9 \vdash F_7 \otimes F_8}{- : !\Upsilon 2, \Delta_{10}, \Delta_9 \vdash F_7 \otimes F_8} C \\
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_6 : \Delta_7 \vdash F_8 \quad h_6 : \Delta_{10}, !F_4, \text{contract}(n_5, !F_4) \vdash F_9}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_7, \Delta_{10} \vdash F_8 \otimes F_9} \otimes_R \\
\hline
- : !\Upsilon 2, \Delta_7, \Delta_{10} \vdash F_8 \otimes F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{- : \Delta_7 \vdash F_8}{- : \Delta_7 \vdash F_8} \text{ax} \quad \frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2 \vdash !F_4} \text{ax} \quad \frac{h_6 : \Delta_{10}, !F_4, \text{contract}(n_5, !F_4) \vdash F_9}{- : !\Upsilon 2, \Delta_{10} \vdash F_9} \text{ax}}{- : !\Upsilon 2, \Delta_{10}, \Delta_7 \vdash F_8 \otimes F_9} \otimes_R \quad \text{hCut}
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_6 : \Delta_9, !F_4, \text{contract}(n_5, !F_4) \vdash F_8}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_9, !F_7 \vdash F_8} W}{- : !\Upsilon 2, \Delta_9, !F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2, \Delta_9 \vdash F_8} \text{ax} \quad \frac{h_6 : \Delta_9, !F_4, \text{contract}(n_5, !F_4) \vdash F_8}{- : !\Upsilon 2, \Delta_9 \vdash F_8} \text{ax}}{- : !\Upsilon 2, \Delta_9, !F_7 \vdash F_8} \text{hCut} \\
\frac{}{- : !\Upsilon 2, \Delta_9, !F_7 \vdash F_8} W
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_7}{\bullet h_1 : !\Upsilon 2 \vdash !F_7} !R \quad \frac{h_6 : \Delta_4, \text{contract}(n_5, !F_7) \vdash F_8}{\bullet h_6 : \text{contract}(sn_5, !F_7), \Delta_4 \vdash F_8} W}{- : !\Upsilon 2, \Delta_4 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_7}{- : !\Upsilon 2, \Delta_4 \vdash F_8} \text{ax} \quad \frac{h_6 : \Delta_4, \text{contract}(n_5, !F_7) \vdash F_8}{- : !\Upsilon 2, \Delta_4 \vdash F_8} \text{ax}}{- : !\Upsilon 2, \Delta_4 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_6 : \Delta_9, !F_4, !F_7, !F_7, \text{contract}(n_5, !F_4) \vdash F_8}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_9, !F_7 \vdash F_8} C}{- : !\Upsilon 2, \Delta_9, !F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2, \Delta_9, !F_7, !F_7 \vdash F_8} \text{ax} \quad \frac{h_6 : \Delta_9, !F_4, !F_7, !F_7, \text{contract}(n_5, !F_4) \vdash F_8}{- : !\Upsilon 2, \Delta_9, !F_7 \vdash F_8} C}{- : !\Upsilon 2, \Delta_9, !F_7 \vdash F_8} \text{hCut} \\
\frac{}{- : !\Upsilon 2, \Delta_9, !F_7 \vdash F_8} C
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_7}{\bullet h_1 : !\Upsilon 2 \vdash !F_7} !R \quad \frac{h_6 : \Delta_4, !F_7, !F_7, \text{contract}(n_5, !F_7) \vdash F_8}{\bullet h_6 : \text{contract}(sn_5, !F_7), \Delta_4 \vdash F_8} C}{- : !\Upsilon 2, \Delta_4 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_7}{- : !\Upsilon 2, \Delta_4 \vdash F_8} \text{ax} \quad \frac{h_6 : \Delta_4, !F_7, !F_7, \text{contract}(n_5, !F_7) \vdash F_8}{- : !\Upsilon 2, \Delta_4 \vdash F_8} \text{ax}}{- : !\Upsilon 2, \Delta_4 \vdash F_8} \text{hCut}
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_6 : \Delta_9, F_7, !F_4, \text{contract}(n_5, !F_4) \vdash F_8}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_9, !F_7 \vdash F_8} !L}{- : !\Upsilon 2, \Delta_9, !F_7 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2, \Delta_9, F_7 \vdash F_8} \text{ax} \quad \frac{h_6 : \Delta_9, F_7, !F_4, \text{contract}(n_5, !F_4) \vdash F_8}{- : !\Upsilon 2, \Delta_9, !F_7 \vdash F_8} !L}{- : !\Upsilon 2, \Delta_9, !F_7 \vdash F_8} \text{hCut} \\
\frac{}{- : !\Upsilon 2, \Delta_9, !F_7 \vdash F_8} !L
\end{array}$$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_7}{\bullet h_1 : !\Upsilon 2 \vdash !F_7} !R \quad \frac{h_6 : \Delta_4, F_7, \text{contract}(n_5, !F_7) \vdash F_8}{\bullet h_6 : \text{contract}(sn_5, !F_7), \Delta_4 \vdash F_8} !L}{- : !\Upsilon 2, \Delta_4 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_7}{- : !\Upsilon 2, \Delta_4, F_7 \vdash F_8} \text{ax} \quad \frac{h_6 : \Delta_4, F_7, \text{contract}(n_5, !F_7) \vdash F_8}{- : !\Upsilon 2, \Delta_4, F_7 \vdash F_8} !L}{- : !\Upsilon 2, \Delta_4, F_7 \vdash F_8} \text{hCut} \\
\frac{}{- : !\Upsilon 2, \Delta_4, F_7 \vdash F_8} \text{Cut}
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} !R \quad \frac{h_6 : \Delta_{10}, F_8, !F_4, \text{contract}(n_5, !F_4) \vdash F_9}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_{10}, F_7 \& F_8 \vdash F_9} \&_{L2}}{- : !\Upsilon 2, \Delta_{10}, F_7 \& F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : !\Upsilon 2, \Delta_{10}, F_8 \vdash F_9} \text{ax} \quad \frac{h_6 : \Delta_{10}, F_8, !F_4, \text{contract}(n_5, !F_4) \vdash F_9}{- : !\Upsilon 2, \Delta_{10}, F_7 \& F_8 \vdash F_9} \&_{L2}}{- : !\Upsilon 2, \Delta_{10}, F_7 \& F_8 \vdash F_9} \text{hCut} \\
\frac{}{- : !\Upsilon 2, \Delta_{10}, F_7 \& F_8 \vdash F_9} \&_{L2}
\end{array}$$

$$\frac{\frac{\frac{h_1 : !\Upsilon 2 \vdash F_4}{\bullet h_1 : !\Upsilon 2 \vdash !F_4} \text{!R} \quad \frac{h_6 : \Delta_7 \vdash F_8 \quad h_6 : \Delta_{11}, F_9, !F_4, \text{contract}(n_5, !F_4) \vdash F_{10}}{\bullet h_6 : \text{contract}(sn_5, !F_4), \Delta_7, \Delta_{11}, F_8 \multimap F_9 \vdash F_{10}} \multimap_L}{- : !\Upsilon 2, \Delta_7, \Delta_{11}, F_8 \multimap F_9 \vdash F_{10}} \text{Cut}}{\rightarrow} \text{ax} \quad \frac{\frac{\bullet h_1 : !\Upsilon 2 \vdash !F_4}{- : \Delta_7 \vdash F_8} \text{ax} \quad \frac{h_6 : \Delta_{11}, F_9, !F_4, \text{contract}(n_5, !F_4) \vdash F_{10}}{- : !\Upsilon 2, \Delta_{11}, F_9 \vdash F_{10}} \text{ax}}{- : !\Upsilon 2, \Delta_{11}, \Delta_7, F_8 \multimap F_9 \vdash F_{10}} \text{hCut} \multimap_L}$$

- Case rule I

6.2 Status of 1_R : OK

- Case rule $!R$
- Case rule 1_R
- Case rule \top
- Case rule $\&_R$
- Case rule \multimap_R
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule 1_L
- Case rule \otimes_R
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L_2}$
- Case rule $\&_{L_1}$
- Case rule \otimes_L

