

Dyadic system for Linear Logic

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Contents

1	Weakening on the classical context	3
2	Contraction on the classical context	5
3	Measure of derivations	7
4	Invertibility of Rules	9
4.1	Status of $\mathbf{1}$: : Invertible	9
4.2	Status of $!$: : Invertible	10
4.3	Status of $?$: : Invertible	10
4.4	Status of $\$$: : Invertible	12
4.5	Status of $\&$: (Left Premise): Invertible	13
4.6	Status of $\&$ (Right Premise): : Invertible	15
4.7	Status of \oplus_B : : Non invertible	16
4.8	Status of \oplus_A : : Non invertible	17
4.9	Status of \perp : : Invertible	19
4.10	Status of \top : : Invertible	20
4.11	Status of I_3 : : Invertible	21
4.12	Status of \otimes : (Left Premise): Non invertible	22
4.13	Status of \otimes (Right Premise): : Non invertible	24
4.14	Status of I_1 : : Invertible	26
4.15	Status of I_2 : : Invertible	27
4.16	Status of $?_C$: : Non invertible	28
5	Weakening on bang: $\vdash \Gamma, !F$ implies Γ, F.	30
6	Identity-Expansion	32
7	Cut-Elimination	33
7.1	Status of $\mathbf{1}$: OK	33
7.2	Status of $!$: OK	34
7.3	Status of $?$: OK	37
7.4	Status of $\$$: OK	40
7.5	Status of $\&$: OK	44
7.6	Status of \oplus_B : OK	47
7.7	Status of \oplus_A : OK	51
7.8	Status of \perp : OK	54
7.9	Status of \top : OK	58
7.10	Status of I_3 : OK	60

7.11	Status of \otimes : OK	64
7.12	Status of I_1 : OK	68
7.13	Status of I_2 : OK	71
7.14	Status of $?_C$: OK	73
8	Cut-Elimination	76
8.1	Status of $\mathbf{1}$: OK	76
8.2	Status of $!$: OK	76
8.3	Status of $?$: OK	79
8.4	Status of $\$$: OK	80
8.5	Status of $\&$: OK	80
8.6	Status of \oplus_B : OK	81
8.7	Status of \oplus_A : OK	82
8.8	Status of \perp : OK	83
8.9	Status of \top : OK	83
8.10	Status of I_3 : OK	84
8.11	Status of \otimes : OK	85
8.12	Status of I_1 : OK	86
8.13	Status of I_2 : OK	86
8.14	Status of $?_C$: OK	87

1 Weakening on the classical context

- Case(s) rule 1

$$\frac{}{\bullet h_1 \vdash \{\Delta_2\} : 1} \text{1} \rightarrow \frac{}{\bullet h_1 \vdash \{\Delta_2, F_W\} : 1} \text{1}$$

- Case(s) rule !

$$\frac{h_1 \vdash \{\Delta_2\} : F_3}{\bullet h_1 \vdash \{\Delta_2\} : !F_3} \text{!} \rightarrow \frac{\frac{h_1 \vdash \{\Delta_2\} : F_3}{\bullet h_1 \vdash \{\Delta_2, F_W\} : F_3} \text{IH}}{\bullet h_1 \vdash \{\Delta_2, F_W\} : !F_3} \text{!}$$

- Case(s) rule ?

$$\frac{h_1 \vdash \{F_4, \Delta_2\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, ?F_4} \text{?} \rightarrow \frac{\frac{h_1 \vdash \{\Delta_2, F_4\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_2, F_W\} : \Delta_3, ?F_4} \text{IH}}{\bullet h_1 \vdash \{\Delta_2, F_W\} : \Delta_3, ?F_4} \text{?}$$

- Case(s) rule \$

$$\frac{h_1 \vdash \{\Delta_2\} : F_4, F_5, \Delta_3}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4 \$F_5} \$ \rightarrow \frac{\frac{h_1 \vdash \{\Delta_2\} : \Delta_3, F_4, F_5}{\bullet h_1 \vdash \{\Delta_2, F_W\} : \Delta_3, F_4, F_5} \text{IH}}{\bullet h_1 \vdash \{\Delta_2, F_W\} : \Delta_3, F_4 \$F_5} \$$$

- Case(s) rule &

$$\frac{h_1 \vdash \{\Delta_2\} : F_4, \Delta_3 \quad h_1 \vdash \{\Delta_2\} : F_5, \Delta_3}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4 \&F_5} \& \rightarrow \frac{\frac{h_1 \vdash \{\Delta_2\} : \Delta_3, F_4}{\bullet h_1 \vdash \{\Delta_2, F_W\} : \Delta_3, F_4} \text{IH} \quad \frac{h_1 \vdash \{\Delta_2\} : \Delta_3, F_5}{\bullet h_1 \vdash \{\Delta_2, F_W\} : \Delta_3, F_5} \text{IH}}{\bullet h_1 \vdash \{\Delta_2, F_W\} : \Delta_3, F_4 \&F_5} \&}$$

- Case(s) rule \oplus_B

$$\frac{h_1 \vdash \{\Delta_2\} : F_5, \Delta_3}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4 \oplus F_5} \oplus_B \rightarrow \frac{\frac{h_1 \vdash \{\Delta_2\} : \Delta_3, F_5}{\bullet h_1 \vdash \{\Delta_2, F_W\} : \Delta_3, F_5} \text{IH}}{\bullet h_1 \vdash \{\Delta_2, F_W\} : \Delta_3, F_4 \oplus F_5} \oplus_B}$$

- Case(s) rule \oplus_A

$$\frac{h_1 \vdash \{\Delta_2\} : F_4, \Delta_3}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4 \oplus F_5} \oplus_A \rightarrow \frac{\frac{h_1 \vdash \{\Delta_2\} : \Delta_3, F_4}{\bullet h_1 \vdash \{\Delta_2, F_W\} : \Delta_3, F_4} \text{IH}}{\bullet h_1 \vdash \{\Delta_2, F_W\} : \Delta_3, F_4 \oplus F_5} \oplus_A}$$

- Case(s) rule \perp

$$\frac{h_1 \vdash \{\Delta_2\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_2\} : \perp, \Delta_3} \perp \rightarrow \frac{\frac{h_1 \vdash \{\Delta_2\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_2, F_W\} : \Delta_3} \text{IH}}{\bullet h_1 \vdash \{\Delta_2, F_W\} : \Delta_3, \perp} \perp}$$

- Case(s) rule \top

$$\frac{}{\bullet h_1 \vdash \{\Delta_2\} : \top, \Delta_3} \top \rightarrow \frac{}{\bullet h_1 \vdash \{\Delta_2, F_W\} : \Delta_3, \top} \top$$

- Case(s) rule I_3

$$\frac{}{\bullet h_1 \vdash \{\Delta_2\} : p(n_3), \hat{\neg}(n_3)} I_3 \rightarrow \frac{}{\bullet h_1 \vdash \{\Delta_2, F_W\} : p(n_3), \hat{\neg}(n_3)} I_3}$$

- Case(s) rule \otimes

$$\frac{\frac{h_1 \vdash \{\Delta_2\} : F_5, \Delta_3 \quad h_1 \vdash \{\Delta_2\} : F_6, \Delta_4}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, \Delta_4, F_5 \otimes F_6} \otimes}{\bullet h_1 \vdash \{\Delta_2, F_W\} : \Delta_3, \Delta_4, F_5 \otimes F_6} \otimes \rightarrow \frac{\frac{h_1 \vdash \{\Delta_2\} : \Delta_3, F_5}{\bullet h_1 \vdash \{\Delta_2, F_W\} : \Delta_3, F_5} \text{ax} \quad \frac{h_1 \vdash \{\Delta_2\} : \Delta_4, F_6}{\bullet h_1 \vdash \{\Delta_2, F_W\} : \Delta_4, F_6} \text{IH}}{\bullet h_1 \vdash \{\Delta_2, F_W\} : \Delta_3, \Delta_4, F_5 \otimes F_6} \text{IH} \otimes} \otimes$$

- Case(s) rule I_1

$$\frac{}{\bullet h_1 \vdash \{\Delta_2, p(n_3)\} : \wedge(n_3)} I_1 \rightarrow \frac{}{\bullet h_1 \vdash \{\Delta_2, F_W, p(n_3)\} : \wedge(n_3)} I_1$$

- Case(s) rule I_2

$$\frac{}{\bullet h_1 \vdash \{\Delta_2, \wedge(n_3)\} : p(n_3)} I_2 \rightarrow \frac{}{\bullet h_1 \vdash \{\Delta_2, F_W, \wedge(n_3)\} : p(n_3)} I_2$$

- Case(s) rule $?_C$

$$\frac{\frac{h_1 \vdash \{F_2, \Delta_3\} : F_2, \Delta_4}{\bullet h_1 \vdash \{F_2, \Delta_3\} : \Delta_4} ?_C}{\bullet h_1 \vdash \{\Delta_3, F_2, F_W\} : \Delta_4} ?_C \rightarrow \frac{\frac{h_1 \vdash \{\Delta_3, F_2\} : \Delta_4, F_2}{\bullet h_1 \vdash \{\Delta_3, F_2, F_W\} : \Delta_4, F_2} \text{ax} \quad \frac{h_1 \vdash \{\Delta_3, F_2, F_W\} : \Delta_4, F_2}{\bullet h_1 \vdash \{\Delta_3, F_2, F_W\} : \Delta_4} ?_C \text{IH}}{\bullet h_1 \vdash \{\Delta_3, F_2, F_W\} : \Delta_4} ?_C} ?_C$$

2 Contraction on the classical context

- Case(s) rule 1

$$\frac{}{\bullet h_3 \vdash \{\Delta_2, F_1, F_1\} : 1} 1 \rightarrow \frac{}{\bullet h_3 \vdash \{\Delta_2, F_1\} : 1} 1$$

- Case(s) rule !

$$\frac{h_3 \vdash \{F_1, F_1, \Delta_2\} : F_4}{\bullet h_3 \vdash \{\Delta_2, F_1, F_1\} : !F_4} ! \rightarrow \frac{\frac{h_3 \vdash \{\Delta_2, F_1, F_1\} : F_4}{h_3 \vdash \{\Delta_2, F_1\} : F_4} \text{IH}}{\bullet h_3 \vdash \{\Delta_2, F_1\} : !F_4} ! \text{ax}}$$

- Case(s) rule ?

$$\frac{h_3 \vdash \{F_1, F_1, F_5, \Delta_2\} : \Delta_4}{\bullet h_3 \vdash \{\Delta_2, F_1, F_1\} : \Delta_4, ?F_5} ? \rightarrow \frac{\frac{h_3 \vdash \{\Delta_2, F_1, F_1, F_5\} : \Delta_4}{h_3 \vdash \{\Delta_2, F_1, F_5\} : \Delta_4} \text{IH}}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, ?F_5} ? \text{ax}}$$

- Case(s) rule \$

$$\frac{h_3 \vdash \{F_1, F_1, \Delta_2\} : F_5, F_6, \Delta_4}{\bullet h_3 \vdash \{\Delta_2, F_1, F_1\} : \Delta_4, F_5 \$F_6} \$ \rightarrow \frac{\frac{h_3 \vdash \{\Delta_2, F_1, F_1\} : \Delta_4, F_5, F_6}{h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_5, F_6} \text{IH}}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_5 \$F_6} \$ \text{ax}}$$

- Case(s) rule &

$$\frac{h_3 \vdash \{F_1, F_1, \Delta_2\} : F_5, \Delta_4}{\bullet h_3 \vdash \{\Delta_2, F_1, F_1\} : \Delta_4, F_5 \&F_6} \& \rightarrow \frac{\frac{h_3 \vdash \{\Delta_2, F_1, F_1\} : \Delta_4, F_5}{h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_5} \text{IH}}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_5 \&F_6} \& \text{ax}}{\frac{h_3 \vdash \{F_1, F_1, \Delta_2\} : F_6, \Delta_4}{h_3 \vdash \{\Delta_2, F_1, F_1\} : \Delta_4, F_6} \text{IH}}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_5 \&F_6} \& \text{ax}}$$

- Case(s) rule \oplus_B

$$\frac{h_3 \vdash \{F_1, F_1, \Delta_2\} : F_6, \Delta_4}{\bullet h_3 \vdash \{\Delta_2, F_1, F_1\} : \Delta_4, F_5 \oplus F_6} \oplus_B \rightarrow \frac{\frac{h_3 \vdash \{\Delta_2, F_1, F_1\} : \Delta_4, F_6}{h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_6} \text{IH}}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_5 \oplus F_6} \oplus_B \text{ax}}$$

- Case(s) rule \oplus_A

$$\frac{h_3 \vdash \{F_1, F_1, \Delta_2\} : F_5, \Delta_4}{\bullet h_3 \vdash \{\Delta_2, F_1, F_1\} : \Delta_4, F_5 \oplus F_6} \oplus_A \rightarrow \frac{\frac{h_3 \vdash \{\Delta_2, F_1, F_1\} : \Delta_4, F_5}{h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_5} \text{IH}}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_5 \oplus F_6} \oplus_A \text{ax}}$$

- Case(s) rule \perp

$$\frac{h_3 \vdash \{F_1, F_1, \Delta_2\} : \Delta_4}{\bullet h_3 \vdash \{\Delta_2, F_1, F_1\} : \perp, \Delta_4} \perp \rightarrow \frac{\frac{h_3 \vdash \{\Delta_2, F_1, F_1\} : \Delta_4}{h_3 \vdash \{\Delta_2, F_1\} : \Delta_4} \text{IH}}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, \perp} \perp \text{ax}}$$

- Case(s) rule \top

$$\frac{}{\bullet h_3 \vdash \{\Delta_2, F_1, F_1\} : \top, \Delta_4} \top \rightarrow \frac{}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, \top} \top$$

- Case(s) rule I_3

$$\frac{}{\bullet h_3 \vdash \{\Delta_2, F_1, F_1\} : p(n_4), \hat{\cdot}(n_4)} I_3 \rightarrow \frac{}{\bullet h_3 \vdash \{\Delta_2, F_1\} : p(n_4), \hat{\cdot}(n_4)} I_3$$

- Case(s) rule \otimes

$$\frac{\frac{h_3 \vdash \{F_1, F_1, \Delta_2\} : F_6, \Delta_4 \quad h_3 \vdash \{F_1, F_1, \Delta_2\} : F_7, \Delta_5}{\bullet h_3 \vdash \{\Delta_2, F_1, F_1\} : \Delta_4, \Delta_5, F_6 \otimes F_7} \otimes}{\bullet h_3 \vdash \{\Delta_2, F_1, F_1\} : \Delta_4, F_6} \text{ax} \quad \frac{h_3 \vdash \{\Delta_2, F_1, F_1\} : \Delta_5, F_7}{h_3 \vdash \{\Delta_2, F_1\} : \Delta_5, F_7} \text{IH}}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, \Delta_5, F_6 \otimes F_7} \otimes \rightarrow$$

- Case(s) rule I_1

$$\frac{}{\bullet h_2 \vdash \{\Delta_1, p(n_3), p(n_3)\} : \wedge(n_3)} I_1 \rightarrow \frac{}{\bullet h_2 \vdash \{\Delta_1, p(n_3)\} : \wedge(n_3)} I_1$$

$$\frac{}{\bullet h_2 \vdash \{(\Delta_4, p(n_3)), F_1, F_1\} : \wedge(n_3)} I_1 \rightarrow \frac{}{\bullet h_2 \vdash \{\Delta_4, F_1, p(n_3)\} : \wedge(n_3)} I_1$$

- Case(s) rule I_2

$$\frac{}{\bullet h_2 \vdash \{\Delta_1, \wedge(n_3), \wedge(n_3)\} : p(n_3)} I_2 \rightarrow \frac{}{\bullet h_2 \vdash \{\Delta_1, \wedge(n_3)\} : p(n_3)} I_2$$

$$\frac{}{\bullet h_2 \vdash \{(\Delta_4, \wedge(n_3)), F_1, F_1\} : p(n_3)} I_2 \rightarrow \frac{}{\bullet h_2 \vdash \{\Delta_4, F_1, \wedge(n_3)\} : p(n_3)} I_2$$

- Case(s) rule $?_C$

$$\frac{\frac{h_2 \vdash \{F_3, F_3, \Delta_1\} : F_3, \Delta_4}{\bullet h_2 \vdash \{\Delta_1, F_3, F_3\} : \Delta_4} ?_C}{\bullet h_2 \vdash \{\Delta_1, F_3, F_3\} : \Delta_4, F_3} \text{ax} \quad \frac{h_2 \vdash \{\Delta_1, F_3, F_3\} : \Delta_4, F_3}{h_2 \vdash \{\Delta_1, F_3\} : \Delta_4, F_3} \text{IH}}{\bullet h_2 \vdash \{\Delta_1, F_3\} : \Delta_4} ?_C \rightarrow$$

$$\frac{\frac{h_2 \vdash \{F_1, F_1, F_3, \Delta_5\} : F_3, \Delta_4}{\bullet h_2 \vdash \{F_3, \Delta_5, F_1, F_1\} : \Delta_4} ?_C}{\bullet h_2 \vdash \{\Delta_5, F_1, F_1, F_3\} : \Delta_4, F_3} \text{ax} \quad \frac{h_2 \vdash \{\Delta_5, F_1, F_1, F_3\} : \Delta_4, F_3}{h_2 \vdash \{\Delta_5, F_1, F_3\} : \Delta_4, F_3} \text{IH}}{\bullet h_2 \vdash \{\Delta_5, F_1, F_3\} : \Delta_4} ?_C \rightarrow$$

3 Measure of derivations

- Case(s) rule 1

$$\frac{}{\bullet h_1 \vdash \{\Delta_2\} : 1} \text{ 1} \rightarrow \frac{}{\bullet \bullet h_1 \vdash \{\Delta_2\} : 1} \text{ 1}$$

- Case(s) rule !

$$\frac{h_1 \vdash \{\Delta_2\} : F_3}{\bullet h_1 \vdash \{\Delta_2\} : !F_3} \text{ !} \rightarrow \frac{\frac{h_1 \vdash \{\Delta_2\} : F_3}{\bullet h_1 \vdash \{\Delta_2\} : F_3} \text{ IH}}{\bullet \bullet h_1 \vdash \{\Delta_2\} : !F_3} \text{ !} \text{ ax}$$

- Case(s) rule ?

$$\frac{h_1 \vdash \{F_4, \Delta_2\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, ?F_4} \text{ ?} \rightarrow \frac{\frac{h_1 \vdash \{\Delta_2, F_4\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_2, F_4\} : \Delta_3} \text{ IH}}{\bullet \bullet h_1 \vdash \{\Delta_2\} : \Delta_3, ?F_4} \text{ ?} \text{ ax}$$

- Case(s) rule \$

$$\frac{h_1 \vdash \{\Delta_2\} : F_4, F_5, \Delta_3}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4 \$F_5} \$ \rightarrow \frac{\frac{h_1 \vdash \{\Delta_2\} : \Delta_3, F_4, F_5}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4, F_5} \text{ IH}}{\bullet \bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4 \$F_5} \$ \text{ ax}$$

- Case(s) rule &

$$\frac{h_1 \vdash \{\Delta_2\} : F_4, \Delta_3 \quad h_1 \vdash \{\Delta_2\} : F_5, \Delta_3}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4 \&F_5} \& \rightarrow \frac{\frac{h_1 \vdash \{\Delta_2\} : \Delta_3, F_4}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4} \text{ IH} \quad \frac{h_1 \vdash \{\Delta_2\} : \Delta_3, F_5}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_5} \text{ IH}}{\bullet \bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4 \&F_5} \& \text{ ax}$$

- Case(s) rule \oplus_B

$$\frac{h_1 \vdash \{\Delta_2\} : F_5, \Delta_3}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4 \oplus F_5} \oplus_B \rightarrow \frac{\frac{h_1 \vdash \{\Delta_2\} : \Delta_3, F_5}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_5} \text{ IH}}{\bullet \bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4 \oplus F_5} \oplus_B \text{ ax}$$

- Case(s) rule \oplus_A

$$\frac{h_1 \vdash \{\Delta_2\} : F_4, \Delta_3}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4 \oplus F_5} \oplus_A \rightarrow \frac{\frac{h_1 \vdash \{\Delta_2\} : \Delta_3, F_4}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4} \text{ IH}}{\bullet \bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4 \oplus F_5} \oplus_A \text{ ax}$$

- Case(s) rule \perp

$$\frac{h_1 \vdash \{\Delta_2\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_2\} : \perp, \Delta_3} \perp \rightarrow \frac{\frac{h_1 \vdash \{\Delta_2\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3} \text{ IH}}{\bullet \bullet h_1 \vdash \{\Delta_2\} : \Delta_3, \perp} \perp \text{ ax}$$

- Case(s) rule \top

$$\frac{}{\bullet h_1 \vdash \{\Delta_2\} : \top, \Delta_3} \top \rightarrow \frac{}{\bullet \bullet h_1 \vdash \{\Delta_2\} : \Delta_3, \top} \top$$

- Case(s) rule I_3

$$\frac{}{\bullet h_1 \vdash \{\Delta_2\} : p(n_3), \wedge(n_3)} \text{ I}_3 \rightarrow \frac{}{\bullet \bullet h_1 \vdash \{\Delta_2\} : p(n_3), \wedge(n_3)} \text{ I}_3$$

- Case(s) rule \otimes

$$\frac{\frac{h_1 \vdash \{\Delta_2\} : F_5, \Delta_3 \quad h_1 \vdash \{\Delta_2\} : F_6, \Delta_4}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, \Delta_4, F_5 \otimes F_6} \otimes}{\bullet \bullet h_1 \vdash \{\Delta_2\} : \Delta_3, \Delta_4, F_5 \otimes F_6} \otimes \rightarrow \frac{\frac{h_1 \vdash \{\Delta_2\} : \Delta_3, F_5}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_5} \text{IH} \quad \frac{h_1 \vdash \{\Delta_2\} : \Delta_4, F_6}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_4, F_6} \text{IH}}{\bullet \bullet h_1 \vdash \{\Delta_2\} : \Delta_3, \Delta_4, F_5 \otimes F_6} \otimes} \otimes$$

- Case(s) rule I_1

$$\frac{}{\bullet h_1 \vdash \{\Delta_2, p(n_3)\} : \hat{\cdot}(n_3)} I_1 \rightarrow \frac{}{\bullet \bullet h_1 \vdash \{\Delta_2, p(n_3)\} : \hat{\cdot}(n_3)} I_1$$

- Case(s) rule I_2

$$\frac{}{\bullet h_1 \vdash \{\Delta_2, \hat{\cdot}(n_3)\} : p(n_3)} I_2 \rightarrow \frac{}{\bullet \bullet h_1 \vdash \{\Delta_2, \hat{\cdot}(n_3)\} : p(n_3)} I_2$$

- Case(s) rule $?_C$

$$\frac{\frac{h_1 \vdash \{F_2, \Delta_3\} : F_2, \Delta_4}{\bullet h_1 \vdash \{F_2, \Delta_3\} : \Delta_4} ?_C}{\bullet \bullet h_1 \vdash \{\Delta_3, F_2\} : \Delta_4, F_2} \text{IH} \rightarrow \frac{\frac{h_1 \vdash \{\Delta_3, F_2\} : \Delta_4, F_2}{\bullet h_1 \vdash \{\Delta_3, F_2\} : \Delta_4, F_2} \text{IH}}{\bullet \bullet h_1 \vdash \{\Delta_3, F_2\} : \Delta_4} ?_C} \text{ax}$$

4 Invertibility of Rules

4.1 Status of $\mathbf{1}$: : Invertible

- Case rule $\mathbf{1}$

$$\frac{}{\bullet h_1 \vdash \{\Delta_2\} : \mathbf{1}} \mathbf{1} \rightarrow \text{trivial}$$

- Case rule $!$
- Case rule $?$
- Case rule $\$$
- Case rule $\&$
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I_3
- Case rule \otimes
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

$$\frac{h_1 \vdash \{F_2, \Delta_3\} : \mathbf{1}, F_2}{\bullet h_1 \vdash \{F_2, \Delta_3\} : \mathbf{1}} ?_C \rightarrow \text{trivial}$$

4.2 Status of !: : Invertible

- Case rule 1
- Case rule !

$$\frac{h_1 \vdash \{\Delta_2\} : F_3}{\bullet h_1 \vdash \{\Delta_2\} :!F_3} ! \rightarrow \frac{\overline{h_1 \vdash \{\Delta_2\} : F_3}^{\text{ax}}}{\bullet h_1 \vdash \{\Delta_2\} : F_3} \text{H}$$

- Case rule ?
- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I_3
- Case rule \otimes
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

$$\frac{h_2 \vdash \{F_3, \Delta_4\} : F_3, !F_1}{\bullet h_2 \vdash \{F_3, \Delta_4\} :!F_1} ?_C \rightarrow \frac{\overline{h_2 \vdash \{\Delta_4, F_3\} : F_3, !F_1}^{\text{ax}}}{\bullet h_2 \vdash \{\Delta_4, F_3\} :!F_1} ?_C \quad \text{WB}$$

4.3 Status of ?: : Invertible

- Case rule 1
- Case rule !

- Case rule ?

$$\frac{h_2 \text{ :- } \{F_4, \Delta_3\} : \Delta_5, ?F_1}{\bullet h_2 \text{ :- } \{\Delta_3\} : (\Delta_5, ?F_1), ?F_4} ? \rightarrow \frac{\overline{h_2 \text{ :- } \{\Delta_3, F_1, F_4\} : \Delta_5}}{\bullet h_2 \text{ :- } \{\Delta_3, F_1\} : \Delta_5, ?F_4} \text{ ax/ind} ?$$

$$\frac{h_1 \text{ :- } \{F_4, \Delta_2\} : \Delta_3}{\bullet h_1 \text{ :- } \{\Delta_2\} : \Delta_3, ?F_4} ? \rightarrow \frac{\overline{h_1 \text{ :- } \{\Delta_2, F_4\} : \Delta_3}}{\bullet h_1 \text{ :- } \{\Delta_2, F_4\} : \Delta_3} \text{ ax} \text{ H}$$

- Case rule \$

$$\frac{h_2 \text{ :- } \{\Delta_3\} : F_4, F_5, \Delta_6, ?F_1}{\bullet h_2 \text{ :- } \{\Delta_3\} : (\Delta_6, ?F_1), F_4 \& F_5} \$ \rightarrow \frac{\overline{h_2 \text{ :- } \{\Delta_3, F_1\} : \Delta_6, F_4, F_5}}{\bullet h_2 \text{ :- } \{\Delta_3, F_1\} : \Delta_6, F_4 \& F_5} \text{ ax/ind} \$$$

- Case rule &

$$\frac{h_2 \text{ :- } \{\Delta_3\} : F_4, \Delta_6, ?F_1 \quad h_2 \text{ :- } \{\Delta_3\} : F_5, \Delta_6, ?F_1}{\bullet h_2 \text{ :- } \{\Delta_3\} : (\Delta_6, ?F_1), F_4 \& F_5} \& \rightarrow \frac{\overline{h_2 \text{ :- } \{\Delta_3, F_1\} : \Delta_6, F_4} \text{ ax/ind} \quad \overline{h_2 \text{ :- } \{\Delta_3, F_1\} : \Delta_6, F_5} \text{ ax/ind}}{\bullet h_2 \text{ :- } \{\Delta_3, F_1\} : \Delta_6, F_4 \& F_5} \&$$

- Case rule \oplus_B

$$\frac{h_2 \text{ :- } \{\Delta_3\} : F_5, \Delta_6, ?F_1}{\bullet h_2 \text{ :- } \{\Delta_3\} : (\Delta_6, ?F_1), F_4 \oplus F_5} \oplus_B \rightarrow \frac{\overline{h_2 \text{ :- } \{\Delta_3, F_1\} : \Delta_6, F_5} \text{ ax/ind}}{\bullet h_2 \text{ :- } \{\Delta_3, F_1\} : \Delta_6, F_4 \oplus F_5} \oplus_B$$

- Case rule \oplus_A

$$\frac{h_2 \text{ :- } \{\Delta_3\} : F_4, \Delta_6, ?F_1}{\bullet h_2 \text{ :- } \{\Delta_3\} : (\Delta_6, ?F_1), F_4 \oplus F_5} \oplus_A \rightarrow \frac{\overline{h_2 \text{ :- } \{\Delta_3, F_1\} : \Delta_6, F_4} \text{ ax/ind}}{\bullet h_2 \text{ :- } \{\Delta_3, F_1\} : \Delta_6, F_4 \oplus F_5} \oplus_A$$

- Case rule \perp

$$\frac{h_2 \text{ :- } \{\Delta_3\} : \Delta_4, ?F_1}{\bullet h_2 \text{ :- } \{\Delta_3\} : \perp, \Delta_4, ?F_1} \perp \rightarrow \frac{\overline{h_2 \text{ :- } \{\Delta_3, F_1\} : \Delta_4} \text{ ax/ind}}{\bullet h_2 \text{ :- } \{\Delta_3, F_1\} : \Delta_4, \perp} \perp$$

- Case rule \top

$$\frac{\overline{\bullet h_2 \text{ :- } \{\Delta_3\} : \top, \Delta_4, ?F_1}}{\bullet h_2 \text{ :- } \{\Delta_3, F_1\} : \Delta_4, \top} \top \rightarrow \frac{\overline{\bullet h_2 \text{ :- } \{\Delta_3, F_1\} : \Delta_4, \top}}{\bullet h_2 \text{ :- } \{\Delta_3, F_1\} : \Delta_4, \top} \top$$

- Case rule I_3

- Case rule \otimes

$$\frac{h_2 \text{ :- } \{\Delta_3\} : F_5, \Delta_7, ?F_1 \quad h_2 \text{ :- } \{\Delta_3\} : F_6, \Delta_4}{\bullet h_2 \text{ :- } \{\Delta_3\} : (\Delta_7, ?F_1), \Delta_4, F_5 \otimes F_6} \otimes \rightarrow \frac{\overline{h_2 \text{ :- } \{\Delta_3, F_1\} : \Delta_7, F_5} \text{ ax/ind} \quad \overline{h_2 \text{ :- } \{\Delta_3\} : \Delta_4, F_6} \text{ ax}}{\bullet h_2 \text{ :- } \{\Delta_3, F_1\} : \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \text{ W}$$

$$\frac{h_2 \text{ :- } \{\Delta_3\} : F_5, \Delta_4 \quad h_2 \text{ :- } \{\Delta_3\} : F_6, \Delta_7, ?F_1}{\bullet h_2 \text{ :- } \{\Delta_3\} : \Delta_4, (\Delta_7, ?F_1), F_5 \otimes F_6} \otimes \rightarrow \frac{\overline{h_2 \text{ :- } \{\Delta_3\} : \Delta_4, F_5} \text{ ax}}{\bullet h_2 \text{ :- } \{\Delta_3, F_1\} : \Delta_4, F_5} \text{ W} \quad \frac{\overline{h_2 \text{ :- } \{\Delta_3, F_1\} : \Delta_7, F_6} \text{ ax/ind}}{\bullet h_2 \text{ :- } \{\Delta_3, F_1\} : \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \text{ ax/ind}$$

- Case rule I_1
- Case rule I_2
- Case rule $?_C$

$$\frac{h_3 \vdash \{F_4, \Delta_5\} : F_4, \Delta_1, ?F_2}{\bullet h_3 \vdash \{F_4, \Delta_5\} : \Delta_1, ?F_2} ?_C \rightarrow \frac{\overline{h_3 \vdash \{\Delta_5, F_2, F_4\} : \Delta_1, F_4}}{\bullet h_3 \vdash \{\Delta_5, F_2, F_4\} : \Delta_1} \text{ax/ind} ?_C$$

4.4 Status of $\$$: : Invertible

- Case rule $\mathbf{1}$
- Case rule $!$
- Case rule $?$

$$\frac{h_3 \vdash \{F_5, \Delta_4\} : \Delta_6, F_1 \$F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_6, F_1 \$F_2), ?F_5} ? \rightarrow \frac{\overline{h_3 \vdash \{\Delta_4, F_5\} : \Delta_6, F_1, F_2}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_6, F_1, F_2, ?F_5} \text{ax/ind} ?$$

- Case rule $\$$

$$\frac{h_3 \vdash \{\Delta_4\} : F_5, F_6, \Delta_7, F_1 \$F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, F_1 \$F_2), F_5 \$F_6} \$ \rightarrow \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_2, F_5, F_6}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_2, F_5 \$F_6} \text{ax/ind} \$$$

$$\frac{h_1 \vdash \{\Delta_2\} : F_4, F_5, \Delta_3}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4 \$F_5} \$ \rightarrow \frac{\overline{h_1 \vdash \{\Delta_2\} : \Delta_3, F_4, F_5}}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4, F_5} \text{ax} \text{H}$$

- Case rule $\&$

$$\frac{h_3 \vdash \{\Delta_4\} : F_5, \Delta_7, F_1 \$F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, F_1 \$F_2), F_5 \&F_6} \& \rightarrow \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_2, F_5}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_2, F_5 \&F_6} \text{ax/ind} \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_2, F_6}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_2, F_5 \&F_6} \text{ax/ind} \&$$

- Case rule \oplus_B

$$\frac{h_3 \vdash \{\Delta_4\} : F_6, \Delta_7, F_1 \$F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, F_1 \$F_2), F_5 \oplus F_6} \oplus_B \rightarrow \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_2, F_6}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_2, F_5 \oplus F_6} \text{ax/ind} \oplus_B$$

- Case rule \oplus_A

$$\frac{h_3 \vdash \{\Delta_4\} : F_5, \Delta_7, F_1 \$F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, F_1 \$F_2), F_5 \oplus F_6} \oplus_A \rightarrow \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_2, F_5}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_2, F_5 \oplus F_6} \text{ax/ind} \oplus_A$$

- Case rule \perp

$$\frac{h_3 \vdash \{\Delta_4\} : \Delta_5, F_1 \S F_2}{\bullet h_3 \vdash \{\Delta_4\} : \perp, \Delta_5, F_1 \S F_2} \perp \rightarrow \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_5, F_1, F_2} \text{ ax/ind}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, F_1, F_2, \perp} \perp$$

- Case rule \top

$$\frac{}{\bullet h_3 \vdash \{\Delta_4\} : \top, \Delta_5, F_1 \S F_2} \top \rightarrow \frac{}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, F_1, F_2, \top} \top$$

- Case rule I_3

- Case rule \otimes

$$\frac{h_3 \vdash \{\Delta_4\} : F_6, \Delta_8, F_1 \S F_2 \quad h_3 \vdash \{\Delta_4\} : F_7, \Delta_5}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_8, F_1 \S F_2), \Delta_5, F_6 \otimes F_7} \otimes \rightarrow \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_8, F_1, F_2, F_6} \text{ ax/ind} \quad \overline{h_3 \vdash \{\Delta_4\} : \Delta_5, F_7} \text{ ax}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_1, F_2, F_6 \otimes F_7} \otimes$$

$$\frac{h_3 \vdash \{\Delta_4\} : F_6, \Delta_5 \quad h_3 \vdash \{\Delta_4\} : F_7, \Delta_8, F_1 \S F_2}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, (\Delta_8, F_1 \S F_2), F_6 \otimes F_7} \otimes \rightarrow \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_5, F_6} \text{ ax} \quad \overline{h_3 \vdash \{\Delta_4\} : \Delta_8, F_1, F_2, F_7} \text{ ax/ind}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_1, F_2, F_6 \otimes F_7} \otimes$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\frac{h_4 \vdash \{F_5, \Delta_6\} : F_5, \Delta_1, F_2 \S F_3}{\bullet h_4 \vdash \{F_5, \Delta_6\} : \Delta_1, F_2 \S F_3} ?_C \rightarrow \frac{\overline{h_4 \vdash \{\Delta_6, F_5\} : \Delta_1, F_2, F_3, F_5} \text{ ax/ind}}{\bullet h_4 \vdash \{\Delta_6, F_5\} : \Delta_1, F_2, F_3} ?_C$$

4.5 Status of $\&$: (Left Premise): Invertible

- Case rule 1

- Case rule !

- Case rule ?

$$\frac{h_3 \vdash \{F_5, \Delta_4\} : \Delta_6, F_1 \& F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_6, F_1 \& F_2), ?F_5} ? \rightarrow \frac{\overline{h_3 \vdash \{\Delta_4, F_5\} : \Delta_6, F_1} \text{ ax/ind}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_6, F_1, ?F_5} ?$$

- Case rule $\$$

$$\frac{h_3 \vdash \{\Delta_4\} : F_5, F_6, \Delta_7, F_1 \& F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, F_1 \& F_2), F_5 \S F_6} \$ \rightarrow \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5, F_6} \text{ ax/ind}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5 \S F_6} \$$$

- Case rule $\&$

$$\frac{\frac{h_3 \vdash \{\Delta_4\} : F_5, \Delta_7, F_1 \& F_2 \quad h_3 \vdash \{\Delta_4\} : F_6, \Delta_7, F_1 \& F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, F_1 \& F_2), F_5 \& F_6} \&}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5} \text{ax/ind} \quad \frac{h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5 \& F_6} \text{ax/ind} \&$$

$$\frac{\frac{h_1 \vdash \{\Delta_2\} : F_4, \Delta_3 \quad h_1 \vdash \{\Delta_2\} : F_5, \Delta_3}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4 \& F_5} \&}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4} \text{H} \quad \frac{h_1 \vdash \{\Delta_2\} : \Delta_3, F_4}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4} \text{ax}$$

- Case rule \oplus_B

$$\frac{\frac{h_3 \vdash \{\Delta_4\} : F_6, \Delta_7, F_1 \& F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, F_1 \& F_2), F_5 \oplus F_6} \oplus_B}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5 \oplus F_6} \text{ax/ind} \oplus_B$$

- Case rule \oplus_A

$$\frac{\frac{h_3 \vdash \{\Delta_4\} : F_5, \Delta_7, F_1 \& F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, F_1 \& F_2), F_5 \oplus F_6} \oplus_A}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5 \oplus F_6} \text{ax/ind} \oplus_A$$

- Case rule \perp

$$\frac{\frac{h_3 \vdash \{\Delta_4\} : \Delta_5, F_1 \& F_2}{\bullet h_3 \vdash \{\Delta_4\} : \perp, \Delta_5, F_1 \& F_2} \perp}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, F_1, \perp} \text{ax/ind} \perp$$

- Case rule \top

$$\frac{\bullet h_3 \vdash \{\Delta_4\} : \top, \Delta_5, F_1 \& F_2}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, F_1, \top} \top$$

- Case rule I_3

- Case rule \otimes

$$\frac{\frac{h_3 \vdash \{\Delta_4\} : F_6, \Delta_8, F_1 \& F_2 \quad h_3 \vdash \{\Delta_4\} : F_7, \Delta_5}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_8, F_1 \& F_2), \Delta_5, F_6 \otimes F_7} \otimes}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_1, F_6 \otimes F_7} \text{ax/ind} \otimes$$

$$\frac{\frac{h_3 \vdash \{\Delta_4\} : F_6, \Delta_5 \quad h_3 \vdash \{\Delta_4\} : F_7, \Delta_8, F_1 \& F_2}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, (\Delta_8, F_1 \& F_2), F_6 \otimes F_7} \otimes}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_1, F_6 \otimes F_7} \text{ax} \otimes$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\frac{\frac{h_4 \vdash \{F_5, \Delta_6\} : F_5, \Delta_1, F_2 \& F_3}{\bullet h_4 \vdash \{F_5, \Delta_6\} : \Delta_1, F_2 \& F_3} ?_C}{\bullet h_4 \vdash \{\Delta_6, F_5\} : \Delta_1, F_2} \text{ax/ind} ?_C$$

4.6 Status of & (Right Premise): : Invertible

- Case rule 1
- Case rule !
- Case rule ?

$$\frac{h_3 \vdash \{F_5, \Delta_4\} : \Delta_6, F_1 \& F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_6, F_1 \& F_2), ?F_5} ? \rightarrow \frac{\overline{h_3 \vdash \{\Delta_4, F_5\} : \Delta_6, F_2}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_6, F_2, ?F_5} \text{ax/ind} ?$$

- Case rule \$

$$\frac{h_3 \vdash \{\Delta_4\} : F_5, F_6, \Delta_7, F_1 \& F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, F_1 \& F_2), F_5 \$ F_6} \$ \rightarrow \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_7, F_2, F_5, F_6}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_2, F_5 \$ F_6} \text{ax/ind} \$$$

- Case rule &

$$\frac{\frac{h_3 \vdash \{\Delta_4\} : F_5, \Delta_7, F_1 \& F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, F_1 \& F_2), F_5 \& F_6} \& \quad h_3 \vdash \{\Delta_4\} : F_6, \Delta_7, F_1 \& F_2}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_2, F_5 \& F_6} \& \rightarrow \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_7, F_2, F_5} \text{ax/ind}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_2, F_5 \& F_6} \& \quad \frac{h_3 \vdash \{\Delta_4\} : \Delta_7, F_2, F_6}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_2, F_5 \& F_6} \text{ax/ind} \&$$

$$\frac{\frac{h_1 \vdash \{\Delta_2\} : F_4, \Delta_3 \quad h_1 \vdash \{\Delta_2\} : F_5, \Delta_3}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4 \& F_5} \&}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_5} \text{ax} \rightarrow \frac{\overline{h_1 \vdash \{\Delta_2\} : \Delta_3, F_5}}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_5} \text{H}$$

- Case rule \oplus_B

$$\frac{h_3 \vdash \{\Delta_4\} : F_6, \Delta_7, F_1 \& F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, F_1 \& F_2), F_5 \oplus F_6} \oplus_B \rightarrow \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_7, F_2, F_6} \text{ax/ind}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_2, F_5 \oplus F_6} \oplus_B$$

- Case rule \oplus_A

$$\frac{h_3 \vdash \{\Delta_4\} : F_5, \Delta_7, F_1 \& F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, F_1 \& F_2), F_5 \oplus F_6} \oplus_A \rightarrow \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_7, F_2, F_5} \text{ax/ind}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_2, F_5 \oplus F_6} \oplus_A$$

- Case rule \perp

$$\frac{h_3 \vdash \{\Delta_4\} : \Delta_5, F_1 \& F_2}{\bullet h_3 \vdash \{\Delta_4\} : \perp, \Delta_5, F_1 \& F_2} \perp \rightarrow \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_5, F_2} \text{ax/ind}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, F_2, \perp} \perp$$

- Case rule \top

$$\frac{\overline{\bullet h_3 \vdash \{\Delta_4\} : \top, \Delta_5, F_1 \& F_2} \top}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, F_2, \top} \top \rightarrow \frac{\overline{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, F_2} \top}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, F_2, \top} \top$$

- Case rule I_3

- Case rule \otimes

$$\frac{h_3 \vdash \{\Delta_4\} : F_6, \Delta_8, F_1 \& F_2 \quad h_3 \vdash \{\Delta_4\} : F_7, \Delta_5}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_8, F_1 \& F_2), \Delta_5, F_6 \otimes F_7} \otimes \rightarrow \frac{h_3 \vdash \{\Delta_4\} : \Delta_8, F_2, F_6 \quad \text{ax/ind} \quad h_3 \vdash \{\Delta_4\} : \Delta_5, F_7}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_2, F_6 \otimes F_7} \otimes$$

$$\frac{h_3 \vdash \{\Delta_4\} : F_6, \Delta_5 \quad h_3 \vdash \{\Delta_4\} : F_7, \Delta_8, F_1 \& F_2}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, (\Delta_8, F_1 \& F_2), F_6 \otimes F_7} \otimes \rightarrow \frac{h_3 \vdash \{\Delta_4\} : \Delta_5, F_6 \quad \text{ax} \quad h_3 \vdash \{\Delta_4\} : \Delta_8, F_2, F_7}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_2, F_6 \otimes F_7} \otimes \text{ax/ind}$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\frac{h_4 \vdash \{F_5, \Delta_6\} : F_5, \Delta_1, F_2 \& F_3}{\bullet h_4 \vdash \{F_5, \Delta_6\} : \Delta_1, F_2 \& F_3} ?_C \rightarrow \frac{h_4 \vdash \{\Delta_6, F_5\} : \Delta_1, F_3, F_5}{\bullet h_4 \vdash \{\Delta_6, F_5\} : \Delta_1, F_3} \text{ax/ind} ?_C$$

4.7 Status of \oplus_B : : Non invertible

- Case rule 1

- Case rule !