- Case rule \oplus_L
- Case rule $\neg\circ_L$
- Case rule I

6.3 Status of \top : OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule \top
- Case rule $\&_R$
- Case rule $\neg\circ_R$
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L_2}$
- Case rule $\&_{L_1}$
- Case rule \otimes_L
- Case rule \oplus_L

- Case rule $\neg\circ_L$
- Case rule I

6.4 Status of $\&_R$: OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule \top
- Case rule $\&_R$
- Case rule $\neg\circ_R$
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L_2}$
- Case rule $\&_{L_1}$
- Case rule \otimes_L
- Case rule \oplus_L
- Case rule $\neg\circ_L$
- Case rule I

6.5 Status of $\neg\circ_R$: OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule \top
- Case rule $\&_R$
- Case rule $\neg\circ_R$
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L_2}$
- Case rule $\&_{L_1}$
- Case rule \otimes_L
- Case rule \oplus_L
- Case rule $\neg\circ_L$
- Case rule I

6.6 Status of \oplus_{R_2} : OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule \top
- Case rule $\&_R$
- Case rule $\neg\circ_R$
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L_2}$
- Case rule $\&_{L_1}$
- Case rule \otimes_L
- Case rule \oplus_L
- Case rule $\neg\circ_L$
- Case rule I

6.7 Status of \oplus_{R_1} : OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule \top
- Case rule $\&_R$
- Case rule $\neg\circ_R$
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L_2}$
- Case rule $\&_{L_1}$
- Case rule \otimes_L
- Case rule \oplus_L
- Case rule $\neg\circ_L$
- Case rule I

6.8 Status of 1_L : OK

- Case rule $!R$
- Case rule 1_R
- Case rule \top
- Case rule $\&_R$
- Case rule \multimap_R
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule 1_L
- Case rule \otimes_R
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L_2}$
- Case rule $\&_{L_1}$
- Case rule \otimes_L
- Case rule \oplus_L
- Case rule \multimap_L
- Case rule I

6.9 Status of \otimes_R : OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule \top
- Case rule $\&_R$
- Case rule $\neg\circ_R$
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L_2}$
- Case rule $\&_{L_1}$
- Case rule \otimes_L
- Case rule \oplus_L
- Case rule $\neg\circ_L$
- Case rule I

6.10 Status of W : OK

- Case rule $!R$

$$\frac{\frac{\frac{h_2 : !\Upsilon_3 \vdash !F_6}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_6} W \quad \frac{h_8 : !\Upsilon_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9}{\bullet h_8 : \text{contract}(sn_7, !F_6), !\Upsilon_5 \vdash !F_9} !R}{- : (!\Upsilon_3, !F_4), !\Upsilon_5 \vdash !F_9} \text{Cut}}{\frac{\frac{h_2 : !\Upsilon_3 \vdash !F_6}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_6} \text{ax} \quad \frac{h_8 : !\Upsilon_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9}{\bullet h_8 : \text{contract}(n_7, !F_6) \vdash F_9} \text{ax}}{- : !\Upsilon_3, !\Upsilon_5, !F_4 \vdash F_9} \text{hCut}}{- : !\Upsilon_3, !\Upsilon_5, !F_4 \vdash !F_9} !R$$

- Case rule 1_R

- Case rule \top

$$\frac{\frac{\frac{h_2 : !\Upsilon_3 \vdash !F_6}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_6} W \quad \frac{\bullet h_8 : \text{contract}(sn_7, !F_6), \Delta_5 \vdash \top}{\bullet h_8 : \text{contract}(sn_7, !F_6), \Delta_5 \vdash \top} \top}{- : (!\Upsilon_3, !F_4), \Delta_5 \vdash \top} \text{Cut}}{\frac{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_6}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_6} \text{ax} \quad \frac{\bullet h_8 : \text{contract}(sn_7, !F_6), \Delta_5 \vdash \top}{\bullet h_8 : \text{contract}(sn_7, !F_6), \Delta_5 \vdash \top} \top}{- : !\Upsilon_3, \Delta_5, !F_4 \vdash \top} \top$$

- Case rule $\&_R$

$$\frac{\frac{\frac{h_2 : !\Upsilon_3 \vdash !F_6}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_6} W \quad \frac{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9 \quad h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10}}{\bullet h_8 : \text{contract}(sn_7, !F_6), \Delta_5 \vdash F_9 \& F_{10}} \&_R}{- : (!\Upsilon_3, !F_4), \Delta_5 \vdash F_9 \& F_{10}} \text{Cut}}{\frac{\frac{h_2 : !\Upsilon_3 \vdash !F_6}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_6} \text{ax} \quad \frac{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9 \quad h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10}}{\bullet h_8 : \text{contract}(n_7, !F_6) \vdash F_9 \& F_{10}} \&_R}{- : !\Upsilon_3, \Delta_5, !F_4 \vdash F_9 \& F_{10}} \text{hCut}}{- : !\Upsilon_3, \Delta_5, !F_4 \vdash F_9 \& F_{10}} W$$

- Case rule $\neg\circ_R$

$$\frac{\frac{\frac{h_2 : !\Upsilon_3 \vdash !F_6}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_6} W \quad \frac{h_8 : \Delta_5, F_9, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10}}{\bullet h_8 : \text{contract}(sn_7, !F_6), \Delta_5 \vdash F_9 \neg\circ F_{10}} \neg\circ_R}{- : (!\Upsilon_3, !F_4), \Delta_5 \vdash F_9 \neg\circ F_{10}} \text{Cut}}{\frac{\frac{h_2 : !\Upsilon_3 \vdash !F_6}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_6} \text{ax} \quad \frac{h_8 : \Delta_5, F_9, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10}}{\bullet h_8 : \text{contract}(n_7, !F_6) \vdash F_{10}} \text{ax}}{- : !\Upsilon_3, \Delta_5, F_9, !F_4 \vdash F_{10}} \text{hCut}}{- : !\Upsilon_3, \Delta_5, !F_4 \vdash F_9 \neg\circ F_{10}} \neg\circ_R$$

- Case rule \oplus_{R_2}

$$\frac{\frac{\frac{h_2 : !\Upsilon_3 \vdash !F_6}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_6} W \quad \frac{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10}}{\bullet h_8 : \text{contract}(sn_7, !F_6), \Delta_5 \vdash F_9 \oplus F_{10}} \oplus_{R_2}}{- : (!\Upsilon_3, !F_4), \Delta_5 \vdash F_9 \oplus F_{10}} \text{Cut}}{\frac{\frac{h_2 : !\Upsilon_3 \vdash !F_6}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_6} \text{ax} \quad \frac{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10}}{\bullet h_8 : \text{contract}(n_7, !F_6) \vdash F_{10}} \text{ax}}{- : !\Upsilon_3, \Delta_5, !F_4 \vdash F_{10}} \text{hCut}}{- : !\Upsilon_3, \Delta_5, !F_4 \vdash F_9 \oplus F_{10}} \oplus_{R_2}$$

- Case rule \oplus_{R_1}

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} W \quad \frac{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9}{\bullet h_8 : \text{contract}(sn_7, !F_6), \Delta_5 \vdash F_9 \oplus F_{10}} \oplus_{R_1} \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_5 \vdash F_9 \oplus F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{ax} \quad \frac{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9 \oplus F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9 \oplus F_{10}} \text{hCut} \oplus_{R_1}
\end{array}$$