- Case rule ?

$$\frac{h_3 \vdash \{F_5, \Delta_4\} : \Delta_6, F_1 \oplus F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_6, F_1 \oplus F_2), ?F_5} ? \rightarrow \frac{h_3 \vdash \{\Delta_4, F_5\} : \Delta_6, F_2}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_6, F_2, ?F_5} \text{ax/ind} ?$$

- Case rule \$

$$\frac{h_3 \vdash \{\Delta_4\} : F_5, F_6, \Delta_7, F_1 \oplus F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, F_1 \oplus F_2), F_5 \$ F_6} \$ \rightarrow \frac{h_3 \vdash \{\Delta_4\} : \Delta_7, F_2, F_5, F_6}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_2, F_5 \$ F_6} \text{ax/ind} \$$$

- Case rule &

$$\frac{h_3 \vdash \{\Delta_4\} : F_5, \Delta_7, F_1 \oplus F_2 \quad h_3 \vdash \{\Delta_4\} : F_6, \Delta_7, F_1 \oplus F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, F_1 \oplus F_2), F_5 \& F_6} \& \rightarrow \frac{h_3 \vdash \{\Delta_4\} : \Delta_7, F_2, F_5 \quad \text{ax/ind} \quad h_3 \vdash \{\Delta_4\} : \Delta_7, F_2, F_6}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_2, F_5 \& F_6} \& \text{ax/ind}$$

- Case rule \oplus_B

$$\frac{h_3 \vdash \{\Delta_4\} : F_6, \Delta_7, F_1 \oplus F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, F_1 \oplus F_2), F_5 \oplus F_6} \oplus_B \rightarrow \frac{h_3 \vdash \{\Delta_4\} : \Delta_7, F_2, F_6}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_2, F_5 \oplus F_6} \oplus_B \text{ax/ind}$$

$$\frac{h_1 \vdash \{\Delta_2\} : F_5, \Delta_3}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4 \oplus F_5} \oplus_B \rightarrow \frac{h_1 \vdash \{\Delta_2\} : \Delta_3, F_5}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_5} \text{ax} \text{H}$$

- Case rule \oplus_A

$$\frac{h_3 \text{ :- } \{\Delta_4\} : F_5, \Delta_7, F_1 \oplus F_2}{\bullet h_3 \text{ :- } \{\Delta_4\} : (\Delta_7, F_1 \oplus F_2), F_5 \oplus F_6} \oplus_A \rightarrow \frac{h_3 \text{ :- } \{\Delta_4\} : \Delta_7, F_2, F_5}{\bullet h_3 \text{ :- } \{\Delta_4\} : \Delta_7, F_2, F_5 \oplus F_6} \text{ ax/ind} \oplus_A$$

$$\frac{h_1 \text{ :- } \{\Delta_2\} : F_4, \Delta_3}{\bullet h_1 \text{ :- } \{\Delta_2\} : \Delta_3, F_4 \oplus F_5} \oplus_A \rightarrow \bullet h_1 \text{ :- } \{\Delta_2\} : F_5, \Delta_3 \text{ fail}$$

- Case rule \perp

$$\frac{h_3 \text{ :- } \{\Delta_4\} : \Delta_5, F_1 \oplus F_2}{\bullet h_3 \text{ :- } \{\Delta_4\} : \perp, \Delta_5, F_1 \oplus F_2} \perp \rightarrow \frac{h_3 \text{ :- } \{\Delta_4\} : \Delta_5, F_2}{\bullet h_3 \text{ :- } \{\Delta_4\} : \Delta_5, F_2, \perp} \text{ ax/ind} \perp$$

- Case rule \top

$$\frac{}{\bullet h_3 \text{ :- } \{\Delta_4\} : \top, \Delta_5, F_1 \oplus F_2} \top \rightarrow \frac{}{\bullet h_3 \text{ :- } \{\Delta_4\} : \Delta_5, F_2, \top} \top$$

- Case rule I_3

- Case rule \otimes

$$\frac{h_3 \text{ :- } \{\Delta_4\} : F_6, \Delta_8, F_1 \oplus F_2 \quad h_3 \text{ :- } \{\Delta_4\} : F_7, \Delta_5}{\bullet h_3 \text{ :- } \{\Delta_4\} : (\Delta_8, F_1 \oplus F_2), \Delta_5, F_6 \otimes F_7} \otimes \rightarrow \frac{h_3 \text{ :- } \{\Delta_4\} : \Delta_8, F_2, F_6}{\bullet h_3 \text{ :- } \{\Delta_4\} : \Delta_5, \Delta_8, F_2, F_6 \otimes F_7} \text{ ax/ind} \frac{h_3 \text{ :- } \{\Delta_4\} : \Delta_5, F_7}{\bullet h_3 \text{ :- } \{\Delta_4\} : \Delta_5, F_7} \text{ ax} \otimes$$

$$\frac{h_3 \text{ :- } \{\Delta_4\} : F_6, \Delta_5 \quad h_3 \text{ :- } \{\Delta_4\} : F_7, \Delta_8, F_1 \oplus F_2}{\bullet h_3 \text{ :- } \{\Delta_4\} : \Delta_5, (\Delta_8, F_1 \oplus F_2), F_6 \otimes F_7} \otimes \rightarrow \frac{h_3 \text{ :- } \{\Delta_4\} : \Delta_5, F_6}{\bullet h_3 \text{ :- } \{\Delta_4\} : \Delta_5, \Delta_8, F_2, F_6 \otimes F_7} \text{ ax} \frac{h_3 \text{ :- } \{\Delta_4\} : \Delta_8, F_2, F_7}{\bullet h_3 \text{ :- } \{\Delta_4\} : \Delta_8, F_2, F_7} \text{ ax/ind} \otimes$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\frac{h_4 \text{ :- } \{F_5, \Delta_6\} : F_5, \Delta_1, F_2 \oplus F_3}{\bullet h_4 \text{ :- } \{F_5, \Delta_6\} : \Delta_1, F_2 \oplus F_3} ?_C \rightarrow \frac{h_4 \text{ :- } \{\Delta_6, F_5\} : \Delta_1, F_3, F_5}{\bullet h_4 \text{ :- } \{\Delta_6, F_5\} : \Delta_1, F_3} \text{ ax/ind} ?_C$$

4.8 Status of \oplus_A : Non invertible

- Case rule 1

- Case rule !

- Case rule ?

$$\frac{h_3 \text{ :- } \{F_5, \Delta_4\} : \Delta_6, F_1 \oplus F_2}{\bullet h_3 \text{ :- } \{\Delta_4\} : (\Delta_6, F_1 \oplus F_2), ?F_5} ? \rightarrow \frac{h_3 \text{ :- } \{\Delta_4, F_5\} : \Delta_6, F_1}{\bullet h_3 \text{ :- } \{\Delta_4\} : \Delta_6, F_1, ?F_5} \text{ ax/ind} ?$$

- Case rule \$

$$\frac{h_3 \vdash \{\Delta_4\} : F_5, F_6, \Delta_7, F_1 \oplus F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, F_1 \oplus F_2), F_5 \& F_6} \$ \rightarrow \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5, F_6} \text{ ax/ind}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5 \& F_6} \$$$

- Case rule &

$$\frac{h_3 \vdash \{\Delta_4\} : F_5, \Delta_7, F_1 \oplus F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, F_1 \oplus F_2), F_5 \& F_6} \& \frac{h_3 \vdash \{\Delta_4\} : F_6, \Delta_7, F_1 \oplus F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, F_1 \oplus F_2), F_5 \& F_6} \& \rightarrow \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5} \text{ ax/ind}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5 \& F_6} \& \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_6} \text{ ax/ind}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5 \& F_6} \&$$

- Case rule \oplus_B

$$\frac{h_3 \vdash \{\Delta_4\} : F_6, \Delta_7, F_1 \oplus F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, F_1 \oplus F_2), F_5 \oplus F_6} \oplus_B \rightarrow \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_6} \text{ ax/ind}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5 \oplus F_6} \oplus_B$$

$$\frac{h_1 \vdash \{\Delta_2\} : F_5, \Delta_3}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4 \oplus F_5} \oplus_B \rightarrow \frac{\overline{h_1 \vdash \{\Delta_2\} : F_4, \Delta_3} \text{ fail}}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4 \oplus F_5} \oplus_B$$

- Case rule \oplus_A

$$\frac{h_3 \vdash \{\Delta_4\} : F_5, \Delta_7, F_1 \oplus F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, F_1 \oplus F_2), F_5 \oplus F_6} \oplus_A \rightarrow \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5} \text{ ax/ind}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_7, F_1, F_5 \oplus F_6} \oplus_A$$

$$\frac{h_1 \vdash \{\Delta_2\} : F_4, \Delta_3}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4 \oplus F_5} \oplus_A \rightarrow \frac{\overline{h_1 \vdash \{\Delta_2\} : \Delta_3, F_4} \text{ ax}}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, F_4} \text{ H}$$

- Case rule \perp

$$\frac{h_3 \vdash \{\Delta_4\} : \Delta_5, F_1 \oplus F_2}{\bullet h_3 \vdash \{\Delta_4\} : \perp, \Delta_5, F_1 \oplus F_2} \perp \rightarrow \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_5, F_1} \text{ ax/ind}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, F_1, \perp} \perp$$

- Case rule \top

$$\frac{}{\bullet h_3 \vdash \{\Delta_4\} : \top, \Delta_5, F_1 \oplus F_2} \top \rightarrow \frac{}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, F_1, \top} \top$$

- Case rule I_3

- Case rule \otimes

$$\frac{h_3 \vdash \{\Delta_4\} : F_6, \Delta_8, F_1 \oplus F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_8, F_1 \oplus F_2), \Delta_5, F_6 \otimes F_7} \otimes \frac{h_3 \vdash \{\Delta_4\} : F_7, \Delta_5}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_8, F_1 \oplus F_2), \Delta_5, F_6 \otimes F_7} \otimes \rightarrow \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_8, F_1, F_6} \text{ ax/ind}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_1, F_6 \otimes F_7} \otimes \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_5, F_7} \text{ ax}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_1, F_6 \otimes F_7} \otimes$$

$$\frac{h_3 \vdash \{\Delta_4\} : F_6, \Delta_5}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, (\Delta_8, F_1 \oplus F_2), F_6 \otimes F_7} \otimes \frac{h_3 \vdash \{\Delta_4\} : F_7, \Delta_8, F_1 \oplus F_2}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, (\Delta_8, F_1 \oplus F_2), F_6 \otimes F_7} \otimes \rightarrow \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_5, F_6} \text{ ax}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_1, F_6 \otimes F_7} \otimes \frac{\overline{h_3 \vdash \{\Delta_4\} : \Delta_8, F_1, F_7} \text{ ax/ind}}{\bullet h_3 \vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_1, F_6 \otimes F_7} \otimes$$

- Case rule I_1

- Case rule I_2
- Case rule $?_C$

$$\frac{h_4 : \vdash \{F_5, \Delta_6\} : F_5, \Delta_1, F_2 \oplus F_3}{\bullet h_4 : \vdash \{F_5, \Delta_6\} : \Delta_1, F_2 \oplus F_3} ?_C \rightarrow \frac{\overline{h_4 : \vdash \{\Delta_6, F_5\} : \Delta_1, F_2, F_5}}{ax/ind} ?_C$$

4.9 Status of \perp : : Invertible

- Case rule 1
- Case rule !
- Case rule ?

$$\frac{h_1 : \vdash \{F_3, \Delta_2\} : \perp, \Delta_4}{\bullet h_1 : \vdash \{\Delta_2\} : (\perp, \Delta_4), ?F_3} ? \rightarrow \frac{\overline{h_1 : \vdash \{\Delta_2, F_3\} : \Delta_4}}{ax/ind} ?$$

- Case rule \$

$$\frac{h_1 : \vdash \{\Delta_2\} : \perp, F_3, F_4, \Delta_5}{\bullet h_1 : \vdash \{\Delta_2\} : (\perp, \Delta_5), F_3 \& F_4} \$ \rightarrow \frac{\overline{h_1 : \vdash \{\Delta_2\} : \Delta_5, F_3, F_4}}{ax/ind} \$$$

- Case rule &

$$\frac{h_1 : \vdash \{\Delta_2\} : \perp, F_3, \Delta_5 \quad h_1 : \vdash \{\Delta_2\} : \perp, F_4, \Delta_5}{\bullet h_1 : \vdash \{\Delta_2\} : (\perp, \Delta_5), F_3 \& F_4} \& \rightarrow \frac{\overline{h_1 : \vdash \{\Delta_2\} : \Delta_5, F_3}}{ax/ind} \quad \overline{h_1 : \vdash \{\Delta_2\} : \Delta_5, F_4}}{ax/ind}}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_5, F_3 \& F_4} \&$$

- Case rule \oplus_B

$$\frac{h_1 : \vdash \{\Delta_2\} : \perp, F_4, \Delta_5}{\bullet h_1 : \vdash \{\Delta_2\} : (\perp, \Delta_5), F_3 \oplus F_4} \oplus_B \rightarrow \frac{\overline{h_1 : \vdash \{\Delta_2\} : \Delta_5, F_4}}{ax/ind}}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_5, F_3 \oplus F_4} \oplus_B$$

- Case rule \oplus_A

$$\frac{h_1 : \vdash \{\Delta_2\} : \perp, F_3, \Delta_5}{\bullet h_1 : \vdash \{\Delta_2\} : (\perp, \Delta_5), F_3 \oplus F_4} \oplus_A \rightarrow \frac{\overline{h_1 : \vdash \{\Delta_2\} : \Delta_5, F_3}}{ax/ind}}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_5, F_3 \oplus F_4} \oplus_A$$

- Case rule \perp

$$\frac{h_1 : \vdash \{\Delta_2\} : \Delta_3}{\bullet h_1 : \vdash \{\Delta_2\} : \perp, \Delta_3} \perp \rightarrow \frac{\overline{h_1 : \vdash \{\Delta_2\} : \Delta_3}}{ax}}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3} H$$

- Case rule \top

$$\overline{\bullet h_1 : \vdash \{\Delta_2\} : \top, \perp, \Delta_3} \top \rightarrow \overline{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, \top} \top$$

- Case rule I_3

- Case rule \otimes

$$\frac{h_1 \vdash \{\Delta_2\} : \perp, F_4, \Delta_6 \quad h_1 \vdash \{\Delta_2\} : F_5, \Delta_3}{\bullet h_1 \vdash \{\Delta_2\} : (\perp, \Delta_6), \Delta_3, F_4 \otimes F_5} \otimes \rightarrow \frac{\frac{h_1 \vdash \{\Delta_2\} : \Delta_6, F_4}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, \Delta_6, F_4 \otimes F_5} \text{ax/ind} \quad \frac{h_1 \vdash \{\Delta_2\} : \Delta_3, F_5}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, \Delta_6, F_4 \otimes F_5} \text{ax}}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes$$

$$\frac{h_1 \vdash \{\Delta_2\} : F_4, \Delta_3 \quad h_1 \vdash \{\Delta_2\} : \perp, F_5, \Delta_6}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, (\perp, \Delta_6), F_4 \otimes F_5} \otimes \rightarrow \frac{\frac{h_1 \vdash \{\Delta_2\} : \Delta_3, F_4}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, \Delta_6, F_4 \otimes F_5} \text{ax} \quad \frac{h_1 \vdash \{\Delta_2\} : \Delta_6, F_5}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, \Delta_6, F_4 \otimes F_5} \text{ax/ind}}{\bullet h_1 \vdash \{\Delta_2\} : \Delta_3, \Delta_6, F_4 \otimes F_5} \otimes$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\frac{h_2 \vdash \{F_3, \Delta_4\} : \perp, F_3, \Delta_1}{\bullet h_2 \vdash \{F_3, \Delta_4\} : \perp, \Delta_1} ?_C \rightarrow \frac{\frac{h_2 \vdash \{\Delta_4, F_3\} : \Delta_1, F_3}{\bullet h_2 \vdash \{\Delta_4, F_3\} : \Delta_1} \text{ax/ind}}{\bullet h_2 \vdash \{\Delta_4, F_3\} : \Delta_1} ?_C$$

4.10 Status of \top : : Invertible

- Case rule 1

- Case rule !

- Case rule ?

$$\frac{h_1 \vdash \{F_3, \Delta_2\} : \top, \Delta_4}{\bullet h_1 \vdash \{\Delta_2\} : (\top, \Delta_4), ?F_3} ? \rightarrow \text{trivial}$$

- Case rule \$

$$\frac{h_1 \vdash \{\Delta_2\} : \top, F_3, F_4, \Delta_5}{\bullet h_1 \vdash \{\Delta_2\} : (\top, \Delta_5), F_3 \& F_4} \$ \rightarrow \text{trivial}$$

- Case rule &

$$\frac{h_1 \vdash \{\Delta_2\} : \top, F_3, \Delta_5 \quad h_1 \vdash \{\Delta_2\} : \top, F_4, \Delta_5}{\bullet h_1 \vdash \{\Delta_2\} : (\top, \Delta_5), F_3 \& F_4} \& \rightarrow \text{trivial}$$

- Case rule \oplus_B

$$\frac{h_1 \vdash \{\Delta_2\} : \top, F_4, \Delta_5}{\bullet h_1 \vdash \{\Delta_2\} : (\top, \Delta_5), F_3 \oplus F_4} \oplus_B \rightarrow \text{trivial}$$

- Case rule \oplus_A

$$\frac{h_1 : \vdash \{\Delta_2\} : \top, F_3, \Delta_5}{\bullet h_1 : \vdash \{\Delta_2\} : (\top, \Delta_5), F_3 \oplus F_4} \oplus_A \rightarrow \text{trivial}$$

- Case rule \perp

$$\frac{h_1 : \vdash \{\Delta_2\} : \top, \Delta_3}{\bullet h_1 : \vdash \{\Delta_2\} : \perp, \top, \Delta_3} \perp \rightarrow \text{trivial}$$

- Case rule \top

$$\frac{}{\bullet h_1 : \vdash \{\Delta_2\} : \top, \Delta_3} \top \rightarrow \text{trivial}$$

- Case rule I_3

- Case rule \otimes

$$\frac{h_1 : \vdash \{\Delta_2\} : \top, F_4, \Delta_6 \quad h_1 : \vdash \{\Delta_2\} : F_5, \Delta_3}{\bullet h_1 : \vdash \{\Delta_2\} : (\top, \Delta_6), \Delta_3, F_4 \otimes F_5} \otimes \rightarrow \text{trivial}$$

$$\frac{h_1 : \vdash \{\Delta_2\} : F_4, \Delta_3 \quad h_1 : \vdash \{\Delta_2\} : \top, F_5, \Delta_6}{\bullet h_1 : \vdash \{\Delta_2\} : \Delta_3, (\top, \Delta_6), F_4 \otimes F_5} \otimes \rightarrow \text{trivial}$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\frac{h_2 : \vdash \{F_3, \Delta_4\} : \top, F_3, \Delta_1}{\bullet h_2 : \vdash \{F_3, \Delta_4\} : \top, \Delta_1} ?_C \rightarrow \text{trivial}$$

4.11 Status of I_3 : : Invertible

- Case rule 1

- Case rule !

- Case rule ?

- Case rule \$

- Case rule &

- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I_3

$$\frac{}{\bullet h_1 \vdash \{\Delta_2\} : p(n_3), \hat{\cdot}(n_3)} I_3 \rightarrow \text{trivial}$$

- Case rule \otimes
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

$$\frac{h_2 \vdash \{F_3, \Delta_4\} : F_3, p(n_1), \hat{\cdot}(n_1)}{\bullet h_2 \vdash \{F_3, \Delta_4\} : p(n_1), \hat{\cdot}(n_1)} ?_C \rightarrow \text{trivial}$$

4.12 Status of \otimes : (Left Premise): Non invertible

- Case rule 1
- Case rule !
- Case rule ?

$$\frac{h_4 \vdash \{F_6, \Delta_5\} : \Delta_1, \Delta_7, F_2 \otimes F_3}{\bullet h_4 \vdash \{\Delta_5\} : (\Delta_1, \Delta_7, F_2 \otimes F_3), ?F_6} ? \rightarrow \frac{}{h_4 \vdash \{\Delta_5, F_6\} : \Delta_7, F_2} \text{ax/ind} \quad \frac{}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_7, F_2, ?F_6} ?$$

$$\frac{h_4 \vdash \{F_6, \Delta_5\} : \Delta_1, \Delta_7, F_2 \otimes F_3}{\bullet h_4 \vdash \{\Delta_5\} : (\Delta_1, \Delta_7, F_2 \otimes F_3), ?F_6} ? \rightarrow \frac{}{\bullet h_4 \vdash \{\Delta_5\} : F_2, \Delta_1} \text{fail}$$

- Case rule \$

$$\frac{h_4 \vdash \{\Delta_5\} : F_6, F_7, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 \vdash \{\Delta_5\} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \$F_7} \$ \rightarrow \frac{}{h_4 \vdash \{\Delta_5\} : \Delta_8, F_2, F_6, F_7} \text{ax/ind} \quad \frac{}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_8, F_2, F_6 \$F_7} \$$$

$$\frac{h_4 \vdash \{\Delta_5\} : F_6, F_7, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 \vdash \{\Delta_5\} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \$F_7} \$ \rightarrow \frac{}{h_4 \vdash \{\Delta_5\} : \Delta_1, F_2} \text{ax/ind} \quad \frac{}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_1, F_2} H$$

- Case rule $\&$

$$\frac{h_4 \vdash \{\Delta_5\} : F_6, \Delta_1, \Delta_8, F_2 \otimes F_3 \quad h_4 \vdash \{\Delta_5\} : F_7, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 \vdash \{\Delta_5\} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \& F_7} \& \rightarrow \frac{\frac{h_4 \vdash \{\Delta_5\} : \Delta_8, F_2, F_6}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_8, F_2, F_6 \& F_7} \text{ax/ind} \quad \frac{h_4 \vdash \{\Delta_5\} : \Delta_8, F_2, F_7}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_8, F_2, F_7} \text{ax/ind}}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_8, F_2, F_6 \& F_7} \&$$

$$\frac{h_4 \vdash \{\Delta_5\} : F_6, \Delta_1, \Delta_8, F_2 \otimes F_3 \quad h_4 \vdash \{\Delta_5\} : F_7, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 \vdash \{\Delta_5\} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \& F_7} \& \rightarrow \frac{\frac{h_4 \vdash \{\Delta_5\} : \Delta_1, F_2}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_1, F_2} \text{ax/ind}}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_1, F_2} \text{H}}$$

- Case rule \oplus_B

$$\frac{h_4 \vdash \{\Delta_5\} : F_7, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 \vdash \{\Delta_5\} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \oplus F_7} \oplus_B \rightarrow \frac{\frac{h_4 \vdash \{\Delta_5\} : \Delta_8, F_2, F_7}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_8, F_2, F_6 \oplus F_7} \text{ax/ind}}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_8, F_2, F_6 \oplus F_7} \oplus_B$$

$$\frac{h_4 \vdash \{\Delta_5\} : F_7, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 \vdash \{\Delta_5\} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \oplus F_7} \oplus_B \rightarrow \frac{\frac{h_4 \vdash \{\Delta_5\} : \Delta_1, F_2}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_1, F_2} \text{ax/ind}}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_1, F_2} \text{H}}$$

- Case rule \oplus_A

$$\frac{h_4 \vdash \{\Delta_5\} : F_6, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 \vdash \{\Delta_5\} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \oplus F_7} \oplus_A \rightarrow \frac{\frac{h_4 \vdash \{\Delta_5\} : \Delta_8, F_2, F_6}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_8, F_2, F_6 \oplus F_7} \text{ax/ind}}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_8, F_2, F_6 \oplus F_7} \oplus_A$$

$$\frac{h_4 \vdash \{\Delta_5\} : F_6, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 \vdash \{\Delta_5\} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \oplus F_7} \oplus_A \rightarrow \frac{\frac{h_4 \vdash \{\Delta_5\} : \Delta_1, F_2}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_1, F_2} \text{ax/ind}}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_1, F_2} \text{H}}$$

- Case rule \perp

$$\frac{h_4 \vdash \{\Delta_5\} : \Delta_1, \Delta_6, F_2 \otimes F_3}{\bullet h_4 \vdash \{\Delta_5\} : \perp, \Delta_1, \Delta_6, F_2 \otimes F_3} \perp \rightarrow \frac{\frac{h_4 \vdash \{\Delta_5\} : \Delta_6, F_2}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_6, F_2, \perp} \text{ax/ind}}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_6, F_2, \perp} \perp$$

$$\frac{h_4 \vdash \{\Delta_5\} : \Delta_1, \Delta_6, F_2 \otimes F_3}{\bullet h_4 \vdash \{\Delta_5\} : \perp, \Delta_1, \Delta_6, F_2 \otimes F_3} \perp \rightarrow \frac{\frac{h_4 \vdash \{\Delta_5\} : \Delta_1, F_2}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_1, F_2} \text{ax/ind}}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_1, F_2} \text{H}}$$

- Case rule \top

$$\frac{}{\bullet h_4 \vdash \{\Delta_5\} : \top, \Delta_1, \Delta_6, F_2 \otimes F_3} \top \rightarrow \frac{}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_6, F_2, \top} \top$$

$$\frac{}{\bullet h_4 \vdash \{\Delta_5\} : \top, \Delta_1, \Delta_6, F_2 \otimes F_3} \top \rightarrow \frac{}{\bullet h_4 \vdash \{\Delta_5\} : F_2, \Delta_1} \text{fail}$$

- Case rule I_3

- Case rule \otimes

$$\frac{h_3 \vdash \{\Delta_4\} : F_5, \Delta_7, \Delta_8, F_1 \otimes F_2 \quad h_3 \vdash \{\Delta_4\} : F_6, \Delta_9, \Delta_{10}}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, F_1 \otimes F_2), (\Delta_9, \Delta_{10}), F_5 \otimes F_6} \otimes \rightarrow \frac{}{\bullet h_3 \vdash \{\Delta_4\} : F_1, \Delta_7, \Delta_9, F_5 \otimes F_6} \text{fail}$$

$$\frac{h_3 \vdash \{\Delta_4\} : F_5, \Delta_7, \Delta_8 \quad h_3 \vdash \{\Delta_4\} : F_6, \Delta_9, \Delta_{10}, F_1 \otimes F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, F_1 \otimes F_2), F_5 \otimes F_6} \otimes \rightarrow \frac{}{\bullet h_3 \vdash \{\Delta_4\} : F_1, \Delta_7, \Delta_9, F_5 \otimes F_6} \text{fail}$$

$$\frac{h_3 \vdash \{\Delta_4\} : F_5, \Delta_7, \Delta_8, F_1 \otimes F_2 \quad h_3 \vdash \{\Delta_4\} : F_6, \Delta_9, \Delta_{10}}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, F_1 \otimes F_2), (\Delta_9, \Delta_{10}), F_5 \otimes F_6}} \otimes \rightarrow \frac{}{\bullet h_3 \vdash \{\Delta_4\} : F_1, \Delta_7, \Delta_9} \text{ fail}$$

$$\frac{h_3 \vdash \{\Delta_4\} : F_5, \Delta_7, \Delta_8 \quad h_3 \vdash \{\Delta_4\} : F_6, \Delta_9, \Delta_{10}, F_1 \otimes F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, F_1 \otimes F_2), F_5 \otimes F_6}} \otimes \rightarrow \frac{}{\bullet h_3 \vdash \{\Delta_4\} : F_1, \Delta_7, \Delta_9} \text{ fail}$$

$$\frac{h_1 \vdash \{\Delta_2\} : F_3, \Delta_5, \Delta_6 \quad h_1 \vdash \{\Delta_2\} : F_4, \Delta_7, \Delta_8}{\bullet h_1 \vdash \{\Delta_2\} : (\Delta_5, \Delta_6), (\Delta_7, \Delta_8), F_3 \otimes F_4}} \otimes \rightarrow \frac{}{\bullet h_1 \vdash \{\Delta_2\} : F_3, \Delta_5, \Delta_7} \text{ fail}$$

- Case rule I_1
- Case rule I_2
- Case rule $?_C$

$$\frac{h_5 \vdash \{F_6, \Delta_7\} : F_6, \Delta_1, \Delta_2, F_3 \otimes F_4}{\bullet h_5 \vdash \{F_6, \Delta_7\} : \Delta_1, \Delta_2, F_3 \otimes F_4}} ?_C \rightarrow \frac{h_5 \vdash \{\Delta_7, F_6\} : \Delta_1, F_3, F_6}{\bullet h_5 \vdash \{\Delta_7, F_6\} : \Delta_1, F_3}} \text{ ax/ind} \quad ?_C$$

4.13 Status of \otimes (Right Premise): : Non invertible

- Case rule 1
- Case rule !
- Case rule ?

$$\frac{h_4 \vdash \{F_6, \Delta_5\} : \Delta_1, \Delta_7, F_2 \otimes F_3}{\bullet h_4 \vdash \{\Delta_5\} : (\Delta_1, \Delta_7, F_2 \otimes F_3), ?F_6}} ? \rightarrow \frac{}{\bullet h_4 \vdash \{\Delta_5\} : F_3, \Delta_1} \text{ fail}$$

$$\frac{h_4 \vdash \{F_6, \Delta_5\} : \Delta_1, \Delta_7, F_2 \otimes F_3}{\bullet h_4 \vdash \{\Delta_5\} : (\Delta_1, \Delta_7, F_2 \otimes F_3), ?F_6}} ? \rightarrow \frac{h_4 \vdash \{\Delta_5, F_6\} : \Delta_7, F_3}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_7, F_3, ?F_6}} \text{ ax/ind} \quad ?$$

- Case rule \$

$$\frac{h_4 \vdash \{\Delta_5\} : F_6, F_7, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 \vdash \{\Delta_5\} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \$ F_7}} \$ \rightarrow \frac{h_4 \vdash \{\Delta_5\} : \Delta_1, F_3}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_1, F_3}} \text{ ax/ind} \quad H$$

$$\frac{h_4 \vdash \{\Delta_5\} : F_6, F_7, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 \vdash \{\Delta_5\} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \$ F_7}} \$ \rightarrow \frac{h_4 \vdash \{\Delta_5\} : \Delta_8, F_3, F_6, F_7}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_8, F_3, F_6 \$ F_7}} \text{ ax/ind} \quad \$$$

- Case rule &

$$\frac{h_4 \vdash \{\Delta_5\} : F_6, \Delta_1, \Delta_8, F_2 \otimes F_3 \quad h_4 \vdash \{\Delta_5\} : F_7, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 \vdash \{\Delta_5\} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \& F_7}} \& \rightarrow \frac{h_4 \vdash \{\Delta_5\} : \Delta_1, F_3}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_1, F_3}} \text{ ax/ind} \quad H$$

$$\frac{h_4 \vdash \{\Delta_5\} : F_6, \Delta_1, \Delta_8, F_2 \otimes F_3 \quad h_4 \vdash \{\Delta_5\} : F_7, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 \vdash \{\Delta_5\} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \& F_7}} \& \rightarrow \frac{h_4 \vdash \{\Delta_5\} : \Delta_8, F_3, F_6}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_8, F_3, F_6 \& F_7}} \text{ ax/ind} \quad \frac{h_4 \vdash \{\Delta_5\} : \Delta_8, F_3, F_7}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_8, F_3, F_7}} \text{ ax/ind} \quad \&$$

- Case rule \oplus_B

$$\frac{h_4 \vdash \{\Delta_5\} : F_7, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 \vdash \{\Delta_5\} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \oplus F_7} \oplus_B \rightarrow \frac{h_4 \vdash \{\Delta_5\} : \Delta_1, F_3}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_1, F_3} \text{ax/ind} \text{H}$$

$$\frac{h_4 \vdash \{\Delta_5\} : F_7, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 \vdash \{\Delta_5\} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \oplus F_7} \oplus_B \rightarrow \frac{h_4 \vdash \{\Delta_5\} : \Delta_8, F_3, F_7}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_8, F_3, F_6 \oplus F_7} \text{ax/ind} \oplus_B$$

- Case rule \oplus_A

$$\frac{h_4 \vdash \{\Delta_5\} : F_6, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 \vdash \{\Delta_5\} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \oplus F_7} \oplus_A \rightarrow \frac{h_4 \vdash \{\Delta_5\} : \Delta_1, F_3}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_1, F_3} \text{ax/ind} \text{H}$$

$$\frac{h_4 \vdash \{\Delta_5\} : F_6, \Delta_1, \Delta_8, F_2 \otimes F_3}{\bullet h_4 \vdash \{\Delta_5\} : (\Delta_1, \Delta_8, F_2 \otimes F_3), F_6 \oplus F_7} \oplus_A \rightarrow \frac{h_4 \vdash \{\Delta_5\} : \Delta_8, F_3, F_6}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_8, F_3, F_6 \oplus F_7} \text{ax/ind} \oplus_A$$

- Case rule \perp

$$\frac{h_4 \vdash \{\Delta_5\} : \Delta_1, \Delta_6, F_2 \otimes F_3}{\bullet h_4 \vdash \{\Delta_5\} : \perp, \Delta_1, \Delta_6, F_2 \otimes F_3} \perp \rightarrow \frac{h_4 \vdash \{\Delta_5\} : \Delta_1, F_3}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_1, F_3} \text{ax/ind} \text{H}$$

$$\frac{h_4 \vdash \{\Delta_5\} : \Delta_1, \Delta_6, F_2 \otimes F_3}{\bullet h_4 \vdash \{\Delta_5\} : \perp, \Delta_1, \Delta_6, F_2 \otimes F_3} \perp \rightarrow \frac{h_4 \vdash \{\Delta_5\} : \Delta_6, F_3}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_6, F_3, \perp} \text{ax/ind} \perp$$

- Case rule \top

$$\frac{}{\bullet h_4 \vdash \{\Delta_5\} : \top, \Delta_1, \Delta_6, F_2 \otimes F_3} \top \rightarrow \frac{}{\bullet h_4 \vdash \{\Delta_5\} : F_3, \Delta_1} \text{fail}$$

$$\frac{}{\bullet h_4 \vdash \{\Delta_5\} : \top, \Delta_1, \Delta_6, F_2 \otimes F_3} \top \rightarrow \frac{}{\bullet h_4 \vdash \{\Delta_5\} : \Delta_6, F_3, \top} \top$$

- Case rule I_3

- Case rule \otimes

$$\frac{h_3 \vdash \{\Delta_4\} : F_5, \Delta_7, \Delta_8, F_1 \otimes F_2 \quad h_3 \vdash \{\Delta_4\} : F_6, \Delta_9, \Delta_{10}}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, F_1 \otimes F_2), (\Delta_9, \Delta_{10}), F_5 \otimes F_6} \otimes \rightarrow \frac{}{\bullet h_3 \vdash \{\Delta_4\} : F_2, \Delta_8, \Delta_{10}} \text{fail}$$

$$\frac{h_3 \vdash \{\Delta_4\} : F_5, \Delta_7, \Delta_8 \quad h_3 \vdash \{\Delta_4\} : F_6, \Delta_9, \Delta_{10}, F_1 \otimes F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, F_1 \otimes F_2), F_5 \otimes F_6} \otimes \rightarrow \frac{}{\bullet h_3 \vdash \{\Delta_4\} : F_2, \Delta_8, \Delta_{10}} \text{fail}$$

$$\frac{h_3 \vdash \{\Delta_4\} : F_5, \Delta_7, \Delta_8, F_1 \otimes F_2 \quad h_3 \vdash \{\Delta_4\} : F_6, \Delta_9, \Delta_{10}}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, \Delta_8, F_1 \otimes F_2), (\Delta_9, \Delta_{10}), F_5 \otimes F_6} \otimes \rightarrow \frac{}{\bullet h_3 \vdash \{\Delta_4\} : F_2, \Delta_8, \Delta_{10}, F_5 \otimes F_6} \text{fail}$$

$$\frac{h_3 \vdash \{\Delta_4\} : F_5, \Delta_7, \Delta_8 \quad h_3 \vdash \{\Delta_4\} : F_6, \Delta_9, \Delta_{10}, F_1 \otimes F_2}{\bullet h_3 \vdash \{\Delta_4\} : (\Delta_7, \Delta_8), (\Delta_9, \Delta_{10}, F_1 \otimes F_2), F_5 \otimes F_6} \otimes \rightarrow \frac{}{\bullet h_3 \vdash \{\Delta_4\} : F_2, \Delta_8, \Delta_{10}, F_5 \otimes F_6} \text{fail}$$

$$\frac{h_1 \vdash \{\Delta_2\} : F_3, \Delta_5, \Delta_6 \quad h_1 \vdash \{\Delta_2\} : F_4, \Delta_7, \Delta_8}{\bullet h_1 \vdash \{\Delta_2\} : (\Delta_5, \Delta_6), (\Delta_7, \Delta_8), F_3 \otimes F_4} \otimes \rightarrow \frac{}{\bullet h_1 \vdash \{\Delta_2\} : F_4, \Delta_6, \Delta_8} \text{fail}$$

- Case rule I_1
- Case rule I_2
- Case rule $?_C$

$$\frac{h_5 \text{ :- } \{F_6, \Delta_7\} : F_6, \Delta_1, \Delta_2, F_3 \otimes F_4}{\bullet h_5 \text{ :- } \{F_6, \Delta_7\} : \Delta_1, \Delta_2, F_3 \otimes F_4} ?_C \rightarrow \frac{\overline{h_5 \text{ :- } \{\Delta_7, F_6\} : \Delta_2, F_4, F_6}}{\bullet h_5 \text{ :- } \{\Delta_7, F_6\} : \Delta_2, F_4} \text{ ax/ind} ?_C$$

4.14 Status of I_1 : : Invertible

- Case rule 1
- Case rule !
- Case rule ?
- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I_3
- Case rule \otimes
- Case rule I_1

$$\overline{\bullet h_1 \text{ :- } \{\Delta_2, p(n_3)\} : \ulcorner(n_3)} I_1 \rightarrow \text{trivial}$$

- Case rule I_2

- Case rule $?_C$

$$\frac{h_2 := \{\Delta_3, p(n_1)\} : p(n_1), \wedge(n_1)}{\bullet h_2 := \{p(n_1), \Delta_3\} : \wedge(n_1)} ?_C \rightarrow \text{trivial}$$

$$\frac{h_2 := \{F_3, \Delta_4, p(n_1)\} : F_3, \wedge(n_1)}{\bullet h_2 := \{F_3, \Delta_4, p(n_1)\} : \wedge(n_1)} ?_C \rightarrow \text{trivial}$$

4.15 Status of I_2 : : Invertible

- Case rule **1**
- Case rule **!**
- Case rule **?**
- Case rule **\$**
- Case rule **&**
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I_3
- Case rule \otimes
- Case rule I_1
- Case rule I_2

$$\frac{}{\bullet h_1 := \{\Delta_2, \wedge(n_3)\} : p(n_3)} I_2 \rightarrow \text{trivial}$$

- Case rule $?_C$

$$\frac{h_2 := \{\Delta_3, \wedge(n_1)\} : p(n_1), \wedge(n_1)}{\bullet h_2 := \{\wedge(n_1), \Delta_3\} : p(n_1)} ?_C \rightarrow \text{trivial}$$

$$\frac{h_2 := \{F_3, \Delta_4, \wedge(n_1)\} : F_3, p(n_1)}{\bullet h_2 := \{F_3, \Delta_4, \wedge(n_1)\} : p(n_1)} ?_C \rightarrow \text{trivial}$$

4.16 Status of $?_C$: : Non invertible

- Case rule 1

$$\frac{}{\bullet h_3 \vdash \{F_1, \Delta_2\} : 1} \text{ 1} \rightarrow \frac{}{\bullet h_3 \vdash \{F_1, \Delta_2\} : 1, F_1} \text{ fail}$$

- Case rule !

$$\frac{h_3 \vdash \{F_1, \Delta_2\} : F_4}{\bullet h_3 \vdash \{F_1, \Delta_2\} : !F_4} \text{ !} \rightarrow \frac{}{\bullet h_3 \vdash \{F_1, \Delta_2\} : F_1, !F_4} \text{ fail}$$

- Case rule ?

$$\frac{h_3 \vdash \{F_1, F_5, \Delta_2\} : \Delta_4}{\bullet h_3 \vdash \{F_1, \Delta_2\} : \Delta_4, ?F_5} \text{ ?} \rightarrow \frac{h_3 \vdash \{\Delta_2, F_1, F_5\} : \Delta_4, F_1}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_1, ?F_5} \text{ ax/ind ?}$$

- Case rule \$

$$\frac{h_3 \vdash \{F_1, \Delta_2\} : F_5, F_6, \Delta_4}{\bullet h_3 \vdash \{F_1, \Delta_2\} : \Delta_4, F_5 \$F_6} \text{ \$} \rightarrow \frac{h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_1, F_5, F_6}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_1, F_5 \$F_6} \text{ ax/ind \$}$$

- Case rule &

$$\frac{h_3 \vdash \{F_1, \Delta_2\} : F_5, \Delta_4}{\bullet h_3 \vdash \{F_1, \Delta_2\} : \Delta_4, F_5 \&F_6} \text{ h}_3 \vdash \{F_1, \Delta_2\} : F_6, \Delta_4 \text{ \&} \rightarrow \frac{h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_1, F_5}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_1, F_5 \&F_6} \text{ ax/ind } \frac{h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_1, F_6}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_1, F_5 \&F_6} \text{ ax/ind \&}$$