- Case rule 1_L

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{\bullet h_7 : \text{contract}(sn_6, !F_5), \mathbf{1}, \Delta_9 \vdash F_8} 1_L \\
\hline
- : (!\Upsilon 3, !F_4), \mathbf{1}, \Delta_9 \vdash F_8 \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_9, !F_4 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{- : \mathbf{1}, !\Upsilon 3, \Delta_9, !F_4 \vdash F_8} \text{ax}}{- : \mathbf{1}, !\Upsilon 3, \Delta_9, !F_4 \vdash F_8} \text{hCut} 1_L
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9 \quad h_7 : \Delta_8 \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10}} \otimes_R \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{11}, !F_4 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : \Delta_8 \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, \Delta_8, !F_4 \vdash F_9 \otimes F_{10}} \text{hCut} \otimes_R
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_{10}, \text{contract}(n_6, !F_5) \vdash F_8 \quad h_7 : \Delta_{11}, !F_5 \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \otimes_R \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9 \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3 \vdash !F_5}{- : !\Upsilon 3, \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \text{ax} \quad \frac{h_7 : \Delta_{10}, \text{contract}(n_6, !F_5) \vdash F_8 \quad h_7 : \Delta_{11}, !F_5 \vdash F_9}{- : !\Upsilon 3, \Delta_{10}, \Delta_{11}, !F_4 \vdash F_8 \otimes F_9} \text{ax}}{- : !\Upsilon 3, \Delta_{10}, \Delta_{11}, !F_4 \vdash F_8 \otimes F_9} \text{hCut} \otimes_R
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_{10}, !F_5 \vdash F_8 \quad h_7 : \Delta_{11}, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \otimes_R \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9 \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3 \vdash !F_5}{- : !\Upsilon 3, \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5 \vdash F_8 \quad h_7 : \Delta_{11}, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 3, \Delta_{10}, \Delta_{11}, !F_4 \vdash F_8 \otimes F_9} \text{ax}}{- : !\Upsilon 3, \Delta_{10}, \Delta_{11}, !F_4 \vdash F_8 \otimes F_9} \text{hCut} \otimes_R
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10}} \otimes_R \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10} \\
\rightarrow \\
\frac{- : \Delta_8 \vdash F_9 \quad \frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{11}, !F_4 \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, !F_4 \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, \Delta_8, !F_4 \vdash F_9 \otimes F_{10}} \text{hCut} \otimes_R
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_{10}, !F_5, \mathit{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \mathit{contract}(sn_6, !F_5), \Delta_{10}, !F_8 \vdash F_9} W}{- : (!\Upsilon 3, !F_4), \Delta_{10}, !F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5, \mathit{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 3, \Delta_{10}, !F_4 \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8 \vdash F_9} \text{hCut} \\
\frac{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8 \vdash F_9}{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8 \vdash F_9} W \\
\frac{h_2 : !\Upsilon 3 \vdash !F_8}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_8} W \quad \frac{h_7 : \Delta_5, \mathit{contract}(n_6, !F_8) \vdash F_9}{\bullet h_7 : \mathit{contract}(sn_6, !F_8), \Delta_5 \vdash F_9} W}{- : (!\Upsilon 3, !F_4), \Delta_5 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3 \vdash !F_8}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_8} \text{ax} \quad \frac{h_7 : \Delta_5, \mathit{contract}(n_6, !F_8) \vdash F_9}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{hCut}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_{10}, !F_5, !F_8, \mathit{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \mathit{contract}(sn_6, !F_5), \Delta_{10}, !F_8 \vdash F_9} C}{- : (!\Upsilon 3, !F_4), \Delta_{10}, !F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5, !F_8, \mathit{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8, !F_8 \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8 \vdash F_9} \text{hCut} \\
\frac{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8, !F_8 \vdash F_9}{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8 \vdash F_9} C \\
\frac{h_2 : !\Upsilon 3 \vdash !F_8}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_8} W \quad \frac{h_7 : \Delta_5, !F_8, \mathit{contract}(n_6, !F_8) \vdash F_9}{\bullet h_7 : \mathit{contract}(sn_6, !F_8), \Delta_5 \vdash F_9} C}{- : (!\Upsilon 3, !F_4), \Delta_5 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3 \vdash !F_8}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_8} \text{ax} \quad \frac{h_7 : \Delta_5, !F_8, \mathit{contract}(n_6, !F_8) \vdash F_9}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{hCut}
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_{10}, F_8, !F_5, \mathit{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \mathit{contract}(sn_6, !F_5), \Delta_{10}, !F_8 \vdash F_9} !L}{- : (!\Upsilon 3, !F_4), \Delta_{10}, !F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{10}, F_8, !F_5, \mathit{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 3, \Delta_{10}, F_8, !F_4 \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8 \vdash F_9} \text{hCut} \\
\frac{- : !\Upsilon 3, \Delta_{10}, F_8, !F_4 \vdash F_9}{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8 \vdash F_9} !L \\
\frac{h_2 : !\Upsilon 3 \vdash !F_8}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_8} W \quad \frac{h_7 : \Delta_5, F_8, \mathit{contract}(n_6, !F_8) \vdash F_9}{\bullet h_7 : \mathit{contract}(sn_6, !F_8), \Delta_5 \vdash F_9} !L}{- : (!\Upsilon 3, !F_4), \Delta_5 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3 \vdash !F_8}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_8} \text{ax} \quad \frac{h_7 : \Delta_5, F_8, \mathit{contract}(n_6, !F_8) \vdash F_9}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} !L}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{hCut} \\
\frac{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} W
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_{11}, F_9, !F_5, \mathit{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \mathit{contract}(sn_6, !F_5), \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \&_{L2}}{- : (!\Upsilon 3, !F_4), \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_9, !F_5, \mathit{contract}(n_6, !F_5) \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, F_9, !F_4 \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, F_9, !F_4 \vdash F_{10}} \text{hCut} \\
\frac{- : !\Upsilon 3, \Delta_{11}, F_9, !F_4 \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \& F_9 \vdash F_{10}} \&_{L2}
\end{array}$$

- Case rule $\&_{L1}$

$$\frac{\frac{h_2 : !\Upsilon_3 \vdash !F_5}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \&_{L1}}{\rightarrow : (!\Upsilon_3, !F_4), \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \text{Cut}}{\frac{\frac{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_5}{\rightarrow} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\rightarrow} \text{ax}}{\rightarrow : !\Upsilon_3, \Delta_{11}, F_8, !F_4 \vdash F_{10}} \text{hCut}}{\rightarrow : !\Upsilon_3, \Delta_{11}, !F_4, F_8 \& F_9 \vdash F_{10}} \&_{L1}} \text{Cut}$$

- Case rule \otimes_L

$$\frac{\frac{h_2 : !\Upsilon_3 \vdash !F_5}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_{11}, F_8, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \otimes F_9 \vdash F_{10}} \otimes_L}}{\rightarrow : (!\Upsilon_3, !F_4), \Delta_{11}, F_8 \otimes F_9 \vdash F_{10}} \text{Cut}}{\frac{\frac{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_5}{\rightarrow} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\rightarrow} \text{ax}}{\rightarrow : !\Upsilon_3, \Delta_{11}, F_8, F_9, !F_4 \vdash F_{10}} \text{hCut}}{\rightarrow : !\Upsilon_3, \Delta_{11}, !F_4, F_8 \otimes F_9 \vdash F_{10}} \otimes_L}} \text{Cut}$$

- Case rule \oplus_L

$$\frac{\frac{h_2 : !\Upsilon_3 \vdash !F_5}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}} \oplus_L}}{\rightarrow : (!\Upsilon_3, !F_4), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}} \text{Cut}}{\frac{\frac{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_5}{\rightarrow} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\rightarrow} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\rightarrow} \text{ax}}{\rightarrow : !\Upsilon_3, \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}} \text{hCut}}{\rightarrow : !\Upsilon_3, \Delta_{11}, !F_4, F_8 \oplus F_9 \vdash F_{10}} W \oplus_L}} \text{Cut}$$