- Case rule \oplus_B

$$\frac{h_3 \vdash \{F_1, \Delta_2\} : F_6, \Delta_4}{\bullet h_3 \vdash \{F_1, \Delta_2\} : \Delta_4, F_5 \oplus_B F_6} \text{ \oplus}_B \rightarrow \frac{h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_1, F_6}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_1, F_5 \oplus_B F_6} \text{ ax/ind \oplus}_B$$

- Case rule \oplus_A

$$\frac{h_3 \vdash \{F_1, \Delta_2\} : F_5, \Delta_4}{\bullet h_3 \vdash \{F_1, \Delta_2\} : \Delta_4, F_5 \oplus_A F_6} \text{ \oplus}_A \rightarrow \frac{h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_1, F_5}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_1, F_5 \oplus_A F_6} \text{ ax/ind \oplus}_A$$

- Case rule \perp

$$\frac{h_3 \vdash \{F_1, \Delta_2\} : \Delta_4}{\bullet h_3 \vdash \{F_1, \Delta_2\} : \perp, \Delta_4} \text{ \perp} \rightarrow \frac{h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_1}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_1, \perp} \text{ ax/ind \perp}$$

- Case rule \top

$$\frac{}{\bullet h_3 \vdash \{F_1, \Delta_2\} : \top, \Delta_4} \text{ \top} \rightarrow \frac{}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_1, \top} \text{ \top}$$

- Case rule I_3

$$\frac{}{\bullet h_3 \vdash \{F_1, \Delta_2\} : p(n_4), \hat{\cdot}(n_4)} I_3 \rightarrow \frac{}{\bullet h_3 \vdash \{F_1, \Delta_2\} : F_1, p(n_4), \hat{\cdot}(n_4)} \text{fail}$$

- Case rule \otimes

$$\frac{h_3 \vdash \{F_1, \Delta_2\} : F_6, \Delta_4 \quad h_3 \vdash \{F_1, \Delta_2\} : F_7, \Delta_5}{\bullet h_3 \vdash \{F_1, \Delta_2\} : \Delta_4, \Delta_5, F_6 \otimes F_7} \otimes \rightarrow \frac{h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, F_6 \quad \text{ax} \quad h_3 \vdash \{\Delta_2, F_1\} : \Delta_5, F_1, F_7}{\bullet h_3 \vdash \{\Delta_2, F_1\} : \Delta_4, \Delta_5, F_1, F_6 \otimes F_7} \otimes \text{ax/ind}$$

- Case rule I_1

$$\frac{}{\bullet h_1 \vdash \{\Delta_2, p(n_3)\} : \hat{\cdot}(n_3)} I_1 \rightarrow \frac{}{\bullet h_1 \vdash \{\Delta_2, p(n_3)\} : p(n_3), \hat{\cdot}(n_3)} I_3$$

$$\frac{}{\bullet h_2 \vdash \{(F_1, \Delta_4), p(n_3)\} : \hat{\cdot}(n_3)} I_1 \rightarrow \frac{}{\bullet h_2 \vdash \{F_1, \Delta_4, p(n_3)\} : F_1, \hat{\cdot}(n_3)} \text{fail}$$

- Case rule I_2

$$\frac{}{\bullet h_1 \vdash \{\Delta_2, \hat{\cdot}(n_3)\} : p(n_3)} I_2 \rightarrow \frac{}{\bullet h_1 \vdash \{\Delta_2, \hat{\cdot}(n_3)\} : p(n_3), \hat{\cdot}(n_3)} I_3$$

$$\frac{}{\bullet h_2 \vdash \{(F_1, \Delta_4), \hat{\cdot}(n_3)\} : p(n_3)} I_2 \rightarrow \frac{}{\bullet h_2 \vdash \{F_1, \Delta_4, \hat{\cdot}(n_3)\} : F_1, p(n_3)} \text{fail}$$

- Case rule $?_C$

$$\frac{h_1 \vdash \{F_2, \Delta_3\} : F_2, \Delta_4}{\bullet h_1 \vdash \{F_2, \Delta_3\} : \Delta_4} ?_C \rightarrow \frac{h_1 \vdash \{\Delta_3, F_2\} : \Delta_4, F_2, F_2 \quad \text{ax/ind}}{\bullet h_1 \vdash \{\Delta_3, F_2\} : \Delta_4, F_2} ?_C$$

$$\frac{h_2 \vdash \{F_1, F_3, \Delta_5\} : F_3, \Delta_4}{\bullet h_2 \vdash \{F_3, F_1, \Delta_5\} : \Delta_4} ?_C \rightarrow \frac{h_2 \vdash \{\Delta_5, F_1, F_3\} : \Delta_4, F_1, F_3 \quad \text{ax/ind}}{\bullet h_2 \vdash \{\Delta_5, F_1, F_3\} : \Delta_4, F_1} ?_C$$

5 Weakening on bang: $\vdash \Gamma, !F$ implies Γ, F .

- Case(s) rule 1
- Case(s) rule !

$$\frac{h_1 : \vdash \{\Delta_2\} : F_3}{\bullet h_1 : \vdash \{\Delta_2\} : !F_3, * } ! \rightarrow \frac{\overline{h_1 : \vdash \{\Delta_2\} : F_3}^{\text{ax}}}{\bullet h_1 : \vdash \{\Delta_2\} : F_3} H$$

- Case(s) rule ?

$$\frac{h_2 : \vdash \{F_4, \Delta_3\} : \Delta_5, !F_1}{\bullet h_2 : \vdash \{\Delta_3\} : !F_1, \Delta_5, ?F_4} ? \rightarrow \frac{\overline{h_2 : \vdash \{\Delta_3, F_4\} : \Delta_5, !F_1}^{\text{ax}}}{h_2 : \vdash \{\Delta_3, F_4\} : \Delta_5, F_1} \text{IH} \quad \frac{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_5, F_1, ?F_4}^{\text{ax}}}{\bullet h_2 : \vdash \{\Delta_3\} : \Delta_5, F_1, ?F_4} ?$$

- Case(s) rule \$

$$\frac{h_2 : \vdash \{\Delta_3\} : F_4, F_5, \Delta_6, !F_1}{\bullet h_2 : \vdash \{\Delta_3\} : !F_1, \Delta_6, F_4 \$F_5} \$ \rightarrow \frac{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_6, F_4, F_5, !F_1}^{\text{ax}}}{h_2 : \vdash \{\Delta_3\} : \Delta_6, F_1, F_4, F_5} \text{IH} \quad \frac{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_6, F_1, F_4 \$F_5}^{\text{ax}}}{\bullet h_2 : \vdash \{\Delta_3\} : \Delta_6, F_1, F_4 \$F_5} \$$$

- Case(s) rule &

$$\frac{h_2 : \vdash \{\Delta_3\} : F_4, \Delta_6, !F_1 \quad h_2 : \vdash \{\Delta_3\} : F_5, \Delta_6, !F_1}{\bullet h_2 : \vdash \{\Delta_3\} : !F_1, \Delta_6, F_4 \&F_5} \& \rightarrow \frac{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_6, F_4, !F_1}^{\text{ax}}}{h_2 : \vdash \{\Delta_3\} : \Delta_6, F_1, F_4} \text{IH} \quad \frac{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_6, F_5, !F_1}^{\text{ax}}}{h_2 : \vdash \{\Delta_3\} : \Delta_6, F_1, F_5} \text{IH}}{\bullet h_2 : \vdash \{\Delta_3\} : \Delta_6, F_1, F_4 \&F_5} \&$$

- Case(s) rule \oplus_B

$$\frac{h_2 : \vdash \{\Delta_3\} : F_5, \Delta_6, !F_1}{\bullet h_2 : \vdash \{\Delta_3\} : !F_1, \Delta_6, F_4 \oplus_B F_5} \oplus_B \rightarrow \frac{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_6, F_5, !F_1}^{\text{ax}}}{h_2 : \vdash \{\Delta_3\} : \Delta_6, F_1, F_5} \text{IH}}{\bullet h_2 : \vdash \{\Delta_3\} : \Delta_6, F_1, F_4 \oplus_B F_5} \oplus_B$$

- Case(s) rule \oplus_A

$$\frac{h_2 : \vdash \{\Delta_3\} : F_4, \Delta_6, !F_1}{\bullet h_2 : \vdash \{\Delta_3\} : !F_1, \Delta_6, F_4 \oplus_A F_5} \oplus_A \rightarrow \frac{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_6, F_4, !F_1}^{\text{ax}}}{h_2 : \vdash \{\Delta_3\} : \Delta_6, F_1, F_4} \text{IH}}{\bullet h_2 : \vdash \{\Delta_3\} : \Delta_6, F_1, F_4 \oplus_A F_5} \oplus_A$$

- Case(s) rule \perp

$$\frac{h_2 : \vdash \{\Delta_3\} : \Delta_4, !F_1}{\bullet h_2 : \vdash \{\Delta_3\} : !F_1, \perp, \Delta_4} \perp \rightarrow \frac{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_4, !F_1}^{\text{ax}}}{h_2 : \vdash \{\Delta_3\} : \Delta_4, F_1} \text{IH}}{\bullet h_2 : \vdash \{\Delta_3\} : \Delta_4, F_1, \perp} \perp$$

- Case(s) rule \top

$$\frac{\overline{\bullet h_2 : \vdash \{\Delta_3\} : !F_1, \top, \Delta_4}^{\top}}{\bullet h_2 : \vdash \{\Delta_3\} : \Delta_4, F_1, \top} \top$$

- Case(s) rule I_3

- Case(s) rule \otimes

$$\frac{h_2 : \vdash \{\Delta_3\} : F_5, \Delta_7, !F_1 \quad h_2 : \vdash \{\Delta_3\} : F_6, \Delta_4}{\bullet h_2 : \vdash \{\Delta_3\} : !F_1, \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \rightarrow \frac{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_7, F_5, !F_1}^{\text{ax}}}{h_2 : \vdash \{\Delta_3\} : \Delta_7, F_1, F_5} \text{IH} \quad \frac{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_4, F_6}^{\text{ax}}}{h_2 : \vdash \{\Delta_3\} : \Delta_4, F_1, F_6} \text{IH}}{\bullet h_2 : \vdash \{\Delta_3\} : \Delta_4, \Delta_7, F_1, F_5 \otimes F_6} \otimes$$

$$\frac{h_2 : \vdash \{\Delta_3\} : F_5, \Delta_4 \quad h_2 : \vdash \{\Delta_3\} : F_6, \Delta_7, !F_1}{\bullet h_2 : \vdash \{\Delta_3\} : !F_1, \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \rightarrow \frac{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_4, F_5}^{\text{ax}}}{h_2 : \vdash \{\Delta_3\} : \Delta_4, F_1, F_5} \text{IH} \quad \frac{\overline{h_2 : \vdash \{\Delta_3\} : \Delta_7, F_6, !F_1}^{\text{ax}}}{h_2 : \vdash \{\Delta_3\} : \Delta_7, F_1, F_6} \text{IH}}{\bullet h_2 : \vdash \{\Delta_3\} : \Delta_4, \Delta_7, F_1, F_5 \otimes F_6} \otimes$$

- Case(s) rule I_1
- Case(s) rule I_2
- Case(s) rule $?_C$

$$\frac{h_3 \vdash \{F_4, \Delta_5\} : F_4, \Delta_1, !F_2}{\bullet h_3 \vdash \{F_4, \Delta_5\} : !F_2, \Delta_1} ?_C \rightarrow \frac{\frac{h_3 \vdash \{\Delta_5, F_4\} : \Delta_1, F_4, !F_2}{h_3 \vdash \{\Delta_5, F_4\} : \Delta_1, F_2, F_4} \text{IH}}{\bullet h_3 \vdash \{\Delta_5, F_4\} : \Delta_1, F_2} \text{ax} ?_C$$

6 Identity-Expansion

$$\begin{array}{c}
\overline{- : \vdash \{*\} : \mathbf{0}, \top} \top \\
\\
\overline{- : \vdash \{*\} : \mathbf{1}} \mathbf{1} \\
\overline{- : \vdash \{*\} : \mathbf{1}, \perp} \perp \\
\\
\frac{\overline{- : \vdash \{*\} : \mathbf{F}_0, dual(\mathbf{F}_0)} \text{IH} \quad \overline{- : \vdash \{*\} : \mathbf{F}_1, dual(\mathbf{F}_1)} \text{IH}}{\overline{- : \vdash \{*\} : dual(\mathbf{F}_0), \mathbf{F}_0 \oplus \mathbf{F}_1} \oplus_A \quad \overline{- : \vdash \{*\} : dual(\mathbf{F}_1), \mathbf{F}_0 \oplus \mathbf{F}_1} \oplus_B} \oplus_B \\
\overline{- : \vdash \{*\} : dual(\mathbf{F}_0) \& dual(\mathbf{F}_1), \mathbf{F}_0 \oplus \mathbf{F}_1} \& \\
\\
\frac{\overline{- : \vdash \{*\} : \mathbf{F}_0, dual(\mathbf{F}_0)} \text{IH} \quad \overline{- : \vdash \{*\} : \mathbf{F}_1, dual(\mathbf{F}_1)} \text{IH}}{\overline{- : \vdash \{*\} : dual(\mathbf{F}_0), dual(\mathbf{F}_1), \mathbf{F}_0 \otimes \mathbf{F}_1} \otimes} \otimes \\
\overline{- : \vdash \{*\} : dual(\mathbf{F}_0) \$ dual(\mathbf{F}_1), \mathbf{F}_0 \otimes \mathbf{F}_1} \$ \\
\\
\frac{\overline{- : \vdash \{*\} : \mathbf{F}_0, dual(\mathbf{F}_0)} \text{IH} \quad \overline{- : \vdash \{*\} : \mathbf{F}_1, dual(\mathbf{F}_1)} \text{IH}}{\overline{- : \vdash \{*\} : \mathbf{F}_0, dual(\mathbf{F}_0) \oplus dual(\mathbf{F}_1)} \oplus_A \quad \overline{- : \vdash \{*\} : \mathbf{F}_1, dual(\mathbf{F}_0) \oplus dual(\mathbf{F}_1)} \oplus_B} \oplus_B \\
\overline{- : \vdash \{*\} : \mathbf{F}_0 \& \mathbf{F}_1, dual(\mathbf{F}_0) \oplus dual(\mathbf{F}_1)} \& \\
\\
\frac{\overline{- : \vdash \{*\} : \mathbf{F}_0, dual(\mathbf{F}_0)} \text{IH} \quad \overline{- : \vdash \{*\} : \mathbf{F}_1, dual(\mathbf{F}_1)} \text{IH}}{\overline{- : \vdash \{*\} : \mathbf{F}_0, \mathbf{F}_1, dual(\mathbf{F}_0) \otimes dual(\mathbf{F}_1)} \otimes} \otimes \\
\overline{- : \vdash \{*\} : \mathbf{F}_0 \$ \mathbf{F}_1, dual(\mathbf{F}_0) \otimes dual(\mathbf{F}_1)} \$ \\
\\
\frac{\overline{- : \vdash \{*\} : \mathbf{F}_0, dual(\mathbf{F}_0)} \text{IH}}{\overline{- : \vdash \{\mathbf{F}_0\} : \mathbf{F}_0, dual(\mathbf{F}_0)} W} W \\
\overline{- : \vdash \{\mathbf{F}_0\} : dual(\mathbf{F}_0)} ?_C \\
\overline{- : \vdash \{\mathbf{F}_0\} : !dual(\mathbf{F}_0)} ! \\
\overline{- : \vdash \{*\} : !dual(\mathbf{F}_0), ?\mathbf{F}_0} ? \\
\\
\overline{- : \vdash \{*\} : \mathbf{1}} \mathbf{1} \\
\overline{- : \vdash \{*\} : \mathbf{1}, \perp} \perp \\
\\
\overline{- : \vdash \{*\} : \mathbf{0}, \top} \top \\
\\
\frac{\overline{- : \vdash \{*\} : \mathbf{F}_0, dual(\mathbf{F}_0)} \text{IH}}{\overline{- : \vdash \{dual(\mathbf{F}_0)\} : \mathbf{F}_0, dual(\mathbf{F}_0)} W} W \\
\overline{- : \vdash \{dual(\mathbf{F}_0)\} : \mathbf{F}_0} ?_C \\
\overline{- : \vdash \{dual(\mathbf{F}_0)\} : !\mathbf{F}_0} ! \\
\overline{- : \vdash \{*\} : !\mathbf{F}_0, ?dual(\mathbf{F}_0)} ?
\end{array}$$

7 Cut-Elimination

7.1 Status of 1: OK

- Case rule 1
- Case rule !
- Case rule ?

$$\frac{\frac{}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}, *}}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}, *} \mathbf{1} \quad \frac{h_3 \vdash \{F_5, \Delta_4\} : \perp, \Delta_6}{\bullet h_3 \vdash \{\Delta_4\} : dual(\mathbf{1}), \Delta_6, ?F_5} ?}{- \vdash \{\Delta_4\} : *, \Delta_6, ?F_5} \text{Cut}$$

$$\rightarrow$$

$$\frac{\frac{}{\bullet h_1 \vdash \{\Delta_4, F_5\} : \mathbf{1}}{\bullet h_1 \vdash \{\Delta_4, F_5\} : \mathbf{1}} \mathbf{1} \quad \frac{h_3 \vdash \{\Delta_4, F_5\} : \Delta_6, \perp}{h_3 \vdash \{\Delta_4, F_5\} : \Delta_6, \perp} \text{ax}}{- \vdash \{\Delta_4, F_5\} : \Delta_6} \text{hCut}}{- \vdash \{\Delta_4\} : \Delta_6, ?F_5} ?$$

- Case rule \$

$$\frac{\frac{}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}, *} \mathbf{1} \quad \frac{h_3 \vdash \{\Delta_4\} : \perp, F_5, F_6, \Delta_7}{\bullet h_3 \vdash \{\Delta_4\} : dual(\mathbf{1}), \Delta_7, F_5 \& F_6} \$}{- \vdash \{\Delta_4\} : *, \Delta_7, F_5 \& F_6} \text{Cut}}{\rightarrow}$$

$$\frac{\frac{}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}} \text{ax} \quad \frac{h_3 \vdash \{\Delta_4\} : \Delta_7, F_5, F_6, \perp}{h_3 \vdash \{\Delta_4\} : \Delta_7, F_5, F_6, \perp} \text{ax}}{- \vdash \{\Delta_4\} : \Delta_7, F_5, F_6} \text{hCut}}{- \vdash \{\Delta_4\} : \Delta_7, F_5 \& F_6} \$$$

- Case rule &

$$\frac{\frac{}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}, *} \mathbf{1} \quad \frac{h_3 \vdash \{\Delta_4\} : \perp, F_5, \Delta_7 \quad h_3 \vdash \{\Delta_4\} : \perp, F_6, \Delta_7}{\bullet h_3 \vdash \{\Delta_4\} : dual(\mathbf{1}), \Delta_7, F_5 \& F_6} \&}{- \vdash \{\Delta_4\} : *, \Delta_7, F_5 \& F_6} \text{Cut}}{\rightarrow}$$

$$\frac{\frac{}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}} \text{ax} \quad \frac{h_3 \vdash \{\Delta_4\} : \Delta_7, F_5, \perp}{h_3 \vdash \{\Delta_4\} : \Delta_7, F_5, \perp} \text{ax}}{- \vdash \{\Delta_4\} : \Delta_7, F_5} \text{hCut} \quad \frac{\frac{}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}} \text{ax} \quad \frac{h_3 \vdash \{\Delta_4\} : \Delta_7, F_6, \perp}{h_3 \vdash \{\Delta_4\} : \Delta_7, F_6, \perp} \text{ax}}{- \vdash \{\Delta_4\} : \Delta_7, F_6} \text{hCut}}{- \vdash \{\Delta_4\} : \Delta_7, F_5 \& F_6} \&}$$

- Case rule \oplus_B

$$\frac{\frac{}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}, *} \mathbf{1} \quad \frac{h_3 \vdash \{\Delta_4\} : \perp, F_6, \Delta_7}{\bullet h_3 \vdash \{\Delta_4\} : dual(\mathbf{1}), \Delta_7, F_5 \oplus F_6} \oplus_B}{- \vdash \{\Delta_4\} : *, \Delta_7, F_5 \oplus F_6} \text{Cut}}{\rightarrow}$$

$$\frac{\frac{}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}} \text{ax} \quad \frac{h_3 \vdash \{\Delta_4\} : \Delta_7, F_6, \perp}{h_3 \vdash \{\Delta_4\} : \Delta_7, F_6, \perp} \text{ax}}{- \vdash \{\Delta_4\} : \Delta_7, F_6} \text{hCut}}{- \vdash \{\Delta_4\} : \Delta_7, F_5 \oplus F_6} \oplus_B$$

- Case rule \oplus_A

$$\frac{\frac{}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}, *} \mathbf{1} \quad \frac{h_3 \vdash \{\Delta_4\} : \perp, F_5, \Delta_7}{\bullet h_3 \vdash \{\Delta_4\} : dual(\mathbf{1}), \Delta_7, F_5 \oplus F_6} \oplus_A}{- \vdash \{\Delta_4\} : *, \Delta_7, F_5 \oplus F_6} \text{Cut}}{\rightarrow}$$

$$\frac{\frac{}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}} \text{ax} \quad \frac{h_3 \vdash \{\Delta_4\} : \Delta_7, F_5, \perp}{h_3 \vdash \{\Delta_4\} : \Delta_7, F_5, \perp} \text{ax}}{- \vdash \{\Delta_4\} : \Delta_7, F_5} \text{hCut}}{- \vdash \{\Delta_4\} : \Delta_7, F_5 \oplus F_6} \oplus_A$$

- Case rule \perp

$$\frac{\frac{\frac{}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}, *}}{\vdash \{\Delta_4\} : *, \Delta_5} \mathbf{1} \quad \frac{h_3 \vdash \{\Delta_4\} : \Delta_5}{\bullet h_3 \vdash \{\Delta_4\} : dual(\mathbf{1}), \Delta_5} \perp}{\vdash \{\Delta_4\} : *, \Delta_5} \text{Cut}}{\vdash \{\Delta_4\} : \Delta_5} \text{ax}$$

- Case rule \top

$$\frac{\frac{\frac{}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}, *}}{\vdash \{\Delta_4\} : *, \top, \Delta_5} \mathbf{1} \quad \frac{}{\bullet h_3 \vdash \{\Delta_4\} : dual(\mathbf{1}), \top, \Delta_5} \top}{\vdash \{\Delta_4\} : *, \top, \Delta_5} \text{Cut}}{\vdash \{\Delta_4\} : \Delta_5, \top} \top$$

- Case rule I_3

- Case rule \otimes

$$\frac{\frac{\frac{\frac{}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}, *}}{\vdash \{\Delta_4\} : *, \Delta_5, \Delta_8, F_6 \otimes F_7} \mathbf{1} \quad \frac{h_3 \vdash \{\Delta_4\} : \perp, F_6, \Delta_8 \quad h_3 \vdash \{\Delta_4\} : F_7, \Delta_5}{\bullet h_3 \vdash \{\Delta_4\} : dual(\mathbf{1}), \Delta_5, \Delta_8, F_6 \otimes F_7} \otimes}{\vdash \{\Delta_4\} : *, \Delta_5, \Delta_8, F_6 \otimes F_7} \text{Cut}}{\vdash \{\Delta_4\} : \Delta_8, F_6} \text{ax} \quad \frac{\frac{}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}}{\vdash \{\Delta_4\} : \Delta_5, F_7} \text{ax}}{\vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_6 \otimes F_7} \text{hCut} \quad \frac{}{\vdash \{\Delta_4\} : \Delta_5, F_7} \text{ax}}{\vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_6 \otimes F_7} \otimes$$

$$\frac{\frac{\frac{\frac{}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}, *}}{\vdash \{\Delta_4\} : *, \Delta_5, \Delta_8, F_6 \otimes F_7} \mathbf{1} \quad \frac{h_3 \vdash \{\Delta_4\} : F_6, \Delta_5 \quad h_3 \vdash \{\Delta_4\} : \perp, F_7, \Delta_8}{\bullet h_3 \vdash \{\Delta_4\} : dual(\mathbf{1}), \Delta_5, \Delta_8, F_6 \otimes F_7} \otimes}{\vdash \{\Delta_4\} : *, \Delta_5, \Delta_8, F_6 \otimes F_7} \text{Cut}}{\vdash \{\Delta_4\} : \Delta_5, F_6} \text{ax} \quad \frac{\frac{}{\bullet h_1 \vdash \{\Delta_4\} : \mathbf{1}}{\vdash \{\Delta_4\} : \Delta_8, F_7} \text{ax}}{\vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_6 \otimes F_7} \text{hCut} \quad \frac{}{\vdash \{\Delta_4\} : \Delta_8, F_7} \text{ax}}{\vdash \{\Delta_4\} : \Delta_5, \Delta_8, F_6 \otimes F_7} \otimes$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\frac{\frac{\frac{\frac{}{\bullet h_1 \vdash \{F_5, \Delta_6\} : \mathbf{1}, *}}{\vdash \{F_5, \Delta_6\} : *, \Delta_3} \mathbf{1} \quad \frac{h_4 \vdash \{F_5, \Delta_6\} : \perp, F_5, \Delta_3}{\bullet h_4 \vdash \{F_5, \Delta_6\} : dual(\mathbf{1}), \Delta_3} ?_C}{\vdash \{F_5, \Delta_6\} : *, \Delta_3} \text{Cut}}{\vdash \{\Delta_6, F_5\} : \Delta_3, F_5} \text{ax} \quad \frac{\frac{}{\bullet h_1 \vdash \{\Delta_6, F_5\} : \mathbf{1}}{\vdash \{\Delta_6, F_5\} : \Delta_3, F_5} \text{ax}}{\vdash \{\Delta_6, F_5\} : \Delta_3} \text{hCut} \quad \frac{}{\vdash \{\Delta_6, F_5\} : \Delta_3} ?_C}}{\vdash \{\Delta_6, F_5\} : \Delta_3} ?_C$$

7.2 Status of !: OK

- Case rule $\mathbf{1}$

- Case rule !

- Case rule ?

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \{\Delta_6\} : F_4}{\bullet h_1 : \vdash \{\Delta_6\} : !F_4, *} \quad ! \quad \frac{h_5 : \vdash \{F_7, \Delta_6\} : \Delta_8, ?dual(F_4)}{\bullet h_5 : \vdash \{\Delta_6\} : dual(!F_4), \Delta_8, ?F_7} ?}{- : \vdash \{\Delta_6\} : *, \Delta_8, ?F_7} \text{Cut}}{\rightarrow} \\
\frac{\frac{\frac{\bullet h_1 : \vdash \{\Delta_6\} : !F_4}{\bullet h_1 : \vdash \{\Delta_6, F_7\} : !F_4} \text{ax} \quad W \quad \frac{h_5 : \vdash \{\Delta_6, F_7\} : \Delta_8, ?dual(F_4)}{h_5 : \vdash \{\Delta_6, F_7\} : \Delta_8, ?dual(F_4)} \text{ax}}{- : \vdash \{\Delta_6, F_7\} : \Delta_8} \text{hCut}}{- : \vdash \{\Delta_6\} : \Delta_8, ?F_7} ?} \\
\frac{\frac{h_1 : \vdash \{\Delta_6\} : F_4}{\bullet h_1 : \vdash \{\Delta_6\} : !F_4, *} \quad ! \quad \frac{h_5 : \vdash \{\Delta_6, dual(F_4)\} : \Delta_7}{\bullet h_5 : \vdash \{\Delta_6\} : dual(!F_4), \Delta_7} ?}{- : \vdash \{\Delta_6\} : *, \Delta_7} \text{Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_1 : \vdash \{\Delta_6\} : !F_4}{\bullet h_1 : \vdash \{\Delta_6\} : !F_4} \text{ax} \quad \frac{h_5 : \vdash \{\Delta_6, dual(F_4)\} : \Delta_7}{h_5 : \vdash \{\Delta_6, dual(F_4)\} : \Delta_7} \text{ax}}{- : \vdash \{\Delta_6\} : \Delta_7} \text{mCut}
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \{\Delta_6\} : F_4}{\bullet h_1 : \vdash \{\Delta_6\} : !F_4, *} \quad ! \quad \frac{h_5 : \vdash \{\Delta_6\} : F_7, F_8, \Delta_9, ?dual(F_4)}{\bullet h_5 : \vdash \{\Delta_6\} : dual(!F_4), \Delta_9, F_7\$F_8} \$}{- : \vdash \{\Delta_6\} : *, \Delta_9, F_7\$F_8} \text{Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_1 : \vdash \{\Delta_6\} : !F_4}{\bullet h_1 : \vdash \{\Delta_6\} : !F_4} \text{ax} \quad \frac{h_5 : \vdash \{\Delta_6\} : \Delta_9, F_7, F_8, ?dual(F_4)}{h_5 : \vdash \{\Delta_6\} : \Delta_9, F_7, F_8, ?dual(F_4)} \text{ax}}{- : \vdash \{\Delta_6\} : \Delta_9, F_7, F_8} \text{hCut}}{- : \vdash \{\Delta_6\} : \Delta_9, F_7\$F_8} \$
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \{\Delta_6\} : F_4}{\bullet h_1 : \vdash \{\Delta_6\} : !F_4, *} \quad ! \quad \frac{h_5 : \vdash \{\Delta_6\} : F_7, \Delta_9, ?dual(F_4)}{\bullet h_5 : \vdash \{\Delta_6\} : dual(!F_4), \Delta_9, F_7\&F_8} \quad \frac{h_5 : \vdash \{\Delta_6\} : F_8, \Delta_9, ?dual(F_4)}{\bullet h_5 : \vdash \{\Delta_6\} : dual(!F_4), \Delta_9, F_7\&F_8} \&}{- : \vdash \{\Delta_6\} : *, \Delta_9, F_7\&F_8} \text{Cut}}{\rightarrow} \\
\frac{\frac{\frac{\bullet h_1 : \vdash \{\Delta_6\} : !F_4}{\bullet h_1 : \vdash \{\Delta_6\} : !F_4} \text{ax} \quad \frac{h_5 : \vdash \{\Delta_6\} : \Delta_9, F_7, ?dual(F_4)}{h_5 : \vdash \{\Delta_6\} : \Delta_9, F_7, ?dual(F_4)} \text{ax}}{- : \vdash \{\Delta_6\} : \Delta_9, F_7} \text{hCut} \quad \frac{\frac{\bullet h_1 : \vdash \{\Delta_6\} : !F_4}{\bullet h_1 : \vdash \{\Delta_6\} : !F_4} \text{ax} \quad \frac{h_5 : \vdash \{\Delta_6\} : \Delta_9, F_8, ?dual(F_4)}{h_5 : \vdash \{\Delta_6\} : \Delta_9, F_8, ?dual(F_4)} \text{ax}}{- : \vdash \{\Delta_6\} : \Delta_9, F_8} \text{hCut}}{- : \vdash \{\Delta_6\} : \Delta_9, F_7\&F_8} \&
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{\frac{h_1 : \vdash \{\Delta_6\} : F_4}{\bullet h_1 : \vdash \{\Delta_6\} : !F_4, *} \quad ! \quad \frac{h_5 : \vdash \{\Delta_6\} : F_8, \Delta_9, ?dual(F_4)}{\bullet h_5 : \vdash \{\Delta_6\} : dual(!F_4), \Delta_9, F_7 \oplus F_8} \oplus_B}{- : \vdash \{\Delta_6\} : *, \Delta_9, F_7 \oplus F_8} \text{Cut}}{\rightarrow} \\
\frac{\frac{\bullet h_1 : \vdash \{\Delta_6\} : !F_4}{\bullet h_1 : \vdash \{\Delta_6\} : !F_4} \text{ax} \quad \frac{h_5 : \vdash \{\Delta_6\} : \Delta_9, F_8, ?dual(F_4)}{h_5 : \vdash \{\Delta_6\} : \Delta_9, F_8, ?dual(F_4)} \text{ax}}{- : \vdash \{\Delta_6\} : \Delta_9, F_8} \text{hCut}}{- : \vdash \{\Delta_6\} : \Delta_9, F_7 \oplus F_8} \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_6\} : F_4}{\bullet h_1 \vdash \{\Delta_6\} : !F_4, *} \quad ! \quad \frac{h_5 \vdash \{\Delta_6\} : F_7, \Delta_9, ?dual(F_4)}{\bullet h_5 \vdash \{\Delta_6\} : dual(!F_4), \Delta_9, F_7 \oplus F_8} \oplus^A \\
\text{Cut} \\
\frac{}{- \vdash \{\Delta_6\} : *, \Delta_9, F_7 \oplus F_8} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_6\} : !F_4}{- \vdash \{\Delta_6\} : \Delta_9, F_7} \text{ax} \quad \frac{h_5 \vdash \{\Delta_6\} : \Delta_9, F_7, ?dual(F_4)}{- \vdash \{\Delta_6\} : \Delta_9, F_7 \oplus F_8} \text{ax}}{- \vdash \{\Delta_6\} : \Delta_9, F_7 \oplus F_8} \text{hCut} \oplus^A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_6\} : F_4}{\bullet h_1 \vdash \{\Delta_6\} : !F_4, *} \quad ! \quad \frac{h_5 \vdash \{\Delta_6\} : \Delta_7, ?dual(F_4)}{\bullet h_5 \vdash \{\Delta_6\} : dual(!F_4), \perp, \Delta_7} \perp \\
\text{Cut} \\
\frac{}{- \vdash \{\Delta_6\} : *, \perp, \Delta_7} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_6\} : !F_4}{- \vdash \{\Delta_6\} : \Delta_7} \text{ax} \quad \frac{h_5 \vdash \{\Delta_6\} : \Delta_7, ?dual(F_4)}{- \vdash \{\Delta_6\} : \Delta_7, \perp} \text{ax}}{- \vdash \{\Delta_6\} : \Delta_7, \perp} \text{hCut} \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_6\} : F_4}{\bullet h_1 \vdash \{\Delta_6\} : !F_4, *} \quad ! \quad \frac{h_5 \vdash \{\Delta_6\} : dual(!F_4), \top, \Delta_7}{\bullet h_5 \vdash \{\Delta_6\} : dual(!F_4), \top, \Delta_7} \top \\
\text{Cut} \\
\frac{}{- \vdash \{\Delta_6\} : *, \top, \Delta_7} \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_6\} : \Delta_7, \top} \top
\end{array}$$

- Case rule I_3

- Case rule \otimes

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_6\} : F_4}{\bullet h_1 \vdash \{\Delta_6\} : !F_4, *} \quad ! \quad \frac{h_5 \vdash \{\Delta_6\} : F_8, \Delta_{10}, ?dual(F_4) \quad h_5 \vdash \{\Delta_6\} : F_9, \Delta_7}{\bullet h_5 \vdash \{\Delta_6\} : dual(!F_4), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes \\
\text{Cut} \\
\frac{}{- \vdash \{\Delta_6\} : *, \Delta_7, \Delta_{10}, F_8 \otimes F_9} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_6\} : !F_4}{- \vdash \{\Delta_6\} : \Delta_{10}, F_8} \text{ax} \quad \frac{h_5 \vdash \{\Delta_6\} : \Delta_{10}, F_8, ?dual(F_4)}{- \vdash \{\Delta_6\} : \Delta_7, F_9} \text{ax}}{- \vdash \{\Delta_6\} : \Delta_{10}, \Delta_7, F_8 \otimes F_9} \text{hCut} \otimes \\
\frac{h_1 \vdash \{\Delta_6\} : F_4}{\bullet h_1 \vdash \{\Delta_6\} : !F_4, *} \quad ! \quad \frac{h_5 \vdash \{\Delta_6\} : F_8, \Delta_7 \quad h_5 \vdash \{\Delta_6\} : F_9, \Delta_{10}, ?dual(F_4)}{\bullet h_5 \vdash \{\Delta_6\} : dual(!F_4), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes \\
\text{Cut} \\
\frac{}{- \vdash \{\Delta_6\} : *, \Delta_7, \Delta_{10}, F_8 \otimes F_9} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_6\} : !F_4}{- \vdash \{\Delta_6\} : \Delta_7, F_8} \text{ax} \quad \frac{h_5 \vdash \{\Delta_6\} : \Delta_{10}, F_9, ?dual(F_4)}{- \vdash \{\Delta_6\} : \Delta_{10}, F_9} \text{ax}}{- \vdash \{\Delta_6\} : \Delta_{10}, \Delta_7, F_8 \otimes F_9} \text{hCut} \otimes
\end{array}$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\begin{array}{c}
\frac{h_1 \vdash \{F_7, \Delta_8\} : F_5 \quad h_6 \vdash \{F_7, \Delta_8\} : F_7, \Delta_4, ?dual(F_5)}{\bullet h_1 \vdash \{F_7, \Delta_8\} : !F_5, * \quad \bullet h_6 \vdash \{F_7, \Delta_8\} : dual(!F_5), \Delta_4} \text{?}_C \text{Cut} \\
\frac{}{- \vdash \{F_7, \Delta_8\} : *, \Delta_4} \rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_8, F_7\} : !F_5 \quad ax \quad h_6 \vdash \{\Delta_8, F_7\} : \Delta_4, F_7, ?dual(F_5) \quad ax}{- \vdash \{\Delta_8, F_7\} : \Delta_4, F_7} \text{hCut} \\
\frac{}{- \vdash \{\Delta_8, F_7\} : \Delta_4} \text{?}_C
\end{array}$$

7.3 Status of ?: OK

- Case rule 1

- Case rule !

$$\begin{array}{c}
\frac{h_1 \vdash \{F_5, \Delta_7\} : \Delta_3 \quad ? \quad h_6 \vdash \{\Delta_7\} : dual(F_5)}{\bullet h_1 \vdash \{\Delta_7\} : ?F_5, \Delta_3 \quad \bullet h_6 \vdash \{\Delta_7\} : dual(?F_5), *} \text{!} \text{Cut} \\
\frac{}{- \vdash \{\Delta_7\} : \Delta_3, *} \rightarrow \\
\frac{h_1 \vdash \{\Delta_7, F_5\} : \Delta_3 \quad ax \quad h_6 \vdash \{\Delta_7\} : dual(F_5) \quad ax}{\bullet h_1 \vdash \{\Delta_7, F_5\} : \Delta_3 \quad \bullet h_6 \vdash \{\Delta_7\} : !dual(F_5) \quad !} \text{mCut} \\
\frac{}{- \vdash \{\Delta_7\} : \Delta_3}
\end{array}$$

- Case rule ?

$$\begin{array}{c}
\frac{h_1 \vdash \{F_5, \Delta_7\} : \Delta_3 \quad ? \quad h_6 \vdash \{F_8, \Delta_7\} : \Delta_9, !dual(F_5)}{\bullet h_1 \vdash \{\Delta_7\} : ?F_5, \Delta_3 \quad \bullet h_6 \vdash \{\Delta_7\} : dual(?F_5), \Delta_9, ?F_8} \text{?} \text{Cut} \\
\frac{}{- \vdash \{\Delta_7\} : \Delta_3, \Delta_9, ?F_8} \rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_7\} : \Delta_3, ?F_5 \quad ax \quad h_6 \vdash \{\Delta_7, F_8\} : \Delta_9, !dual(F_5) \quad ax}{\bullet h_1 \vdash \{\Delta_7, F_8\} : \Delta_3, ?F_5 \quad W \quad h_6 \vdash \{\Delta_7, F_8\} : \Delta_9, !dual(F_5)} \text{hCut} \\
\frac{}{- \vdash \{\Delta_7, F_8\} : \Delta_3, \Delta_9} \text{?} \\
\frac{}{- \vdash \{\Delta_7\} : \Delta_3, \Delta_9, ?F_8} \\
\frac{h_2 \vdash \{F_4, \Delta_8\} : F_6, \Delta_5 \quad ? \quad h_7 \vdash \{F_9, \Delta_8\} : \Delta_{10}, dual(F_6)}{\bullet h_2 \vdash \{\Delta_8\} : F_6, \Delta_5, ?F_4 \quad \bullet h_7 \vdash \{\Delta_8\} : dual(F_6), \Delta_{10}, ?F_9} \text{?} \text{Cut} \\
\frac{}{- \vdash \{\Delta_8\} : (\Delta_5, ?F_4), \Delta_{10}, ?F_9} \rightarrow \\
\frac{h_2 \vdash \{\Delta_8, F_4\} : \Delta_5, F_6 \quad ax \quad \bullet h_7 \vdash \{\Delta_8\} : \Delta_{10}, ?F_9, dual(F_6) \quad ax}{h_2 \vdash \{\Delta_8, F_4\} : \Delta_5, F_6 \quad \bullet h_7 \vdash \{\Delta_8, F_4\} : \Delta_{10}, ?F_9, dual(F_6)} \text{W} \text{hCut} \\
\frac{}{- \vdash \{\Delta_8, F_4\} : \Delta_{10}, \Delta_5, ?F_9} \text{?} \\
\frac{}{- \vdash \{\Delta_8\} : \Delta_{10}, \Delta_5, ?F_4, ?F_9}
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{h_1 \vdash \{F_5, \Delta_7\} : \Delta_3 \quad ? \quad h_6 \vdash \{\Delta_7\} : F_8, F_9, \Delta_{10}, !dual(F_5)}{\bullet h_1 \vdash \{\Delta_7\} : ?F_5, \Delta_3 \quad \bullet h_6 \vdash \{\Delta_7\} : dual(?F_5), \Delta_{10}, F_8 \$F_9} \$ \text{Cut} \\
\frac{}{- \vdash \{\Delta_7\} : \Delta_3, \Delta_{10}, F_8 \$F_9} \rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_7\} : \Delta_3, ?F_5 \quad ax \quad h_6 \vdash \{\Delta_7\} : \Delta_{10}, F_8, F_9, !dual(F_5) \quad ax}{\bullet h_1 \vdash \{\Delta_7\} : \Delta_3, ?F_5 \quad h_6 \vdash \{\Delta_7\} : \Delta_{10}, F_8, F_9, !dual(F_5)} \text{hCut} \\
\frac{}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_8, F_9} \$ \\
\frac{}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_8 \$F_9}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash \{F_4, \Delta_8\} : F_6, \Delta_5}{\bullet h_2 \vdash \{\Delta_8\} : F_6, \Delta_5, ?F_4} ? \quad \frac{h_7 \vdash \{\Delta_8\} : F_9, F_{10}, \Delta_{11}, dual(F_6)}{\bullet h_7 \vdash \{\Delta_8\} : dual(F_6), \Delta_{11}, F_9 \& F_{10}} \$ \\
\hline
- \vdash \{\Delta_8\} : (\Delta_5, ?F_4), \Delta_{11}, F_9 \& F_{10} \\
\rightarrow \\
\frac{\bullet h_2 \vdash \{\Delta_8\} : \Delta_5, F_6, ?F_4}{- \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, F_{10}, F_9, ?F_4} ax \quad \frac{h_7 \vdash \{\Delta_8\} : \Delta_{11}, F_{10}, F_9, dual(F_6)}{- \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, ?F_4, F_9 \& F_{10}} ax \\
\hline
- \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, ?F_4, F_9 \& F_{10} \quad hCut \quad \$
\end{array}$$