- Case rule \multimap_L

$$\frac{\frac{h_2 : !\Upsilon_3 \vdash !F_5}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_{12}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9 \quad h_7 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11}} \multimap_L}}{\rightarrow : (!\Upsilon_3, !F_4), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11}} \text{Cut}}{\frac{\frac{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_5}{\rightarrow} \text{ax} \quad \frac{h_7 : \Delta_{12}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\rightarrow} \text{ax}}{\rightarrow : !\Upsilon_3, \Delta_{12}, !F_4 \vdash F_9} \text{hCut} \quad \frac{h_7 : \Delta_8, F_{10} \vdash F_{11}}{\rightarrow} \text{ax}}{\rightarrow : !\Upsilon_3, \Delta_{12}, \Delta_8, !F_4, F_9 \multimap F_{10} \vdash F_{11}} \multimap_L}} \text{Cut}$$

$$\frac{\frac{h_2 : !\Upsilon_3 \vdash !F_5}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_{11}, \text{contract}(n_6, !F_5) \vdash F_8 \quad h_7 : \Delta_{12}, F_9, !F_5 \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \multimap_L}}{\rightarrow : (!\Upsilon_3, !F_4), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \text{Cut}}{\frac{\frac{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_5}{\rightarrow} \text{ax} \quad \frac{h_7 : \Delta_{11}, \text{contract}(n_6, !F_5) \vdash F_8}{\rightarrow} \text{ax} \quad \frac{h_7 : \Delta_{12}, F_9, !F_5 \vdash F_{10}}{\rightarrow} \text{ax}}{\rightarrow : !\Upsilon_3, \Delta_{11}, \Delta_{12}, !F_5, F_8 \multimap F_9, \text{contract}(n_6, !F_5) \vdash F_{10}} \text{hCut}}{\rightarrow : !\Upsilon_3, \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} W \multimap_L}} \text{Cut}$$

$$\frac{\frac{h_2 : !\Upsilon_3 \vdash !F_5}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_{11}, !F_5 \vdash F_8 \quad h_7 : \Delta_{12}, F_9, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \multimap_L}}{\rightarrow : (!\Upsilon_3, !F_4), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \text{Cut}}{\frac{\frac{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_5}{\rightarrow} \text{ax} \quad \frac{h_7 : \Delta_{11}, !F_5 \vdash F_8}{\rightarrow} \text{ax} \quad \frac{h_7 : \Delta_{12}, F_9, \text{contract}(n_6, !F_5) \vdash F_{10}}{\rightarrow} \text{ax}}{\rightarrow : !\Upsilon_3, \Delta_{11}, \Delta_{12}, !F_5, F_8 \multimap F_9, \text{contract}(n_6, !F_5) \vdash F_{10}} \text{hCut}}{\rightarrow : !\Upsilon_3, \Delta_{11}, \Delta_{12}, !F_4, F_8 \multimap F_9 \vdash F_{10}} W \multimap_L}} \text{Cut}$$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon_3 \vdash !F_5}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_5} W \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{12}, F_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_{11}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11}} \multimap_L \\
\hline
- : (!\Upsilon_3, !F_4), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11} \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \Delta_8 \vdash F_9} \text{ax} \quad \frac{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_5 \quad \text{ax} \quad h_7 : \Delta_{12}, F_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_{11} \quad \text{ax}}{- : !\Upsilon_3, \Delta_{12}, F_{10}, !F_4 \vdash F_{11}} \text{hCut} \\
\hline
- : !\Upsilon_3, \Delta_{12}, \Delta_8, !F_4, F_9 \multimap F_{10} \vdash F_{11} \quad \multimap_L
\end{array}$$

- Case rule I

6.11 Status of C : OK

- Case rule $!R$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon_3, !F_4, !F_4 \vdash !F_6}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_6} C \quad \frac{h_8 : !\Upsilon_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9}{\bullet h_8 : \text{contract}(sn_7, !F_6), !\Upsilon_5 \vdash F_9} !R \\
\hline
- : (!\Upsilon_3, !F_4), !\Upsilon_5 \vdash F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_6 \quad \text{ax} \quad h_8 : !\Upsilon_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9 \quad \text{ax}}{- : !\Upsilon_3, !\Upsilon_5, !F_4 \vdash F_9} \text{hCut} \\
\hline
- : !\Upsilon_3, !\Upsilon_5, !F_4 \vdash F_9 \quad !R \\
- : !\Upsilon_3, !\Upsilon_5, !F_4 \vdash F_9
\end{array}$$

- Case rule 1_R

- Case rule \top

$$\begin{array}{c}
\frac{h_2 : !\Upsilon_3, !F_4, !F_4 \vdash !F_6}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_6} C \quad \frac{}{\bullet h_8 : \text{contract}(sn_7, !F_6), \Delta_5 \vdash \top} \top \\
\hline
- : (!\Upsilon_3, !F_4), \Delta_5 \vdash \top \quad \text{Cut} \\
\rightarrow \\
- : !\Upsilon_3, \Delta_5, !F_4 \vdash \top \quad \top
\end{array}$$

- Case rule $\&_R$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon_3, !F_4, !F_4 \vdash !F_6}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_6} C \quad \frac{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9 \quad h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10}}{\bullet h_8 : \text{contract}(sn_7, !F_6), \Delta_5 \vdash F_9 \& F_{10}} \&_R \\
\hline
- : (!\Upsilon_3, !F_4), \Delta_5 \vdash F_9 \& F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{h_2 : !\Upsilon_3, !F_4, !F_4 \vdash !F_6 \quad \text{ax} \quad h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9 \quad \text{ax} \quad h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10} \quad \text{ax}}{- : !\Upsilon_3, \Delta_5, !F_4, !F_4 \vdash F_9 \& F_{10}} \text{hCut} \\
\hline
- : !\Upsilon_3, \Delta_5, !F_4, !F_4 \vdash F_9 \& F_{10} \quad C \\
- : !\Upsilon_3, \Delta_5, !F_4 \vdash F_9 \& F_{10}
\end{array}$$

- Case rule \multimap_R

$$\begin{array}{c}
\frac{h_2 : !\Upsilon_3, !F_4, !F_4 \vdash !F_6}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_6} C \quad \frac{h_8 : \Delta_5, F_9, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10}}{\bullet h_8 : \text{contract}(sn_7, !F_6), \Delta_5 \vdash F_9 \multimap F_{10}} \multimap_R \\
\hline
- : (!\Upsilon_3, !F_4), \Delta_5 \vdash F_9 \multimap F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_6 \quad \text{ax} \quad h_8 : \Delta_5, F_9, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10} \quad \text{ax}}{- : !\Upsilon_3, \Delta_5, F_9, !F_4 \vdash F_{10}} \text{hCut} \\
\hline
- : !\Upsilon_3, \Delta_5, !F_4 \vdash F_9 \multimap F_{10} \quad \multimap_R \\
- : !\Upsilon_3, \Delta_5, !F_4 \vdash F_9 \multimap F_{10}
\end{array}$$

- Case rule \oplus_{R_2}

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} C \quad \frac{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10}}{\bullet h_8 : \text{contract}(sn_7, !F_6), \Delta_5 \vdash F_9 \oplus F_{10}} \oplus_{R_2} \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_5 \vdash F_9 \oplus F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} \text{ax} \quad \frac{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10}}{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_{10}} \text{hCut} \\
\hline
- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9 \oplus F_{10} \oplus_{R_2}
\end{array}$$

- Case rule \oplus_{R_1}

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} C \quad \frac{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9}{\bullet h_8 : \text{contract}(sn_7, !F_6), \Delta_5 \vdash F_9 \oplus F_{10}} \oplus_{R_1} \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_5 \vdash F_9 \oplus F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} \text{ax} \quad \frac{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9}{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{hCut} \\
\hline
- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9 \oplus F_{10} \oplus_{R_1}
\end{array}$$