• Case rule $\&$

$$\begin{array}{c}
\frac{h_1 \vdash \{F_5, \Delta_7\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_7\} : ?F_5, \Delta_3} ? \quad \frac{h_6 \vdash \{\Delta_7\} : F_8, \Delta_{10}, !dual(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : dual(?F_5), \Delta_{10}, F_8 \& F_9} hCut \\
\hline
- \vdash \{\Delta_7\} : \Delta_3, \Delta_{10}, F_8 \& F_9 \\
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_7\} : \Delta_3, ?F_5}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_8} ax \quad \frac{h_6 \vdash \{\Delta_7\} : \Delta_{10}, F_8, !dual(F_5)}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_8} hCut \\
\hline
- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_8 \& F_9 \quad \& \\
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_7\} : \Delta_3, ?F_5}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_9} ax \quad \frac{h_6 \vdash \{\Delta_7\} : \Delta_{10}, F_9, !dual(F_5)}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_9} hCut \\
\hline
- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_8 \& F_9 \quad \&
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash \{F_4, \Delta_8\} : F_6, \Delta_5}{\bullet h_2 \vdash \{\Delta_8\} : F_6, \Delta_5, ?F_4} ? \quad \frac{h_7 \vdash \{\Delta_8\} : F_9, \Delta_{11}, dual(F_6)}{\bullet h_7 \vdash \{\Delta_8\} : dual(F_6), \Delta_{11}, F_9 \& F_{10}} Cut \\
\hline
- \vdash \{\Delta_8\} : (\Delta_5, ?F_4), \Delta_{11}, F_9 \& F_{10} \\
\rightarrow \\
\frac{h_2 \vdash \{\Delta_8, F_4\} : \Delta_5, F_6}{- \vdash \{\Delta_8, F_4\} : \Delta_{11}, \Delta_5, F_9 \& F_{10}} ax \quad \frac{\bullet h_7 \vdash \{\Delta_8\} : \Delta_{11}, dual(F_6), F_9 \& F_{10}}{\bullet h_7 \vdash \{\Delta_8, F_4\} : \Delta_{11}, dual(F_6), F_9 \& F_{10}} W \\
\hline
- \vdash \{\Delta_8, F_4\} : \Delta_{11}, \Delta_5, F_9 \& F_{10} \quad hCut \\
- \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, ?F_4, F_9 \& F_{10} ?
\end{array}$$

• Case rule \oplus_B

$$\begin{array}{c}
\frac{h_1 \vdash \{F_5, \Delta_7\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_7\} : ?F_5, \Delta_3} ? \quad \frac{h_6 \vdash \{\Delta_7\} : F_9, \Delta_{10}, !dual(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : dual(?F_5), \Delta_{10}, F_8 \oplus F_9} \oplus_B \\
\hline
- \vdash \{\Delta_7\} : \Delta_3, \Delta_{10}, F_8 \oplus F_9 \\
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_7\} : \Delta_3, ?F_5}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_9} ax \quad \frac{h_6 \vdash \{\Delta_7\} : \Delta_{10}, F_9, !dual(F_5)}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_9} hCut \\
\hline
- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_9 \oplus_B \\
- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_8 \oplus F_9 \oplus_B
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash \{F_4, \Delta_8\} : F_6, \Delta_5}{\bullet h_2 \vdash \{\Delta_8\} : F_6, \Delta_5, ?F_4} ? \quad \frac{h_7 \vdash \{\Delta_8\} : F_{10}, \Delta_{11}, dual(F_6)}{\bullet h_7 \vdash \{\Delta_8\} : dual(F_6), \Delta_{11}, F_9 \oplus F_{10}} \oplus_B \\
\hline
- \vdash \{\Delta_8\} : (\Delta_5, ?F_4), \Delta_{11}, F_9 \oplus F_{10} \\
\rightarrow \\
\frac{\bullet h_2 \vdash \{\Delta_8\} : \Delta_5, F_6, ?F_4}{- \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, F_{10}, ?F_4} ax \quad \frac{h_7 \vdash \{\Delta_8\} : \Delta_{11}, F_{10}, dual(F_6)}{- \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, ?F_4, F_9 \oplus F_{10}} hCut \\
\hline
- \vdash \{\Delta_8\} : \Delta_{11}, \Delta_5, ?F_4, F_9 \oplus F_{10} \oplus_B
\end{array}$$

• Case rule \oplus_A

$$\begin{array}{c}
\frac{h_1 \vdash \{F_5, \Delta_7\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_7\} : ?F_5, \Delta_3} ? \quad \frac{h_6 \vdash \{\Delta_7\} : F_8, \Delta_{10}, !dual(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : dual(?F_5), \Delta_{10}, F_8 \oplus F_9} \oplus_A \\
\hline
- \vdash \{\Delta_7\} : \Delta_3, \Delta_{10}, F_8 \oplus F_9 \\
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_7\} : \Delta_3, ?F_5}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_8} ax \quad \frac{h_6 \vdash \{\Delta_7\} : \Delta_{10}, F_8, !dual(F_5)}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_8} hCut \\
\hline
- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_8 \oplus_A \\
- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_3, F_8 \oplus F_9 \oplus_A
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \text{ :- } \{F_4, \Delta_8\} : F_6, \Delta_5}{\bullet h_2 \text{ :- } \{\Delta_8\} : F_6, \Delta_5, ?F_4} ? \quad \frac{h_7 \text{ :- } \{\Delta_8\} : F_9, \Delta_{11}, \text{dual}(F_6)}{\bullet h_7 \text{ :- } \{\Delta_8\} : \text{dual}(F_6), \Delta_{11}, F_9 \oplus F_{10}} \oplus^A \\
\text{Cut} \\
\text{-- :- } \{\Delta_8\} : (\Delta_5, ?F_4), \Delta_{11}, F_9 \oplus F_{10} \\
\rightarrow \\
\frac{\bullet h_2 \text{ :- } \{\Delta_8\} : \Delta_5, F_6, ?F_4 \quad \text{ax} \quad h_7 \text{ :- } \{\Delta_8\} : \Delta_{11}, F_9, \text{dual}(F_6) \quad \text{ax}}{\text{-- :- } \{\Delta_8\} : \Delta_{11}, \Delta_5, F_9, ?F_4} \text{hCut} \\
\oplus^A \\
\text{-- :- } \{\Delta_8\} : \Delta_{11}, \Delta_5, ?F_4, F_9 \oplus F_{10}
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_1 \text{ :- } \{F_5, \Delta_7\} : \Delta_3}{\bullet h_1 \text{ :- } \{\Delta_7\} : ?F_5, \Delta_3} ? \quad \frac{h_6 \text{ :- } \{\Delta_7\} : \Delta_8, !\text{dual}(F_5)}{\bullet h_6 \text{ :- } \{\Delta_7\} : \text{dual}(?F_5), \perp, \Delta_8} \perp \\
\text{Cut} \\
\text{-- :- } \{\Delta_7\} : \Delta_3, \perp, \Delta_8 \\
\rightarrow \\
\frac{\bullet h_1 \text{ :- } \{\Delta_7\} : \Delta_3, ?F_5 \quad \text{ax} \quad h_6 \text{ :- } \{\Delta_7\} : \Delta_8, !\text{dual}(F_5) \quad \text{ax}}{\text{-- :- } \{\Delta_7\} : \Delta_3, \Delta_8} \text{hCut} \\
\perp \\
\text{-- :- } \{\Delta_7\} : \Delta_3, \Delta_8, \perp \\
\frac{h_2 \text{ :- } \{F_4, \Delta_8\} : F_6, \Delta_5}{\bullet h_2 \text{ :- } \{\Delta_8\} : F_6, \Delta_5, ?F_4} ? \quad \frac{h_7 \text{ :- } \{\Delta_8\} : \Delta_9, \text{dual}(F_6)}{\bullet h_7 \text{ :- } \{\Delta_8\} : \text{dual}(F_6), \perp, \Delta_9} \perp \\
\text{Cut} \\
\text{-- :- } \{\Delta_8\} : (\Delta_5, ?F_4), \perp, \Delta_9 \\
\rightarrow \\
\frac{\bullet h_2 \text{ :- } \{\Delta_8\} : \Delta_5, F_6, ?F_4 \quad \text{ax} \quad h_7 \text{ :- } \{\Delta_8\} : \Delta_9, \text{dual}(F_6) \quad \text{ax}}{\text{-- :- } \{\Delta_8\} : \Delta_5, \Delta_9, ?F_4} \text{hCut} \\
\perp \\
\text{-- :- } \{\Delta_8\} : \Delta_5, \Delta_9, \perp, ?F_4
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_1 \text{ :- } \{F_5, \Delta_7\} : \Delta_3}{\bullet h_1 \text{ :- } \{\Delta_7\} : ?F_5, \Delta_3} ? \quad \frac{\bullet h_6 \text{ :- } \{\Delta_7\} : \text{dual}(?F_5), \top, \Delta_8}{\text{-- :- } \{\Delta_7\} : \Delta_3, \top, \Delta_8} \top \\
\text{Cut} \\
\text{-- :- } \{\Delta_7\} : \Delta_3, \top, \Delta_8 \\
\rightarrow \\
\text{-- :- } \{\Delta_7\} : \Delta_3, \Delta_8, \top \quad \top \\
\frac{h_2 \text{ :- } \{F_4, \Delta_8\} : F_6, \Delta_5}{\bullet h_2 \text{ :- } \{\Delta_8\} : F_6, \Delta_5, ?F_4} ? \quad \frac{\bullet h_7 \text{ :- } \{\Delta_8\} : \text{dual}(F_6), \top, \Delta_9}{\text{-- :- } \{\Delta_8\} : (\Delta_5, ?F_4), \top, \Delta_9} \top \\
\text{Cut} \\
\text{-- :- } \{\Delta_8\} : (\Delta_5, ?F_4), \top, \Delta_9 \\
\rightarrow \\
\text{-- :- } \{\Delta_8\} : \Delta_5, \Delta_9, \top, ?F_4 \quad \top
\end{array}$$

- Case rule I_3

- Case rule \otimes

$$\begin{array}{c}
\frac{h_1 \text{ :- } \{F_5, \Delta_7\} : \Delta_3}{\bullet h_1 \text{ :- } \{\Delta_7\} : ?F_5, \Delta_3} ? \quad \frac{h_6 \text{ :- } \{\Delta_7\} : F_9, \Delta_{11}, !\text{dual}(F_5) \quad h_6 \text{ :- } \{\Delta_7\} : F_{10}, \Delta_8}{\bullet h_6 \text{ :- } \{\Delta_7\} : \text{dual}(?F_5), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\text{Cut} \\
\text{-- :- } \{\Delta_7\} : \Delta_3, \Delta_8, \Delta_{11}, F_9 \otimes F_{10} \\
\rightarrow \\
\frac{\bullet h_1 \text{ :- } \{\Delta_7\} : \Delta_3, ?F_5 \quad \text{ax} \quad h_6 \text{ :- } \{\Delta_7\} : \Delta_{11}, F_9, !\text{dual}(F_5) \quad \text{ax}}{\text{-- :- } \{\Delta_7\} : \Delta_{11}, \Delta_3, F_9} \text{hCut} \quad \frac{\text{-- :- } \{\Delta_7\} : \Delta_8, F_{10} \quad \text{ax}}{\text{-- :- } \{\Delta_7\} : \Delta_{11}, \Delta_3, \Delta_8, F_9 \otimes F_{10}} \otimes \\
\frac{h_1 \text{ :- } \{F_5, \Delta_7\} : \Delta_3}{\bullet h_1 \text{ :- } \{\Delta_7\} : ?F_5, \Delta_3} ? \quad \frac{h_6 \text{ :- } \{\Delta_7\} : F_9, \Delta_8 \quad h_6 \text{ :- } \{\Delta_7\} : F_{10}, \Delta_{11}, !\text{dual}(F_5)}{\bullet h_6 \text{ :- } \{\Delta_7\} : \text{dual}(?F_5), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes \\
\text{Cut} \\
\text{-- :- } \{\Delta_7\} : \Delta_3, \Delta_8, \Delta_{11}, F_9 \otimes F_{10} \\
\rightarrow \\
\frac{\text{-- :- } \{\Delta_7\} : \Delta_8, F_9 \quad \text{ax} \quad \bullet h_1 \text{ :- } \{\Delta_7\} : \Delta_3, ?F_5 \quad \text{ax} \quad h_6 \text{ :- } \{\Delta_7\} : \Delta_{11}, F_{10}, !\text{dual}(F_5) \quad \text{ax}}{\text{-- :- } \{\Delta_7\} : \Delta_{11}, \Delta_3, F_{10}} \text{hCut} \\
\otimes \\
\text{-- :- } \{\Delta_7\} : \Delta_{11}, \Delta_3, \Delta_8, F_9 \otimes F_{10}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash \{F_4, \Delta_8\} : F_6, \Delta_5}{\bullet h_2 \vdash \{\Delta_8\} : F_6, \Delta_5, ?F_4} ? \quad \frac{h_7 \vdash \{\Delta_8\} : F_{10}, \Delta_{12}, dual(F_6) \quad h_7 \vdash \{\Delta_8\} : F_{11}, \Delta_9}{\bullet h_7 \vdash \{\Delta_8\} : dual(F_6), \Delta_9, \Delta_{12}, F_{10} \otimes F_{11}} \otimes \\
\hline
- \vdash \{\Delta_8\} : (\Delta_5, ?F_4), \Delta_9, \Delta_{12}, F_{10} \otimes F_{11} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \vdash \{\Delta_8, F_4\} : \Delta_5, F_6}{- \vdash \{\Delta_8, F_4\} : \Delta_{12}, \Delta_5, \Delta_9, F_{10} \otimes F_{11}} \text{ax} \quad \frac{\bullet h_7 \vdash \{\Delta_8\} : \Delta_{12}, \Delta_9, dual(F_6), F_{10} \otimes F_{11}}{\bullet h_7 \vdash \{\Delta_8, F_4\} : \Delta_{12}, \Delta_9, dual(F_6), F_{10} \otimes F_{11}} \text{ax}}{- \vdash \{\Delta_8, F_4\} : \Delta_{12}, \Delta_5, \Delta_9, ?F_4, F_{10} \otimes F_{11}} ? \quad \text{hCut} \\
- \vdash \{\Delta_8\} : \Delta_{12}, \Delta_5, \Delta_9, ?F_4, F_{10} \otimes F_{11} \\
? \\
\frac{h_2 \vdash \{F_4, \Delta_8\} : F_6, \Delta_5}{\bullet h_2 \vdash \{\Delta_8\} : F_6, \Delta_5, ?F_4} ? \quad \frac{h_7 \vdash \{\Delta_8\} : F_{10}, \Delta_9 \quad h_7 \vdash \{\Delta_8\} : F_{11}, \Delta_{12}, dual(F_6)}{\bullet h_7 \vdash \{\Delta_8\} : dual(F_6), \Delta_9, \Delta_{12}, F_{10} \otimes F_{11}} \otimes \\
\hline
- \vdash \{\Delta_8\} : (\Delta_5, ?F_4), \Delta_9, \Delta_{12}, F_{10} \otimes F_{11} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \vdash \{\Delta_8, F_4\} : \Delta_5, F_6}{- \vdash \{\Delta_8, F_4\} : \Delta_{12}, \Delta_5, \Delta_9, F_{10} \otimes F_{11}} \text{ax} \quad \frac{\bullet h_7 \vdash \{\Delta_8\} : \Delta_{12}, \Delta_9, dual(F_6), F_{10} \otimes F_{11}}{\bullet h_7 \vdash \{\Delta_8, F_4\} : \Delta_{12}, \Delta_9, dual(F_6), F_{10} \otimes F_{11}} \text{ax}}{- \vdash \{\Delta_8, F_4\} : \Delta_{12}, \Delta_5, \Delta_9, ?F_4, F_{10} \otimes F_{11}} ? \quad \text{hCut} \\
- \vdash \{\Delta_8\} : \Delta_{12}, \Delta_5, \Delta_9, ?F_4, F_{10} \otimes F_{11} \\
?
\end{array}$$

- Case rule I_1
- Case rule I_2
- Case rule $?_C$

$$\begin{array}{c}
\frac{h_1 \vdash \{F_6, F_8, \Delta_9\} : \Delta_3}{\bullet h_1 \vdash \{F_8, \Delta_9\} : ?F_6, \Delta_3} ? \quad \frac{h_7 \vdash \{F_8, \Delta_9\} : F_8, \Delta_5, !dual(F_6)}{\bullet h_7 \vdash \{F_8, \Delta_9\} : dual(?F_6), \Delta_5} ?_C \\
\hline
- \vdash \{F_8, \Delta_9\} : \Delta_3, \Delta_5 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_9, F_8\} : \Delta_3, ?F_6}{- \vdash \{\Delta_9, F_8\} : \Delta_3, \Delta_5, F_8} \text{ax} \quad \frac{h_7 \vdash \{\Delta_9, F_8\} : \Delta_5, F_8, !dual(F_6)}{\bullet h_7 \vdash \{\Delta_9, F_8\} : \Delta_5, F_8, !dual(F_6)} \text{ax}}{- \vdash \{\Delta_9, F_8\} : \Delta_3, \Delta_5} ?_C \quad \text{hCut} \\
- \vdash \{\Delta_9, F_8\} : \Delta_3, \Delta_5 \\
?_C \\
\frac{h_2 \vdash \{F_4, F_9, \Delta_{10}\} : F_7, \Delta_5}{\bullet h_2 \vdash \{F_9, \Delta_{10}\} : F_7, \Delta_5, ?F_4} ? \quad \frac{h_8 \vdash \{F_9, \Delta_{10}\} : F_9, \Delta_6, dual(F_7)}{\bullet h_8 \vdash \{F_9, \Delta_{10}\} : dual(F_7), \Delta_6} ?_C \\
\hline
- \vdash \{F_9, \Delta_{10}\} : (\Delta_5, ?F_4), \Delta_6 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \vdash \{\Delta_{10}, F_9\} : \Delta_5, F_7, ?F_4}{- \vdash \{\Delta_{10}, F_9\} : \Delta_5, \Delta_6, F_9, ?F_4} \text{ax} \quad \frac{h_8 \vdash \{\Delta_{10}, F_9\} : \Delta_6, F_9, dual(F_7)}{\bullet h_8 \vdash \{\Delta_{10}, F_9\} : \Delta_6, F_9, dual(F_7)} \text{ax}}{- \vdash \{\Delta_{10}, F_9\} : \Delta_5, \Delta_6, ?F_4} ?_C \quad \text{hCut} \\
- \vdash \{\Delta_{10}, F_9\} : \Delta_5, \Delta_6, ?F_4 \\
?_C
\end{array}$$

7.4 Status of \$: OK

- Case rule 1
- Case rule !
- Case rule ?

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_6, F_7, \Delta_3 \quad \bullet h_8 \vdash \{F_{10}, \Delta_9\} : \Delta_{11}, dual(F_6) \otimes dual(F_7) \quad ?}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \S F_7, \Delta_3 \quad \bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \S F_7), \Delta_{11}, ?F_{10} \quad ?} \text{Cut} \\
\quad \quad \quad \rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \S F_7 \quad ax}{\bullet h_1 \vdash \{\Delta_9, F_{10}\} : \Delta_3, F_6 \S F_7} \quad W \quad \frac{h_8 \vdash \{\Delta_9, F_{10}\} : \Delta_{11}, dual(F_6) \otimes dual(F_7) \quad ax}{- \vdash \{\Delta_9, F_{10}\} : \Delta_{11}, \Delta_3 \quad ?} \quad hCut}{- \vdash \{\Delta_9\} : \Delta_{11}, \Delta_3, ?F_{10} \quad ?} \\
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, F_5, \Delta_6 \quad \bullet h_8 \vdash \{F_{10}, \Delta_9\} : \Delta_{11}, dual(F_7) \quad ?}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \S F_5 \quad \bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{11}, ?F_{10} \quad ?} \text{Cut} \\
\quad \quad \quad \rightarrow \\
\frac{\frac{h_2 \vdash \{\Delta_9\} : \Delta_6, F_4, F_5, F_7 \quad ax \quad \bullet h_8 \vdash \{\Delta_9\} : \Delta_{11}, ?F_{10}, dual(F_7) \quad ax}{- \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, F_4, F_5, ?F_{10} \quad \S} \quad hCut}{- \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, ?F_{10}, F_4 \S F_5} \text{Cut}
\end{array}$$

- Case rule \S

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_6, F_7, \Delta_3 \quad \bullet h_8 \vdash \{\Delta_9\} : F_{10}, F_{11}, \Delta_{12}, dual(F_6) \otimes dual(F_7) \quad \S}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \S F_7, \Delta_3 \quad \bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \S F_7), \Delta_{12}, F_{10} \S F_{11} \quad ?} \text{Cut} \\
\quad \quad \quad \rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \S F_7 \quad ax \quad h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, F_{11}, dual(F_6) \otimes dual(F_7) \quad ax}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10}, F_{11} \quad \S} \quad hCut}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10} \S F_{11} \quad \S} \\
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, F_5, \Delta_6 \quad \bullet h_8 \vdash \{\Delta_9\} : F_{10}, F_{11}, \Delta_{12}, dual(F_7) \quad \S}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \S F_5 \quad \bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{12}, F_{10} \S F_{11} \quad \S} \text{Cut} \\
\quad \quad \quad \rightarrow \\
\frac{\frac{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \S F_5 \quad ax \quad h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, F_{11}, dual(F_7) \quad ax}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10}, F_{11}, F_4 \S F_5 \quad \S} \quad hCut}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10} \S F_{11}, F_4 \S F_5} \text{Cut}
\end{array}$$

- Case rule $\&$

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_6, F_7, \Delta_3 \quad \bullet h_8 \vdash \{\Delta_9\} : F_{10}, \Delta_{12}, dual(F_6) \otimes dual(F_7) \quad h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{12}, dual(F_6) \otimes dual(F_7) \quad \&}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \S F_7, \Delta_3 \quad \bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \S F_7), \Delta_{12}, F_{10} \& F_{11} \quad ?} \text{Cut} \\
\quad \quad \quad \rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \S F_7 \quad ax \quad h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, dual(F_6) \otimes dual(F_7) \quad ax}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10} \quad \S} \quad hCut \quad \frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \S F_7 \quad ax \quad h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{11}, dual(F_6) \otimes dual(F_7) \quad ax}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{11} \quad \&}}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10} \& F_{11} \quad \&}} \\
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, F_5, \Delta_6 \quad \bullet h_8 \vdash \{\Delta_9\} : F_{10}, \Delta_{12}, dual(F_7) \quad h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{12}, dual(F_7) \quad \&}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \S F_5 \quad \bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{12}, F_{10} \& F_{11} \quad ?} \text{Cut} \\
\quad \quad \quad \rightarrow \\
\frac{\frac{h_2 \vdash \{\Delta_9\} : \Delta_6, F_4, F_5, F_7 \quad ax \quad \bullet h_8 \vdash \{\Delta_9\} : \Delta_{12}, dual(F_7), F_{10} \& F_{11} \quad ax}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_4, F_5, F_{10} \& F_{11} \quad \S} \quad hCut}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_4 \S F_5, F_{10} \& F_{11} \quad \&}}
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, F_5, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \S F_5} \S \quad \frac{}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \top, \Delta_{10}} \top \\
\hline
- \vdash \{\Delta_9\} : (\Delta_6, F_4 \S F_5), \top, \Delta_{10} \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \top, F_4 \S F_5} \top \\
\text{Cut}
\end{array}$$

- Case rule I_3

- Case rule \otimes

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_6, F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \S F_7, \Delta_3} \S \quad \frac{h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{13}, dual(F_6) \otimes dual(F_7) \quad h_8 \vdash \{\Delta_9\} : F_{12}, \Delta_{10}}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \S F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes \\
\hline
- \vdash \{\Delta_9\} : \Delta_3, \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12} \\
\rightarrow \\
\frac{\frac{h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \S F_7}{- \vdash \{\Delta_9\} : \Delta_{13}, \Delta_3, F_{11}} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{13}, F_{11}, dual(F_6) \otimes dual(F_7)}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_3, F_{11} \otimes F_{12}} \text{hCut}}{- \vdash \{\Delta_9\} : \Delta_{10}, F_{12}} \text{ax} \\
\otimes
\end{array}$$

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_6, F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \S F_7, \Delta_3} \S \quad \frac{h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{10} \quad h_8 \vdash \{\Delta_9\} : F_{12}, \Delta_{13}, dual(F_6) \otimes dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \S F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes \\
\hline
- \vdash \{\Delta_9\} : \Delta_3, \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12} \\
\rightarrow \\
\frac{\frac{h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \S F_7}{- \vdash \{\Delta_9\} : \Delta_{10}, F_{11}} \text{ax} \quad \frac{\frac{h_8 \vdash \{\Delta_9\} : \Delta_{13}, F_{12}, dual(F_6) \otimes dual(F_7)}{- \vdash \{\Delta_9\} : \Delta_{13}, \Delta_3, F_{12}} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_3, F_{11} \otimes F_{12}} \text{hCut}}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_3, F_{11} \otimes F_{12}} \otimes
\end{array}$$

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_6, F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \S F_7, \Delta_3} \S \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{10}, dual(F_6) \quad h_8 \vdash \{\Delta_9\} : \Delta_{11}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \S F_7), \Delta_{10}, \Delta_{11}} \otimes \\
\hline
- \vdash \{\Delta_9\} : \Delta_3, \Delta_{10}, \Delta_{11} \\
\rightarrow \\
\frac{\frac{h_1 \vdash \{\Delta_9\} : \Delta_3, F_6, F_7}{- \vdash \{\Delta_9\} : \Delta_{11}, \Delta_3, F_6} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{11}, dual(F_7)}{- \vdash \{\Delta_9\} : \Delta_{10}, dual(F_6)} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{11}, \Delta_3} \text{sCut} \\
\text{sCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, F_5, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \S F_5} \S \quad \frac{h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{13}, dual(F_7) \quad h_8 \vdash \{\Delta_9\} : F_{12}, \Delta_{10}}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes \\
\hline
- \vdash \{\Delta_9\} : (\Delta_6, F_4 \S F_5), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12} \\
\rightarrow \\
\frac{\frac{h_2 \vdash \{\Delta_9\} : \Delta_6, F_4, F_5, F_7}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_4, F_5, F_{11} \otimes F_{12}} \text{ax} \quad \frac{\bullet h_8 \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, dual(F_7), F_{11} \otimes F_{12}}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_4 \S F_5, F_{11} \otimes F_{12}} \text{hCut}}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_4 \S F_5, F_{11} \otimes F_{12}} \S
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, F_5, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \S F_5} \S \quad \frac{h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{10} \quad h_8 \vdash \{\Delta_9\} : F_{12}, \Delta_{13}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes \\
\hline
- \vdash \{\Delta_9\} : (\Delta_6, F_4 \S F_5), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12} \\
\rightarrow \\
\frac{\frac{h_2 \vdash \{\Delta_9\} : \Delta_6, F_4, F_5, F_7}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_4, F_5, F_{11} \otimes F_{12}} \text{ax} \quad \frac{\bullet h_8 \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, dual(F_7), F_{11} \otimes F_{12}}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_4 \S F_5, F_{11} \otimes F_{12}} \text{hCut}}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_4 \S F_5, F_{11} \otimes F_{12}} \S
\end{array}$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\begin{array}{c}
\frac{h_1 \vdash \{F_{10}, \Delta_{11}\} : F_7, F_8, \Delta_3 \quad \bullet h_1 \vdash \{F_{10}, \Delta_{11}\} : F_7 \S F_8, \Delta_3}{\bullet h_1 \vdash \{F_{10}, \Delta_{11}\} : F_7 \S F_8, \Delta_3} \S \quad \frac{h_9 \vdash \{F_{10}, \Delta_{11}\} : F_{10}, \Delta_6, \text{dual}(F_7) \otimes \text{dual}(F_8)}{\bullet h_9 \vdash \{F_{10}, \Delta_{11}\} : \text{dual}(F_7 \S F_8), \Delta_6} ?_C}{\vdash \{F_{10}, \Delta_{11}\} : \Delta_3, \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, F_7 \S F_8 \quad \text{ax} \quad \frac{h_9 \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, F_{10}, \text{dual}(F_7) \otimes \text{dual}(F_8)}{\vdash \{\Delta_{11}, F_{10}\} : \Delta_3, \Delta_6, F_{10}} \text{ax}}{\vdash \{\Delta_{11}, F_{10}\} : \Delta_3, \Delta_6} \text{hCut} \\
\vdash \{\Delta_{11}, F_{10}\} : \Delta_3, \Delta_6} ?_C \\
\vdash \{\Delta_{11}, F_{10}\} : \Delta_3, \Delta_6} \\
\frac{h_2 \vdash \{F_{10}, \Delta_{11}\} : F_8, F_4, F_5, \Delta_6 \quad \bullet h_2 \vdash \{F_{10}, \Delta_{11}\} : F_8, \Delta_6, F_4 \S F_5}{\bullet h_2 \vdash \{F_{10}, \Delta_{11}\} : F_8, \Delta_6, F_4 \S F_5} \S \quad \frac{h_9 \vdash \{F_{10}, \Delta_{11}\} : F_{10}, \Delta_7, \text{dual}(F_8)}{\bullet h_9 \vdash \{F_{10}, \Delta_{11}\} : \text{dual}(F_8), \Delta_7} ?_C}{\vdash \{F_{10}, \Delta_{11}\} : (\Delta_6, F_4 \S F_5), \Delta_7} \text{Cut} \\
\rightarrow \\
\frac{h_2 \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, F_4, F_5, F_8 \quad \text{ax} \quad \bullet h_9 \vdash \{\Delta_{11}, F_{10}\} : \Delta_7, \text{dual}(F_8)}{\vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_7, F_4, F_5} \text{hCut} \\
\vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_7, F_4, F_5} \S \\
\vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_7, F_4 \S F_5}
\end{array}$$

7.5 Status of &: OK

- Case rule 1
- Case rule !
- Case rule ?

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_6, \Delta_3 \quad h_1 \vdash \{\Delta_9\} : F_7, \Delta_3 \quad \bullet h_1 \vdash \{\Delta_9\} : F_6 \& F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \& F_7, \Delta_3} \& \quad \frac{h_8 \vdash \{F_{10}, \Delta_9\} : \Delta_{11}, \text{dual}(F_6) \oplus \text{dual}(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : \text{dual}(F_6 \& F_7), \Delta_{11}, ?F_{10}} ?}{\vdash \{\Delta_9\} : \Delta_3, \Delta_{11}, ?F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \& F_7 \quad \text{ax} \quad \frac{h_8 \vdash \{F_{10}, \Delta_9\} : \Delta_{11}, \text{dual}(F_6) \oplus \text{dual}(F_7)}{\bullet h_8 \vdash \{\Delta_9, F_{10}\} : \Delta_{11}, \text{dual}(F_6) \oplus \text{dual}(F_7)} \text{ax}}{\bullet h_1 \vdash \{\Delta_9, F_{10}\} : \Delta_3, F_6 \& F_7 \quad W \quad \bullet h_8 \vdash \{\Delta_9, F_{10}\} : \Delta_{11}, \text{dual}(F_6) \oplus \text{dual}(F_7)} \text{hCut} \\
\vdash \{\Delta_9, F_{10}\} : \Delta_{11}, \Delta_3} ? \\
\vdash \{\Delta_9\} : \Delta_{11}, \Delta_3, ?F_{10}} \\
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, \Delta_6 \quad h_2 \vdash \{\Delta_9\} : F_7, F_5, \Delta_6 \quad \bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \& F_5}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \& F_5} \& \quad \frac{h_8 \vdash \{F_{10}, \Delta_9\} : \Delta_{11}, \text{dual}(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : \text{dual}(F_7), \Delta_{11}, ?F_{10}} ?}{\vdash \{\Delta_9\} : (\Delta_6, F_4 \& F_5), \Delta_{11}, ?F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \& F_5 \quad \text{ax} \quad \frac{h_8 \vdash \{F_{10}, \Delta_9\} : \Delta_{11}, \text{dual}(F_7)}{\bullet h_8 \vdash \{\Delta_9, F_{10}\} : \Delta_{11}, \text{dual}(F_7)} \text{ax}}{\bullet h_2 \vdash \{\Delta_9, F_{10}\} : \Delta_6, F_7, F_4 \& F_5 \quad W \quad \bullet h_8 \vdash \{\Delta_9, F_{10}\} : \Delta_{11}, \text{dual}(F_7)} \text{hCut} \\
\vdash \{\Delta_9, F_{10}\} : \Delta_{11}, \Delta_6, F_4 \& F_5} ? \\
\vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, ?F_{10}, F_4 \& F_5}
\end{array}$$

- Case rule §

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_6, \Delta_3 \quad h_1 \vdash \{\Delta_9\} : F_7, \Delta_3 \quad \bullet h_1 \vdash \{\Delta_9\} : F_6 \& F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \& F_7, \Delta_3} \& \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, F_{11}, \Delta_{12}, \text{dual}(F_6) \oplus \text{dual}(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : \text{dual}(F_6 \& F_7), \Delta_{12}, F_{10} \S F_{11}} \S}{\vdash \{\Delta_9\} : \Delta_3, \Delta_{12}, F_{10} \S F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \& F_7 \quad \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, F_{11}, \Delta_{12}, \text{dual}(F_6) \oplus \text{dual}(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, F_{11}, \text{dual}(F_6) \oplus \text{dual}(F_7)} \text{ax}}{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \& F_7 \quad \text{ax} \quad \bullet h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, F_{11}, \text{dual}(F_6) \oplus \text{dual}(F_7)} \text{hCut} \\
\vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10}, F_{11}} \S \\
\vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10} \S F_{11}}
\end{array}$$

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_6, \Delta_3 \quad h_1 \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \& F_7, \Delta_3} \& \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{10}, \text{dual}(F_6)}{\bullet h_8 \vdash \{\Delta_9\} : \text{dual}(F_6 \& F_7), \Delta_{10}} \oplus^A \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{- \vdash \{\Delta_9\} : \Delta_3, F_6} \text{ax} \quad \frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \text{dual}(F_6)} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3} \text{sCut} \\
\\
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, \Delta_6 \quad h_2 \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \& F_5} \& \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, \Delta_{12}, \text{dual}(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : \text{dual}(F_7), \Delta_{12}, F_{10} \oplus F_{11}} \oplus^A \\
\frac{}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \& F_5), \Delta_{12}, F_{10} \oplus F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \& F_5} \text{ax} \quad \frac{}{h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, \text{dual}(F_7)} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10}, F_4 \& F_5} \text{hCut} \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_4 \& F_5, F_{10} \oplus F_{11}} \oplus^A \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_4 \& F_5, F_{10} \oplus F_{11}} \oplus^A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_6, \Delta_3 \quad h_1 \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \& F_7, \Delta_3} \& \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{10}, \text{dual}(F_6) \oplus \text{dual}(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : \text{dual}(F_6 \& F_7), \perp, \Delta_{10}} \perp \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_3, \perp, \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \& F_7} \text{ax} \quad \frac{}{h_8 \vdash \{\Delta_9\} : \Delta_{10}, \text{dual}(F_6) \oplus \text{dual}(F_7)} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3} \text{hCut} \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3, \perp} \perp \\
\\
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, \Delta_6 \quad h_2 \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \& F_5} \& \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{10}, \text{dual}(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : \text{dual}(F_7), \perp, \Delta_{10}} \perp \\
\frac{}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \& F_5), \perp, \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \& F_5} \text{ax} \quad \frac{}{h_8 \vdash \{\Delta_9\} : \Delta_{10}, \text{dual}(F_7)} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, F_4 \& F_5} \text{hCut} \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \perp, F_4 \& F_5} \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_6, \Delta_3 \quad h_1 \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \& F_7, \Delta_3} \& \quad \frac{}{\bullet h_8 \vdash \{\Delta_9\} : \text{dual}(F_6 \& F_7), \top, \Delta_{10}} \top \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_3, \top, \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3, \top} \top \\
\\
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, \Delta_6 \quad h_2 \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \& F_5} \& \quad \frac{}{\bullet h_8 \vdash \{\Delta_9\} : \text{dual}(F_7), \top, \Delta_{10}} \top \\
\frac{}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \& F_5), \top, \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \top, F_4 \& F_5} \top
\end{array}$$

- Case rule I_3

- Case rule \otimes

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_6, \Delta_3 \quad h_1 \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \& F_7, \Delta_3} \& \quad \frac{h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{13}, \text{dual}(F_6) \oplus \text{dual}(F_7) \quad h_8 \vdash \{\Delta_9\} : F_{12}, \Delta_{10}}{\bullet h_8 \vdash \{\Delta_9\} : \text{dual}(F_6 \& F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \& F_7} \text{ax} \quad \frac{}{h_8 \vdash \{\Delta_9\} : \Delta_{13}, F_{11}, \text{dual}(F_6) \oplus \text{dual}(F_7)} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{13}, \Delta_3, F_{11}} \text{hCut} \quad \frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, F_{12}} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_3, F_{11} \otimes F_{12}} \otimes
\end{array}$$

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_6, \Delta_3 \quad h_1 \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \& F_7, \Delta_3} \& \quad \frac{h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{10} \quad h_8 \vdash \{\Delta_9\} : F_{12}, \Delta_{13}, \text{dual}(F_6) \oplus \text{dual}(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : \text{dual}(F_6 \& F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \text{Cut} \\
\rightarrow \\
\frac{\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, F_{11}} \text{ax} \quad \frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \& F_7}{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \& F_7} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{13}, F_{12}, \text{dual}(F_6) \oplus \text{dual}(F_7)}{h_8 \vdash \{\Delta_9\} : \Delta_{13}, F_{12}, \text{dual}(F_6) \oplus \text{dual}(F_7)} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_3, F_{11} \otimes F_{12}} \otimes \text{hCut} \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_3, F_{11} \otimes F_{12}} \otimes \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_3, F_{11} \otimes F_{12}} \otimes \\
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, \Delta_6 \quad h_2 \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \& F_5} \& \quad \frac{h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{13}, \text{dual}(F_7) \quad h_8 \vdash \{\Delta_9\} : F_{12}, \Delta_{10}}{\bullet h_8 \vdash \{\Delta_9\} : \text{dual}(F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes \\
\frac{}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \& F_5), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \& F_5}{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \& F_5} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{13}, F_{11}, \text{dual}(F_7)}{h_8 \vdash \{\Delta_9\} : \Delta_{13}, F_{11}, \text{dual}(F_7)} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{13}, \Delta_6, F_{11}, F_4 \& F_5} \text{hCut} \quad \frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, F_{12}} \text{ax} \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_4 \& F_5, F_{11} \otimes F_{12}} \otimes \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_4 \& F_5, F_{11} \otimes F_{12}} \otimes \\
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, \Delta_6 \quad h_2 \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \& F_5} \& \quad \frac{h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{10} \quad h_8 \vdash \{\Delta_9\} : F_{12}, \Delta_{13}, \text{dual}(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : \text{dual}(F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes \\
\frac{}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \& F_5), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \& F_5}{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \& F_5} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{13}, F_{12}, \text{dual}(F_7)}{h_8 \vdash \{\Delta_9\} : \Delta_{13}, F_{12}, \text{dual}(F_7)} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{13}, \Delta_6, F_{12}, F_4 \& F_5} \text{hCut} \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, F_{11}} \text{ax} \quad \frac{}{- \vdash \{\Delta_9\} : \Delta_{13}, \Delta_6, F_{12}, F_4 \& F_5} \otimes \\
\frac{}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_4 \& F_5, F_{11} \otimes F_{12}} \otimes
\end{array}$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\begin{array}{c}
\frac{h_1 \vdash \{F_{10}, \Delta_{11}\} : F_7, \Delta_3 \quad h_1 \vdash \{F_{10}, \Delta_{11}\} : F_8, \Delta_3}{\bullet h_1 \vdash \{F_{10}, \Delta_{11}\} : F_7 \& F_8, \Delta_3} \& \quad \frac{h_9 \vdash \{F_{10}, \Delta_{11}\} : F_{10}, \Delta_6, \text{dual}(F_7) \oplus \text{dual}(F_8)}{\bullet h_9 \vdash \{F_{10}, \Delta_{11}\} : \text{dual}(F_7 \& F_8), \Delta_6} \otimes \\
\frac{}{- \vdash \{F_{10}, \Delta_{11}\} : \Delta_3, \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, F_7 \& F_8}{\bullet h_1 \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, F_7 \& F_8} \text{ax} \quad \frac{h_9 \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, F_{10}, \text{dual}(F_7) \oplus \text{dual}(F_8)}{h_9 \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, F_{10}, \text{dual}(F_7) \oplus \text{dual}(F_8)} \text{ax}}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, \Delta_6, F_{10}} \text{hCut} \\
\frac{}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, \Delta_6} \text{?}_C \\
\frac{}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, \Delta_6} \text{?}_C \\
\frac{h_2 \vdash \{F_{10}, \Delta_{11}\} : F_8, F_4, \Delta_6 \quad h_2 \vdash \{F_{10}, \Delta_{11}\} : F_8, F_5, \Delta_6}{\bullet h_2 \vdash \{F_{10}, \Delta_{11}\} : F_8, \Delta_6, F_4 \& F_5} \& \quad \frac{h_9 \vdash \{F_{10}, \Delta_{11}\} : F_{10}, \Delta_7, \text{dual}(F_8)}{\bullet h_9 \vdash \{F_{10}, \Delta_{11}\} : \text{dual}(F_8), \Delta_7} \otimes \\
\frac{}{- \vdash \{F_{10}, \Delta_{11}\} : (\Delta_6, F_4 \& F_5), \Delta_7} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, F_8, F_4 \& F_5}{\bullet h_2 \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, F_8, F_4 \& F_5} \text{ax} \quad \frac{h_9 \vdash \{\Delta_{11}, F_{10}\} : \Delta_7, F_{10}, \text{dual}(F_8)}{h_9 \vdash \{\Delta_{11}, F_{10}\} : \Delta_7, F_{10}, \text{dual}(F_8)} \text{ax}}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_7, F_{10}, F_4 \& F_5} \text{hCut} \\
\frac{}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_7, F_{10}, F_4 \& F_5} \text{?}_C \\
\frac{}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_7, F_4 \& F_5} \text{?}_C
\end{array}$$

7.6 Status of \oplus_B : OK

- Case rule 1

- Case rule !

- Case rule ?

$$\frac{\frac{h_1 \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_B \quad \frac{h_8 \vdash \{F_{10}, \Delta_9\} : \Delta_{11}, dual(F_6) \& dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{11}, ?F_{10}} ?}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{11}, ?F_{10}} \text{Cut}$$

$$\frac{\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7}{\bullet h_1 \vdash \{\Delta_9, F_{10}\} : \Delta_3, F_6 \oplus F_7} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9, F_{10}\} : \Delta_{11}, dual(F_6) \& dual(F_7)}{h_8 \vdash \{\Delta_9, F_{10}\} : \Delta_{11}, \Delta_3} \text{ax}}{- \vdash \{\Delta_9, F_{10}\} : \Delta_{11}, \Delta_3} \text{hCut}$$

$$\frac{\frac{h_2 \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus_B \quad \frac{h_8 \vdash \{F_{10}, \Delta_9\} : \Delta_{11}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{11}, ?F_{10}} ?}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \Delta_{11}, ?F_{10}} \text{Cut}$$

$$\frac{\frac{h_2 \vdash \{\Delta_9\} : \Delta_6, F_5, F_7}{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_5, F_7} \text{ax} \quad \frac{\bullet h_8 \vdash \{\Delta_9\} : \Delta_{11}, ?F_{10}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, ?F_{10}} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, ?F_{10}} \text{hCut}$$

• Case rule \$

$$\frac{\frac{h_1 \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_B \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, F_{11}, \Delta_{12}, dual(F_6) \& dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{12}, F_{10} \$ F_{11}} \$}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{12}, F_{10} \$ F_{11}} \text{Cut}$$

$$\frac{\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7}{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, F_{11}, dual(F_6) \& dual(F_7)}{h_8 \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10}, F_{11}} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10}, F_{11}} \text{hCut}$$

$$\frac{\frac{h_2 \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus_B \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, F_{11}, \Delta_{12}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{12}, F_{10} \$ F_{11}} \$}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \Delta_{12}, F_{10} \$ F_{11}} \text{Cut}$$

$$\frac{\frac{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \oplus F_5}{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \oplus F_5} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, F_{11}, dual(F_7)}{h_8 \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10}, F_{11}, F_4 \oplus F_5} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10} \$ F_{11}, F_4 \oplus F_5} \text{hCut}$$

• Case rule &

$$\frac{\frac{h_1 \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_B \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, \Delta_{12}, dual(F_6) \& dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{12}, F_{10} \& F_{11}} \text{Cut} \quad \frac{h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{12}, dual(F_6) \& dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{12}, F_{10} \& F_{11}} \&}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{12}, F_{10} \& F_{11}} \text{Cut}$$

$$\frac{\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7}{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, dual(F_6) \& dual(F_7)}{h_8 \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10}} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10}} \text{hCut}$$

$$\frac{\frac{h_1 \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_B \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{10}, dual(F_6)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{10}} \text{Cut} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{10}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{10}} \&}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{10}} \text{Cut}$$

$$\frac{\frac{- \vdash \{\Delta_9\} : \Delta_3, F_7}{- \vdash \{\Delta_9\} : \Delta_3, F_7} \text{ax} \quad \frac{- \vdash \{\Delta_9\} : \Delta_{10}, dual(F_7)}{- \vdash \{\Delta_9\} : \Delta_{10}, dual(F_7)} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3} \text{sCut}$$

$$\frac{\frac{h_2 \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus_B \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, \Delta_{12}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{12}, F_{10} \& F_{11}} \text{Cut} \quad \frac{h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{12}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{12}, F_{10} \& F_{11}} \&}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \Delta_{12}, F_{10} \& F_{11}} \text{Cut}$$

$$\frac{\frac{h_2 \vdash \{\Delta_9\} : \Delta_6, F_5, F_7}{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_5, F_7} \text{ax} \quad \frac{\bullet h_8 \vdash \{\Delta_9\} : \Delta_{12}, dual(F_7), F_{10} \& F_{11}}{\bullet h_8 \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_5, F_{10} \& F_{11}} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_5, F_{10} \& F_{11}} \text{hCut}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_B \quad \frac{h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{12}, dual(F_6) \& dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{12}, F_{10} \oplus F_{11}} \oplus_B}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{12}, F_{10} \oplus F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{11}} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{11}, dual(F_6) \& dual(F_7)}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10} \oplus F_{11}} \oplus_B}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10} \oplus F_{11}} \oplus_B \text{hCut}} \\
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus_B \quad \frac{h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{12}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{12}, F_{10} \oplus F_{11}} \oplus_B}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \Delta_{12}, F_{10} \oplus F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \oplus F_5}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{11}, F_4 \oplus F_5} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{11}, dual(F_7)}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10} \oplus F_{11}, F_4 \oplus F_5} \oplus_B}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10} \oplus F_{11}, F_4 \oplus F_5} \oplus_B \text{hCut}}
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_B \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, \Delta_{12}, dual(F_6) \& dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{12}, F_{10} \oplus F_{11}} \oplus_A}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{12}, F_{10} \oplus F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10}} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, dual(F_6) \& dual(F_7)}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10} \oplus F_{11}} \oplus_A}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10} \oplus F_{11}} \oplus_A \text{hCut}} \\
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus_B \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, \Delta_{12}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{12}, F_{10} \oplus F_{11}} \oplus_A}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \Delta_{12}, F_{10} \oplus F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \vdash \{\Delta_9\} : \Delta_6, F_5, F_7}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_5, F_{10} \oplus F_{11}} \text{ax} \quad \frac{\bullet h_8 \vdash \{\Delta_9\} : \Delta_{12}, dual(F_7), F_{10} \oplus F_{11}}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10} \oplus F_{11}, F_4 \oplus F_5} \oplus_B}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10} \oplus F_{11}, F_4 \oplus F_5} \oplus_B \text{hCut}}
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_B \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{10}, dual(F_6) \& dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \perp, \Delta_{10}} \perp}{- \vdash \{\Delta_9\} : \Delta_3, \perp, \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{10}, dual(F_6) \& dual(F_7)}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3, \perp} \perp}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3, \perp} \perp \text{hCut}} \\
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus_B \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{10}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \perp, \Delta_{10}} \perp}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \perp, \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \vdash \{\Delta_9\} : \Delta_6, F_5, F_7}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, F_5, \perp} \text{ax} \quad \frac{\bullet h_8 \vdash \{\Delta_9\} : \Delta_{10}, \perp, dual(F_7)}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \perp, F_4 \oplus F_5} \oplus_B}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \perp, F_4 \oplus F_5} \oplus_B \text{hCut}}
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_1 : \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 : \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_B \frac{}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \top, \Delta_{10}} \top \\
\frac{}{- : \vdash \{\Delta_9\} : \Delta_3, \top, \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3, \top} \top \\
\\
\frac{h_2 : \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 : \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus_B \frac{}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_7), \top, \Delta_{10}} \top \\
\frac{}{- : \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \top, \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \top, F_4 \oplus F_5} \top
\end{array}$$

• Case rule I_3

• Case rule \otimes

$$\begin{array}{c}
\frac{h_1 : \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 : \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_B \frac{h_8 : \vdash \{\Delta_9\} : F_{11}, \Delta_{13}, dual(F_6) \& dual(F_7) \quad h_8 : \vdash \{\Delta_9\} : F_{12}, \Delta_{10}}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes \\
\frac{}{- : \vdash \{\Delta_9\} : \Delta_3, \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 : \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7 \quad \text{ax} \quad h_8 : \vdash \{\Delta_9\} : \Delta_{13}, F_{11}, dual(F_6) \& dual(F_7) \quad \text{ax}}{- : \vdash \{\Delta_9\} : \Delta_{13}, \Delta_3, F_{11}} \text{hCut} \frac{}{- : \vdash \{\Delta_9\} : \Delta_{10}, F_{12}} \text{ax} \\
\frac{}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_3, F_{11} \otimes F_{12}} \otimes \\
\\
\frac{h_1 : \vdash \{\Delta_9\} : F_7, \Delta_3}{\bullet h_1 : \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_B \frac{h_8 : \vdash \{\Delta_9\} : F_{11}, \Delta_{10} \quad h_8 : \vdash \{\Delta_9\} : F_{12}, \Delta_{13}, dual(F_6) \& dual(F_7)}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes \\
\frac{}{- : \vdash \{\Delta_9\} : \Delta_3, \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \text{Cut} \\
\rightarrow \\
\frac{\text{ax} \quad \bullet h_1 : \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7 \quad \text{ax} \quad h_8 : \vdash \{\Delta_9\} : \Delta_{13}, F_{12}, dual(F_6) \& dual(F_7) \quad \text{ax}}{- : \vdash \{\Delta_9\} : \Delta_{10}, F_{11}} \text{hCut} \frac{}{- : \vdash \{\Delta_9\} : \Delta_{13}, \Delta_3, F_{12}} \text{hCut} \\
\frac{}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_3, F_{11} \otimes F_{12}} \otimes \\
\\
\frac{h_2 : \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 : \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus_B \frac{h_8 : \vdash \{\Delta_9\} : F_{11}, \Delta_{13}, dual(F_7) \quad h_8 : \vdash \{\Delta_9\} : F_{12}, \Delta_{10}}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes \\
\frac{}{- : \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \text{Cut} \\
\rightarrow \\
\frac{h_2 : \vdash \{\Delta_9\} : \Delta_6, F_5, F_7 \quad \text{ax} \quad \bullet h_8 : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, dual(F_7), F_{11} \otimes F_{12} \quad \text{ax}}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_5, F_{11} \otimes F_{12}} \text{hCut} \\
\frac{}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_{11} \otimes F_{12}, F_4 \oplus F_5} \oplus_B \\
\\
\frac{h_2 : \vdash \{\Delta_9\} : F_7, F_5, \Delta_6}{\bullet h_2 : \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus_B \frac{h_8 : \vdash \{\Delta_9\} : F_{11}, \Delta_{10} \quad h_8 : \vdash \{\Delta_9\} : F_{12}, \Delta_{13}, dual(F_7)}{\bullet h_8 : \vdash \{\Delta_9\} : dual(F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes \\
\frac{}{- : \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \text{Cut} \\
\rightarrow \\
\frac{h_2 : \vdash \{\Delta_9\} : \Delta_6, F_5, F_7 \quad \text{ax} \quad \bullet h_8 : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, dual(F_7), F_{11} \otimes F_{12} \quad \text{ax}}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_5, F_{11} \otimes F_{12}} \text{hCut} \\
\frac{}{- : \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_{11} \otimes F_{12}, F_4 \oplus F_5} \oplus_B
\end{array}$$

• Case rule I_1

• Case rule I_2

• Case rule $?_C$

$$\begin{array}{c}
\frac{h_1 \vdash \{F_{10}, \Delta_{11}\} : F_8, \Delta_3}{\bullet h_1 \vdash \{F_{10}, \Delta_{11}\} : F_7 \oplus F_8, \Delta_3} \oplus_B \quad \frac{h_9 \vdash \{F_{10}, \Delta_{11}\} : F_{10}, \Delta_6, dual(F_7) \& dual(F_8)}{\bullet h_9 \vdash \{F_{10}, \Delta_{11}\} : dual(F_7 \oplus F_8), \Delta_6} ?_C}{- \vdash \{F_{10}, \Delta_{11}\} : \Delta_3, \Delta_6} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, F_7 \oplus F_8}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, \Delta_6, F_{10}} \text{ax} \quad \frac{h_9 \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, F_{10}, dual(F_7) \& dual(F_8)}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, \Delta_6} \text{ax}}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, \Delta_6} \text{hCut} \\
?_C \\
\frac{h_2 \vdash \{F_{10}, \Delta_{11}\} : F_8, F_5, \Delta_6}{\bullet h_2 \vdash \{F_{10}, \Delta_{11}\} : F_8, \Delta_6, F_4 \oplus F_5} \oplus_B \quad \frac{h_9 \vdash \{F_{10}, \Delta_{11}\} : F_{10}, \Delta_7, dual(F_8)}{\bullet h_9 \vdash \{F_{10}, \Delta_{11}\} : dual(F_8), \Delta_7} ?_C}{- \vdash \{F_{10}, \Delta_{11}\} : (\Delta_6, F_4 \oplus F_5), \Delta_7} \text{Cut} \\
\rightarrow \\
\frac{h_2 \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, F_5, F_8}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_7, F_5} \text{ax} \quad \frac{\bullet h_9 \vdash \{\Delta_{11}, F_{10}\} : \Delta_7, dual(F_8)}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_7, F_4 \oplus F_5} \text{ax}}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_7, F_4 \oplus F_5} \text{hCut} \\
\oplus_B
\end{array}$$

7.7 Status of \oplus_A : OK

- Case rule 1
- Case rule !
- Case rule ?

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_6, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_A \quad \frac{h_8 \vdash \{F_{10}, \Delta_9\} : \Delta_{11}, dual(F_6) \& dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{11}, ?F_{10}} ?}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{11}, ?F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7}{\bullet h_1 \vdash \{\Delta_9, F_{10}\} : \Delta_3, F_6 \oplus F_7} \text{ax} \quad \frac{h_8 \vdash \{F_{10}, \Delta_9\} : \Delta_{11}, dual(F_6) \& dual(F_7)}{h_8 \vdash \{\Delta_9, F_{10}\} : \Delta_{11}, dual(F_6) \& dual(F_7)} \text{ax}}{- \vdash \{\Delta_9, F_{10}\} : \Delta_{11}, \Delta_3} \text{hCut} \\
? \\
- \vdash \{\Delta_9\} : \Delta_{11}, \Delta_3, ?F_{10} \\
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus_A \quad \frac{h_8 \vdash \{F_{10}, \Delta_9\} : \Delta_{11}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{11}, ?F_{10}} ?}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \Delta_{11}, ?F_{10}} \text{Cut} \\
\rightarrow \\
\frac{h_2 \vdash \{\Delta_9\} : \Delta_6, F_4, F_7}{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_4, F_7} \text{ax} \quad \frac{\bullet h_8 \vdash \{\Delta_9\} : \Delta_{11}, ?F_{10}, dual(F_7)}{- \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, F_4, ?F_{10}} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{11}, \Delta_6, ?F_{10}, F_4 \oplus F_5} \text{hCut} \\
\oplus_A
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_6, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_A \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, F_{11}, \Delta_{12}, dual(F_6) \& dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{12}, F_{10} \$ F_{11}} \$}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{12}, F_{10} \$ F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7}{\bullet h_1 \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10}, F_{11}} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, F_{11}, \Delta_{12}, dual(F_7)}{h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, F_{11}, dual(F_6) \& dual(F_7)} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10}, F_{11}} \text{hCut} \\
\$ \\
- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_3, F_{10} \$ F_{11} \\
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus_A \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, F_{11}, \Delta_{12}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{12}, F_{10} \$ F_{11}} \$}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \Delta_{12}, F_{10} \$ F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \oplus F_5}{\bullet h_2 \vdash \{\Delta_9\} : \Delta_6, F_7, F_4 \oplus F_5} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : F_{10}, F_{11}, \Delta_{12}, dual(F_7)}{h_8 \vdash \{\Delta_9\} : \Delta_{12}, F_{10}, F_{11}, dual(F_7)} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10}, F_{11}, F_4 \oplus F_5} \text{hCut} \\
\$ \\
- \vdash \{\Delta_9\} : \Delta_{12}, \Delta_6, F_{10} \$ F_{11}, F_4 \oplus F_5
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_6, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_A \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{10}, dual(F_6) \& dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \perp, \Delta_{10}} \perp}{- \vdash \{\Delta_9\} : \Delta_3, \perp, \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{10}, dual(F_6) \& dual(F_7)}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3, \perp} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3, \perp} \text{hCut} \\
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus_A \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{10}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \perp, \Delta_{10}} \perp}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \perp, \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{h_2 \vdash \{\Delta_9\} : \Delta_6, F_4, F_7}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, F_4, \perp} \text{ax} \quad \frac{\bullet h_8 \vdash \{\Delta_9\} : \Delta_{10}, \perp, dual(F_7)}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \perp, F_4 \oplus F_5} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \perp, F_4 \oplus F_5} \text{hCut} \oplus_A
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_6, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_A \quad \frac{h_8 \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \top, \Delta_{10}}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \top, \Delta_{10}} \top}{- \vdash \{\Delta_9\} : \Delta_3, \top, \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3, \top}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_3, \top} \top \\
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus_A \quad \frac{h_8 \vdash \{\Delta_9\} : dual(F_7), \top, \Delta_{10}}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \top, \Delta_{10}} \top}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \top, \Delta_{10}} \text{Cut} \\
\rightarrow \\
\frac{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \top, F_4 \oplus F_5}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_6, \top, F_4 \oplus F_5} \top
\end{array}$$

- Case rule I_3

- Case rule \otimes

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_9\} : F_6, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_A \quad \frac{h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{13}, dual(F_6) \& dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes \quad \frac{h_8 \vdash \{\Delta_9\} : F_{12}, \Delta_{10}}{\bullet h_8 \vdash \{\Delta_9\} : F_{12}, \Delta_{10}} \otimes}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \text{Cut} \\
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7}{- \vdash \{\Delta_9\} : \Delta_{13}, \Delta_3, F_{11}} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{13}, F_{11}, dual(F_6) \& dual(F_7)}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_3, F_{11} \otimes F_{12}} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_3, F_{11} \otimes F_{12}} \text{hCut} \otimes \\
\frac{h_1 \vdash \{\Delta_9\} : F_6, \Delta_3}{\bullet h_1 \vdash \{\Delta_9\} : F_6 \oplus F_7, \Delta_3} \oplus_A \quad \frac{h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{10}}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_6 \oplus F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes \quad \frac{h_8 \vdash \{\Delta_9\} : F_{12}, \Delta_{13}, dual(F_6) \& dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : F_{12}, \Delta_{13}, dual(F_6) \& dual(F_7)} \otimes}{- \vdash \{\Delta_9\} : \Delta_3, \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \text{Cut} \\
\rightarrow \\
\frac{- \vdash \{\Delta_9\} : \Delta_{10}, F_{11}}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_3, F_{11} \otimes F_{12}} \text{ax} \quad \frac{\bullet h_1 \vdash \{\Delta_9\} : \Delta_3, F_6 \oplus F_7}{- \vdash \{\Delta_9\} : \Delta_{13}, \Delta_3, F_{12}} \text{ax} \quad \frac{h_8 \vdash \{\Delta_9\} : \Delta_{13}, F_{12}, dual(F_6) \& dual(F_7)}{- \vdash \{\Delta_9\} : \Delta_{13}, \Delta_3, F_{12}} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_3, F_{11} \otimes F_{12}} \text{hCut} \otimes \\
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus_A \quad \frac{h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{13}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes \quad \frac{h_8 \vdash \{\Delta_9\} : F_{12}, \Delta_{10}}{\bullet h_8 \vdash \{\Delta_9\} : F_{12}, \Delta_{10}} \otimes}{- \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \text{Cut} \\
\rightarrow \\
\frac{h_2 \vdash \{\Delta_9\} : \Delta_6, F_4, F_7}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_4, F_{11} \otimes F_{12}} \text{ax} \quad \frac{\bullet h_8 \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, dual(F_7), F_{11} \otimes F_{12}}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_4, F_{11} \otimes F_{12}} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_4, F_{11} \otimes F_{12}} \text{hCut} \oplus_A
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash \{\Delta_9\} : F_7, F_4, \Delta_6}{\bullet h_2 \vdash \{\Delta_9\} : F_7, \Delta_6, F_4 \oplus F_5} \oplus A \quad \frac{h_8 \vdash \{\Delta_9\} : F_{11}, \Delta_{10} \quad h_8 \vdash \{\Delta_9\} : F_{12}, \Delta_{13}, dual(F_7)}{\bullet h_8 \vdash \{\Delta_9\} : dual(F_7), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \otimes \\
\hline
- \vdash \{\Delta_9\} : (\Delta_6, F_4 \oplus F_5), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \vdash \{\Delta_9\} : \Delta_6, F_4, F_7}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_4, F_{11} \otimes F_{12}} \text{ax} \quad \frac{\bullet h_8 \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, dual(F_7), F_{11} \otimes F_{12}}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_{11} \otimes F_{12}} \text{ax}}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_{11} \otimes F_{12}, F_4 \oplus F_5} \oplus A}{- \vdash \{\Delta_9\} : \Delta_{10}, \Delta_{13}, \Delta_6, F_{11} \otimes F_{12}, F_4 \oplus F_5} \text{hCut}
\end{array}$$

• Case rule I_1

• Case rule I_2

• Case rule $?_C$

$$\begin{array}{c}
\frac{h_1 \vdash \{F_{10}, \Delta_{11}\} : F_7, \Delta_3}{\bullet h_1 \vdash \{F_{10}, \Delta_{11}\} : F_7 \oplus F_8, \Delta_3} \oplus A \quad \frac{h_9 \vdash \{F_{10}, \Delta_{11}\} : F_{10}, \Delta_6, dual(F_7) \& dual(F_8)}{\bullet h_9 \vdash \{F_{10}, \Delta_{11}\} : dual(F_7 \oplus F_8), \Delta_6} ?_C \\
\hline
- \vdash \{F_{10}, \Delta_{11}\} : \Delta_3, \Delta_6 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, F_7 \oplus F_8}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, \Delta_6, F_{10}} \text{ax} \quad \frac{h_9 \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, F_{10}, dual(F_7) \& dual(F_8)}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, \Delta_6} ?_C}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, \Delta_6} \text{hCut} \\
- \vdash \{\Delta_{11}, F_{10}\} : \Delta_3, \Delta_6
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash \{F_{10}, \Delta_{11}\} : F_8, F_4, \Delta_6}{\bullet h_2 \vdash \{F_{10}, \Delta_{11}\} : F_8, \Delta_6, F_4 \oplus F_5} \oplus A \quad \frac{h_9 \vdash \{F_{10}, \Delta_{11}\} : F_{10}, \Delta_7, dual(F_8)}{\bullet h_9 \vdash \{F_{10}, \Delta_{11}\} : dual(F_8), \Delta_7} ?_C \\
\hline
- \vdash \{F_{10}, \Delta_{11}\} : (\Delta_6, F_4 \oplus F_5), \Delta_7 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, F_4, F_8}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_7, F_4} \text{ax} \quad \frac{\bullet h_9 \vdash \{\Delta_{11}, F_{10}\} : \Delta_7, dual(F_8)}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_7, F_4} \text{ax}}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_7, F_4 \oplus F_5} \oplus A}{- \vdash \{\Delta_{11}, F_{10}\} : \Delta_6, \Delta_7, F_4 \oplus F_5} \text{hCut}
\end{array}$$

7.8 Status of \perp : OK

• Case rule 1

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_5\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_5\} : \perp, \Delta_3} \perp \quad \frac{}{\bullet h_4 \vdash \{\Delta_5\} : dual(\perp), *} \mathbf{1} \\
\hline
- \vdash \{\Delta_5\} : \Delta_3, * \quad \text{Cut} \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_5\} : \Delta_3} \text{ax}
\end{array}$$

• Case rule !

• Case rule ?

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_5\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_5\} : \perp, \Delta_3} \perp \quad \frac{h_4 \vdash \{F_6, \Delta_5\} : \mathbf{1}, \Delta_7}{\bullet h_4 \vdash \{\Delta_5\} : dual(\perp), \Delta_7, ?F_6} ? \\
\hline
- \vdash \{\Delta_5\} : \Delta_3, \Delta_7, ?F_6 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_5\} : \Delta_3, \perp}{- \vdash \{\Delta_5, F_6\} : \Delta_3, \Delta_7} \text{ax} \quad \frac{h_4 \vdash \{\Delta_5, F_6\} : \mathbf{1}, \Delta_7}{- \vdash \{\Delta_5, F_6\} : \Delta_3, \Delta_7} \text{ax}}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_7, ?F_6} ? \quad \text{hCut} \\
- \vdash \{\Delta_5\} : \Delta_3, \Delta_7, ?F_6
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash \{\Delta_7\} : F_5, \Delta_4}{\bullet h_2 \vdash \{\Delta_7\} : F_5, \perp, \Delta_4} \perp \quad \frac{h_6 \vdash \{F_8, \Delta_7\} : \Delta_9, dual(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : dual(F_5), \Delta_9, ?F_8} ? \\
\hline
- \vdash \{\Delta_7\} : (\perp, \Delta_4), \Delta_9, ?F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \vdash \{\Delta_7\} : \Delta_4, F_5}{- \vdash \{\Delta_7\} : \Delta_4, \Delta_9, ?F_8} \text{ax} \quad \frac{\bullet h_6 \vdash \{\Delta_7\} : \Delta_9, ?F_8, dual(F_5)}{- \vdash \{\Delta_7\} : \Delta_4, \Delta_9, \perp, ?F_8} \text{hCut}}{- \vdash \{\Delta_7\} : \Delta_4, \Delta_9, \perp, ?F_8} \perp
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_5\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_5\} : \perp, \Delta_3} \perp \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{1}, F_6, F_7, \Delta_8}{\bullet h_4 \vdash \{\Delta_5\} : dual(\perp), \Delta_8, F_6\$F_7} \$ \\
\hline
- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6\$F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_5\} : \Delta_3, \perp}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6, F_7} \text{ax} \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{1}, \Delta_8, F_6, F_7}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6\$F_7} \text{hCut}}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6\$F_7} \$ \\
\frac{h_2 \vdash \{\Delta_7\} : F_5, \Delta_4}{\bullet h_2 \vdash \{\Delta_7\} : F_5, \perp, \Delta_4} \perp \quad \frac{h_6 \vdash \{\Delta_7\} : F_8, F_9, \Delta_{10}, dual(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : dual(F_5), \Delta_{10}, F_8\$F_9} \$ \\
\hline
- \vdash \{\Delta_7\} : (\perp, \Delta_4), \Delta_{10}, F_8\$F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \vdash \{\Delta_7\} : \Delta_4, F_5, \perp}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, F_8, F_9, \perp} \text{ax} \quad \frac{h_6 \vdash \{\Delta_7\} : \Delta_{10}, F_8, F_9, dual(F_5)}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \perp, F_8\$F_9} \text{hCut}}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \perp, F_8\$F_9} \$
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_5\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_5\} : \perp, \Delta_3} \perp \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{1}, F_6, \Delta_8 \quad h_4 \vdash \{\Delta_5\} : \mathbf{1}, F_7, \Delta_8}{\bullet h_4 \vdash \{\Delta_5\} : dual(\perp), \Delta_8, F_6\&F_7} \& \\
\hline
- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6\&F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_5\} : \Delta_3, \perp}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6} \text{ax} \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{1}, \Delta_8, F_6}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6\&F_7} \text{hCut} \quad \frac{\bullet h_1 \vdash \{\Delta_5\} : \Delta_3, \perp}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_7} \text{ax} \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{1}, \Delta_8, F_7}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_7} \text{hCut}}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6\&F_7} \& \\
\frac{h_2 \vdash \{\Delta_7\} : F_5, \Delta_4}{\bullet h_2 \vdash \{\Delta_7\} : F_5, \perp, \Delta_4} \perp \quad \frac{h_6 \vdash \{\Delta_7\} : F_8, \Delta_{10}, dual(F_5) \quad h_6 \vdash \{\Delta_7\} : F_9, \Delta_{10}, dual(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : dual(F_5), \Delta_{10}, F_8\&F_9} \& \\
\hline
- \vdash \{\Delta_7\} : (\perp, \Delta_4), \Delta_{10}, F_8\&F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \vdash \{\Delta_7\} : \Delta_4, F_5}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, F_8\&F_9} \text{ax} \quad \frac{\bullet h_6 \vdash \{\Delta_7\} : \Delta_{10}, dual(F_5), F_8\&F_9}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \perp, F_8\&F_9} \text{hCut}}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \perp, F_8\&F_9} \perp
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_5\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_5\} : \perp, \Delta_3} \perp \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{1}, F_7, \Delta_8}{\bullet h_4 \vdash \{\Delta_5\} : dual(\perp), \Delta_8, F_6 \oplus F_7} \oplus_B \\
\hline
- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \oplus F_7 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_5\} : \Delta_3, \perp}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_7} \text{ax} \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{1}, \Delta_8, F_7}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \oplus F_7} \text{hCut}}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \oplus F_7} \oplus_B
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \vdash \{\Delta_7\} : F_5, \Delta_4}{\bullet h_2 : \vdash \{\Delta_7\} : F_5, \perp, \Delta_4} \perp \quad \frac{h_6 : \vdash \{\Delta_7\} : F_9, \Delta_{10}, \text{dual}(F_5)}{\bullet h_6 : \vdash \{\Delta_7\} : \text{dual}(F_5), \Delta_{10}, F_8 \oplus F_9} \oplus_B \\
\hline
- : \vdash \{\Delta_7\} : (\perp, \Delta_4), \Delta_{10}, F_8 \oplus F_9 \\
\rightarrow \\
\frac{\frac{\bullet h_2 : \vdash \{\Delta_7\} : \Delta_4, F_5, \perp}{- : \vdash \{\Delta_7\} : \Delta_4, F_5, \perp} \text{ax} \quad \frac{h_6 : \vdash \{\Delta_7\} : \Delta_{10}, F_9, \text{dual}(F_5)}{h_6 : \vdash \{\Delta_7\} : \Delta_{10}, F_9, \perp} \text{ax}}{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, F_9, \perp} \text{hCut}}{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \perp, F_8 \oplus F_9} \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{h_1 : \vdash \{\Delta_5\} : \Delta_3}{\bullet h_1 : \vdash \{\Delta_5\} : \perp, \Delta_3} \perp \quad \frac{h_4 : \vdash \{\Delta_5\} : \mathbf{1}, F_6, \Delta_8}{\bullet h_4 : \vdash \{\Delta_5\} : \text{dual}(\perp), \Delta_8, F_6 \oplus F_7} \oplus_A \\
\hline
- : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \oplus F_7 \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \{\Delta_5\} : \Delta_3, \perp}{- : \vdash \{\Delta_5\} : \Delta_3, \perp} \text{ax} \quad \frac{h_4 : \vdash \{\Delta_5\} : \mathbf{1}, \Delta_8, F_6}{h_4 : \vdash \{\Delta_5\} : \mathbf{1}, \Delta_8, F_6} \text{ax}}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6} \text{hCut}}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \oplus F_7} \oplus_A \\
\\
\frac{h_2 : \vdash \{\Delta_7\} : F_5, \Delta_4}{\bullet h_2 : \vdash \{\Delta_7\} : F_5, \perp, \Delta_4} \perp \quad \frac{h_6 : \vdash \{\Delta_7\} : F_8, \Delta_{10}, \text{dual}(F_5)}{\bullet h_6 : \vdash \{\Delta_7\} : \text{dual}(F_5), \Delta_{10}, F_8 \oplus F_9} \oplus_A \\
\hline
- : \vdash \{\Delta_7\} : (\perp, \Delta_4), \Delta_{10}, F_8 \oplus F_9 \\
\rightarrow \\
\frac{\frac{\bullet h_2 : \vdash \{\Delta_7\} : \Delta_4, F_5, \perp}{- : \vdash \{\Delta_7\} : \Delta_4, F_5, \perp} \text{ax} \quad \frac{h_6 : \vdash \{\Delta_7\} : \Delta_{10}, F_8, \text{dual}(F_5)}{h_6 : \vdash \{\Delta_7\} : \Delta_{10}, F_8, \perp} \text{ax}}{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, F_8, \perp} \text{hCut}}{- : \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \perp, F_8 \oplus F_9} \oplus_A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_1 : \vdash \{\Delta_5\} : \Delta_3}{\bullet h_1 : \vdash \{\Delta_5\} : \perp, \Delta_3} \perp \quad \frac{h_4 : \vdash \{\Delta_5\} : \mathbf{1}, \Delta_6}{\bullet h_4 : \vdash \{\Delta_5\} : \text{dual}(\perp), \perp, \Delta_6} \perp \\
\hline
- : \vdash \{\Delta_5\} : \Delta_3, \perp, \Delta_6 \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \{\Delta_5\} : \Delta_3, \perp}{- : \vdash \{\Delta_5\} : \Delta_3, \perp} \text{ax} \quad \frac{h_4 : \vdash \{\Delta_5\} : \mathbf{1}, \Delta_6}{h_4 : \vdash \{\Delta_5\} : \mathbf{1}, \Delta_6} \text{ax}}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_6} \text{hCut}}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \perp} \perp \\
\\
\frac{h_2 : \vdash \{\Delta_7\} : F_5, \Delta_4}{\bullet h_2 : \vdash \{\Delta_7\} : F_5, \perp, \Delta_4} \perp \quad \frac{h_6 : \vdash \{\Delta_7\} : \Delta_8, \text{dual}(F_5)}{\bullet h_6 : \vdash \{\Delta_7\} : \text{dual}(F_5), \perp, \Delta_8} \perp \\
\hline
- : \vdash \{\Delta_7\} : (\perp, \Delta_4), \perp, \Delta_8 \\
\rightarrow \\
\frac{\frac{h_2 : \vdash \{\Delta_7\} : \Delta_4, F_5}{- : \vdash \{\Delta_7\} : \Delta_4, F_5} \text{ax} \quad \frac{\bullet h_6 : \vdash \{\Delta_7\} : \Delta_8, \perp, \text{dual}(F_5)}{\bullet h_6 : \vdash \{\Delta_7\} : \Delta_8, \perp, \text{dual}(F_5)} \text{ax}}{- : \vdash \{\Delta_7\} : \Delta_4, \Delta_8, \perp} \text{hCut}}{- : \vdash \{\Delta_7\} : \Delta_4, \Delta_8, \perp, \perp} \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_1 : \vdash \{\Delta_5\} : \Delta_3}{\bullet h_1 : \vdash \{\Delta_5\} : \perp, \Delta_3} \perp \quad \frac{}{\bullet h_4 : \vdash \{\Delta_5\} : \text{dual}(\perp), \top, \Delta_6} \top \\
\hline
- : \vdash \{\Delta_5\} : \Delta_3, \top, \Delta_6 \\
\rightarrow \\
- : \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \top \quad \top \\
\\
\frac{h_2 : \vdash \{\Delta_7\} : F_5, \Delta_4}{\bullet h_2 : \vdash \{\Delta_7\} : F_5, \perp, \Delta_4} \perp \quad \frac{}{\bullet h_6 : \vdash \{\Delta_7\} : \text{dual}(F_5), \top, \Delta_8} \top \\
\hline
- : \vdash \{\Delta_7\} : (\perp, \Delta_4), \top, \Delta_8 \\
\rightarrow \\
- : \vdash \{\Delta_7\} : \Delta_4, \Delta_8, \perp, \top \quad \top
\end{array}$$

- Case rule I_3

- Case rule \otimes

$$\begin{array}{c}
\frac{\frac{h_1 \vdash \{\Delta_5\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_5\} : \perp, \Delta_3} \perp \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{1}, F_7, \Delta_9 \quad h_4 \vdash \{\Delta_5\} : F_8, \Delta_6}{\bullet h_4 \vdash \{\Delta_5\} : dual(\perp), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 \vdash \{\Delta_5\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_5\} : \Delta_3, \perp} \text{ax} \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{1}, \Delta_9, F_7}{h_4 \vdash \{\Delta_5\} : \mathbf{1}, \Delta_9, F_7} \text{ax}}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_9, F_7} \text{hCut} \quad \frac{}{- \vdash \{\Delta_5\} : \Delta_6, F_8} \text{ax}}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes \\
\frac{\frac{h_1 \vdash \{\Delta_5\} : \Delta_3}{\bullet h_1 \vdash \{\Delta_5\} : \perp, \Delta_3} \perp \quad \frac{h_4 \vdash \{\Delta_5\} : F_7, \Delta_6 \quad h_4 \vdash \{\Delta_5\} : \mathbf{1}, F_8, \Delta_9}{\bullet h_4 \vdash \{\Delta_5\} : dual(\perp), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{Cut} \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_5\} : \Delta_6, F_7} \text{ax} \quad \frac{\frac{h_1 \vdash \{\Delta_5\} : \Delta_3, \perp}{\bullet h_1 \vdash \{\Delta_5\} : \Delta_3, \perp} \text{ax} \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{1}, \Delta_9, F_8}{h_4 \vdash \{\Delta_5\} : \mathbf{1}, \Delta_9, F_8} \text{ax}}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_9, F_8} \text{hCut}}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes \\
\frac{\frac{h_2 \vdash \{\Delta_7\} : F_5, \Delta_4}{\bullet h_2 \vdash \{\Delta_7\} : F_5, \perp, \Delta_4} \perp \quad \frac{h_6 \vdash \{\Delta_7\} : F_9, \Delta_{11}, dual(F_5) \quad h_6 \vdash \{\Delta_7\} : F_{10}, \Delta_8}{\bullet h_6 \vdash \{\Delta_7\} : dual(F_5), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- \vdash \{\Delta_7\} : (\perp, \Delta_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \vdash \{\Delta_7\} : \Delta_4, F_5}{\bullet h_2 \vdash \{\Delta_7\} : \Delta_4, F_5} \text{ax} \quad \frac{h_6 \vdash \{\Delta_7\} : \Delta_{11}, \Delta_8, dual(F_5), F_9 \otimes F_{10}}{\bullet h_6 \vdash \{\Delta_7\} : \Delta_{11}, \Delta_8, dual(F_5), F_9 \otimes F_{10}} \text{ax}}{- \vdash \{\Delta_7\} : \Delta_{11}, \Delta_4, \Delta_8, F_9 \otimes F_{10}} \text{hCut} \quad \perp}{- \vdash \{\Delta_7\} : \Delta_{11}, \Delta_4, \Delta_8, \perp, F_9 \otimes F_{10}} \perp \\
\frac{\frac{h_2 \vdash \{\Delta_7\} : F_5, \Delta_4}{\bullet h_2 \vdash \{\Delta_7\} : F_5, \perp, \Delta_4} \perp \quad \frac{h_6 \vdash \{\Delta_7\} : F_9, \Delta_8 \quad h_6 \vdash \{\Delta_7\} : F_{10}, \Delta_{11}, dual(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : dual(F_5), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- \vdash \{\Delta_7\} : (\perp, \Delta_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \vdash \{\Delta_7\} : \Delta_4, F_5}{\bullet h_2 \vdash \{\Delta_7\} : \Delta_4, F_5} \text{ax} \quad \frac{h_6 \vdash \{\Delta_7\} : \Delta_{11}, \Delta_8, dual(F_5), F_9 \otimes F_{10}}{\bullet h_6 \vdash \{\Delta_7\} : \Delta_{11}, \Delta_8, dual(F_5), F_9 \otimes F_{10}} \text{ax}}{- \vdash \{\Delta_7\} : \Delta_{11}, \Delta_4, \Delta_8, F_9 \otimes F_{10}} \text{hCut} \quad \perp}{- \vdash \{\Delta_7\} : \Delta_{11}, \Delta_4, \Delta_8, \perp, F_9 \otimes F_{10}} \perp
\end{array}$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\begin{array}{c}
\frac{\frac{h_1 \vdash \{F_6, \Delta_7\} : \Delta_3}{\bullet h_1 \vdash \{F_6, \Delta_7\} : \perp, \Delta_3} \perp \quad \frac{h_5 \vdash \{F_6, \Delta_7\} : \mathbf{1}, F_6, \Delta_4}{\bullet h_5 \vdash \{F_6, \Delta_7\} : dual(\perp), \Delta_4} ?_C}{- \vdash \{F_6, \Delta_7\} : \Delta_3, \Delta_4} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_1 \vdash \{\Delta_7, F_6\} : \Delta_3, \perp}{\bullet h_1 \vdash \{\Delta_7, F_6\} : \Delta_3, \perp} \text{ax} \quad \frac{h_5 \vdash \{\Delta_7, F_6\} : \mathbf{1}, \Delta_4, F_6}{h_5 \vdash \{\Delta_7, F_6\} : \mathbf{1}, \Delta_4, F_6} \text{ax}}{- \vdash \{\Delta_7, F_6\} : \Delta_3, \Delta_4, F_6} \text{hCut} \quad ?_C}{- \vdash \{\Delta_7, F_6\} : \Delta_3, \Delta_4} ?_C \\
\frac{\frac{h_2 \vdash \{F_8, \Delta_9\} : F_6, \Delta_4}{\bullet h_2 \vdash \{F_8, \Delta_9\} : F_6, \perp, \Delta_4} \perp \quad \frac{h_7 \vdash \{F_8, \Delta_9\} : F_8, \Delta_5, dual(F_6)}{\bullet h_7 \vdash \{F_8, \Delta_9\} : dual(F_6), \Delta_5} ?_C}{- \vdash \{F_8, \Delta_9\} : (\perp, \Delta_4), \Delta_5} \text{Cut} \\
\rightarrow \\
\frac{\frac{h_2 \vdash \{\Delta_9, F_8\} : \Delta_4, F_6}{\bullet h_2 \vdash \{\Delta_9, F_8\} : \Delta_4, F_6} \text{ax} \quad \frac{h_7 \vdash \{\Delta_9, F_8\} : \Delta_5, dual(F_6)}{\bullet h_7 \vdash \{\Delta_9, F_8\} : \Delta_5, dual(F_6)} \text{ax}}{- \vdash \{\Delta_9, F_8\} : \Delta_4, \Delta_5} \text{hCut} \quad \perp}{- \vdash \{\Delta_9, F_8\} : \Delta_4, \Delta_5, \perp} \perp
\end{array}$$

7.9 Status of \top : OK

- Case rule 1
- Case rule !
- Case rule ?