- Case rule 1_L

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} C \quad \frac{h_7 : \Delta_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{\bullet h_7 : \text{contract}(sn_6, !F_5), \mathbf{1}, \Delta_9 \vdash F_8} \mathbf{1}_L \\
\hline
- : (!\Upsilon 3, !F_4), \mathbf{1}, \Delta_9 \vdash F_8 \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_8}{h_7 : \Delta_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_8} \text{ax}}{- : !\Upsilon 3, \Delta_9, !F_4 \vdash F_8} \text{hCut} \\
\hline
- : \mathbf{1}, !\Upsilon 3, \Delta_9, !F_4 \vdash F_8 \mathbf{1}_L
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} C \quad \frac{h_7 : \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9 \quad h_7 : \Delta_8 \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10}} \otimes_R \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{h_7 : \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, !F_4 \vdash F_9} \text{hCut} \quad \frac{- : \Delta_8 \vdash F_{10}}{- : \Delta_8 \vdash F_{10}} \text{ax} \\
\hline
- : !\Upsilon 3, \Delta_{11}, \Delta_8, !F_4 \vdash F_9 \otimes F_{10} \otimes_R
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} C \quad \frac{h_7 : \Delta_{10}, \text{contract}(n_6, !F_5) \vdash F_8 \quad h_7 : \Delta_{11}, !F_5 \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \otimes_R \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9 \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{10}, \text{contract}(n_6, !F_5) \vdash F_8}{h_7 : \Delta_{10}, \text{contract}(n_6, !F_5) \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_{11}, !F_5 \vdash F_9}{h_7 : \Delta_{11}, !F_5 \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_{10}, \Delta_{11}, !F_4, !F_4 \vdash F_8 \otimes F_9} \text{hCut} \\
\hline
- : !\Upsilon 3, \Delta_{10}, \Delta_{11}, !F_4 \vdash F_8 \otimes F_9 C
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} C \quad \frac{h_7 : \Delta_{10}, !F_5 \vdash F_8 \quad h_7 : \Delta_{11}, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \otimes_R \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9 \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5 \vdash F_8}{h_7 : \Delta_{10}, !F_5 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_{11}, \text{contract}(n_6, !F_5) \vdash F_9}{h_7 : \Delta_{11}, \text{contract}(n_6, !F_5) \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_{10}, \Delta_{11}, !F_4, !F_4 \vdash F_8 \otimes F_9} \text{hCut} \\
\hline
- : !\Upsilon 3, \Delta_{10}, \Delta_{11}, !F_4 \vdash F_8 \otimes F_9 C
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} C \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10}} \otimes_R \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{}{- : \Delta_8 \vdash F_9} \text{ax}}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} \text{ax} \quad \frac{\frac{}{- : \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}} h_7 : \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, !F_4 \vdash F_{10}} \text{hCut}}{- : !\Upsilon 3, \Delta_{11}, \Delta_8, !F_4 \vdash F_9 \otimes F_{10}} \otimes_R
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} C \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, !F_8 \vdash F_9} W \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{10}, !F_8 \vdash F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{}{- : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9} h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_{10}, !F_4 \vdash F_9} \text{hCut}}{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8 \vdash F_9} W \\
\frac{\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_8}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_8} C \quad \frac{h_7 : \Delta_5, \text{contract}(n_6, !F_8) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_8), \Delta_5 \vdash F_9} W}{- : (!\Upsilon 3, !F_4), \Delta_5 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{}{- : \Delta_5, \text{contract}(n_6, !F_8) \vdash F_9} h_7 : \Delta_5, \text{contract}(n_6, !F_8) \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{hCut}}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} C \quad \frac{h_7 : \Delta_{10}, !F_5, !F_8, !F_8, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, !F_8 \vdash F_9} C \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{10}, !F_8 \vdash F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{\frac{}{- : \Delta_{10}, !F_5, !F_8, !F_8, \text{contract}(n_6, !F_5) \vdash F_9} h_7 : \Delta_{10}, !F_5, !F_8, !F_8, \text{contract}(n_6, !F_5) \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8, !F_8 \vdash F_9} \text{hCut}}{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8 \vdash F_9} C}}{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8 \vdash F_9} \\
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_8}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_8} C \quad \frac{h_7 : \Delta_5, !F_8, !F_8, \text{contract}(n_6, !F_8) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_8), \Delta_5 \vdash F_9} C \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_5 \vdash F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{\frac{}{- : \Delta_5, !F_8, !F_8, \text{contract}(n_6, !F_8) \vdash F_9} h_7 : \Delta_5, !F_8, !F_8, \text{contract}(n_6, !F_8) \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{hCut}}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9}
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} C \quad \frac{h_7 : \Delta_{10}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, !F_8 \vdash F_9} !L \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{10}, !F_8 \vdash F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{\frac{}{- : \Delta_{10}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_9} h_7 : \Delta_{10}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_{10}, F_8, !F_4 \vdash F_9} \text{hCut}}{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8 \vdash F_9} !L}}{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8 \vdash F_9} \\
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_8}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_8} C \quad \frac{h_7 : \Delta_5, F_8, \text{contract}(n_6, !F_8) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_8), \Delta_5 \vdash F_9} !L \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_5 \vdash F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\frac{\frac{}{- : \Delta_5, F_8, \text{contract}(n_6, !F_8) \vdash F_9} h_7 : \Delta_5, F_8, \text{contract}(n_6, !F_8) \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_5, !F_4, !F_8 \vdash F_9} !L}}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{hCut}}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} C
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} C \quad \frac{h_7 : \Delta_{11}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \&_{L2} \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{11}, F_8 \& F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \& F_9 \vdash F_{10}} \text{hCut} \\
\&_{L2} \\
- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \& F_9 \vdash F_{10}
\end{array}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} C \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \&_{L1} \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{11}, F_8 \& F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, F_8, !F_4 \vdash F_{10}} \text{hCut} \\
\&_{L1} \\
- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \& F_9 \vdash F_{10}
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} C \quad \frac{h_7 : \Delta_{11}, F_8, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \otimes F_9 \vdash F_{10}} \otimes_L \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{11}, F_8 \otimes F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \otimes F_9 \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, F_8, F_9, !F_4 \vdash F_{10}} \text{hCut} \\
\otimes_L \\
- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \otimes F_9 \vdash F_{10}
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} C \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}} \oplus_L \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \Delta_{11}, !F_5, F_8 \oplus F_9, \text{contract}(n_6, !F_5) \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, !F_4, !F_4, F_8 \oplus F_9 \vdash F_{10}} \text{hCut} \\
\oplus_L \\
\frac{- : !\Upsilon 3, \Delta_{11}, !F_4, !F_4, F_8 \oplus F_9 \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \oplus F_9 \vdash F_{10}} C
\end{array}$$

- Case rule \multimap_L

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} C \quad \frac{h_7 : \Delta_{12}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9 \quad h_7 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11}} \multimap_L \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11} \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{12}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \Delta_{11}, !F_5, F_8 \oplus F_9, \text{contract}(n_6, !F_5) \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{12}, !F_4 \vdash F_9} \text{hCut} \quad \frac{- : \Delta_8, F_{10} \vdash F_{11}}{- : \Delta_8, F_{10} \vdash F_{11}} \text{ax} \\
\multimap_L \\
- : !\Upsilon 3, \Delta_{12}, \Delta_8, !F_4, F_9 \multimap F_{10} \vdash F_{11}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} C \quad \frac{h_7 : \Delta_{11}, \text{contract}(n_6, !F_5) \vdash F_8 \quad h_7 : \Delta_{12}, F_9, !F_5 \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \multimap_L \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10} \\
\rightarrow \\
\frac{\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} \text{ax} \quad \frac{h_7 : \Delta_{11}, \text{contract}(n_6, !F_5) \vdash F_8}{\bullet h_7 : \Delta_{11}, \Delta_{12}, !F_5, F_8 \multimap F_9, \text{contract}(n_6, !F_5) \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, \Delta_{12}, !F_4, !F_4, F_8 \multimap F_9 \vdash F_{10}} \text{hCut} \\
\multimap_L \\
\frac{- : !\Upsilon 3, \Delta_{11}, \Delta_{12}, !F_4, !F_4, F_8 \multimap F_9 \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, \Delta_{12}, !F_4, F_8 \multimap F_9 \vdash F_{10}} C
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} C \quad \frac{h_7 : \Delta_{11}, !F_5 \vdash F_8 \quad h_7 : \Delta_{12}, F_9, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \multimap_L}{- : (!\Upsilon 3, !F_4), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \text{Cut}}{\rightarrow} \\
\frac{\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} \text{ax} \quad \frac{\frac{h_7 : \Delta_{11}, !F_5 \vdash F_8}{\bullet h_7 : \Delta_{11}, \Delta_{12}, !F_5, F_8 \multimap F_9, \text{contract}(n_6, !F_5) \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{12}, F_9, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9, \text{contract}(n_6, !F_5) \vdash F_{10}} \multimap_L}}{- : !\Upsilon 3, \Delta_{11}, \Delta_{12}, !F_4, F_8 \multimap F_9 \vdash F_{10}} \text{hCut}}{- : !\Upsilon 3, \Delta_{11}, \Delta_{12}, !F_4, F_8 \multimap F_9 \vdash F_{10}} C} \\
\frac{\frac{h_2 : !\Upsilon 3, !F_4, !F_4 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} C \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{12}, F_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_{11}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11}} \multimap_L}{- : (!\Upsilon 3, !F_4), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11}} \text{Cut}}{\rightarrow} \\
\frac{\frac{- : \Delta_8 \vdash F_9}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} \text{ax} \quad \frac{\frac{h_7 : \Delta_{12}, F_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_{11}}{\bullet h_2 : !\Upsilon 3, \Delta_{12}, F_{10}, !F_4 \vdash F_{11}} \text{ax} \quad \frac{- : !\Upsilon 3, \Delta_{12}, F_{10}, !F_4 \vdash F_{11}}{- : !\Upsilon 3, \Delta_{12}, \Delta_8, !F_4, F_9 \multimap F_{10} \vdash F_{11}} \multimap_L}}{- : !\Upsilon 3, \Delta_{12}, \Delta_8, !F_4, F_9 \multimap F_{10} \vdash F_{11}} \text{hCut}}{- : !\Upsilon 3, \Delta_{12}, \Delta_8, !F_4, F_9 \multimap F_{10} \vdash F_{11}} \multimap_L}
\end{array}$$

- Case rule I

6.12 Status of $!L$: OK

- Case rule $!R$

$$\begin{array}{c}
\frac{\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} !L \quad \frac{h_8 : !\Upsilon 5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9}{\bullet h_8 : \text{contract}(sn_7, !F_6), !\Upsilon 5 \vdash !F_9} !R}{- : (!\Upsilon 3, !F_4), !\Upsilon 5 \vdash !F_9} \text{Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} \text{ax} \quad \frac{h_8 : !\Upsilon 5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9}{\bullet h_8 : \text{contract}(sn_7, !F_6), !\Upsilon 5 \vdash !F_9} \text{ax}}{- : !\Upsilon 3, !\Upsilon 5, !F_4 \vdash F_9} \text{hCut}}{- : !\Upsilon 3, !\Upsilon 5, !F_4 \vdash F_9} !R}
\end{array}$$

- Case rule 1_R

- Case rule \top

$$\frac{\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} !L \quad \frac{\text{---}}{\bullet h_8 : \text{contract}(sn_7, !F_6), \Delta_5 \vdash \top} \top}{- : (!\Upsilon 3, !F_4), \Delta_5 \vdash \top} \text{Cut}}{\rightarrow} \\
\frac{\text{---}}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash \top} \top}$$

- Case rule $\&_R$

$$\frac{\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} !L \quad \frac{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9 \quad h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10}}{\bullet h_8 : \text{contract}(sn_7, !F_6), \Delta_5 \vdash F_9 \& F_{10}} \&_R}{- : (!\Upsilon 3, !F_4), \Delta_5 \vdash F_9 \& F_{10}} \text{Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6} \text{ax} \quad \frac{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_9}{\bullet h_2 : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{ax} \quad \frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_6}{\bullet h_2 : !\Upsilon 3, \Delta_5, !F_4 \vdash F_{10}} \text{ax} \quad \frac{h_8 : \Delta_5, !F_6, \text{contract}(n_7, !F_6) \vdash F_{10}}{\bullet h_2 : !\Upsilon 3, \Delta_5, !F_4 \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_{10}} \text{hCut}}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9 \& F_{10}} \&_R}$$

- Case rule \multimap_R

$$\begin{array}{c}
\frac{h_2 : F_4, !\Upsilon_3 \vdash !F_6}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_6} !L \quad \frac{h_8 : \Delta_5, F_9, !F_6, \mathit{contract}(n_7, !F_6) \vdash F_{10}}{\bullet h_8 : \mathit{contract}(sn_7, !F_6), \Delta_5 \vdash F_9 \multimap F_{10}} \multimap_R}{\vdash : (!\Upsilon_3, !F_4), \Delta_5 \vdash F_9 \multimap F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_6}{\vdash : !\Upsilon_3, \Delta_5, F_9, !F_4 \vdash F_{10}} \text{ax} \quad \frac{h_8 : \Delta_5, F_9, !F_6, \mathit{contract}(n_7, !F_6) \vdash F_{10}}{\vdash : !\Upsilon_3, \Delta_5, !F_4 \vdash F_9 \multimap F_{10}} \text{ax}}{\vdash : !\Upsilon_3, \Delta_5, !F_4 \vdash F_9 \multimap F_{10}} \text{hCut} \multimap_R
\end{array}$$

- Case rule \oplus_{R_2}

$$\begin{array}{c}
\frac{h_2 : F_4, !\Upsilon_3 \vdash !F_6}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_6} !L \quad \frac{h_8 : \Delta_5, !F_6, \mathit{contract}(n_7, !F_6) \vdash F_{10}}{\bullet h_8 : \mathit{contract}(sn_7, !F_6), \Delta_5 \vdash F_9 \oplus F_{10}} \oplus_{R_2}}{\vdash : (!\Upsilon_3, !F_4), \Delta_5 \vdash F_9 \oplus F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_6}{\vdash : !\Upsilon_3, \Delta_5, !F_4 \vdash F_{10}} \text{ax} \quad \frac{h_8 : \Delta_5, !F_6, \mathit{contract}(n_7, !F_6) \vdash F_{10}}{\vdash : !\Upsilon_3, \Delta_5, !F_4 \vdash F_9 \oplus F_{10}} \text{ax}}{\vdash : !\Upsilon_3, \Delta_5, !F_4 \vdash F_9 \oplus F_{10}} \text{hCut} \oplus_{R_2}
\end{array}$$

- Case rule \oplus_{R_1}

$$\begin{array}{c}
\frac{h_2 : F_4, !\Upsilon_3 \vdash !F_6}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_6} !L \quad \frac{h_8 : \Delta_5, !F_6, \mathit{contract}(n_7, !F_6) \vdash F_9}{\bullet h_8 : \mathit{contract}(sn_7, !F_6), \Delta_5 \vdash F_9 \oplus F_{10}} \oplus_{R_1}}{\vdash : (!\Upsilon_3, !F_4), \Delta_5 \vdash F_9 \oplus F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_6}{\vdash : !\Upsilon_3, \Delta_5, !F_4 \vdash F_9} \text{ax} \quad \frac{h_8 : \Delta_5, !F_6, \mathit{contract}(n_7, !F_6) \vdash F_9}{\vdash : !\Upsilon_3, \Delta_5, !F_4 \vdash F_9 \oplus F_{10}} \text{ax}}{\vdash : !\Upsilon_3, \Delta_5, !F_4 \vdash F_9 \oplus F_{10}} \text{hCut} \oplus_{R_1}
\end{array}$$

- Case rule $\mathbf{1}_L$

$$\begin{array}{c}
\frac{h_2 : F_4, !\Upsilon_3 \vdash !F_5}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_9, !F_5, \mathit{contract}(n_6, !F_5) \vdash F_8}{\bullet h_7 : \mathit{contract}(sn_6, !F_5), \mathbf{1}, \Delta_9 \vdash F_8} \mathbf{1}_L}{\vdash : (!\Upsilon_3, !F_4), \mathbf{1}, \Delta_9 \vdash F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_5}{\vdash : !\Upsilon_3, \Delta_9, !F_4 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_9, !F_5, \mathit{contract}(n_6, !F_5) \vdash F_8}{\vdash : \mathbf{1}, !\Upsilon_3, \Delta_9, !F_4 \vdash F_8} \text{ax}}{\vdash : \mathbf{1}, !\Upsilon_3, \Delta_9, !F_4 \vdash F_8} \text{hCut} \mathbf{1}_L
\end{array}$$