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_5\} : \top, \Delta_3} \top \quad \frac{h_4 \vdash \{F_6, \Delta_5\} : \mathbf{0}, \Delta_7}{\bullet h_4 \vdash \{\Delta_5\} : dual(\top), \Delta_7, ?F_6} ?}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_7, ?F_6} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_5, F_6\} : \Delta_3, \top}{- \vdash \{\Delta_5, F_6\} : \Delta_3, \Delta_7} \text{ax} \quad \frac{h_4 \vdash \{\Delta_5, F_6\} : \mathbf{0}, \Delta_7}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_7, ?F_6} \text{hCut}}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_7, ?F_6} ? \\
\frac{}{\bullet h_2 \vdash \{\Delta_7\} : F_5, \top, \Delta_4} \top \quad \frac{h_6 \vdash \{F_8, \Delta_7\} : \Delta_9, dual(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : dual(F_5), \Delta_9, ?F_8} ?}{- \vdash \{\Delta_7\} : (\top, \Delta_4), \Delta_9, ?F_8} \text{Cut} \\
\rightarrow \\
- \vdash \{\Delta_7\} : \Delta_4, \Delta_9, \top, ?F_8 \top
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_5\} : \top, \Delta_3} \top \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{0}, F_6, F_7, \Delta_8}{\bullet h_4 \vdash \{\Delta_5\} : dual(\top), \Delta_8, F_6\$F_7} \$}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6\$F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_5\} : \Delta_3, \top}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6, F_7} \text{ax} \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{0}, \Delta_8, F_6, F_7}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6\$F_7} \text{hCut}}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6\$F_7} \$ \\
\frac{}{\bullet h_2 \vdash \{\Delta_7\} : F_5, \top, \Delta_4} \top \quad \frac{h_6 \vdash \{\Delta_7\} : F_8, F_9, \Delta_{10}, dual(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : dual(F_5), \Delta_{10}, F_8\$F_9} \$}{- \vdash \{\Delta_7\} : (\top, \Delta_4), \Delta_{10}, F_8\$F_9} \text{Cut} \\
\rightarrow \\
- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8\$F_9 \top
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_5\} : \top, \Delta_3} \top \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{0}, F_6, \Delta_8 \quad h_4 \vdash \{\Delta_5\} : \mathbf{0}, F_7, \Delta_8}{\bullet h_4 \vdash \{\Delta_5\} : dual(\top), \Delta_8, F_6\&F_7} \&}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6\&F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_5\} : \Delta_3, \top}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6} \text{ax} \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{0}, \Delta_8, F_6}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6\&F_7} \text{hCut} \quad \frac{\bullet h_1 \vdash \{\Delta_5\} : \Delta_3, \top}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_7} \text{ax} \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{0}, \Delta_8, F_7}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_7} \text{hCut}}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6\&F_7} \& \\
\frac{}{\bullet h_2 \vdash \{\Delta_7\} : F_5, \top, \Delta_4} \top \quad \frac{h_6 \vdash \{\Delta_7\} : F_8, \Delta_{10}, dual(F_5) \quad h_6 \vdash \{\Delta_7\} : F_9, \Delta_{10}, dual(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : dual(F_5), \Delta_{10}, F_8\&F_9} \&}{- \vdash \{\Delta_7\} : (\top, \Delta_4), \Delta_{10}, F_8\&F_9} \text{Cut} \\
\rightarrow \\
- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8\&F_9 \top
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 \vdash \{\Delta_5\} : \top, \Delta_3} \top \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{0}, F_7, \Delta_8}{\bullet h_4 \vdash \{\Delta_5\} : \mathit{dual}(\top), \Delta_8, F_6 \oplus F_7} \oplus_B}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \oplus F_7} \text{Cut} \\
\frac{\frac{\frac{}{\bullet h_1 \vdash \{\Delta_5\} : \Delta_3, \top} \top \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{0}, \Delta_8, F_7}{\bullet h_4 \vdash \{\Delta_5\} : \mathbf{0}, \Delta_8, F_7} \text{ax}}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_7} \text{hCut}}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \oplus F_7} \oplus_B \\
\frac{}{\bullet h_2 \vdash \{\Delta_7\} : F_5, \top, \Delta_4} \top \quad \frac{h_6 \vdash \{\Delta_7\} : F_9, \Delta_{10}, \mathit{dual}(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : \mathit{dual}(F_5), \Delta_{10}, F_8 \oplus F_9} \oplus_B}{- \vdash \{\Delta_7\} : (\top, \Delta_4), \Delta_{10}, F_8 \oplus F_9} \text{Cut} \\
\frac{}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8 \oplus F_9} \top
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 \vdash \{\Delta_5\} : \top, \Delta_3} \top \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{0}, F_6, \Delta_8}{\bullet h_4 \vdash \{\Delta_5\} : \mathit{dual}(\top), \Delta_8, F_6 \oplus F_7} \oplus_A}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \oplus F_7} \text{Cut} \\
\frac{\frac{\frac{}{\bullet h_1 \vdash \{\Delta_5\} : \Delta_3, \top} \top \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{0}, \Delta_8, F_6}{\bullet h_4 \vdash \{\Delta_5\} : \mathbf{0}, \Delta_8, F_6} \text{ax}}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6} \text{hCut}}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_8, F_6 \oplus F_7} \oplus_A \\
\frac{}{\bullet h_2 \vdash \{\Delta_7\} : F_5, \top, \Delta_4} \top \quad \frac{h_6 \vdash \{\Delta_7\} : F_8, \Delta_{10}, \mathit{dual}(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : \mathit{dual}(F_5), \Delta_{10}, F_8 \oplus F_9} \oplus_A}{- \vdash \{\Delta_7\} : (\top, \Delta_4), \Delta_{10}, F_8 \oplus F_9} \text{Cut} \\
\frac{}{- \vdash \{\Delta_7\} : \Delta_{10}, \Delta_4, \top, F_8 \oplus F_9} \top
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 \vdash \{\Delta_5\} : \top, \Delta_3} \top \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{0}, \Delta_6}{\bullet h_4 \vdash \{\Delta_5\} : \mathit{dual}(\top), \perp, \Delta_6} \perp}{- \vdash \{\Delta_5\} : \Delta_3, \perp, \Delta_6} \text{Cut} \\
\frac{\frac{\frac{}{\bullet h_1 \vdash \{\Delta_5\} : \Delta_3, \perp, \top} \top \quad \frac{h_4 \vdash \{\Delta_5\} : \mathbf{0}, \Delta_6}{\bullet h_4 \vdash \{\Delta_5\} : \mathbf{0}, \Delta_6} \text{ax}}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \perp} \text{hCut}}{- \vdash \{\Delta_5\} : \Delta_3, \perp, \Delta_6} \\
\frac{}{\bullet h_2 \vdash \{\Delta_7\} : F_5, \top, \Delta_4} \top \quad \frac{h_6 \vdash \{\Delta_7\} : \Delta_8, \mathit{dual}(F_5)}{\bullet h_6 \vdash \{\Delta_7\} : \mathit{dual}(F_5), \perp, \Delta_8} \perp}{- \vdash \{\Delta_7\} : (\top, \Delta_4), \perp, \Delta_8} \text{Cut} \\
\frac{}{- \vdash \{\Delta_7\} : \Delta_4, \Delta_8, \perp, \top} \top
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{\frac{}{\bullet h_1 \vdash \{\Delta_5\} : \top, \Delta_3} \top \quad \frac{h_4 \vdash \{\Delta_5\} : \mathit{dual}(\top), \top, \Delta_6}{\bullet h_4 \vdash \{\Delta_5\} : \mathit{dual}(\top), \top, \Delta_6} \top}{- \vdash \{\Delta_5\} : \Delta_3, \top, \Delta_6} \text{Cut} \\
\frac{}{- \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \top} \top \\
\frac{}{\bullet h_2 \vdash \{\Delta_7\} : F_5, \top, \Delta_4} \top \quad \frac{h_6 \vdash \{\Delta_7\} : \mathit{dual}(F_5), \top, \Delta_8}{\bullet h_6 \vdash \{\Delta_7\} : \mathit{dual}(F_5), \top, \Delta_8} \top}{- \vdash \{\Delta_7\} : (\top, \Delta_4), \top, \Delta_8} \text{Cut} \\
\frac{}{- \vdash \{\Delta_7\} : \Delta_4, \Delta_8, \top, \top} \top
\end{array}$$

- Case rule I_3

- Case rule \otimes

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_1 : \vdash \{\Delta_5\} : \top, \Delta_3} \top \quad \frac{h_4 : \vdash \{\Delta_5\} : \mathbf{O}, F_7, \Delta_9 \quad h_4 : \vdash \{\Delta_5\} : F_8, \Delta_6}{\bullet h_4 : \vdash \{\Delta_5\} : dual(\top), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{Cut}}{\rightarrow} \\
\frac{\frac{\frac{}{\bullet h_1 : \vdash \{\Delta_5\} : \Delta_3, \top} \text{ax} \quad \frac{h_4 : \vdash \{\Delta_5\} : \mathbf{O}, \Delta_9, F_7}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_9, F_7} \text{hCut}}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes}{- : \vdash \{\Delta_5\} : \Delta_6, F_8} \text{ax}}{\otimes} \\
\frac{\frac{\frac{}{\bullet h_1 : \vdash \{\Delta_5\} : \top, \Delta_3} \top \quad \frac{h_4 : \vdash \{\Delta_5\} : F_7, \Delta_6 \quad h_4 : \vdash \{\Delta_5\} : \mathbf{O}, F_8, \Delta_9}{\bullet h_4 : \vdash \{\Delta_5\} : dual(\top), \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \Delta_9, F_7 \otimes F_8} \text{Cut}}{\rightarrow} \\
\frac{\frac{\frac{}{- : \vdash \{\Delta_5\} : \Delta_6, F_7} \text{ax} \quad \frac{\frac{\bullet h_1 : \vdash \{\Delta_5\} : \Delta_3, \top}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_9, F_8} \top \quad \frac{h_4 : \vdash \{\Delta_5\} : \mathbf{O}, \Delta_9, F_8}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_9, F_8} \text{hCut}}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes}{- : \vdash \{\Delta_5\} : \Delta_3, \Delta_6, \Delta_9, F_7 \otimes F_8} \otimes} \\
\frac{\frac{\frac{}{\bullet h_2 : \vdash \{\Delta_7\} : F_5, \top, \Delta_4} \top \quad \frac{h_6 : \vdash \{\Delta_7\} : F_9, \Delta_{11}, dual(F_5) \quad h_6 : \vdash \{\Delta_7\} : F_{10}, \Delta_8}{\bullet h_6 : \vdash \{\Delta_7\} : dual(F_5), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- : \vdash \{\Delta_7\} : (\top, \Delta_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut}}{\rightarrow} \\
\frac{- : \vdash \{\Delta_7\} : \Delta_{11}, \Delta_4, \Delta_8, \top, F_9 \otimes F_{10}}{\top} \\
\frac{\frac{\frac{}{\bullet h_2 : \vdash \{\Delta_7\} : F_5, \top, \Delta_4} \top \quad \frac{h_6 : \vdash \{\Delta_7\} : F_9, \Delta_8 \quad h_6 : \vdash \{\Delta_7\} : F_{10}, \Delta_{11}, dual(F_5)}{\bullet h_6 : \vdash \{\Delta_7\} : dual(F_5), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \otimes}{- : \vdash \{\Delta_7\} : (\top, \Delta_4), \Delta_8, \Delta_{11}, F_9 \otimes F_{10}} \text{Cut}}{\rightarrow} \\
\frac{- : \vdash \{\Delta_7\} : \Delta_{11}, \Delta_4, \Delta_8, \top, F_9 \otimes F_{10}}{\top}
\end{array}$$

- Case rule I_1

- Case rule I_2

- Case rule $?_C$

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_1 : \vdash \{F_6, \Delta_7\} : \top, \Delta_3} \top \quad \frac{h_5 : \vdash \{F_6, \Delta_7\} : \mathbf{O}, F_6, \Delta_4}{\bullet h_5 : \vdash \{F_6, \Delta_7\} : dual(\top), \Delta_4} ?_C}{- : \vdash \{F_6, \Delta_7\} : \Delta_3, \Delta_4} \text{Cut}}{\rightarrow} \\
\frac{\frac{\frac{}{\bullet h_1 : \vdash \{\Delta_7, F_6\} : \Delta_3, \top} \top \quad \frac{h_5 : \vdash \{\Delta_7, F_6\} : \mathbf{O}, \Delta_4, F_6}{- : \vdash \{\Delta_7, F_6\} : \Delta_3, \Delta_4, F_6} \text{ax}}{- : \vdash \{\Delta_7, F_6\} : \Delta_3, \Delta_4} ?_C}{- : \vdash \{\Delta_7, F_6\} : \Delta_3, \Delta_4} \text{hCut}}{\otimes} \\
\frac{\frac{\frac{}{\bullet h_2 : \vdash \{F_8, \Delta_9\} : F_6, \top, \Delta_4} \top \quad \frac{h_7 : \vdash \{F_8, \Delta_9\} : F_8, \Delta_5, dual(F_6)}{\bullet h_7 : \vdash \{F_8, \Delta_9\} : dual(F_6), \Delta_5} ?_C}{- : \vdash \{F_8, \Delta_9\} : (\top, \Delta_4), \Delta_5} \text{Cut}}{\rightarrow} \\
\frac{- : \vdash \{\Delta_9, F_8\} : \Delta_4, \Delta_5, \top}{\top}
\end{array}$$

7.10 Status of I_3 : OK

- Case rule 1

- Case rule !

- Case rule ?

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_1 \vdash \{\Delta_6\} : \hat{\wedge}(n_4), p(n_4)}{I_3} \quad \frac{h_5 \vdash \{F_7, \Delta_6\} : \Delta_8, p(n_4)}{\bullet h_5 \vdash \{\Delta_6\} : dual(\hat{\wedge}(n_4)), \Delta_8, ?F_7} ?}{\vdash \{\Delta_6\} : p(n_4), \Delta_8, ?F_7} ?}{\rightarrow} \text{Cut} \\
\frac{\frac{\frac{}{\vdash \{\Delta_6, F_7\} : \Delta_8, p(n_4)}{ax} \quad \frac{}{\vdash \{\Delta_6\} : \Delta_8, ?F_7, p(n_4)} ?}{\vdash \{\Delta_6\} : \Delta_8, ?F_7, p(n_4)} ?}{} \\
\frac{\frac{\frac{}{\bullet h_1 \vdash \{\Delta_6\} : p(n_4), \hat{\wedge}(n_4)}{I_3} \quad \frac{h_5 \vdash \{F_7, \Delta_6\} : \Delta_8, \hat{\wedge}(n_4)}{\bullet h_5 \vdash \{\Delta_6\} : dual(p(n_4)), \Delta_8, ?F_7} ?}{\vdash \{\Delta_6\} : \hat{\wedge}(n_4), \Delta_8, ?F_7} ?}{\rightarrow} \text{Cut} \\
\frac{\frac{\frac{}{\vdash \{\Delta_6, F_7\} : \Delta_8, \hat{\wedge}(n_4)}{ax} \quad \frac{}{\vdash \{\Delta_6\} : \Delta_8, ?F_7, \hat{\wedge}(n_4)} ?}{\vdash \{\Delta_6\} : \Delta_8, ?F_7, \hat{\wedge}(n_4)} ?}{}
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_1 \vdash \{\Delta_6\} : \hat{\wedge}(n_4), p(n_4)}{I_3} \quad \frac{h_5 \vdash \{\Delta_6\} : F_7, F_8, \Delta_9, p(n_4)}{\bullet h_5 \vdash \{\Delta_6\} : dual(\hat{\wedge}(n_4)), \Delta_9, F_7\$F_8} \$}{\vdash \{\Delta_6\} : p(n_4), \Delta_9, F_7\$F_8} \$}{\rightarrow} \text{Cut} \\
\frac{\frac{\frac{}{\vdash \{\Delta_6\} : \Delta_9, F_7, F_8, p(n_4)}{ax} \quad \frac{}{\vdash \{\Delta_6\} : \Delta_9, p(n_4), F_7\$F_8} \$}{\vdash \{\Delta_6\} : \Delta_9, p(n_4), F_7\$F_8} \$}{} \\
\frac{\frac{\frac{}{\bullet h_1 \vdash \{\Delta_6\} : p(n_4), \hat{\wedge}(n_4)}{I_3} \quad \frac{h_5 \vdash \{\Delta_6\} : F_7, F_8, \Delta_9, \hat{\wedge}(n_4)}{\bullet h_5 \vdash \{\Delta_6\} : dual(p(n_4)), \Delta_9, F_7\$F_8} \$}{\vdash \{\Delta_6\} : \hat{\wedge}(n_4), \Delta_9, F_7\$F_8} \$}{\rightarrow} \text{Cut} \\
\frac{\frac{\frac{}{\vdash \{\Delta_6\} : \Delta_9, F_7, F_8, \hat{\wedge}(n_4)}{ax} \quad \frac{}{\vdash \{\Delta_6\} : \Delta_9, \hat{\wedge}(n_4), F_7\$F_8} \$}{\vdash \{\Delta_6\} : \Delta_9, \hat{\wedge}(n_4), F_7\$F_8} \$}{}
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_1 \vdash \{\Delta_6\} : \hat{\wedge}(n_4), p(n_4)}{I_3} \quad \frac{h_5 \vdash \{\Delta_6\} : F_7, \Delta_9, p(n_4) \quad h_5 \vdash \{\Delta_6\} : F_8, \Delta_9, p(n_4)}{\bullet h_5 \vdash \{\Delta_6\} : dual(\hat{\wedge}(n_4)), \Delta_9, F_7\&F_8} \&}{\vdash \{\Delta_6\} : p(n_4), \Delta_9, F_7\&F_8} \&}{\rightarrow} \text{Cut} \\
\frac{\frac{\frac{}{\vdash \{\Delta_6\} : \Delta_9, F_7, p(n_4)}{ax} \quad \frac{}{\vdash \{\Delta_6\} : \Delta_9, F_8, p(n_4)}{ax} \quad \frac{}{\vdash \{\Delta_6\} : \Delta_9, p(n_4), F_7\&F_8} \&}{\vdash \{\Delta_6\} : \Delta_9, p(n_4), F_7\&F_8} \&}{} \\
\frac{\frac{\frac{}{\bullet h_1 \vdash \{\Delta_6\} : p(n_4), \hat{\wedge}(n_4)}{I_3} \quad \frac{h_5 \vdash \{\Delta_6\} : F_7, \Delta_9, \hat{\wedge}(n_4) \quad h_5 \vdash \{\Delta_6\} : F_8, \Delta_9, \hat{\wedge}(n_4)}{\bullet h_5 \vdash \{\Delta_6\} : dual(p(n_4)), \Delta_9, F_7\&F_8} \&}{\vdash \{\Delta_6\} : \hat{\wedge}(n_4), \Delta_9, F_7\&F_8} \&}{\rightarrow} \text{Cut} \\
\frac{\frac{\frac{}{\vdash \{\Delta_6\} : \Delta_9, F_7, \hat{\wedge}(n_4)}{ax} \quad \frac{}{\vdash \{\Delta_6\} : \Delta_9, F_8, \hat{\wedge}(n_4)}{ax} \quad \frac{}{\vdash \{\Delta_6\} : \Delta_9, \hat{\wedge}(n_4), F_7\&F_8} \&}{\vdash \{\Delta_6\} : \Delta_9, \hat{\wedge}(n_4), F_7\&F_8} \&}{}
\end{array}$$

- Case rule \oplus_B

$$\frac{\frac{\frac{}{\bullet h_1 \vdash \{\Delta_6\} : \hat{\wedge}(n_4), p(n_4)}{I_3} \quad \frac{h_5 \vdash \{\Delta_6\} : F_8, \Delta_9, p(n_4)}{\bullet h_5 \vdash \{\Delta_6\} : dual(\hat{\wedge}(n_4)), \Delta_9, F_7 \oplus F_8} \oplus_B}{\vdash \{\Delta_6\} : p(n_4), \Delta_9, F_7 \oplus F_8} \oplus_B}{\rightarrow} \text{Cut} \\
\frac{\frac{\frac{}{\vdash \{\Delta_6\} : \Delta_9, F_8, p(n_4)}{ax} \quad \frac{}{\vdash \{\Delta_6\} : \Delta_9, p(n_4), F_7 \oplus F_8} \oplus_B}{\vdash \{\Delta_6\} : \Delta_9, p(n_4), F_7 \oplus F_8} \oplus_B}{}$$

$$\begin{array}{c}
\frac{\bullet h_1 \vdash \{\Delta_6\} : p(\mathbf{n}_4), \hat{(\mathbf{n}_4)}}{I_3} \quad \frac{h_5 \vdash \{\Delta_6\} : F_8, \Delta_9, \hat{(\mathbf{n}_4)}}{\bullet h_5 \vdash \{\Delta_6\} : dual(p(\mathbf{n}_4)), \Delta_9, F_7 \oplus F_8} \oplus_B \\
\hline
- \vdash \{\Delta_6\} : \hat{(\mathbf{n}_4)}, \Delta_9, F_7 \oplus F_8 \\
\rightarrow \\
\frac{- \vdash \{\Delta_6\} : \Delta_9, F_8, \hat{(\mathbf{n}_4)}}{ax} \\
\hline
- \vdash \{\Delta_6\} : \Delta_9, \hat{(\mathbf{n}_4)}, F_7 \oplus F_8 \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{\bullet h_1 \vdash \{\Delta_6\} : \hat{(\mathbf{n}_4)}, p(\mathbf{n}_4)}{I_3} \quad \frac{h_5 \vdash \{\Delta_6\} : F_7, \Delta_9, p(\mathbf{n}_4)}{\bullet h_5 \vdash \{\Delta_6\} : dual(\hat{(\mathbf{n}_4)}), \Delta_9, F_7 \oplus F_8} \oplus_A \\
\hline
- \vdash \{\Delta_6\} : p(\mathbf{n}_4), \Delta_9, F_7 \oplus F_8 \\
\rightarrow \\
\frac{- \vdash \{\Delta_6\} : \Delta_9, F_7, p(\mathbf{n}_4)}{ax} \\
\hline
- \vdash \{\Delta_6\} : \Delta_9, p(\mathbf{n}_4), F_7 \oplus F_8 \oplus_A
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_1 \vdash \{\Delta_6\} : p(\mathbf{n}_4), \hat{(\mathbf{n}_4)}}{I_3} \quad \frac{h_5 \vdash \{\Delta_6\} : F_7, \Delta_9, \hat{(\mathbf{n}_4)}}{\bullet h_5 \vdash \{\Delta_6\} : dual(p(\mathbf{n}_4)), \Delta_9, F_7 \oplus F_8} \oplus_A \\
\hline
- \vdash \{\Delta_6\} : \hat{(\mathbf{n}_4)}, \Delta_9, F_7 \oplus F_8 \\
\rightarrow \\
\frac{- \vdash \{\Delta_6\} : \Delta_9, F_7, \hat{(\mathbf{n}_4)}}{ax} \\
\hline
- \vdash \{\Delta_6\} : \Delta_9, \hat{(\mathbf{n}_4)}, F_7 \oplus F_8 \oplus_A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{\bullet h_1 \vdash \{\Delta_6\} : \hat{(\mathbf{n}_4)}, p(\mathbf{n}_4)}{I_3} \quad \frac{h_5 \vdash \{\Delta_6\} : \Delta_7, p(\mathbf{n}_4)}{\bullet h_5 \vdash \{\Delta_6\} : dual(\hat{(\mathbf{n}_4)}), \perp, \Delta_7} \perp \\
\hline
- \vdash \{\Delta_6\} : p(\mathbf{n}_4), \perp, \Delta_7 \\
\rightarrow \\
\frac{- \vdash \{\Delta_6\} : \Delta_7, p(\mathbf{n}_4)}{ax} \\
\hline
- \vdash \{\Delta_6\} : \Delta_7, \perp, p(\mathbf{n}_4) \perp
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_1 \vdash \{\Delta_6\} : p(\mathbf{n}_4), \hat{(\mathbf{n}_4)}}{I_3} \quad \frac{h_5 \vdash \{\Delta_6\} : \Delta_7, \hat{(\mathbf{n}_4)}}{\bullet h_5 \vdash \{\Delta_6\} : dual(p(\mathbf{n}_4)), \perp, \Delta_7} \perp \\
\hline
- \vdash \{\Delta_6\} : \hat{(\mathbf{n}_4)}, \perp, \Delta_7 \\
\rightarrow \\
\frac{- \vdash \{\Delta_6\} : \Delta_7, \hat{(\mathbf{n}_4)}}{ax} \\
\hline
- \vdash \{\Delta_6\} : \Delta_7, \perp, \hat{(\mathbf{n}_4)} \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{\bullet h_1 \vdash \{\Delta_6\} : \hat{(\mathbf{n}_4)}, p(\mathbf{n}_4)}{I_3} \quad \frac{\top}{\bullet h_5 \vdash \{\Delta_6\} : dual(\hat{(\mathbf{n}_4)}), \top, \Delta_7} \top \\
\hline
- \vdash \{\Delta_6\} : p(\mathbf{n}_4), \top, \Delta_7 \\
\rightarrow \\
\frac{- \vdash \{\Delta_6\} : \Delta_7, \top, p(\mathbf{n}_4)}{\top}
\end{array}$$

$$\begin{array}{c}
\frac{\bullet h_1 \vdash \{\Delta_6\} : p(\mathbf{n}_4), \hat{(\mathbf{n}_4)}}{I_3} \quad \frac{\top}{\bullet h_5 \vdash \{\Delta_6\} : dual(p(\mathbf{n}_4)), \top, \Delta_7} \top \\
\hline
- \vdash \{\Delta_6\} : \hat{(\mathbf{n}_4)}, \top, \Delta_7 \\
\rightarrow \\
\frac{- \vdash \{\Delta_6\} : \Delta_7, \top, \hat{(\mathbf{n}_4)}}{\top}
\end{array}$$

- Case rule I_3

$$\begin{array}{c}
\frac{\bullet h_1 \vdash \{\Delta_5\} : \hat{(\mathbf{n}_6)}, p(\mathbf{n}_6)}{I_3} \quad \frac{\bullet h_4 \vdash \{\Delta_5\} : dual(\hat{(\mathbf{n}_6)}), \hat{(\mathbf{n}_6)}}{I_3} \\
\hline
- \vdash \{\Delta_5\} : p(\mathbf{n}_6), \hat{(\mathbf{n}_6)} \\
\rightarrow \\
- \vdash \{\Delta_5\} : p(\mathbf{n}_6), \hat{(\mathbf{n}_6)} I_3
\end{array}$$

$$\frac{\frac{\bullet h_1 \vdash \{\Delta_5\} : p(\mathbf{n}_6), \wedge(\mathbf{n}_6)}{- \vdash \{\Delta_5\} : \wedge(\mathbf{n}_6), p(\mathbf{n}_6)} I_3 \quad \frac{\bullet h_4 \vdash \{\Delta_5\} : dual(p(\mathbf{n}_6)), p(\mathbf{n}_6)}{Cut} I_3}{\rightarrow} \frac{}{- \vdash \{\Delta_5\} : p(\mathbf{n}_6), \wedge(\mathbf{n}_6)} I_3$$

- Case rule \otimes

$$\frac{\frac{\bullet h_1 \vdash \{\Delta_6\} : \wedge(\mathbf{n}_4), p(\mathbf{n}_4)}{- \vdash \{\Delta_6\} : p(\mathbf{n}_4), \Delta_7, \Delta_{10}, F_8 \otimes F_9} I_3 \quad \frac{h_5 \vdash \{\Delta_6\} : F_8, \Delta_{10}, p(\mathbf{n}_4) \quad h_5 \vdash \{\Delta_6\} : F_9, \Delta_7}{\bullet h_5 \vdash \{\Delta_6\} : dual(\wedge(\mathbf{n}_4)), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes}{\rightarrow} \frac{}{- \vdash \{\Delta_6\} : p(\mathbf{n}_4), \Delta_7, \Delta_{10}, F_8 \otimes F_9} Cut}{\rightarrow} \frac{\frac{}{- \vdash \{\Delta_6\} : \Delta_{10}, F_8, p(\mathbf{n}_4)} ax \quad \frac{}{- \vdash \{\Delta_6\} : \Delta_7, F_9} ax}{- \vdash \{\Delta_6\} : \Delta_{10}, \Delta_7, p(\mathbf{n}_4), F_8 \otimes F_9} \otimes} \otimes$$

$$\frac{\frac{\bullet h_1 \vdash \{\Delta_6\} : \wedge(\mathbf{n}_4), p(\mathbf{n}_4)}{- \vdash \{\Delta_6\} : p(\mathbf{n}_4), \Delta_7, \Delta_{10}, F_8 \otimes F_9} I_3 \quad \frac{h_5 \vdash \{\Delta_6\} : F_8, \Delta_7 \quad h_5 \vdash \{\Delta_6\} : F_9, \Delta_{10}, p(\mathbf{n}_4)}{\bullet h_5 \vdash \{\Delta_6\} : dual(\wedge(\mathbf{n}_4)), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes}{\rightarrow} \frac{}{- \vdash \{\Delta_6\} : p(\mathbf{n}_4), \Delta_7, \Delta_{10}, F_8 \otimes F_9} Cut}{\rightarrow} \frac{\frac{}{- \vdash \{\Delta_6\} : \Delta_7, F_8} ax \quad \frac{}{- \vdash \{\Delta_6\} : \Delta_{10}, F_9, p(\mathbf{n}_4)} ax}{- \vdash \{\Delta_6\} : \Delta_{10}, \Delta_7, p(\mathbf{n}_4), F_8 \otimes F_9} \otimes} \otimes$$

$$\frac{\frac{\bullet h_1 \vdash \{\Delta_6\} : p(\mathbf{n}_4), \wedge(\mathbf{n}_4)}{- \vdash \{\Delta_6\} : \wedge(\mathbf{n}_4), \Delta_7, \Delta_{10}, F_8 \otimes F_9} I_3 \quad \frac{h_5 \vdash \{\Delta_6\} : F_8, \Delta_{10}, \wedge(\mathbf{n}_4) \quad h_5 \vdash \{\Delta_6\} : F_9, \Delta_7}{\bullet h_5 \vdash \{\Delta_6\} : dual(p(\mathbf{n}_4)), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes}{\rightarrow} \frac{}{- \vdash \{\Delta_6\} : \wedge(\mathbf{n}_4), \Delta_7, \Delta_{10}, F_8 \otimes F_9} Cut}{\rightarrow} \frac{\frac{}{- \vdash \{\Delta_6\} : \Delta_{10}, F_8, \wedge(\mathbf{n}_4)} ax \quad \frac{}{- \vdash \{\Delta_6\} : \Delta_7, F_9} ax}{- \vdash \{\Delta_6\} : \Delta_{10}, \Delta_7, \wedge(\mathbf{n}_4), F_8 \otimes F_9} \otimes} \otimes$$

$$\frac{\frac{\bullet h_1 \vdash \{\Delta_6\} : p(\mathbf{n}_4), \wedge(\mathbf{n}_4)}{- \vdash \{\Delta_6\} : \wedge(\mathbf{n}_4), \Delta_7, \Delta_{10}, F_8 \otimes F_9} I_3 \quad \frac{h_5 \vdash \{\Delta_6\} : F_8, \Delta_7 \quad h_5 \vdash \{\Delta_6\} : F_9, \Delta_{10}, \wedge(\mathbf{n}_4)}{\bullet h_5 \vdash \{\Delta_6\} : dual(p(\mathbf{n}_4)), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes}{\rightarrow} \frac{}{- \vdash \{\Delta_6\} : \wedge(\mathbf{n}_4), \Delta_7, \Delta_{10}, F_8 \otimes F_9} Cut}{\rightarrow} \frac{\frac{}{- \vdash \{\Delta_6\} : \Delta_7, F_8} ax \quad \frac{}{- \vdash \{\Delta_6\} : \Delta_{10}, F_9, \wedge(\mathbf{n}_4)} ax}{- \vdash \{\Delta_6\} : \Delta_{10}, \Delta_7, \wedge(\mathbf{n}_4), F_8 \otimes F_9} \otimes} \otimes$$

- Case rule I_1

$$\frac{\frac{\bullet h_1 \vdash \{\Delta_5, p(\mathbf{n}_6)\} : p(\mathbf{n}_6), \wedge(\mathbf{n}_6)}{- \vdash \{\Delta_5, p(\mathbf{n}_6)\} : \wedge(\mathbf{n}_6), *} I_3 \quad \frac{\bullet h_4 \vdash \{\Delta_5, p(\mathbf{n}_6)\} : dual(p(\mathbf{n}_6)), *}{Cut} I_1}{\rightarrow} \frac{}{- \vdash \{\Delta_5, p(\mathbf{n}_6)\} : \wedge(\mathbf{n}_6), *} I_1$$

- Case rule I_2

$$\frac{\frac{\bullet h_1 \vdash \{\Delta_5, \wedge(\mathbf{n}_6)\} : \wedge(\mathbf{n}_6), p(\mathbf{n}_6)}{- \vdash \{\Delta_5, \wedge(\mathbf{n}_6)\} : p(\mathbf{n}_6), *} I_3 \quad \frac{\bullet h_4 \vdash \{\Delta_5, \wedge(\mathbf{n}_6)\} : dual(\wedge(\mathbf{n}_6)), *}{Cut} I_2}{\rightarrow} \frac{}{- \vdash \{\Delta_5, \wedge(\mathbf{n}_6)\} : p(\mathbf{n}_6), *} I_2$$

- Case rule $?_C$

$$\frac{\frac{\bullet h_1 \vdash \{F_7, \Delta_8\} : \wedge(\mathbf{n}_5), p(\mathbf{n}_5)}{- \vdash \{F_7, \Delta_8\} : p(\mathbf{n}_5), \Delta_4} I_3 \quad \frac{h_6 \vdash \{F_7, \Delta_8\} : F_7, \Delta_4, p(\mathbf{n}_5)}{\bullet h_6 \vdash \{F_7, \Delta_8\} : dual(\wedge(\mathbf{n}_5)), \Delta_4} ?_C}{\rightarrow} \frac{}{- \vdash \{F_7, \Delta_8\} : p(\mathbf{n}_5), \Delta_4} Cut}{\rightarrow} \frac{\frac{}{- \vdash \{\Delta_8, F_7\} : \Delta_4, F_7, p(\mathbf{n}_5)} ax \quad \frac{}{- \vdash \{\Delta_8, F_7\} : \Delta_4, p(\mathbf{n}_5)} ?_C} \otimes} \otimes$$

$$\begin{array}{c}
\frac{\bullet h_1 \vdash \{F_7, \Delta_8\} : p(n_5), \hat{\wedge}(n_5)}{I_3} \quad \frac{h_6 \vdash \{F_7, \Delta_8\} : F_7, \Delta_4, \hat{\wedge}(n_5)}{?_C} \\
\frac{\bullet h_6 \vdash \{F_7, \Delta_8\} : dual(p(n_5)), \Delta_4}{\text{Cut}} \\
\frac{}{- \vdash \{F_7, \Delta_8\} : \hat{\wedge}(n_5), \Delta_4} \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_8, F_7\} : \Delta_4, F_7, \hat{\wedge}(n_5)}{ax} \\
\frac{}{- \vdash \{\Delta_8, F_7\} : \Delta_4, \hat{\wedge}(n_5)}{?_C}
\end{array}$$

7.11 Status of \otimes : OK

- Case rule 1
- Case rule !
- Case rule ?

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_{10}\} : F_7, \Delta_3 \quad h_1 \vdash \{\Delta_{10}\} : F_8, \Delta_4 \quad h_9 \vdash \{F_{11}, \Delta_{10}\} : \Delta_{12}, dual(F_7)\$dual(F_8)}{\bullet h_1 \vdash \{\Delta_{10}\} : F_7 \otimes F_8, \Delta_3, \Delta_4} \otimes \frac{h_9 \vdash \{F_{11}, \Delta_{10}\} : \Delta_{12}, dual(F_7)\$dual(F_8)}{\bullet h_9 \vdash \{\Delta_{10}\} : dual(F_7 \otimes F_8), \Delta_{12}, ?F_{11}} ? \\
\frac{}{- \vdash \{\Delta_{10}\} : (\Delta_3, \Delta_4), \Delta_{12}, ?F_{11}}{\text{Cut}} \\
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_{10}\} : \Delta_3, \Delta_4, F_7 \otimes F_8}{\bullet h_1 \vdash \{\Delta_{10}, F_{11}\} : \Delta_3, \Delta_4, F_7 \otimes F_8} ax \\
\frac{}{- \vdash \{\Delta_{10}, F_{11}\} : \Delta_{12}, \Delta_3, \Delta_4} W \\
\frac{h_9 \vdash \{F_{11}, \Delta_{10}\} : \Delta_{12}, dual(F_7)\$dual(F_8)}{\bullet h_9 \vdash \{\Delta_{10}, F_{11}\} : \Delta_{12}, dual(F_7)\$dual(F_8)} ax \\
\frac{}{- \vdash \{\Delta_{10}, F_{11}\} : \Delta_{12}, \Delta_3, \Delta_4} hCut \\
\frac{}{- \vdash \{\Delta_{10}\} : \Delta_{12}, \Delta_3, \Delta_4, ?F_{11}} ?
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash \{\Delta_{10}\} : F_8, F_5, \Delta_7 \quad h_2 \vdash \{\Delta_{10}\} : F_6, \Delta_4 \quad h_9 \vdash \{F_{11}, \Delta_{10}\} : \Delta_{12}, dual(F_8)}{\bullet h_2 \vdash \{\Delta_{10}\} : F_8, \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \frac{h_9 \vdash \{F_{11}, \Delta_{10}\} : \Delta_{12}, dual(F_8)}{\bullet h_9 \vdash \{\Delta_{10}\} : dual(F_8), \Delta_{12}, ?F_{11}} ? \\
\frac{}{- \vdash \{\Delta_{10}\} : (\Delta_4, \Delta_7, F_5 \otimes F_6), \Delta_{12}, ?F_{11}}{\text{Cut}} \\
\rightarrow \\
\frac{\bullet h_2 \vdash \{\Delta_{10}\} : \Delta_4, \Delta_7, F_8, F_5 \otimes F_6}{\bullet h_2 \vdash \{\Delta_{10}, F_{11}\} : \Delta_4, \Delta_7, F_8, F_5 \otimes F_6} ax \\
\frac{}{- \vdash \{\Delta_{10}, F_{11}\} : \Delta_{12}, \Delta_4, \Delta_7, F_5 \otimes F_6} W \\
\frac{h_9 \vdash \{F_{11}, \Delta_{10}\} : \Delta_{12}, dual(F_8)}{\bullet h_9 \vdash \{\Delta_{10}, F_{11}\} : \Delta_{12}, dual(F_8)} ax \\
\frac{}{- \vdash \{\Delta_{10}, F_{11}\} : \Delta_{12}, \Delta_4, \Delta_7, F_5 \otimes F_6} hCut \\
\frac{}{- \vdash \{\Delta_{10}\} : \Delta_{12}, \Delta_4, \Delta_7, ?F_{11}, F_5 \otimes F_6} ?
\end{array}$$

$$\begin{array}{c}
\frac{h_2 \vdash \{\Delta_{10}\} : F_5, \Delta_4 \quad h_2 \vdash \{\Delta_{10}\} : F_8, F_6, \Delta_7 \quad h_9 \vdash \{F_{11}, \Delta_{10}\} : \Delta_{12}, dual(F_8)}{\bullet h_2 \vdash \{\Delta_{10}\} : F_8, \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \frac{h_9 \vdash \{F_{11}, \Delta_{10}\} : \Delta_{12}, dual(F_8)}{\bullet h_9 \vdash \{\Delta_{10}\} : dual(F_8), \Delta_{12}, ?F_{11}} ? \\
\frac{}{- \vdash \{\Delta_{10}\} : (\Delta_4, \Delta_7, F_5 \otimes F_6), \Delta_{12}, ?F_{11}}{\text{Cut}} \\
\rightarrow \\
\frac{\bullet h_2 \vdash \{\Delta_{10}\} : \Delta_4, \Delta_7, F_8, F_5 \otimes F_6}{\bullet h_2 \vdash \{\Delta_{10}, F_{11}\} : \Delta_4, \Delta_7, F_8, F_5 \otimes F_6} ax \\
\frac{}{- \vdash \{\Delta_{10}, F_{11}\} : \Delta_{12}, \Delta_4, \Delta_7, F_5 \otimes F_6} W \\
\frac{h_9 \vdash \{F_{11}, \Delta_{10}\} : \Delta_{12}, dual(F_8)}{\bullet h_9 \vdash \{\Delta_{10}, F_{11}\} : \Delta_{12}, dual(F_8)} ax \\
\frac{}{- \vdash \{\Delta_{10}, F_{11}\} : \Delta_{12}, \Delta_4, \Delta_7, F_5 \otimes F_6} hCut \\
\frac{}{- \vdash \{\Delta_{10}\} : \Delta_{12}, \Delta_4, \Delta_7, ?F_{11}, F_5 \otimes F_6} ?
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_{10}\} : F_7, \Delta_3 \quad h_1 \vdash \{\Delta_{10}\} : F_8, \Delta_4 \quad h_9 \vdash \{\Delta_{10}\} : F_{11}, F_{12}, \Delta_{13}, dual(F_7)\$dual(F_8)}{\bullet h_1 \vdash \{\Delta_{10}\} : F_7 \otimes F_8, \Delta_3, \Delta_4} \otimes \frac{h_9 \vdash \{\Delta_{10}\} : F_{11}, F_{12}, \Delta_{13}, dual(F_7)\$dual(F_8)}{\bullet h_9 \vdash \{\Delta_{10}\} : dual(F_7 \otimes F_8), \Delta_{13}, F_{11}\$F_{12}} \$ \\
\frac{}{- \vdash \{\Delta_{10}\} : (\Delta_3, \Delta_4), \Delta_{13}, F_{11}\$F_{12}}{\text{Cut}} \\
\rightarrow \\
\frac{\bullet h_1 \vdash \{\Delta_{10}\} : \Delta_3, \Delta_4, F_7 \otimes F_8}{\bullet h_1 \vdash \{\Delta_{10}\} : \Delta_{13}, F_{11}, F_{12}, dual(F_7)\$dual(F_8)} ax \\
\frac{}{- \vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_3, \Delta_4, F_{11}, F_{12}} W \\
\frac{h_9 \vdash \{\Delta_{10}\} : F_{11}, F_{12}, \Delta_{13}, dual(F_7)\$dual(F_8)}{\bullet h_9 \vdash \{\Delta_{10}, F_{11}\} : \Delta_{13}, F_{11}, F_{12}, dual(F_7)\$dual(F_8)} ax \\
\frac{}{- \vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_3, \Delta_4, F_{11}, F_{12}} hCut \\
\frac{}{- \vdash \{\Delta_{10}\} : \Delta_{13}, \Delta_3, \Delta_4, F_{11}\$F_{12}} \$
\end{array}$$

$$\begin{array}{c}
\frac{h_2 : \vdash \{\Delta_{10}\} : F_5, \Delta_4 \quad h_2 : \vdash \{\Delta_{10}\} : F_8, F_6, \Delta_7}{\bullet h_2 : \vdash \{\Delta_{10}\} : F_8, \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \frac{h_9 : \vdash \{\Delta_{10}\} : \Delta_{11}, dual(F_8)}{\bullet h_9 : \vdash \{\Delta_{10}\} : dual(F_8), \perp, \Delta_{11}} \perp \\
\hline
- : \vdash \{\Delta_{10}\} : (\Delta_4, \Delta_7, F_5 \otimes F_6), \perp, \Delta_{11} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 : \vdash \{\Delta_{10}\} : \Delta_4, \Delta_7, F_8, F_5 \otimes F_6}{- : \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_4, \Delta_7, F_5 \otimes F_6} \text{ax} \quad \frac{h_9 : \vdash \{\Delta_{10}\} : \Delta_{11}, dual(F_8)}{- : \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_4, \Delta_7, \perp, F_5 \otimes F_6} \text{hCut}}{- : \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_4, \Delta_7, \perp, F_5 \otimes F_6} \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_1 : \vdash \{\Delta_{10}\} : F_7, \Delta_3 \quad h_1 : \vdash \{\Delta_{10}\} : F_8, \Delta_4}{\bullet h_1 : \vdash \{\Delta_{10}\} : F_7 \otimes F_8, \Delta_3, \Delta_4} \otimes \frac{}{\bullet h_9 : \vdash \{\Delta_{10}\} : dual(F_7 \otimes F_8), \top, \Delta_{11}} \top \\
\hline
- : \vdash \{\Delta_{10}\} : (\Delta_3, \Delta_4), \top, \Delta_{11} \quad \text{Cut} \\
\rightarrow \\
- : \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_3, \Delta_4, \top \quad \top \\
\hline
\frac{h_2 : \vdash \{\Delta_{10}\} : F_8, F_5, \Delta_7 \quad h_2 : \vdash \{\Delta_{10}\} : F_6, \Delta_4}{\bullet h_2 : \vdash \{\Delta_{10}\} : F_8, \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \frac{}{\bullet h_9 : \vdash \{\Delta_{10}\} : dual(F_8), \top, \Delta_{11}} \top \\
\hline
- : \vdash \{\Delta_{10}\} : (\Delta_4, \Delta_7, F_5 \otimes F_6), \top, \Delta_{11} \quad \text{Cut} \\
\rightarrow \\
- : \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_4, \Delta_7, \top, F_5 \otimes F_6 \quad \top \\
\hline
\frac{h_2 : \vdash \{\Delta_{10}\} : F_5, \Delta_4 \quad h_2 : \vdash \{\Delta_{10}\} : F_8, F_6, \Delta_7}{\bullet h_2 : \vdash \{\Delta_{10}\} : F_8, \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \frac{}{\bullet h_9 : \vdash \{\Delta_{10}\} : dual(F_8), \top, \Delta_{11}} \top \\
\hline
- : \vdash \{\Delta_{10}\} : (\Delta_4, \Delta_7, F_5 \otimes F_6), \top, \Delta_{11} \quad \text{Cut} \\
\rightarrow \\
- : \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_4, \Delta_7, \top, F_5 \otimes F_6 \quad \top
\end{array}$$