- Case rule \otimes_R

$$\begin{array}{c}
\frac{h_2 : F_4, !\Upsilon_3 \vdash !F_5}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{11}, !F_5, \mathit{contract}(n_6, !F_5) \vdash F_9 \quad h_7 : \Delta_8 \vdash F_{10}}{\bullet h_7 : \mathit{contract}(sn_6, !F_5), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10}} \otimes_R}{\vdash : (!\Upsilon_3, !F_4), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_5}{\vdash : !\Upsilon_3, \Delta_{11}, !F_4 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{11}, !F_5, \mathit{contract}(n_6, !F_5) \vdash F_9}{\vdash : \Delta_8 \vdash F_{10}} \text{ax}}{\vdash : !\Upsilon_3, \Delta_{11}, \Delta_8, !F_4 \vdash F_9 \otimes F_{10}} \text{hCut} \otimes_R \\
\frac{h_2 : F_4, !\Upsilon_3 \vdash !F_5}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{10}, \mathit{contract}(n_6, !F_5) \vdash F_8 \quad h_7 : \Delta_{11}, !F_5 \vdash F_9}{\bullet h_7 : \mathit{contract}(sn_6, !F_5), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \otimes_R}{\vdash : (!\Upsilon_3, !F_4), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_5}{\vdash : !\Upsilon_3, \Delta_{10}, !F_4 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_{10}, \mathit{contract}(n_6, !F_5) \vdash F_8}{\vdash : !\Upsilon_3, \Delta_{11}, !F_4 \vdash F_9} \text{ax}}{\vdash : !\Upsilon_3, \Delta_{10}, !F_4 \vdash F_8} \text{hCut} \otimes_R \\
\frac{\frac{\frac{\frac{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_5}{\vdash : !\Upsilon_3, \Delta_{10}, !F_4 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_{10}, \mathit{contract}(n_6, !F_5) \vdash F_8}{\vdash : !\Upsilon_3, \Delta_{11}, !F_4 \vdash F_9} \text{ax}}{\vdash : !\Upsilon_3, \Delta_{10}, \Delta_{11}, !F_4 \vdash F_8 \otimes F_9} \text{hCut} \otimes_R}{\vdash : !\Upsilon_3, \Delta_{10}, \Delta_{11}, !F_4 \vdash F_8 \otimes F_9} \text{C}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{10}, !F_5 \vdash F_8 \quad h_7 : \Delta_{11}, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \otimes_R}{- : (!\Upsilon 3, !F_4), \Delta_{10}, \Delta_{11} \vdash F_8 \otimes F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{10}, !F_4 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5 \vdash F_8}{h_7 : \Delta_{11}, \text{contract}(n_6, !F_5) \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_{10}, !F_4 \vdash F_8 \otimes F_9} \text{hCut} \quad \frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{11}, !F_4 \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_{10}, \Delta_{11}, !F_4 \vdash F_8 \otimes F_9} \otimes_R \\
\text{C} \\
\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{11}, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10}} \otimes_R}{- : (!\Upsilon 3, !F_4), \Delta_8, \Delta_{11} \vdash F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{- : \Delta_8 \vdash F_9}{- : !\Upsilon 3, \Delta_{11}, \Delta_8, !F_4 \vdash F_9 \otimes F_{10}} \text{ax} \quad \frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{11}, !F_4 \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, \Delta_8, !F_4 \vdash F_9 \otimes F_{10}} \text{hCut} \otimes_R
\end{array}$$

- Case rule W

$$\begin{array}{c}
\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, !F_8 \vdash F_9} W}{- : (!\Upsilon 3, !F_4), \Delta_{10}, !F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{10}, !F_4 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8 \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8 \vdash F_9} \text{hCut} \\
W \\
\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_8}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_8} !L \quad \frac{h_7 : \Delta_5, \text{contract}(n_6, !F_8) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_8), \Delta_5 \vdash F_9} W}{- : (!\Upsilon 3, !F_4), \Delta_5 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_8}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_5, \text{contract}(n_6, !F_8) \vdash F_9}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{hCut}
\end{array}$$

- Case rule C

$$\begin{array}{c}
\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{10}, !F_5, !F_8, !F_8, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, !F_8 \vdash F_9} C}{- : (!\Upsilon 3, !F_4), \Delta_{10}, !F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8, !F_8 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{10}, !F_5, !F_8, !F_8, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8 \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8 \vdash F_9} \text{hCut} \\
C \\
\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_8}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_8} !L \quad \frac{h_7 : \Delta_5, !F_8, !F_8, \text{contract}(n_6, !F_8) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_8), \Delta_5 \vdash F_9} C}{- : (!\Upsilon 3, !F_4), \Delta_5 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_8}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_5, !F_8, !F_8, \text{contract}(n_6, !F_8) \vdash F_9}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{hCut}
\end{array}$$

- Case rule $!L$

$$\begin{array}{c}
\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_5}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{10}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{10}, !F_8 \vdash F_9} !L}{- : (!\Upsilon 3, !F_4), \Delta_{10}, !F_8 \vdash F_9} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{10}, F_8, !F_4 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{10}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 3, \Delta_{10}, !F_4, !F_8 \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_{10}, F_8, !F_4 \vdash F_9} \text{hCut} \\
!L
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_8 \quad h_7 : \Delta_5, F_8, \text{contract}(n_6, !F_8) \vdash F_9}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_8} !L \quad \frac{h_7 : \Delta_5, F_8, \text{contract}(n_6, !F_8) \vdash F_9}{\bullet h_7 : \text{contract}(sn_6, !F_8), \Delta_5 \vdash F_9} !L \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_5 \vdash F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{- : !\Upsilon 3, !F_4 \vdash !F_8}{- : !\Upsilon 3, !F_4 \vdash F_8} \text{ax} \quad \frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_8}{- : !\Upsilon 3, \Delta_5, F_8, !F_4 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_5, F_8, \text{contract}(n_6, !F_8) \vdash F_9}{- : !\Upsilon 3, \Delta_5, F_8, !F_4 \vdash F_9} \text{ax}}{- : !\Upsilon 3, \Delta_5, !F_4 \vdash F_9} \text{bInv} \quad \text{hCut} \quad \text{Cut}
\end{array}$$

- Case rule $\&_{L2}$

$$\begin{array}{c}
\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_5 \quad h_7 : \Delta_{11}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{11}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \&_{L2} \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{11}, F_8 \& F_9 \vdash F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{11}, F_9, !F_4 \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \& F_9 \vdash F_{10}} \text{hCut} \quad \&_{L2}
\end{array}$$

- Case rule $\&_{L1}$

$$\begin{array}{c}
\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_5 \quad h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \& F_9 \vdash F_{10}} \&_{L1} \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{11}, F_8 \& F_9 \vdash F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{11}, F_8, !F_4 \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \& F_9 \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \& F_9 \vdash F_{10}} \text{hCut} \quad \&_{L1}
\end{array}$$

- Case rule \otimes_L

$$\begin{array}{c}
\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_5 \quad h_7 : \Delta_{11}, F_8, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{11}, F_8, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \otimes F_9 \vdash F_{10}} \otimes_L \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{11}, F_8 \otimes F_9 \vdash F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{11}, F_8, F_9, !F_4 \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_8, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \otimes F_9 \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \otimes F_9 \vdash F_{10}} \text{hCut} \quad \otimes_L
\end{array}$$

- Case rule \oplus_L

$$\begin{array}{c}
\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_5 \quad h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10} \quad h_7 : \Delta_{11}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10}} \oplus_L \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_{11}, F_8 \oplus F_9 \vdash F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{11}, F_8, !F_4 \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_8, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, F_8, !F_4 \vdash F_{10}} \text{ax} \quad \frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{11}, F_9, !F_4 \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{11}, F_9, !F_5, \text{contract}(n_6, !F_5) \vdash F_{10}}{- : !\Upsilon 3, \Delta_{11}, F_9, !F_4 \vdash F_{10}} \text{ax}}{- : !\Upsilon 3, \Delta_{11}, !F_4, F_8 \oplus F_9 \vdash F_{10}} \text{hCut} \quad \oplus_L
\end{array}$$