- Case rule I_3

- Case rule \otimes

$$\begin{array}{c}
\frac{h_1 : \vdash \{\Delta_{10}\} : F_7, \Delta_3 \quad h_1 : \vdash \{\Delta_{10}\} : F_8, \Delta_4}{\bullet h_1 : \vdash \{\Delta_{10}\} : F_7 \otimes F_8, \Delta_3, \Delta_4} \otimes \frac{h_9 : \vdash \{\Delta_{10}\} : F_{12}, \Delta_{14}, dual(F_7) \$ dual(F_8) \quad h_9 : \vdash \{\Delta_{10}\} : F_{13}, \Delta_{11}}{\bullet h_9 : \vdash \{\Delta_{10}\} : dual(F_7 \otimes F_8), \Delta_{11}, \Delta_{14}, F_{12} \otimes F_{13}} \otimes \\
\hline
- : \vdash \{\Delta_{10}\} : (\Delta_3, \Delta_4), \Delta_{11}, \Delta_{14}, F_{12} \otimes F_{13} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 : \vdash \{\Delta_{10}\} : \Delta_3, \Delta_4, F_7 \otimes F_8}{- : \vdash \{\Delta_{10}\} : \Delta_{14}, \Delta_3, \Delta_4, F_{12}} \text{ax} \quad \frac{h_9 : \vdash \{\Delta_{10}\} : \Delta_{14}, F_{12}, dual(F_7) \$ dual(F_8)}{- : \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_{14}, \Delta_3, \Delta_4, F_{12} \otimes F_{13}} \text{hCut}}{- : \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_{14}, \Delta_3, \Delta_4, F_{12} \otimes F_{13}} \otimes \\
\hline
\frac{h_1 : \vdash \{\Delta_{10}\} : F_7, \Delta_3 \quad h_1 : \vdash \{\Delta_{10}\} : F_8, \Delta_4}{\bullet h_1 : \vdash \{\Delta_{10}\} : F_7 \otimes F_8, \Delta_3, \Delta_4} \otimes \frac{h_9 : \vdash \{\Delta_{10}\} : F_{12}, \Delta_{11} \quad h_9 : \vdash \{\Delta_{10}\} : F_{13}, \Delta_{14}, dual(F_7) \$ dual(F_8)}{\bullet h_9 : \vdash \{\Delta_{10}\} : dual(F_7 \otimes F_8), \Delta_{11}, \Delta_{14}, F_{12} \otimes F_{13}} \otimes \\
\hline
- : \vdash \{\Delta_{10}\} : (\Delta_3, \Delta_4), \Delta_{11}, \Delta_{14}, F_{12} \otimes F_{13} \quad \text{Cut} \\
\rightarrow \\
\frac{}{- : \vdash \{\Delta_{10}\} : \Delta_{11}, F_{12}} \text{ax} \quad \frac{\bullet h_1 : \vdash \{\Delta_{10}\} : \Delta_3, \Delta_4, F_7 \otimes F_8}{- : \vdash \{\Delta_{10}\} : \Delta_{14}, \Delta_3, \Delta_4, F_{13}} \text{ax} \quad \frac{h_9 : \vdash \{\Delta_{10}\} : \Delta_{14}, F_{13}, dual(F_7) \$ dual(F_8)}{- : \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_{14}, \Delta_3, \Delta_4, F_{13}} \text{hCut}}{- : \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_{14}, \Delta_3, \Delta_4, F_{12} \otimes F_{13}} \otimes \\
\hline
\frac{h_2 : \vdash \{\Delta_{10}\} : F_8, F_5, \Delta_7 \quad h_2 : \vdash \{\Delta_{10}\} : F_6, \Delta_4}{\bullet h_2 : \vdash \{\Delta_{10}\} : F_8, \Delta_4, \Delta_7, F_5 \otimes F_6} \otimes \frac{h_9 : \vdash \{\Delta_{10}\} : F_{12}, \Delta_{14}, dual(F_8) \quad h_9 : \vdash \{\Delta_{10}\} : F_{13}, \Delta_{11}}{\bullet h_9 : \vdash \{\Delta_{10}\} : dual(F_8), \Delta_{11}, \Delta_{14}, F_{12} \otimes F_{13}} \otimes \\
\hline
- : \vdash \{\Delta_{10}\} : (\Delta_4, \Delta_7, F_5 \otimes F_6), \Delta_{11}, \Delta_{14}, F_{12} \otimes F_{13} \quad \text{Cut} \\
\rightarrow \\
\frac{\bullet h_2 : \vdash \{\Delta_{10}\} : \Delta_4, \Delta_7, F_8, F_5 \otimes F_6}{- : \vdash \{\Delta_{10}\} : \Delta_{14}, \Delta_4, \Delta_7, F_{12}, F_5 \otimes F_6} \text{ax} \quad \frac{h_9 : \vdash \{\Delta_{10}\} : \Delta_{14}, F_{12}, dual(F_8)}{- : \vdash \{\Delta_{10}\} : \Delta_{11}, F_{13}} \text{hCut}}{- : \vdash \{\Delta_{10}\} : \Delta_{11}, \Delta_{14}, \Delta_4, \Delta_7, F_{12} \otimes F_{13}, F_5 \otimes F_6} \otimes
\end{array}$$

- Case rule !

- Case rule ?

$$\frac{\frac{\frac{}{\bullet \mathbf{h}_1 \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \hat{\wedge}(\mathbf{n}_5), *}}{I_1} \quad \frac{\mathbf{h}_6 \vdash \{\mathbf{F}_7, \Delta_4, p(\mathbf{n}_5)\} : \Delta_8, p(\mathbf{n}_5)}}{\bullet \mathbf{h}_6 \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \mathit{dual}(\hat{\wedge}(\mathbf{n}_5)), \Delta_8, ?\mathbf{F}_7} ?}{\frac{}{- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : *, \Delta_8, ?\mathbf{F}_7} \text{Cut}}{\frac{}{- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \Delta_8, p(\mathbf{n}_5)} \text{ax}}{\frac{}{- \vdash \{\Delta_4, \mathbf{F}_7, p(\mathbf{n}_5)\} : \Delta_8, p(\mathbf{n}_5)} ?_C} \frac{}{- \vdash \{\Delta_4, \mathbf{F}_7, p(\mathbf{n}_5)\} : \Delta_8} ?} \frac{}{- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \Delta_8, ?\mathbf{F}_7} ?$$

- Case rule \$

$$\frac{\frac{\frac{}{\bullet \mathbf{h}_1 \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \hat{\wedge}(\mathbf{n}_5), *}}{I_1} \quad \frac{\mathbf{h}_6 \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \mathbf{F}_7, \mathbf{F}_8, \Delta_9, p(\mathbf{n}_5)}}{\bullet \mathbf{h}_6 \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \mathit{dual}(\hat{\wedge}(\mathbf{n}_5)), \Delta_9, \mathbf{F}_7\$\mathbf{F}_8} \$}{\frac{}{- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : *, \Delta_9, \mathbf{F}_7\$\mathbf{F}_8} \text{Cut}}{\frac{}{- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \Delta_9, \mathbf{F}_7, \mathbf{F}_8, p(\mathbf{n}_5)} \text{ax}}{\frac{}{- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \Delta_9, \mathbf{F}_7, \mathbf{F}_8} ?_C} \frac{}{- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \Delta_9, \mathbf{F}_7\$\mathbf{F}_8} \$}$$

- Case rule &

$$\frac{\frac{\frac{}{\bullet \mathbf{h}_1 \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \hat{\wedge}(\mathbf{n}_5), *}}{I_1} \quad \frac{\mathbf{h}_6 \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \mathbf{F}_7, \Delta_9, p(\mathbf{n}_5)}{\bullet \mathbf{h}_6 \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \mathit{dual}(\hat{\wedge}(\mathbf{n}_5)), \Delta_9, \mathbf{F}_7\&\mathbf{F}_8} \mathbf{h}_6 \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \mathbf{F}_8, \Delta_9, p(\mathbf{n}_5)}}{\frac{}{- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : *, \Delta_9, \mathbf{F}_7\&\mathbf{F}_8} \text{Cut}}{\frac{}{- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \Delta_9, \mathbf{F}_7, p(\mathbf{n}_5)} \text{ax}}{\frac{}{- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \Delta_9, p(\mathbf{n}_5), \mathbf{F}_7\&\mathbf{F}_8} \&} \frac{}{- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \Delta_9, \mathbf{F}_8, p(\mathbf{n}_5)} \text{ax}}{\frac{}{- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \Delta_9, \mathbf{F}_7\&\mathbf{F}_8} ?_C} \frac{}{- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \Delta_9, \mathbf{F}_7\&\mathbf{F}_8} \&}$$

- Case rule \oplus_B

$$\frac{\frac{\frac{}{\bullet \mathbf{h}_1 \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \hat{\wedge}(\mathbf{n}_5), *}}{I_1} \quad \frac{\mathbf{h}_6 \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \mathbf{F}_8, \Delta_9, p(\mathbf{n}_5)}}{\bullet \mathbf{h}_6 \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \mathit{dual}(\hat{\wedge}(\mathbf{n}_5)), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8} \oplus_B}{\frac{}{- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : *, \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8} \text{Cut}}{\frac{}{- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \Delta_9, \mathbf{F}_8, p(\mathbf{n}_5)} \text{ax}}{\frac{}{- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \Delta_9, \mathbf{F}_8} ?_C} \frac{}{- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8} \oplus_B}$$

- Case rule \oplus_A

$$\frac{\frac{\frac{}{\bullet \mathbf{h}_1 \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \hat{\wedge}(\mathbf{n}_5), *}}{I_1} \quad \frac{\mathbf{h}_6 \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \mathbf{F}_7, \Delta_9, p(\mathbf{n}_5)}}{\bullet \mathbf{h}_6 \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \mathit{dual}(\hat{\wedge}(\mathbf{n}_5)), \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8} \oplus_A}{\frac{}{- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : *, \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8} \text{Cut}}{\frac{}{- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \Delta_9, \mathbf{F}_7, p(\mathbf{n}_5)} \text{ax}}{\frac{}{- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \Delta_9, \mathbf{F}_7} ?_C} \frac{}{- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \Delta_9, \mathbf{F}_7 \oplus \mathbf{F}_8} \oplus_A}$$

- Case rule \perp

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \wedge(\mathbf{n}_5), * } I_1 \quad \frac{h_6 \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \Delta_7, p(\mathbf{n}_5)}{\bullet h_6 \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \text{dual}(\wedge(\mathbf{n}_5)), \perp, \Delta_7} \perp \\
\hline
- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : *, \perp, \Delta_7 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \Delta_7, p(\mathbf{n}_5)} \text{ax} \\
\frac{}{- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \Delta_7} ?_C \\
\hline
- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \Delta_7, \perp \quad \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \wedge(\mathbf{n}_5), * } I_1 \quad \frac{}{\bullet h_6 \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \text{dual}(\wedge(\mathbf{n}_5)), \top, \Delta_7} \top \\
\hline
- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : *, \top, \Delta_7 \quad \text{Cut} \\
\rightarrow \\
- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \Delta_7, \top \quad \top
\end{array}$$

- Case rule I_3

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_4, p(\mathbf{n}_6)\} : \wedge(\mathbf{n}_6), * } I_1 \quad \frac{}{\bullet h_5 \vdash \{\Delta_4, p(\mathbf{n}_6)\} : \text{dual}(\wedge(\mathbf{n}_6)), \wedge(\mathbf{n}_6)} I_3 \\
\hline
- \vdash \{\Delta_4, p(\mathbf{n}_6)\} : *, \wedge(\mathbf{n}_6) \quad \text{Cut} \\
\rightarrow \\
- \vdash \{\Delta_4, p(\mathbf{n}_6)\} : \wedge(\mathbf{n}_6) \quad I_1
\end{array}$$

- Case rule \otimes

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \wedge(\mathbf{n}_5), * } I_1 \quad \frac{h_6 \vdash \{\Delta_4, p(\mathbf{n}_5)\} : F_8, \Delta_{10}, p(\mathbf{n}_5) \quad h_6 \vdash \{\Delta_4, p(\mathbf{n}_5)\} : F_9, \Delta_7}{\bullet h_6 \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \text{dual}(\wedge(\mathbf{n}_5)), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes \\
\hline
- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : *, \Delta_7, \Delta_{10}, F_8 \otimes F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \Delta_{10}, F_8, p(\mathbf{n}_5)} \text{ax} \\
\frac{}{- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \Delta_{10}, F_8} ?_C \quad \frac{}{- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \Delta_7, F_9} \text{ax} \\
\hline
- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \Delta_{10}, \Delta_7, F_8 \otimes F_9 \quad \otimes
\end{array}$$

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \wedge(\mathbf{n}_5), * } I_1 \quad \frac{h_6 \vdash \{\Delta_4, p(\mathbf{n}_5)\} : F_8, \Delta_7 \quad h_6 \vdash \{\Delta_4, p(\mathbf{n}_5)\} : F_9, \Delta_{10}, p(\mathbf{n}_5)}{\bullet h_6 \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \text{dual}(\wedge(\mathbf{n}_5)), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes \\
\hline
- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : *, \Delta_7, \Delta_{10}, F_8 \otimes F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \Delta_7, F_8} \text{ax} \quad \frac{}{- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \Delta_{10}, F_9, p(\mathbf{n}_5)} \text{ax} \\
\frac{}{- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \Delta_{10}, F_9} ?_C \\
\hline
- \vdash \{\Delta_4, p(\mathbf{n}_5)\} : \Delta_{10}, \Delta_7, F_8 \otimes F_9 \quad \otimes
\end{array}$$

- Case rule I_1

- Case rule I_2

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{(\Delta_6, \wedge(\mathbf{n}_5)), p(\mathbf{n}_5)\} : \wedge(\mathbf{n}_5), * } I_1 \quad \frac{}{\bullet h_4 \vdash \{(\Delta_6, \wedge(\mathbf{n}_5)), p(\mathbf{n}_5)\} : \text{dual}(\wedge(\mathbf{n}_5)), *} I_2 \\
\hline
- \vdash \{(\Delta_6, \wedge(\mathbf{n}_5)), p(\mathbf{n}_5)\} : *, * \quad \text{Cut} \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_6, p(\mathbf{n}_5), \wedge(\mathbf{n}_5)\} : p(\mathbf{n}_5)} I_2 \\
\frac{}{- \vdash \{\Delta_6, p(\mathbf{n}_5), \wedge(\mathbf{n}_5)\} : *} ?_C
\end{array}$$

- Case rule $?_C$

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_1 \vdash \{\Delta_7, p(n_4)\} : \hat{\wedge}(n_4), *}}{\vdash \{\Delta_7, p(n_4)\} : *, \Delta_5} \quad I_1 \quad \frac{h_6 \vdash \{\Delta_7, p(n_4)\} : \Delta_5, p(n_4), p(n_4)}{\bullet h_6 \vdash \{\Delta_7, p(n_4)\} : dual(\hat{\wedge}(n_4)), \Delta_5} \quad ?_C}{\vdash \{\Delta_7, p(n_4)\} : *, \Delta_5} \quad \text{Cut}}{\vdash \{\Delta_7, p(n_4)\} : \Delta_5} \quad \text{ax} \\
\frac{\frac{\frac{}{\vdash \{\Delta_7, p(n_4)\} : \Delta_5, p(n_4), p(n_4)}}{\vdash \{\Delta_7, p(n_4)\} : \Delta_5, p(n_4)} \quad ?_C}{\vdash \{\Delta_7, p(n_4)\} : \Delta_5} \quad ?_C}{\vdash \{\Delta_7, p(n_4)\} : \Delta_5} \quad ?_C \\
\frac{\frac{\frac{}{\bullet h_1 \vdash \{(F_7, \Delta_8), p(n_4)\} : \hat{\wedge}(n_4), *}}{\vdash \{(F_7, \Delta_8), p(n_4)\} : *, \Delta_5} \quad I_1 \quad \frac{h_6 \vdash \{F_7, \Delta_8, p(n_4)\} : F_7, \Delta_5, p(n_4)}{\bullet h_6 \vdash \{(F_7, \Delta_8), p(n_4)\} : dual(\hat{\wedge}(n_4)), \Delta_5} \quad ?_C}{\vdash \{(F_7, \Delta_8), p(n_4)\} : *, \Delta_5} \quad \text{Cut}}{\vdash \{\Delta_8, F_7, p(n_4)\} : \Delta_5, F_7, p(n_4)} \quad \text{ax} \\
\frac{\frac{\frac{}{\vdash \{\Delta_8, F_7, p(n_4)\} : \Delta_5, F_7, p(n_4)}}{\vdash \{\Delta_8, F_7, p(n_4)\} : \Delta_5, F_7} \quad ?_C}{\vdash \{\Delta_8, F_7, p(n_4)\} : \Delta_5} \quad ?_C}{\vdash \{\Delta_8, F_7, p(n_4)\} : \Delta_5} \quad ?_C
\end{array}$$

7.13 Status of I_2 : OK

- Case rule 1
- Case rule !
- Case rule ?

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_1 \vdash \{\Delta_4, \hat{\wedge}(n_5)\} : p(n_5), *}}{\vdash \{\Delta_4, \hat{\wedge}(n_5)\} : *, \Delta_8, ?F_7} \quad I_2 \quad \frac{h_6 \vdash \{F_7, \Delta_4, \hat{\wedge}(n_5)\} : \Delta_8, \hat{\wedge}(n_5)}{\bullet h_6 \vdash \{\Delta_4, \hat{\wedge}(n_5)\} : dual(p(n_5)), \Delta_8, ?F_7} \quad ?}{\vdash \{\Delta_4, \hat{\wedge}(n_5)\} : *, \Delta_8, ?F_7} \quad \text{Cut}}{\vdash \{\Delta_4, F_7, \hat{\wedge}(n_5)\} : \Delta_8, \hat{\wedge}(n_5)} \quad \text{ax} \\
\frac{\frac{\frac{}{\vdash \{\Delta_4, F_7, \hat{\wedge}(n_5)\} : \Delta_8}}{\vdash \{\Delta_4, F_7, \hat{\wedge}(n_5)\} : \Delta_8} \quad ?}{\vdash \{\Delta_4, \hat{\wedge}(n_5)\} : \Delta_8, ?F_7} \quad ?}{\vdash \{\Delta_4, \hat{\wedge}(n_5)\} : \Delta_8, ?F_7} \quad ?
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_1 \vdash \{\Delta_4, \hat{\wedge}(n_5)\} : p(n_5), *}}{\vdash \{\Delta_4, \hat{\wedge}(n_5)\} : *, \Delta_9, F_7 \& F_8} \quad I_2 \quad \frac{h_6 \vdash \{\Delta_4, \hat{\wedge}(n_5)\} : F_7, F_8, \Delta_9, \hat{\wedge}(n_5)}{\bullet h_6 \vdash \{\Delta_4, \hat{\wedge}(n_5)\} : dual(p(n_5)), \Delta_9, F_7 \& F_8} \quad \$}{\vdash \{\Delta_4, \hat{\wedge}(n_5)\} : *, \Delta_9, F_7 \& F_8} \quad \text{Cut}}{\vdash \{\Delta_4, \hat{\wedge}(n_5)\} : \Delta_9, F_7, F_8, \hat{\wedge}(n_5)} \quad \text{ax} \\
\frac{\frac{\frac{}{\vdash \{\Delta_4, \hat{\wedge}(n_5)\} : \Delta_9, F_7, F_8}}{\vdash \{\Delta_4, \hat{\wedge}(n_5)\} : \Delta_9, F_7, F_8} \quad ?_C}{\vdash \{\Delta_4, \hat{\wedge}(n_5)\} : \Delta_9, F_7 \& F_8} \quad \$}{\vdash \{\Delta_4, \hat{\wedge}(n_5)\} : \Delta_9, F_7 \& F_8} \quad \$
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{\frac{\frac{}{\bullet h_1 \vdash \{\Delta_4, \hat{\wedge}(n_5)\} : p(n_5), *}}{\vdash \{\Delta_4, \hat{\wedge}(n_5)\} : *, \Delta_9, F_7 \& F_8} \quad I_2 \quad \frac{h_6 \vdash \{\Delta_4, \hat{\wedge}(n_5)\} : F_7, \Delta_9, \hat{\wedge}(n_5) \quad h_6 \vdash \{\Delta_4, \hat{\wedge}(n_5)\} : F_8, \Delta_9, \hat{\wedge}(n_5)}{\bullet h_6 \vdash \{\Delta_4, \hat{\wedge}(n_5)\} : dual(p(n_5)), \Delta_9, F_7 \& F_8} \quad \&}{\vdash \{\Delta_4, \hat{\wedge}(n_5)\} : *, \Delta_9, F_7 \& F_8} \quad \text{Cut}}{\vdash \{\Delta_4, \hat{\wedge}(n_5)\} : \Delta_9, F_7, \hat{\wedge}(n_5)} \quad \text{ax} \\
\frac{\frac{\frac{}{\vdash \{\Delta_4, \hat{\wedge}(n_5)\} : \Delta_9, F_7, \hat{\wedge}(n_5)}}{\vdash \{\Delta_4, \hat{\wedge}(n_5)\} : \Delta_9, \hat{\wedge}(n_5), F_7 \& F_8} \quad ?_C}{\vdash \{\Delta_4, \hat{\wedge}(n_5)\} : \Delta_9, F_7 \& F_8} \quad \&}{\vdash \{\Delta_4, \hat{\wedge}(n_5)\} : \Delta_9, F_7 \& F_8} \quad \&
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_4, \hat{(n_5)}\} : p(n_5), *} I_2 \quad \frac{h_6 \vdash \{\Delta_4, \hat{(n_5)}\} : F_8, \Delta_9, \hat{(n_5)}}{\bullet h_6 \vdash \{\Delta_4, \hat{(n_5)}\} : dual(p(n_5)), \Delta_9, F_7 \oplus F_8} \oplus_B \\
\hline
- \vdash \{\Delta_4, \hat{(n_5)}\} : *, \Delta_9, F_7 \oplus F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_4, \hat{(n_5)}\} : \Delta_9, F_8, \hat{(n_5)}} ax \\
\frac{}{- \vdash \{\Delta_4, \hat{(n_5)}\} : \Delta_9, F_8} ?_C \\
\hline
- \vdash \{\Delta_4, \hat{(n_5)}\} : \Delta_9, F_7 \oplus F_8 \quad \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_4, \hat{(n_5)}\} : p(n_5), *} I_2 \quad \frac{h_6 \vdash \{\Delta_4, \hat{(n_5)}\} : F_7, \Delta_9, \hat{(n_5)}}{\bullet h_6 \vdash \{\Delta_4, \hat{(n_5)}\} : dual(p(n_5)), \Delta_9, F_7 \oplus F_8} \oplus_A \\
\hline
- \vdash \{\Delta_4, \hat{(n_5)}\} : *, \Delta_9, F_7 \oplus F_8 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_4, \hat{(n_5)}\} : \Delta_9, F_7, \hat{(n_5)}} ax \\
\frac{}{- \vdash \{\Delta_4, \hat{(n_5)}\} : \Delta_9, F_7} ?_C \\
\hline
- \vdash \{\Delta_4, \hat{(n_5)}\} : \Delta_9, F_7 \oplus F_8 \quad \oplus_A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_4, \hat{(n_5)}\} : p(n_5), *} I_2 \quad \frac{h_6 \vdash \{\Delta_4, \hat{(n_5)}\} : \Delta_7, \hat{(n_5)}}{\bullet h_6 \vdash \{\Delta_4, \hat{(n_5)}\} : dual(p(n_5)), \perp, \Delta_7} \perp \\
\hline
- \vdash \{\Delta_4, \hat{(n_5)}\} : *, \perp, \Delta_7 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_4, \hat{(n_5)}\} : \Delta_7, \hat{(n_5)}} ax \\
\frac{}{- \vdash \{\Delta_4, \hat{(n_5)}\} : \Delta_7} ?_C \\
\hline
- \vdash \{\Delta_4, \hat{(n_5)}\} : \Delta_7, \perp \quad \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_4, \hat{(n_5)}\} : p(n_5), *} I_2 \quad \frac{h_6 \vdash \{\Delta_4, \hat{(n_5)}\} : dual(p(n_5)), \top, \Delta_7}{\bullet h_6 \vdash \{\Delta_4, \hat{(n_5)}\} : dual(p(n_5)), \top, \Delta_7} \top \\
\hline
- \vdash \{\Delta_4, \hat{(n_5)}\} : *, \top, \Delta_7 \quad \text{Cut} \\
\rightarrow \\
- \vdash \{\Delta_4, \hat{(n_5)}\} : \Delta_7, \top \quad \top
\end{array}$$

- Case rule I_3

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_4, \hat{(n_6)}\} : p(n_6), *} I_2 \quad \frac{}{\bullet h_5 \vdash \{\Delta_4, \hat{(n_6)}\} : dual(p(n_6)), p(n_6)} I_3 \\
\hline
- \vdash \{\Delta_4, \hat{(n_6)}\} : *, p(n_6) \quad \text{Cut} \\
\rightarrow \\
- \vdash \{\Delta_4, \hat{(n_6)}\} : p(n_6) \quad I_2
\end{array}$$

- Case rule \otimes

$$\begin{array}{c}
\frac{}{\bullet h_1 \vdash \{\Delta_4, \hat{(n_5)}\} : p(n_5), *} I_2 \quad \frac{h_6 \vdash \{\Delta_4, \hat{(n_5)}\} : F_8, \Delta_{10}, \hat{(n_5)} \quad h_6 \vdash \{\Delta_4, \hat{(n_5)}\} : F_9, \Delta_7}{\bullet h_6 \vdash \{\Delta_4, \hat{(n_5)}\} : dual(p(n_5)), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes \\
\hline
- \vdash \{\Delta_4, \hat{(n_5)}\} : *, \Delta_7, \Delta_{10}, F_8 \otimes F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_4, \hat{(n_5)}\} : \Delta_{10}, F_8, \hat{(n_5)}} ax \\
\frac{}{- \vdash \{\Delta_4, \hat{(n_5)}\} : \Delta_{10}, F_8} ?_C \quad \frac{}{- \vdash \{\Delta_4, \hat{(n_5)}\} : \Delta_7, F_9} ax \\
\hline
- \vdash \{\Delta_4, \hat{(n_5)}\} : \Delta_{10}, \Delta_7, F_8 \otimes F_9 \quad \otimes \\
\frac{}{\bullet h_1 \vdash \{\Delta_4, \hat{(n_5)}\} : p(n_5), *} I_2 \quad \frac{h_6 \vdash \{\Delta_4, \hat{(n_5)}\} : F_8, \Delta_7 \quad h_6 \vdash \{\Delta_4, \hat{(n_5)}\} : F_9, \Delta_{10}, \hat{(n_5)}}{\bullet h_6 \vdash \{\Delta_4, \hat{(n_5)}\} : dual(p(n_5)), \Delta_7, \Delta_{10}, F_8 \otimes F_9} \otimes \\
\hline
- \vdash \{\Delta_4, \hat{(n_5)}\} : *, \Delta_7, \Delta_{10}, F_8 \otimes F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_4, \hat{(n_5)}\} : \Delta_{10}, F_9, \hat{(n_5)}} ax \\
\frac{}{- \vdash \{\Delta_4, \hat{(n_5)}\} : \Delta_7, F_8} ax \quad \frac{}{- \vdash \{\Delta_4, \hat{(n_5)}\} : \Delta_{10}, F_9} ?_C \\
\hline
- \vdash \{\Delta_4, \hat{(n_5)}\} : \Delta_{10}, \Delta_7, F_8 \otimes F_9 \quad \otimes
\end{array}$$

- Case rule I_1

$$\frac{\frac{\bullet h_1 \vdash \{(\Delta_6, p(n_5)), \hat{\cdot}(n_5)\} : p(n_5), *}{- \vdash \{(\Delta_6, p(n_5)), \hat{\cdot}(n_5)\} : *, *}}{I_2} \quad \frac{\bullet h_4 \vdash \{(\Delta_6, p(n_5)), \hat{\cdot}(n_5)\} : dual(p(n_5)), *}{- \vdash \{(\Delta_6, p(n_5)), \hat{\cdot}(n_5)\} : *, *}}{I_1}}{\text{Cut}} \rightarrow \frac{\frac{- \vdash \{(\Delta_6, p(n_5)), \hat{\cdot}(n_5)\} : p(n_5)}{- \vdash \{\Delta_6, p(n_5), \hat{\cdot}(n_5)\} : *}}{?_C} \quad \frac{- \vdash \{(\Delta_6, p(n_5)), \hat{\cdot}(n_5)\} : *, *}}{?_C}}{I_2}$$

- Case rule I_2

- Case rule $?_C$

$$\frac{\frac{\bullet h_1 \vdash \{\Delta_7, \hat{\cdot}(n_4)\} : p(n_4), *}{- \vdash \{\Delta_7, \hat{\cdot}(n_4)\} : *, \Delta_5}}{I_2} \quad \frac{h_6 \vdash \{\Delta_7, \hat{\cdot}(n_4)\} : \Delta_5, \hat{\cdot}(n_4), \hat{\cdot}(n_4)}{\bullet h_6 \vdash \{\Delta_7, \hat{\cdot}(n_4)\} : dual(p(n_4)), \Delta_5}}{?_C}}{\text{Cut}} \rightarrow \frac{\frac{- \vdash \{\Delta_7, \hat{\cdot}(n_4)\} : \Delta_5, \hat{\cdot}(n_4), \hat{\cdot}(n_4)}{- \vdash \{\Delta_7, \hat{\cdot}(n_4)\} : \Delta_5}}{?_C} \quad \frac{- \vdash \{\Delta_7, \hat{\cdot}(n_4)\} : \Delta_5, \hat{\cdot}(n_4), \hat{\cdot}(n_4)}{\text{ax}}}{?_C} \quad \frac{- \vdash \{\Delta_7, \hat{\cdot}(n_4)\} : \Delta_5, \hat{\cdot}(n_4)}{?_C}}{- \vdash \{\Delta_7, \hat{\cdot}(n_4)\} : \Delta_5}$$

$$\frac{\frac{\bullet h_1 \vdash \{(F_7, \Delta_8), \hat{\cdot}(n_4)\} : p(n_4), *}{- \vdash \{(F_7, \Delta_8), \hat{\cdot}(n_4)\} : *, \Delta_5}}{I_2} \quad \frac{h_6 \vdash \{F_7, \Delta_8, \hat{\cdot}(n_4)\} : F_7, \Delta_5, \hat{\cdot}(n_4)}{\bullet h_6 \vdash \{(F_7, \Delta_8), \hat{\cdot}(n_4)\} : dual(p(n_4)), \Delta_5}}{?_C}}{\text{Cut}} \rightarrow \frac{\frac{- \vdash \{\Delta_8, F_7, \hat{\cdot}(n_4)\} : \Delta_5, F_7, \hat{\cdot}(n_4)}{- \vdash \{\Delta_8, F_7, \hat{\cdot}(n_4)\} : \Delta_5, F_7}}{?_C} \quad \frac{- \vdash \{\Delta_8, F_7, \hat{\cdot}(n_4)\} : \Delta_5, F_7, \hat{\cdot}(n_4)}{\text{ax}}}{?_C} \quad \frac{- \vdash \{\Delta_8, F_7, \hat{\cdot}(n_4)\} : \Delta_5, F_7}{?_C}}{- \vdash \{\Delta_8, F_7, \hat{\cdot}(n_4)\} : \Delta_5}$$

7.14 Status of $?_C$: OK

- Case rule 1

- Case rule !

- Case rule ?

$$\frac{\frac{h_3 \vdash \{F_6, \Delta_7\} : F_8, F_6, \Delta_2}{\bullet h_3 \vdash \{F_6, \Delta_7\} : F_8, \Delta_2}}{?_C} \quad \frac{h_9 \vdash \{F_6, F_{10}, \Delta_7\} : \Delta_{11}, dual(F_8)}{\bullet h_9 \vdash \{F_6, \Delta_7\} : dual(F_8), \Delta_{11}, ?F_{10}}}{\text{Cut}} \rightarrow \frac{\frac{- \vdash \{F_6, \Delta_7\} : \Delta_2, \Delta_{11}, ?F_{10}}{- \vdash \{\Delta_7, F_6\} : \Delta_2, F_6, F_8}}{\text{ax}} \quad \frac{- \vdash \{\Delta_7, F_6\} : \Delta_{11}, ?F_{10}, dual(F_8)}{\bullet h_9 \vdash \{\Delta_7, F_6\} : \Delta_{11}, ?F_{10}, dual(F_8)}}{\text{hCut}}}{- \vdash \{\Delta_7, F_6\} : \Delta_{11}, \Delta_2, F_6, ?F_{10}} \quad \frac{- \vdash \{\Delta_7, F_6\} : \Delta_{11}, \Delta_2, F_6, ?F_{10}}{?_C}}$$

- Case rule \$

$$\frac{\frac{h_3 \vdash \{F_6, \Delta_7\} : F_8, F_6, \Delta_2}{\bullet h_3 \vdash \{F_6, \Delta_7\} : F_8, \Delta_2}}{?_C} \quad \frac{h_9 \vdash \{F_6, \Delta_7\} : F_{10}, F_{11}, \Delta_{12}, dual(F_8)}{\bullet h_9 \vdash \{F_6, \Delta_7\} : dual(F_8), \Delta_{12}, F_{10} \$ F_{11}}}{\text{Cut}} \rightarrow \frac{\frac{- \vdash \{F_6, \Delta_7\} : \Delta_2, \Delta_{12}, F_{10} \$ F_{11}}{- \vdash \{\Delta_7, F_6\} : \Delta_2, F_8}}{\text{ax}} \quad \frac{- \vdash \{\Delta_7, F_6\} : \Delta_{12}, F_{10}, F_{11}, dual(F_8)}{\bullet h_9 \vdash \{\Delta_7, F_6\} : \Delta_{12}, F_{10}, F_{11}, dual(F_8)}}{\text{hCut}}}{- \vdash \{\Delta_7, F_6\} : \Delta_{12}, \Delta_2, F_{10}, F_{11}} \quad \frac{- \vdash \{\Delta_7, F_6\} : \Delta_{12}, \Delta_2, F_{10}, F_{11}}{\$}}$$

- Case rule $\&$

$$\begin{array}{c}
\frac{h_3 \vdash \{F_6, \Delta_7\} : F_8, F_6, \Delta_2 \quad \bullet h_3 \vdash \{F_6, \Delta_7\} : F_8, \Delta_2 \quad ?_C}{\bullet h_3 \vdash \{F_6, \Delta_7\} : F_8, \Delta_2} \quad \frac{h_9 \vdash \{F_6, \Delta_7\} : F_{10}, \Delta_{12}, dual(F_8) \quad h_9 \vdash \{F_6, \Delta_7\} : F_{11}, \Delta_{12}, dual(F_8)}{\bullet h_9 \vdash \{F_6, \Delta_7\} : dual(F_8), \Delta_{12}, F_{10} \& F_{11}} \quad \& \\
\hline
- \vdash \{F_6, \Delta_7\} : \Delta_2, \Delta_{12}, F_{10} \& F_{11} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{h_3 \vdash \{\Delta_7, F_6\} : \Delta_2, F_6, F_8 \quad \text{ax}}{- \vdash \{\Delta_7, F_6\} : \Delta_2, F_6, F_8} \quad \frac{\bullet h_9 \vdash \{\Delta_7, F_6\} : \Delta_{12}, dual(F_8), F_{10} \& F_{11} \quad \text{ax}}{- \vdash \{\Delta_7, F_6\} : \Delta_{12}, dual(F_8), F_{10} \& F_{11}} \quad \text{hCut}}{- \vdash \{\Delta_7, F_6\} : \Delta_{12}, \Delta_2, F_6, F_{10} \& F_{11}} \quad ?_C}{- \vdash \{\Delta_7, F_6\} : \Delta_{12}, \Delta_2, F_{10} \& F_{11}} \quad ?_C
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{h_3 \vdash \{F_6, \Delta_7\} : F_8, F_6, \Delta_2 \quad \bullet h_3 \vdash \{F_6, \Delta_7\} : F_8, \Delta_2 \quad ?_C}{\bullet h_3 \vdash \{F_6, \Delta_7\} : F_8, \Delta_2} \quad \frac{h_9 \vdash \{F_6, \Delta_7\} : F_{11}, \Delta_{12}, dual(F_8)}{\bullet h_9 \vdash \{F_6, \Delta_7\} : dual(F_8), \Delta_{12}, F_{10} \oplus F_{11}} \quad \oplus_B \\
\hline
- \vdash \{F_6, \Delta_7\} : \Delta_2, \Delta_{12}, F_{10} \oplus F_{11} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_3 \vdash \{\Delta_7, F_6\} : \Delta_2, F_8 \quad \text{ax}}{- \vdash \{\Delta_7, F_6\} : \Delta_2, F_8} \quad \frac{h_9 \vdash \{\Delta_7, F_6\} : \Delta_{12}, F_{11}, dual(F_8)}{\bullet h_9 \vdash \{\Delta_7, F_6\} : \Delta_{12}, F_{11}, dual(F_8)} \quad \text{ax}}{- \vdash \{\Delta_7, F_6\} : \Delta_{12}, \Delta_2, F_{11}} \quad \text{hCut}}{- \vdash \{\Delta_7, F_6\} : \Delta_{12}, \Delta_2, F_{10} \oplus F_{11}} \quad \oplus_B \\
\rightarrow \\
- \vdash \{\Delta_7, F_6\} : \Delta_{12}, \Delta_2, F_{10} \oplus F_{11} \quad \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{h_3 \vdash \{F_6, \Delta_7\} : F_8, F_6, \Delta_2 \quad \bullet h_3 \vdash \{F_6, \Delta_7\} : F_8, \Delta_2 \quad ?_C}{\bullet h_3 \vdash \{F_6, \Delta_7\} : F_8, \Delta_2} \quad \frac{h_9 \vdash \{F_6, \Delta_7\} : F_{10}, \Delta_{12}, dual(F_8)}{\bullet h_9 \vdash \{F_6, \Delta_7\} : dual(F_8), \Delta_{12}, F_{10} \oplus F_{11}} \quad \oplus_A \\
\hline
- \vdash \{F_6, \Delta_7\} : \Delta_2, \Delta_{12}, F_{10} \oplus F_{11} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_3 \vdash \{\Delta_7, F_6\} : \Delta_2, F_8 \quad \text{ax}}{- \vdash \{\Delta_7, F_6\} : \Delta_2, F_8} \quad \frac{h_9 \vdash \{\Delta_7, F_6\} : \Delta_{12}, F_{10}, dual(F_8)}{\bullet h_9 \vdash \{\Delta_7, F_6\} : \Delta_{12}, F_{10}, dual(F_8)} \quad \text{ax}}{- \vdash \{\Delta_7, F_6\} : \Delta_{12}, \Delta_2, F_{10}} \quad \text{hCut}}{- \vdash \{\Delta_7, F_6\} : \Delta_{12}, \Delta_2, F_{10} \oplus F_{11}} \quad \oplus_A \\
\rightarrow \\
- \vdash \{\Delta_7, F_6\} : \Delta_{12}, \Delta_2, F_{10} \oplus F_{11} \quad \oplus_A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_3 \vdash \{F_6, \Delta_7\} : F_8, F_6, \Delta_2 \quad \bullet h_3 \vdash \{F_6, \Delta_7\} : F_8, \Delta_2 \quad ?_C}{\bullet h_3 \vdash \{F_6, \Delta_7\} : F_8, \Delta_2} \quad \frac{h_9 \vdash \{F_6, \Delta_7\} : \Delta_{10}, dual(F_8)}{\bullet h_9 \vdash \{F_6, \Delta_7\} : dual(F_8), \perp, \Delta_{10}} \quad \perp \\
\hline
- \vdash \{F_6, \Delta_7\} : \Delta_2, \perp, \Delta_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_3 \vdash \{\Delta_7, F_6\} : \Delta_2, F_8 \quad \text{ax}}{- \vdash \{\Delta_7, F_6\} : \Delta_2, F_8} \quad \frac{h_9 \vdash \{\Delta_7, F_6\} : \Delta_{10}, dual(F_8)}{\bullet h_9 \vdash \{\Delta_7, F_6\} : \Delta_{10}, dual(F_8)} \quad \text{ax}}{- \vdash \{\Delta_7, F_6\} : \Delta_{10}, \Delta_2} \quad \text{hCut}}{- \vdash \{\Delta_7, F_6\} : \Delta_{10}, \Delta_2, \perp} \quad \perp \\
\rightarrow \\
- \vdash \{\Delta_7, F_6\} : \Delta_{10}, \Delta_2, \perp \quad \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_3 \vdash \{F_6, \Delta_7\} : F_8, F_6, \Delta_2 \quad \bullet h_3 \vdash \{F_6, \Delta_7\} : F_8, \Delta_2 \quad ?_C}{\bullet h_3 \vdash \{F_6, \Delta_7\} : F_8, \Delta_2} \quad \frac{h_9 \vdash \{F_6, \Delta_7\} : \Delta_{10}, dual(F_8)}{\bullet h_9 \vdash \{F_6, \Delta_7\} : dual(F_8), \top, \Delta_{10}} \quad \top \\
\hline
- \vdash \{F_6, \Delta_7\} : \Delta_2, \top, \Delta_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{- \vdash \{F_6, \Delta_7\} : \Delta_2, \top, \Delta_{10}}{- \vdash \{F_6, \Delta_7\} : \Delta_2, \top, \Delta_{10}} \quad \top \\
\rightarrow \\
- \vdash \{\Delta_7, F_6\} : \Delta_{10}, \Delta_2, \top \quad \top
\end{array}$$