- Case rule \multimap_L

$$\begin{array}{c}
\frac{h_2 : F_4, !\Upsilon 3 \vdash !F_5 \quad h_7 : \Delta_{12}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9 \quad h_7 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{12}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9 \quad h_7 : \Delta_8, F_{10} \vdash F_{11}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11}} \multimap_L \\
\hline
- : (!\Upsilon 3, !F_4), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11} \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 : !\Upsilon 3, !F_4 \vdash !F_5}{- : !\Upsilon 3, \Delta_{12}, !F_4 \vdash F_9} \text{ax} \quad \frac{h_7 : \Delta_{12}, !F_5, \text{contract}(n_6, !F_5) \vdash F_9}{- : !\Upsilon 3, \Delta_{12}, \Delta_8, !F_4, F_9 \multimap F_{10} \vdash F_{11}} \text{ax} \quad \frac{h_7 : \Delta_8, F_{10} \vdash F_{11}}{- : \Delta_8, F_{10} \vdash F_{11}} \text{ax}}{- : !\Upsilon 3, \Delta_{12}, \Delta_8, !F_4, F_9 \multimap F_{10} \vdash F_{11}} \text{hCut} \quad \multimap_L
\end{array}$$

$$\begin{array}{c}
\frac{\frac{h_2 : F_4, !\Upsilon_3 \vdash !F_5}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{11}, \text{contract}(n_6, !F_5) \vdash F_8 \quad h_7 : \Delta_{12}, F_9, !F_5 \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \multimap_L}{- : (!\Upsilon_3, !F_4), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \text{Cut}}{\rightarrow} \\
\frac{\frac{\frac{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_5}{- : !\Upsilon_3, \Delta_{11}, !F_4 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_{11}, \text{contract}(n_6, !F_5) \vdash F_8}{\text{hCut}} \text{ax} \quad \frac{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_5}{- : !\Upsilon_3, \Delta_{12}, F_9, !F_4 \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{12}, F_9, !F_5 \vdash F_{10}}{\text{hCut}} \text{ax}}{\frac{- : !\Upsilon_3, \Delta_{11}, !F_4 \vdash F_8}{- : !\Upsilon_3, \Upsilon_3, \Delta_{11}, \Delta_{12}, !F_4, !F_4, F_8 \multimap F_9 \vdash F_{10}} \multimap_L} \text{C}}{\frac{- : !\Upsilon_3, \Delta_{11}, \Delta_{12}, !F_4, F_8 \multimap F_9 \vdash F_{10}}{- : !\Upsilon_3, \Delta_{11}, \Delta_{12}, !F_4, F_8 \multimap F_9 \vdash F_{10}} \text{C}} \\
\frac{\frac{h_2 : F_4, !\Upsilon_3 \vdash !F_5}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_{11}, !F_5 \vdash F_8 \quad h_7 : \Delta_{12}, F_9, \text{contract}(n_6, !F_5) \vdash F_{10}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \multimap_L}{- : (!\Upsilon_3, !F_4), \Delta_{11}, \Delta_{12}, F_8 \multimap F_9 \vdash F_{10}} \text{Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_5}{- : !\Upsilon_3, \Delta_{11}, !F_4 \vdash F_8} \text{ax} \quad \frac{h_7 : \Delta_{11}, !F_5 \vdash F_8}{\text{hCut}} \text{ax} \quad \frac{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_5}{- : !\Upsilon_3, \Delta_{12}, F_9, !F_4 \vdash F_{10}} \text{ax} \quad \frac{h_7 : \Delta_{12}, F_9, \text{contract}(n_6, !F_5) \vdash F_{10}}{\text{hCut}} \text{ax}}{\frac{- : !\Upsilon_3, \Delta_{11}, !F_4 \vdash F_8}{- : !\Upsilon_3, \Upsilon_3, \Delta_{11}, \Delta_{12}, !F_4, !F_4, F_8 \multimap F_9 \vdash F_{10}} \multimap_L} \text{C}}{\frac{- : !\Upsilon_3, \Delta_{11}, \Delta_{12}, !F_4, F_8 \multimap F_9 \vdash F_{10}}{- : !\Upsilon_3, \Delta_{11}, \Delta_{12}, !F_4, F_8 \multimap F_9 \vdash F_{10}} \text{C}} \\
\frac{\frac{h_2 : F_4, !\Upsilon_3 \vdash !F_5}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_5} !L \quad \frac{h_7 : \Delta_8 \vdash F_9 \quad h_7 : \Delta_{12}, F_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_{11}}{\bullet h_7 : \text{contract}(sn_6, !F_5), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11}} \multimap_L}{- : (!\Upsilon_3, !F_4), \Delta_8, \Delta_{12}, F_9 \multimap F_{10} \vdash F_{11}} \text{Cut}}{\rightarrow} \\
\frac{\frac{- : \Delta_8 \vdash F_9}{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_5} \text{ax} \quad \frac{\bullet h_2 : !\Upsilon_3, !F_4 \vdash !F_5}{- : !\Upsilon_3, \Delta_{12}, F_{10}, !F_4 \vdash F_{11}} \text{ax} \quad \frac{h_7 : \Delta_{12}, F_{10}, !F_5, \text{contract}(n_6, !F_5) \vdash F_{11}}{\text{hCut}} \text{ax}}{\frac{- : \Delta_8 \vdash F_9}{- : !\Upsilon_3, \Delta_{12}, \Delta_8, !F_4, F_9 \multimap F_{10} \vdash F_{11}} \multimap_L} \text{C}}
\end{array}$$

- Case rule I

6.13 Status of $\&_{L2}$: OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule \top
- Case rule $\&_R$
- Case rule \multimap_R
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- Case rule W

- Case rule C
- Case rule $!L$
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule \otimes_L
- Case rule \oplus_L
- Case rule \multimap_L
- Case rule I

6.14 Status of $\&_{L1}$: OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule \top
- Case rule $\&_R$
- Case rule \multimap_R
- Case rule \oplus_{R2}
- Case rule \oplus_{R1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- Case rule W
- Case rule C

- Case rule $!L$
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule \otimes_L
- Case rule \oplus_L
- Case rule $\neg\circ_L$
- Case rule I

6.15 Status of \otimes_L : OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule \top
- Case rule $\&_R$
- Case rule $\neg\circ_R$
- Case rule \oplus_{R2}
- Case rule \oplus_{R1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- Case rule W
- Case rule C
- Case rule $!L$

- Case rule $\&_{L2}$
- Case rule $\&_{L1}$
- Case rule \otimes_L
- Case rule \oplus_L
- Case rule $\neg\circ_L$
- Case rule I

6.16 Status of \oplus_L : OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule \top
- Case rule $\&_R$
- Case rule $\neg\circ_R$
- Case rule \oplus_{R2}
- Case rule \oplus_{R1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L2}$

- Case rule $\&_{L1}$
- Case rule \otimes_L
- Case rule \oplus_L
- Case rule $\neg\circ_L$
- Case rule I

6.17 Status of $\neg\circ_L$: OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule \top
- Case rule $\&_R$
- Case rule $\neg\circ_R$
- Case rule \oplus_{R2}
- Case rule \oplus_{R1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L2}$
- Case rule $\&_{L1}$

- Case rule \otimes_L
- Case rule \oplus_L
- Case rule $\neg\circ_L$
- Case rule I

6.18 Status of I : OK

- Case rule $!R$
- Case rule $\mathbf{1}_R$
- Case rule \top
- Case rule $\&_R$
- Case rule $\neg\circ_R$
- Case rule \oplus_{R_2}
- Case rule \oplus_{R_1}
- Case rule $\mathbf{1}_L$
- Case rule \otimes_R
- Case rule W
- Case rule C
- Case rule $!L$
- Case rule $\&_{L_2}$
- Case rule $\&_{L_1}$
- Case rule \otimes_L

- Case rule \oplus_L
- Case rule $\neg\circ_L$
- Case rule I