- Case rule I_3

- Case rule \otimes

$$\begin{array}{c}
\frac{h_3 \vdash \{F_6, \Delta_7\} : F_8, F_6, \Delta_2}{\bullet h_3 \vdash \{F_6, \Delta_7\} : F_8, \Delta_2} \text{?}_C \quad \frac{h_9 \vdash \{F_6, \Delta_7\} : F_{11}, \Delta_{13}, \mathit{dual}(F_8) \quad h_9 \vdash \{F_6, \Delta_7\} : F_{12}, \Delta_{10}}{\bullet h_9 \vdash \{F_6, \Delta_7\} : \mathit{dual}(F_8), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \text{Cut} \otimes \\
\hline
- \vdash \{F_6, \Delta_7\} : \Delta_2, \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12} \\
\rightarrow \\
\frac{\frac{h_3 \vdash \{\Delta_7, F_6\} : \Delta_2, F_6, F_8}{- \vdash \{\Delta_7, F_6\} : \Delta_{10}, \Delta_{13}, \Delta_2, F_6, F_{11} \otimes F_{12}} \text{?}_C \quad \frac{\bullet h_9 \vdash \{\Delta_7, F_6\} : \Delta_{10}, \Delta_{13}, \mathit{dual}(F_8), F_{11} \otimes F_{12}}{\bullet h_9 \vdash \{\Delta_7, F_6\} : \Delta_{10}, \Delta_{13}, \Delta_2, F_{11} \otimes F_{12}} \text{Cut}}{- \vdash \{\Delta_7, F_6\} : \Delta_{10}, \Delta_{13}, \Delta_2, F_{11} \otimes F_{12}} \text{?}_C \quad \text{ax} \quad \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_3 \vdash \{F_6, \Delta_7\} : F_8, F_6, \Delta_2}{\bullet h_3 \vdash \{F_6, \Delta_7\} : F_8, \Delta_2} \text{?}_C \quad \frac{h_9 \vdash \{F_6, \Delta_7\} : F_{11}, \Delta_{10} \quad h_9 \vdash \{F_6, \Delta_7\} : F_{12}, \Delta_{13}, \mathit{dual}(F_8)}{\bullet h_9 \vdash \{F_6, \Delta_7\} : \mathit{dual}(F_8), \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12}} \text{Cut} \otimes \\
\hline
- \vdash \{F_6, \Delta_7\} : \Delta_2, \Delta_{10}, \Delta_{13}, F_{11} \otimes F_{12} \\
\rightarrow \\
\frac{\frac{h_3 \vdash \{\Delta_7, F_6\} : \Delta_2, F_6, F_8}{- \vdash \{\Delta_7, F_6\} : \Delta_{10}, \Delta_{13}, \Delta_2, F_6, F_{11} \otimes F_{12}} \text{?}_C \quad \frac{\bullet h_9 \vdash \{\Delta_7, F_6\} : \Delta_{10}, \Delta_{13}, \mathit{dual}(F_8), F_{11} \otimes F_{12}}{\bullet h_9 \vdash \{\Delta_7, F_6\} : \Delta_{10}, \Delta_{13}, \Delta_2, F_{11} \otimes F_{12}} \text{Cut}}{- \vdash \{\Delta_7, F_6\} : \Delta_{10}, \Delta_{13}, \Delta_2, F_{11} \otimes F_{12}} \text{?}_C \quad \text{ax} \quad \text{hCut}
\end{array}$$

• Case rule I_1

• Case rule I_2

• Case rule $?_C$

$$\begin{array}{c}
\frac{h_3 \vdash \{F_9, \Delta_{10}\} : F_7, F_9, \Delta_2}{\bullet h_3 \vdash \{F_9, \Delta_{10}\} : F_7, \Delta_2} \text{?}_C \quad \frac{h_8 \vdash \{F_9, \Delta_{10}\} : F_9, \Delta_6, \mathit{dual}(F_7)}{\bullet h_8 \vdash \{F_9, \Delta_{10}\} : \mathit{dual}(F_7), \Delta_6} \text{?}_C \\
\hline
- \vdash \{F_9, \Delta_{10}\} : \Delta_2, \Delta_6 \\
\rightarrow \\
\frac{\frac{h_3 \vdash \{\Delta_{10}, F_9\} : \Delta_2, F_7, F_9}{- \vdash \{\Delta_{10}, F_9\} : \Delta_2, \Delta_6, F_9} \text{?}_C \quad \frac{\bullet h_8 \vdash \{\Delta_{10}, F_9\} : \Delta_6, \mathit{dual}(F_7)}{\bullet h_8 \vdash \{\Delta_{10}, F_9\} : \Delta_2, \Delta_6} \text{Cut}}{- \vdash \{\Delta_{10}, F_9\} : \Delta_2, \Delta_6} \text{?}_C \quad \text{ax} \quad \text{hCut}
\end{array}$$

$$\begin{array}{c}
\frac{h_3 \vdash \{F_6, F_{10}, \Delta_{11}\} : F_8, F_6, \Delta_2}{\bullet h_3 \vdash \{F_6, F_{10}, \Delta_{11}\} : F_8, \Delta_2} \text{?}_C \quad \frac{h_9 \vdash \{F_6, F_{10}, \Delta_{11}\} : F_{10}, \Delta_7, \mathit{dual}(F_8)}{\bullet h_9 \vdash \{F_6, F_{10}, \Delta_{11}\} : \mathit{dual}(F_8), \Delta_7} \text{?}_C \\
\hline
- \vdash \{F_6, F_{10}, \Delta_{11}\} : \Delta_2, \Delta_7 \\
\rightarrow \\
\frac{\frac{\bullet h_3 \vdash \{\Delta_{11}, F_{10}, F_6\} : \Delta_2, F_8}{- \vdash \{\Delta_{11}, F_{10}, F_6\} : \Delta_2, \Delta_7, F_{10}} \text{?}_C \quad \frac{h_9 \vdash \{\Delta_{11}, F_{10}, F_6\} : \Delta_7, F_{10}, \mathit{dual}(F_8)}{\bullet h_9 \vdash \{\Delta_{11}, F_{10}, F_6\} : \Delta_2, \Delta_7} \text{Cut}}{- \vdash \{\Delta_{11}, F_{10}, F_6\} : \Delta_2, \Delta_7} \text{?}_C \quad \text{ax} \quad \text{hCut}
\end{array}$$

8 Cut-Elimination

8.1 Status of 1: OK

- Case rule 1
- Case rule !
- Case rule ?
- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I_3
- Case rule \otimes
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.2 Status of !: OK

- Case rule 1

$$\frac{\frac{\frac{h_1 \vdash \{\Delta_4\} : F_5}{\bullet h_1 \vdash \{\Delta_4\} : !F_5} \quad ! \quad \frac{\bullet h_6 \vdash \{dual(F_5), \Delta_4\} : \mathbf{1}}{\vdash \{\Delta_4\} : \mathbf{1}} \quad \mathbf{1}}{\vdash \{\Delta_4\} : \mathbf{1}} \quad \text{Cut}}{\vdash \{\Delta_4\} : \mathbf{1}} \quad \mathbf{1}$$

- Case rule !

$$\begin{array}{c}
\frac{\frac{h_1 \vdash \{\Delta_4\} : F_5}{\bullet h_1 \vdash \{\Delta_4\} :!F_5} ! \quad \frac{h_6 \vdash \{\Delta_4, dual(F_5)\} : F_7}{\bullet h_6 \vdash \{dual(F_5), \Delta_4\} :!F_7} !}{- \vdash \{\Delta_4\} :!F_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_4\} :!F_5}{- \vdash \{\Delta_4\} : F_7} \text{ax} \quad \frac{h_6 \vdash \{\Delta_4, dual(F_5)\} : F_7}{- \vdash \{\Delta_4\} :!F_7} \text{ax}}{- \vdash \{\Delta_4\} : F_7} \text{hCut} \\
\rightarrow \\
- \vdash \{\Delta_4\} :!F_7
\end{array}$$

- Case rule ?

$$\begin{array}{c}
\frac{\frac{h_1 \vdash \{\Delta_4\} : F_5}{\bullet h_1 \vdash \{\Delta_4\} :!F_5} ! \quad \frac{h_6 \vdash \{F_8, \Delta_4, dual(F_5)\} : \Delta_7}{\bullet h_6 \vdash \{dual(F_5), \Delta_4\} : \Delta_7, ?F_8} ?}{- \vdash \{\Delta_4\} : \Delta_7, ?F_8} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_4\} :!F_5}{\bullet h_1 \vdash \{\Delta_4, F_8\} :!F_5} \text{ax} \quad \frac{h_6 \vdash \{\Delta_4, F_8, dual(F_5)\} : \Delta_7}{- \vdash \{\Delta_4, F_8\} : \Delta_7} \text{ax}}{- \vdash \{\Delta_4\} : \Delta_7, ?F_8} \text{hCut} \\
\rightarrow \\
- \vdash \{\Delta_4\} : \Delta_7, ?F_8
\end{array}$$

- Case rule \$

$$\begin{array}{c}
\frac{\frac{h_1 \vdash \{\Delta_4\} : F_5}{\bullet h_1 \vdash \{\Delta_4\} :!F_5} ! \quad \frac{h_6 \vdash \{\Delta_4, dual(F_5)\} : F_8, F_9, \Delta_7}{\bullet h_6 \vdash \{dual(F_5), \Delta_4\} : \Delta_7, F_8 \$F_9} \$}{- \vdash \{\Delta_4\} : \Delta_7, F_8 \$F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_4\} :!F_5}{- \vdash \{\Delta_4\} : \Delta_7, F_8, F_9} \text{ax} \quad \frac{h_6 \vdash \{\Delta_4, dual(F_5)\} : \Delta_7, F_8, F_9}{- \vdash \{\Delta_4\} : \Delta_7, F_8 \$F_9} \text{ax}}{- \vdash \{\Delta_4\} : \Delta_7, F_8 \$F_9} \text{hCut} \\
\rightarrow \\
- \vdash \{\Delta_4\} : \Delta_7, F_8 \$F_9
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{\frac{h_1 \vdash \{\Delta_4\} : F_5}{\bullet h_1 \vdash \{\Delta_4\} :!F_5} ! \quad \frac{h_6 \vdash \{\Delta_4, dual(F_5)\} : F_8, \Delta_7 \quad h_6 \vdash \{\Delta_4, dual(F_5)\} : F_9, \Delta_7}{\bullet h_6 \vdash \{dual(F_5), \Delta_4\} : \Delta_7, F_8 \&F_9} \&}{- \vdash \{\Delta_4\} : \Delta_7, F_8 \&F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_4\} :!F_5}{- \vdash \{\Delta_4\} : \Delta_7, F_8} \text{ax} \quad \frac{h_6 \vdash \{\Delta_4, dual(F_5)\} : \Delta_7, F_8}{- \vdash \{\Delta_4\} : \Delta_7, F_8} \text{ax}}{- \vdash \{\Delta_4\} : \Delta_7, F_8} \text{hCut} \quad \frac{\frac{\bullet h_1 \vdash \{\Delta_4\} :!F_5}{- \vdash \{\Delta_4\} : \Delta_7, F_9} \text{ax} \quad \frac{h_6 \vdash \{\Delta_4, dual(F_5)\} : \Delta_7, F_9}{- \vdash \{\Delta_4\} : \Delta_7, F_9} \text{ax}}{- \vdash \{\Delta_4\} : \Delta_7, F_9} \text{hCut}}{- \vdash \{\Delta_4\} : \Delta_7, F_8 \&F_9} \&
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{\frac{h_1 \vdash \{\Delta_4\} : F_5}{\bullet h_1 \vdash \{\Delta_4\} :!F_5} ! \quad \frac{h_6 \vdash \{\Delta_4, dual(F_5)\} : F_9, \Delta_7}{\bullet h_6 \vdash \{dual(F_5), \Delta_4\} : \Delta_7, F_8 \oplus F_9} \oplus_B}{- \vdash \{\Delta_4\} : \Delta_7, F_8 \oplus F_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_4\} :!F_5}{- \vdash \{\Delta_4\} : \Delta_7, F_9} \text{ax} \quad \frac{h_6 \vdash \{\Delta_4, dual(F_5)\} : \Delta_7, F_9}{- \vdash \{\Delta_4\} : \Delta_7, F_9} \text{ax}}{- \vdash \{\Delta_4\} : \Delta_7, F_9} \text{hCut}}{- \vdash \{\Delta_4\} : \Delta_7, F_8 \oplus F_9} \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_4\} : F_5}{\bullet h_1 \vdash \{\Delta_4\} : !F_5} ! \quad \frac{h_6 \vdash \{\Delta_4, dual(F_5)\} : F_8, \Delta_7}{\bullet h_6 \vdash \{dual(F_5), \Delta_4\} : \Delta_7, F_8 \oplus F_9} \oplus^A \\
\hline
- \vdash \{\Delta_4\} : \Delta_7, F_8 \oplus F_9 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_4\} : !F_5}{- \vdash \{\Delta_4\} : \Delta_7, F_8} \text{ax} \quad \frac{h_6 \vdash \{\Delta_4, dual(F_5)\} : \Delta_7, F_8}{- \vdash \{\Delta_4\} : \Delta_7, F_8} \text{ax}}{- \vdash \{\Delta_4\} : \Delta_7, F_8 \oplus F_9} \text{hCut} \oplus^A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_4\} : F_5}{\bullet h_1 \vdash \{\Delta_4\} : !F_5} ! \quad \frac{h_6 \vdash \{\Delta_4, dual(F_5)\} : \Delta_7}{\bullet h_6 \vdash \{dual(F_5), \Delta_4\} : \perp, \Delta_7} \perp \\
\hline
- \vdash \{\Delta_4\} : \perp, \Delta_7 \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_4\} : !F_5}{- \vdash \{\Delta_4\} : \Delta_7} \text{ax} \quad \frac{h_6 \vdash \{\Delta_4, dual(F_5)\} : \Delta_7}{- \vdash \{\Delta_4\} : \Delta_7} \text{ax}}{- \vdash \{\Delta_4\} : \Delta_7, \perp} \text{hCut} \perp
\end{array}$$

- Case rule \top

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_4\} : F_5}{\bullet h_1 \vdash \{\Delta_4\} : !F_5} ! \quad \frac{}{\bullet h_6 \vdash \{dual(F_5), \Delta_4\} : \top, \Delta_7} \top \\
\hline
- \vdash \{\Delta_4\} : \top, \Delta_7 \quad \text{Cut} \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_4\} : \Delta_7, \top} \top
\end{array}$$

- Case rule I_3

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_4\} : F_5}{\bullet h_1 \vdash \{\Delta_4\} : !F_5} ! \quad \frac{}{\bullet h_6 \vdash \{dual(F_5), \Delta_4\} : p(\mathbf{n}_7), \hat{(\mathbf{n}_7)}} I_3 \\
\hline
- \vdash \{\Delta_4\} : p(\mathbf{n}_7), \hat{(\mathbf{n}_7)} \quad \text{Cut} \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_4\} : p(\mathbf{n}_7), \hat{(\mathbf{n}_7)}} I_3
\end{array}$$

- Case rule \otimes

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_4\} : F_5}{\bullet h_1 \vdash \{\Delta_4\} : !F_5} ! \quad \frac{h_6 \vdash \{\Delta_4, dual(F_5)\} : F_9, \Delta_7 \quad h_6 \vdash \{\Delta_4, dual(F_5)\} : F_{10}, \Delta_8}{\bullet h_6 \vdash \{dual(F_5), \Delta_4\} : \Delta_7, \Delta_8, F_9 \otimes F_{10}} \otimes \\
\hline
- \vdash \{\Delta_4\} : \Delta_7, \Delta_8, F_9 \otimes F_{10} \quad \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_4\} : !F_5}{- \vdash \{\Delta_4\} : \Delta_7, F_9} \text{ax} \quad \frac{h_6 \vdash \{\Delta_4, dual(F_5)\} : \Delta_7, F_9}{- \vdash \{\Delta_4\} : \Delta_7, F_9} \text{hCut} \quad \frac{\bullet h_1 \vdash \{\Delta_4\} : !F_5}{- \vdash \{\Delta_4\} : \Delta_8, F_{10}} \text{ax} \quad \frac{h_6 \vdash \{\Delta_4, dual(F_5)\} : \Delta_8, F_{10}}{- \vdash \{\Delta_4\} : \Delta_8, F_{10}} \text{hCut}}{- \vdash \{\Delta_4\} : \Delta_7, \Delta_8, F_9 \otimes F_{10}} \otimes
\end{array}$$

- Case rule I_1

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_7, p(\mathbf{n}_6)\} : F_4}{\bullet h_1 \vdash \{\Delta_7, p(\mathbf{n}_6)\} : !F_4} ! \quad \frac{}{\bullet h_5 \vdash \{dual(F_4), \Delta_7, p(\mathbf{n}_6)\} : \hat{(\mathbf{n}_6)}} I_1 \\
\hline
- \vdash \{\Delta_7, p(\mathbf{n}_6)\} : \hat{(\mathbf{n}_6)} \quad \text{Cut} \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_7, p(\mathbf{n}_6)\} : \hat{(\mathbf{n}_6)}} I_1
\end{array}$$

- Case rule I_2

$$\begin{array}{c}
\frac{h_1 \vdash \{\Delta_7, \hat{(\mathbf{n}_6)}\} : F_4}{\bullet h_1 \vdash \{\Delta_7, \hat{(\mathbf{n}_6)}\} : !F_4} ! \quad \frac{}{\bullet h_5 \vdash \{dual(F_4), \Delta_7, \hat{(\mathbf{n}_6)}\} : p(\mathbf{n}_6)} I_2 \\
\hline
- \vdash \{\Delta_7, \hat{(\mathbf{n}_6)}\} : p(\mathbf{n}_6) \quad \text{Cut} \\
\rightarrow \\
\frac{}{- \vdash \{\Delta_7, \hat{(\mathbf{n}_6)}\} : p(\mathbf{n}_6)} I_2
\end{array}$$

- Case rule $?_C$

$$\begin{array}{c}
\frac{\frac{h_1 \vdash \{\Delta_6\} : F_4}{\bullet h_1 \vdash \{\Delta_6\} : !F_4} ! \quad \frac{h_5 \vdash \{\Delta_6, dual(F_4)\} : \Delta_7, dual(F_4)}{\bullet h_5 \vdash \{dual(F_4), \Delta_6\} : \Delta_7} ?_C}{- \vdash \{\Delta_6\} : \Delta_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{- \vdash \{\Delta_6\} : F_4}{- \vdash \{\Delta_6\} : \Delta_7} \text{ax} \quad \frac{\frac{\bullet h_1 \vdash \{\Delta_6\} : !F_4}{- \vdash \{\Delta_6\} : \Delta_7, dual(F_4)} \text{ax} \quad \frac{h_5 \vdash \{\Delta_6, dual(F_4)\} : \Delta_7, dual(F_4)}{- \vdash \{\Delta_6\} : \Delta_7, dual(F_4)} \text{ax}}{- \vdash \{\Delta_6\} : \Delta_7} \text{mCut}}{- \vdash \{\Delta_6\} : \Delta_7} \text{hCut} \\
\\
\frac{\frac{h_1 \vdash \{F_6, \Delta_8\} : F_4}{\bullet h_1 \vdash \{F_6, \Delta_8\} : !F_4} ! \quad \frac{h_5 \vdash \{F_6, \Delta_8, dual(F_4)\} : F_6, \Delta_7}{\bullet h_5 \vdash \{dual(F_4), F_6, \Delta_8\} : \Delta_7} ?_C}{- \vdash \{F_6, \Delta_8\} : \Delta_7} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_1 \vdash \{\Delta_8, F_6\} : !F_4}{- \vdash \{\Delta_8, F_6\} : \Delta_7, F_6} \text{ax} \quad \frac{h_5 \vdash \{\Delta_8, F_6, dual(F_4)\} : \Delta_7, F_6}{- \vdash \{\Delta_8, F_6\} : \Delta_7, F_6} \text{ax}}{- \vdash \{\Delta_8, F_6\} : \Delta_7} \text{hCut}}{- \vdash \{\Delta_8, F_6\} : \Delta_7} ?_C
\end{array}$$

8.3 Status of ?: OK

- Case rule 1
- Case rule !
- Case rule ?
- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I_3
- Case rule \otimes
- Case rule I_1
- Case rule I_2

- Case rule $?_C$

8.4 Status of $\$$: OK

- Case rule **1**
- Case rule **!**
- Case rule **?**
- Case rule $\$$
- Case rule $\&$
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I_3
- Case rule \otimes
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.5 Status of $\&$: OK

- Case rule **1**
- Case rule **!**
- Case rule **?**

- Case rule $\$$
- Case rule $\&$
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I_3
- Case rule \otimes
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.6 Status of \oplus_B : OK

- Case rule $\mathbf{1}$
- Case rule $!$
- Case rule $?$
- Case rule $\$$
- Case rule $\&$
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp

- Case rule \top
- Case rule I_3
- Case rule \otimes
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.7 Status of \oplus_A : OK

- Case rule **1**
- Case rule **!**
- Case rule **?**
- Case rule **\$**
- Case rule **&**
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I_3
- Case rule \otimes
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.8 Status of \perp : OK

- Case rule **1**
- Case rule **!**
- Case rule **?**
- Case rule **\$**
- Case rule **&**
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I_3
- Case rule \otimes
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.9 Status of \top : OK

- Case rule **1**
- Case rule **!**
- Case rule **?**
- Case rule **\$**

- Case rule $\&$
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I_3
- Case rule \otimes
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.10 Status of I_3 : OK

- Case rule 1
- Case rule !
- Case rule ?
- Case rule \$
- Case rule $\&$
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top

- Case rule I_3
- Case rule \otimes
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.11 Status of \otimes : OK

- Case rule 1
- Case rule !
- Case rule ?
- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I_3
- Case rule \otimes
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.12 Status of I_1 : OK

- Case rule 1
- Case rule !
- Case rule ?
- Case rule \$
- Case rule &
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I_3
- Case rule \otimes
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.13 Status of I_2 : OK

- Case rule 1
- Case rule !
- Case rule ?
- Case rule \$

- Case rule $\&$
- Case rule \oplus_B
- Case rule \oplus_A
- Case rule \perp
- Case rule \top
- Case rule I_3
- Case rule \otimes
- Case rule I_1
- Case rule I_2
- Case rule $?_C$

8.14 Status of $?_C$: OK

- Case rule **1**

$$\frac{\frac{h_2 \vdash \{F_5, \Delta_6\} : F_5, !F_7}{\bullet h_2 \vdash \{F_5, \Delta_6\} : !F_7} \quad ?_C \quad \frac{}{\bullet h_8 \vdash \{dual(F_7), F_5, \Delta_6\} : \mathbf{1}} \quad \mathbf{1}}{\vdash \{F_5, \Delta_6\} : \mathbf{1}} \quad \text{Cut}}{\vdash \{F_5, \Delta_6\} : \mathbf{1}} \quad \mathbf{1}$$

- Case rule **!**

$$\frac{\frac{h_2 \vdash \{F_5, \Delta_6\} : F_5, !F_7}{\bullet h_2 \vdash \{F_5, \Delta_6\} : !F_7} \quad ?_C \quad \frac{h_8 \vdash \{F_5, \Delta_6, dual(F_7)\} : F_9}{\bullet h_8 \vdash \{dual(F_7), F_5, \Delta_6\} : !F_9} \quad !}{\vdash \{F_5, \Delta_6\} : !F_9} \quad \text{Cut}}{\vdash \{F_5, \Delta_6\} : !F_9} \quad \mathbf{!}$$

- Case rule **?**

$$\begin{array}{c}
\frac{h_2 \vdash \{F_5, \Delta_6\} : F_5, !F_7 \quad \bullet h_8 \vdash \{F_5, F_{10}, \Delta_6, dual(F_7)\} : \Delta_9}{\bullet h_2 \vdash \{F_5, \Delta_6\} : !F_7} ?_C \quad \frac{h_8 \vdash \{F_5, F_{10}, \Delta_6, dual(F_7)\} : \Delta_9}{\bullet h_8 \vdash \{dual(F_7), F_5, \Delta_6\} : \Delta_9, ?F_{10}} ?}{- \vdash \{F_5, \Delta_6\} : \Delta_9, ?F_{10}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \vdash \{\Delta_6, F_5\} : !F_7}{\bullet h_2 \vdash \{\Delta_6, F_{10}, F_5\} : !F_7} \text{ax} \quad \frac{h_8 \vdash \{\Delta_6, F_{10}, F_5, dual(F_7)\} : \Delta_9}{h_8 \vdash \{\Delta_6, F_{10}, F_5\} : \Delta_9} \text{ax}}{- \vdash \{\Delta_6, F_{10}, F_5\} : \Delta_9} W}{- \vdash \{\Delta_6, F_5\} : \Delta_9, ?F_{10}} \text{hCut}
\end{array}$$

- Case rule §

$$\begin{array}{c}
\frac{h_2 \vdash \{F_5, \Delta_6\} : F_5, !F_7 \quad \bullet h_8 \vdash \{F_5, \Delta_6, dual(F_7)\} : F_{10}, F_{11}, \Delta_9}{\bullet h_2 \vdash \{F_5, \Delta_6\} : !F_7} ?_C \quad \frac{h_8 \vdash \{F_5, \Delta_6, dual(F_7)\} : F_{10}, F_{11}, \Delta_9}{\bullet h_8 \vdash \{dual(F_7), F_5, \Delta_6\} : \Delta_9, F_{10} \S F_{11}} \S}{- \vdash \{F_5, \Delta_6\} : \Delta_9, F_{10} \S F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \vdash \{\Delta_6, F_5\} : !F_7}{\bullet h_2 \vdash \{\Delta_6, F_5\} : \Delta_9, F_{10}, F_{11}} \text{ax} \quad \frac{h_8 \vdash \{\Delta_6, F_5, dual(F_7)\} : \Delta_9, F_{10}, F_{11}}{h_8 \vdash \{\Delta_6, F_5\} : \Delta_9, F_{10} \S F_{11}} \text{ax}}{- \vdash \{\Delta_6, F_5\} : \Delta_9, F_{10}, F_{11}} \text{hCut}}{- \vdash \{\Delta_6, F_5\} : \Delta_9, F_{10} \S F_{11}} \S
\end{array}$$

- Case rule &

$$\begin{array}{c}
\frac{h_2 \vdash \{F_5, \Delta_6\} : F_5, !F_7 \quad \bullet h_8 \vdash \{F_5, \Delta_6, dual(F_7)\} : F_{10}, \Delta_9 \quad h_8 \vdash \{F_5, \Delta_6, dual(F_7)\} : F_{11}, \Delta_9}{\bullet h_2 \vdash \{F_5, \Delta_6\} : !F_7} ?_C \quad \frac{\bullet h_8 \vdash \{dual(F_7), F_5, \Delta_6\} : \Delta_9, F_{10} \& F_{11}}{\bullet h_8 \vdash \{dual(F_7), F_5, \Delta_6\} : \Delta_9, F_{10} \& F_{11}} \&}{- \vdash \{F_5, \Delta_6\} : \Delta_9, F_{10} \& F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \vdash \{\Delta_6, F_5\} : !F_7}{\bullet h_2 \vdash \{\Delta_6, F_5\} : \Delta_9, F_{10}} \text{ax} \quad \frac{h_8 \vdash \{\Delta_6, F_5, dual(F_7)\} : \Delta_9, F_{10}}{h_8 \vdash \{\Delta_6, F_5\} : \Delta_9, F_{10} \& F_{11}} \text{ax}}{- \vdash \{\Delta_6, F_5\} : \Delta_9, F_{10}} \text{hCut} \quad \frac{\bullet h_2 \vdash \{\Delta_6, F_5\} : !F_7}{\bullet h_2 \vdash \{\Delta_6, F_5\} : \Delta_9, F_{11}} \text{ax} \quad \frac{h_8 \vdash \{\Delta_6, F_5, dual(F_7)\} : \Delta_9, F_{11}}{h_8 \vdash \{\Delta_6, F_5\} : \Delta_9, F_{11}} \text{ax}}{- \vdash \{\Delta_6, F_5\} : \Delta_9, F_{11}} \text{hCut}}{- \vdash \{\Delta_6, F_5\} : \Delta_9, F_{10} \& F_{11}} \&
\end{array}$$

- Case rule \oplus_B

$$\begin{array}{c}
\frac{h_2 \vdash \{F_5, \Delta_6\} : F_5, !F_7 \quad \bullet h_8 \vdash \{F_5, \Delta_6, dual(F_7)\} : F_{11}, \Delta_9}{\bullet h_2 \vdash \{F_5, \Delta_6\} : !F_7} ?_C \quad \frac{h_8 \vdash \{F_5, \Delta_6, dual(F_7)\} : F_{11}, \Delta_9}{\bullet h_8 \vdash \{dual(F_7), F_5, \Delta_6\} : \Delta_9, F_{10} \oplus F_{11}} \oplus_B}{- \vdash \{F_5, \Delta_6\} : \Delta_9, F_{10} \oplus F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \vdash \{\Delta_6, F_5\} : !F_7}{\bullet h_2 \vdash \{\Delta_6, F_5\} : \Delta_9, F_{11}} \text{ax} \quad \frac{h_8 \vdash \{\Delta_6, F_5, dual(F_7)\} : \Delta_9, F_{11}}{h_8 \vdash \{\Delta_6, F_5\} : \Delta_9, F_{11}} \text{ax}}{- \vdash \{\Delta_6, F_5\} : \Delta_9, F_{11}} \text{hCut}}{- \vdash \{\Delta_6, F_5\} : \Delta_9, F_{10} \oplus F_{11}} \oplus_B
\end{array}$$

- Case rule \oplus_A

$$\begin{array}{c}
\frac{h_2 \vdash \{F_5, \Delta_6\} : F_5, !F_7 \quad \bullet h_8 \vdash \{F_5, \Delta_6, dual(F_7)\} : F_{10}, \Delta_9}{\bullet h_2 \vdash \{F_5, \Delta_6\} : !F_7} ?_C \quad \frac{h_8 \vdash \{F_5, \Delta_6, dual(F_7)\} : F_{10}, \Delta_9}{\bullet h_8 \vdash \{dual(F_7), F_5, \Delta_6\} : \Delta_9, F_{10} \oplus F_{11}} \oplus_A}{- \vdash \{F_5, \Delta_6\} : \Delta_9, F_{10} \oplus F_{11}} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \vdash \{\Delta_6, F_5\} : !F_7}{\bullet h_2 \vdash \{\Delta_6, F_5\} : \Delta_9, F_{10}} \text{ax} \quad \frac{h_8 \vdash \{\Delta_6, F_5, dual(F_7)\} : \Delta_9, F_{10}}{h_8 \vdash \{\Delta_6, F_5\} : \Delta_9, F_{10}} \text{ax}}{- \vdash \{\Delta_6, F_5\} : \Delta_9, F_{10}} \text{hCut}}{- \vdash \{\Delta_6, F_5\} : \Delta_9, F_{10} \oplus F_{11}} \oplus_A
\end{array}$$

- Case rule \perp

$$\begin{array}{c}
\frac{h_2 \vdash \{F_5, \Delta_6\} : F_5, !F_7 \quad \bullet h_8 \vdash \{F_5, \Delta_6, dual(F_7)\} : \Delta_9}{\bullet h_2 \vdash \{F_5, \Delta_6\} : !F_7} ?_C \quad \frac{h_8 \vdash \{F_5, \Delta_6, dual(F_7)\} : \Delta_9}{\bullet h_8 \vdash \{dual(F_7), F_5, \Delta_6\} : \perp, \Delta_9} \perp}{- \vdash \{F_5, \Delta_6\} : \perp, \Delta_9} \text{Cut} \\
\rightarrow \\
\frac{\frac{\bullet h_2 \vdash \{\Delta_6, F_5\} : !F_7}{\bullet h_2 \vdash \{\Delta_6, F_5\} : \Delta_9} \text{ax} \quad \frac{h_8 \vdash \{\Delta_6, F_5, dual(F_7)\} : \Delta_9}{h_8 \vdash \{\Delta_6, F_5\} : \Delta_9} \text{ax}}{- \vdash \{\Delta_6, F_5\} : \Delta_9} \text{hCut}}{- \vdash \{\Delta_6, F_5\} : \Delta_9, \perp} \perp
\end{array}$$

- Case rule \top

$$\frac{\frac{h_2 \vdash \{F_5, \Delta_6\} : F_5, !F_7}{\bullet h_2 \vdash \{F_5, \Delta_6\} : !F_7} ?_C \quad \frac{}{\bullet h_8 \vdash \{dual(F_7), F_5, \Delta_6\} : \top, \Delta_9} \top}{\frac{}{- \vdash \{F_5, \Delta_6\} : \top, \Delta_9} \rightarrow} \text{Cut}}{\frac{}{- \vdash \{\Delta_6, F_5\} : \Delta_9, \top} \top} \top$$

- Case rule I_3

$$\frac{\frac{h_2 \vdash \{F_5, \Delta_6\} : F_5, !F_7}{\bullet h_2 \vdash \{F_5, \Delta_6\} : !F_7} ?_C \quad \frac{}{\bullet h_8 \vdash \{dual(F_7), F_5, \Delta_6\} : p(\mathbf{n}_9), \hat{\mathbf{n}}_9} I_3}{\frac{}{- \vdash \{F_5, \Delta_6\} : p(\mathbf{n}_9), \hat{\mathbf{n}}_9} \rightarrow} \text{Cut}}{\frac{}{- \vdash \{\Delta_6, F_5\} : p(\mathbf{n}_9), \hat{\mathbf{n}}_9} I_3} \top$$

- Case rule \otimes

$$\frac{\frac{h_2 \vdash \{F_5, \Delta_6\} : F_5, !F_7}{\bullet h_2 \vdash \{F_5, \Delta_6\} : !F_7} ?_C \quad \frac{h_8 \vdash \{F_5, \Delta_6, dual(F_7)\} : F_{11}, \Delta_9 \quad h_8 \vdash \{F_5, \Delta_6, dual(F_7)\} : F_{12}, \Delta_{10}}{\bullet h_8 \vdash \{dual(F_7), F_5, \Delta_6\} : \Delta_9, \Delta_{10}, F_{11} \otimes F_{12}} \otimes}{\frac{}{- \vdash \{F_5, \Delta_6\} : \Delta_9, \Delta_{10}, F_{11} \otimes F_{12}} \rightarrow} \text{Cut}}{\frac{\frac{\bullet h_2 \vdash \{\Delta_6, F_5\} : !F_7}{- \vdash \{\Delta_6, F_5\} : \Delta_9, F_{11}} \text{ax} \quad \frac{h_8 \vdash \{\Delta_6, F_5, dual(F_7)\} : \Delta_9, F_{11}}{hCut} \text{ax}}{- \vdash \{\Delta_6, F_5\} : \Delta_9, F_{11}} \otimes \quad \frac{\bullet h_2 \vdash \{\Delta_6, F_5\} : !F_7}{- \vdash \{\Delta_6, F_5\} : \Delta_{10}, F_{12}} \text{ax} \quad \frac{h_8 \vdash \{\Delta_6, F_5, dual(F_7)\} : \Delta_{10}, F_{12}}{hCut} \text{ax}}{- \vdash \{\Delta_6, F_5\} : \Delta_{10}, \Delta_9, F_{11} \otimes F_{12}} \otimes} \otimes$$

- Case rule I_1

$$\frac{\frac{h_2 \vdash \{p(\mathbf{n}_8), \Delta_5\} : p(\mathbf{n}_8), !F_6}{\bullet h_2 \vdash \{p(\mathbf{n}_8), \Delta_5\} : !F_6} ?_C \quad \frac{}{\bullet h_7 \vdash \{dual(F_6), p(\mathbf{n}_8), \Delta_5\} : \hat{\mathbf{n}}_8} I_1}{\frac{}{- \vdash \{p(\mathbf{n}_8), \Delta_5\} : \hat{\mathbf{n}}_8} \rightarrow} \text{Cut}}{\frac{}{- \vdash \{\Delta_5, p(\mathbf{n}_8)\} : \hat{\mathbf{n}}_8} I_1} \top$$

$$\frac{\frac{h_2 \vdash \{F_5, \Delta_9, p(\mathbf{n}_8)\} : F_5, !F_6}{\bullet h_2 \vdash \{F_5, \Delta_9, p(\mathbf{n}_8)\} : !F_6} ?_C \quad \frac{}{\bullet h_7 \vdash \{dual(F_6), F_5, \Delta_9, p(\mathbf{n}_8)\} : \hat{\mathbf{n}}_8} I_1}{\frac{}{- \vdash \{F_5, \Delta_9, p(\mathbf{n}_8)\} : \hat{\mathbf{n}}_8} \rightarrow} \text{Cut}}{\frac{}{- \vdash \{\Delta_9, F_5, p(\mathbf{n}_8)\} : \hat{\mathbf{n}}_8} I_1} \top$$

- Case rule I_2

$$\frac{\frac{h_2 \vdash \{\hat{\mathbf{n}}_8, \Delta_5\} : \hat{\mathbf{n}}_8, !F_6}{\bullet h_2 \vdash \{\hat{\mathbf{n}}_8, \Delta_5\} : !F_6} ?_C \quad \frac{}{\bullet h_7 \vdash \{dual(F_6), \hat{\mathbf{n}}_8, \Delta_5\} : p(\mathbf{n}_8)} I_2}{\frac{}{- \vdash \{\hat{\mathbf{n}}_8, \Delta_5\} : p(\mathbf{n}_8)} \rightarrow} \text{Cut}}{\frac{}{- \vdash \{\Delta_5, \hat{\mathbf{n}}_8\} : p(\mathbf{n}_8)} I_2} \top$$

$$\frac{\frac{h_2 \vdash \{F_5, \Delta_9, \hat{\mathbf{n}}_8\} : F_5, !F_6}{\bullet h_2 \vdash \{F_5, \Delta_9, \hat{\mathbf{n}}_8\} : !F_6} ?_C \quad \frac{}{\bullet h_7 \vdash \{dual(F_6), F_5, \Delta_9, \hat{\mathbf{n}}_8\} : p(\mathbf{n}_8)} I_2}{\frac{}{- \vdash \{F_5, \Delta_9, \hat{\mathbf{n}}_8\} : p(\mathbf{n}_8)} \rightarrow} \text{Cut}}{\frac{}{- \vdash \{\Delta_9, F_5, \hat{\mathbf{n}}_8\} : p(\mathbf{n}_8)} I_2} \top$$

- Case rule $?_C$

$$\begin{array}{c}
\frac{\frac{h_2 \text{ : } \{F_5, \Delta_6\} : F_5, !F_7}{\bullet h_2 \text{ : } \{F_5, \Delta_6\} : !F_7} \text{ ?}_C \quad \frac{h_8 \text{ : } \{F_5, \Delta_6, dual(F_7)\} : \Delta_9, dual(F_7)}{\bullet h_8 \text{ : } \{dual(F_7), F_5, \Delta_6\} : \Delta_9} \text{ ?}_C}{- \text{ : } \{F_5, \Delta_6\} : \Delta_9} \text{ Cut}}{\rightarrow} \\
\frac{\frac{- \text{ : } \{\Delta_6, F_5\} : !F_7}{- \text{ : } \{\Delta_6, F_5\} : F_7} \text{ ax} \quad \frac{\bullet h_2 \text{ : } \{\Delta_6, F_5\} : !F_7}{- \text{ : } \{\Delta_6, F_5\} : \Delta_9} \text{ ax} \quad \frac{h_8 \text{ : } \{\Delta_6, F_5, dual(F_7)\} : \Delta_9, dual(F_7)}{- \text{ : } \{\Delta_6, F_5\} : \Delta_9, dual(F_7)} \text{ ax}}{- \text{ : } \{\Delta_6, F_5\} : \Delta_9} \text{ mCut} \quad \text{hCut}}{\rightarrow} \\
\frac{\frac{h_2 \text{ : } \{F_8, \Delta_5\} : F_8, !F_6}{\bullet h_2 \text{ : } \{F_8, \Delta_5\} : !F_6} \text{ ?}_C \quad \frac{h_7 \text{ : } \{F_8, \Delta_5, dual(F_6)\} : F_8, \Delta_9}{\bullet h_7 \text{ : } \{dual(F_6), F_8, \Delta_5\} : \Delta_9} \text{ ?}_C}{- \text{ : } \{F_8, \Delta_5\} : \Delta_9} \text{ Cut}}{\rightarrow} \\
\frac{\bullet h_2 \text{ : } \{\Delta_5, F_8\} : !F_6}{- \text{ : } \{\Delta_5, F_8\} : \Delta_9, F_8} \text{ ax} \quad \frac{h_7 \text{ : } \{\Delta_5, F_8, dual(F_6)\} : \Delta_9, F_8}{- \text{ : } \{\Delta_5, F_8\} : \Delta_9} \text{ ax}}{- \text{ : } \{\Delta_5, F_8\} : \Delta_9} \text{ hCut} \quad \text{?}_C \\
\rightarrow} \\
\frac{\frac{h_2 \text{ : } \{F_5, F_8, \Delta_{10}\} : F_5, !F_6}{\bullet h_2 \text{ : } \{F_5, F_8, \Delta_{10}\} : !F_6} \text{ ?}_C \quad \frac{h_7 \text{ : } \{F_5, F_8, \Delta_{10}, dual(F_6)\} : F_8, \Delta_9}{\bullet h_7 \text{ : } \{dual(F_6), F_5, F_8, \Delta_{10}\} : \Delta_9} \text{ ?}_C}{- \text{ : } \{F_5, F_8, \Delta_{10}\} : \Delta_9} \text{ Cut}}{\rightarrow} \\
\frac{\bullet h_2 \text{ : } \{\Delta_{10}, F_5, F_8\} : !F_6}{- \text{ : } \{\Delta_{10}, F_5, F_8\} : \Delta_9, F_8} \text{ ax} \quad \frac{h_7 \text{ : } \{\Delta_{10}, F_5, F_8, dual(F_6)\} : \Delta_9, F_8}{- \text{ : } \{\Delta_{10}, F_5, F_8\} : \Delta_9} \text{ ax}}{- \text{ : } \{\Delta_{10}, F_5, F_8\} : \Delta_9} \text{ hCut} \quad \text{?}_C \\
\rightarrow}
\end{array}$$